Extended Recursive Least Squares With Exponential Forgetting Factor (RELSWEF) of ARMAX (Transfer Function) Systems Based on Equation Error

Equations

Transfer function model

$$A(z)y(t) = z^{-d}B(z)u(t) + C(z)e(t)$$

Where

$$A(z) = z^{na} + a_1 z^{na-1} + \cdots + a_{na}, \quad B(z) = b_0 z^{nb} + b_1 z^{nb-1} + \cdots + b_{nb}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na}z^{-na}, \quad B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb}z^{-nb}$$
$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{nc}z^{-nc}, \quad C(z) = z^{nc} + c_1 z^{nc-1} + \cdots + c_{nc}$$

$e(t) = y(i) - \varphi(i-1)\theta(i-1)$ is a color noise

Writing the difference equation, z is a shift operator $(zy(t) = y(t+1), \ z^{-1}y(t) = y(t-1))$

$$(1 + a_1 z^{-1} + \cdots + a_{na}z^{-na})y(t) = z^{-d}(b_0 + b_1 z^{-1} + \cdots + b_{nb}z^{-nb})u(t) + (1 + c_1 z^{-1} + \cdots + c_{nc}z^{-nc})e(t)$$
$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{na}y(t-na) + b_1 u(t-d-1) + b_2 u(t-d-2) + \cdots$$
$$+ b_{nb}u(t-d-nb) + e(t) + c_1 e(t-1) + \cdots + c_{nc}e(t-nc)$$
$$y(t) = \varphi^T(t-1)\theta^0$$

$$\varphi^T(t-1) = [-y(t-1) \ldots -y(t-na) \quad u(t-d-1) \ldots \quad u(t-d-nb) \quad e(t-1) \ldots e(t-nc(], \qquad \theta^0 = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{na} \\ b_1 \\ b_2 \\ . \\ . \\ . \\ b_{nb} \\ c_1 \\ c_2 \\ . \\ . \\ . \\ c_{nc} \end{bmatrix}$$

Algorithm (note replace each $\varphi(t)$ in the standard RLS algorithm with $\varphi(t-1)$ for the transfer function model)

At $t = 0, P(0) = \alpha I$, where I is the identity matrix $n \times n$ and α is large number. $\theta(0) = 0$. For $t > 0$ do the following

1. $\varphi^T(t-1) = [-y(t-1) \ldots -y(t-na) \quad u(t-d) \ldots \quad u(t-d-nb) \quad e(t-1) \ldots e(t-nc)]$
   Regressors

2. $P(t) = \frac{1}{\lambda}\left(P(t-1) - \frac{P(t-1)\varphi(t-1)}{\left(\lambda+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}\varphi^T(t-1)P(t-1)\right) = \frac{1}{\lambda}\left(I - K(t)\varphi^T(t-1)\right)P(t-1)$ Covariance matrix

3. $K(t) = P(t)\varphi(t) = \dfrac{P(t-1)\varphi(t-1)}{\left(\lambda + \varphi^T(t-1)P(t-1)\varphi(t-1)\right)}$ Weighting (scaling) factor

4. $\varepsilon(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Predicted error

5. $\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$ Estimation

6. $e(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Residual error

Where $\left(I + \varphi^T(t)P(t-1)\varphi(t)\right)$, is scalar for single output system.

na: order of the **numerator**.

nb: order of the **denominator**.

nc: order of the **noise characteristic polynomial**.

d: **delay**.

nu=na+nb+nc+1: number of **parameters** to be estimated.