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Numerical Algorithms:

1) Newton-Raphson:

```
In [3]: def newton_raphson(func, x):
    accuracy = float(input('input the senetivity: '))
    iterations = int(input('input max no. of iterations if it cant reach near: '))
    guess = x
    if func(x)[0] == 0:
        return x
    x_new = x
    f, fprime = func(x)
    iters = 0
    while (func(x_new)[0] > accuracy or func(x_new)[0] < -accuracy) and iters < iterations:
        iters += 1
        x_new = x - (func(x)[0] / func(x)[1])

        print(f'\niteration number ({iters})')
        print(f'Xn-1 = {x}, Xn = {x_new}')
        print(f'f({round(x,3)})= {round(func(x)[0],5)} & f({round(x_new,3)})= {round(func(x_new)[0],5)}')
        print('-----')

        x = x_new

    return f'''The root of the Equation= ({x_new})
    with guess ({guess}) and reached after ({iters} iterations)'''
```

```
In [4]: def ex1(a):
    import sympy
    import numpy as np

    x = sympy.Symbol('x')
    y = x**2 - 10*sympy.cos(x)
    yprime = y.diff(x)
    y_sol = y.evalf(subs={x: a})
    yprime_sol = yprime.evalf(subs={x: a})
    return y_sol, yprime_sol
```

a)

```
In [5]: EX1a = newton_raphson(ex1, -100)
print('\nSolution \n', EX1a)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = -100, Xn = -48.7454384988992
f(-100)= 9991.37681 & f(-48.745)= 2375.61047
```

```
-----
-----
```

```
iteration number (2)
Xn-1 = -48.7454384988992, Xn = -21.5967690939719
f(-48.745)= 2375.61047 & f(-21.597)= 475.65279
```

```
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-----
```

```
iteration number (3)
Xn-1 = -21.5967690939719, Xn = -11.4842195691050
f(-21.597)= 475.65279 & f(-11.484)= 127.19300
```

```
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```

```
iteration number (4)
Xn-1 = -11.4842195691050, Xn = -2.48815834094555
f(-11.484)= 127.19300 & f(-2.488)= 14.13094
```

```
-----
-----
```

```
iteration number (5)
Xn-1 = -2.48815834094555, Xn = -1.20997479565541
f(-2.488)= 14.13094 & f(-1.210)= -2.06639
```

```
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-----
```

```
iteration number (6)
Xn-1 = -1.20997479565541, Xn = -1.38544925227961
f(-1.210)= -2.06639 & f(-1.385)= 0.07659
```

```
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-----
```

```
iteration number (7)
Xn-1 = -1.38544925227961, Xn = -1.37937026950762
f(-1.385)= 0.07659 & f(-1.379)= 0.00007
```

```
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```

```
iteration number (8)
Xn-1 = -1.37937026950762, Xn = -1.37936459422703
f(-1.379)= 0.00007 & f(-1.379)= 0.0
```

```
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-----
```

Solution

```
The root of the Equation= (-1.37936459422703)
with guess (-100) and reached after (8 iterations)
```

b)

```
In [6]: EX1b = newton_raphson(ex1, -50)
print('\nSolution \n',EX1b)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = -50, Xn = -24.4254856569177
f(-50)= 2490.35034 & f(-24.425)= 589.00287
```

```
-----
-----
```

```
iteration number (2)
Xn-1 = -24.4254856569177, Xn = -10.5186473541231
f(-24.425)= 589.00287 & f(-10.519)= 115.23245
```

```
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-----
```

```
iteration number (3)
Xn-1 = -10.5186473541231, Xn = -1.03698932094905
f(-10.519)= 115.23245 & f(-1.037)= -4.01280
```

```
-----
-----
```

```
iteration number (4)
Xn-1 = -1.03698932094905, Xn = -1.41262296149367
f(-1.037)= -4.01280 & f(-1.413)= 0.42036
```

```
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-----
```

```
iteration number (5)
Xn-1 = -1.41262296149367, Xn = -1.37952504038751
f(-1.413)= 0.42036 & f(-1.380)= 0.00202
```

```
-----
-----
```

```
iteration number (6)
Xn-1 = -1.37952504038751, Xn = -1.37936459821508
f(-1.380)= 0.00202 & f(-1.379)= 0.0
```

```
-----
-----
```

Solution

The root of the Equation= (-1.37936459821508)
with guess (-50) and reached after (6 iterations)

c)

```
In [7]: EX1c = newton_raphson(ex1, -25)
print('\nSolution \n', EX1c)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = -25, Xn = -12.3637547270652
f(-25)= 615.08797 & f(-12.364)= 143.06700
```

```
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-----
```

```
iteration number (2)
Xn-1 = -12.3637547270652, Xn = -6.06545725376881
f(-12.364)= 143.06700 & f(-6.065)= 27.02586
```

```
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-----
```

```
iteration number (3)
Xn-1 = -6.06545725376881, Xn = -3.35495500415038
f(-6.065)= 27.02586 & f(-3.355)= 21.02897
```

```
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```

```
iteration number (4)
Xn-1 = -3.35495500415038, Xn = 1.22408725550689
f(-3.355)= 21.02897 & f(1.224)= -1.89966
```

```
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-----
```

```
iteration number (5)
Xn-1 = 1.22408725550689, Xn = 1.38435336424861
f(1.224)= -1.89966 & f(1.384)= 0.06279
```

```
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-----
```

```
iteration number (6)
Xn-1 = 1.38435336424861, Xn = 1.37936841765970
f(1.384)= 0.06279 & f(1.379)= 0.00005
```

```
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-----
```

```
iteration number (7)
Xn-1 = 1.37936841765970, Xn = 1.37936459422430
f(1.379)= 0.00005 & f(1.379)= 0.0
```

```
-----
-----
```

Solution

The root of the Equation= (1.37936459422430)
with guess (-25) and reached after (7 iterations)

d)

```
In [8]: EX1d = newton_raphson(ex1, 25)
print('\nSolution \n', EX1d)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = 25, Xn = 12.3637547270652
f(25)= 615.08797 & f(12.364)= 143.06700
```

```
-----
-----
```

```
iteration number (2)
Xn-1 = 12.3637547270652, Xn = 6.06545725376881
f(12.364)= 143.06700 & f(6.065)= 27.02586
```

```
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-----
```

```
iteration number (3)
Xn-1 = 6.06545725376881, Xn = 3.35495500415038
f(6.065)= 27.02586 & f(3.355)= 21.02897
```

```
-----
-----
```

```
iteration number (4)
Xn-1 = 3.35495500415038, Xn = -1.22408725550689
f(3.355)= 21.02897 & f(-1.224)= -1.89966
```

```
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-----
```

```
iteration number (5)
Xn-1 = -1.22408725550689, Xn = -1.38435336424861
f(-1.224)= -1.89966 & f(-1.384)= 0.06279
```

```
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```

```
iteration number (6)
Xn-1 = -1.38435336424861, Xn = -1.37936841765970
f(-1.384)= 0.06279 & f(-1.379)= 0.00005
```

```
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-----
```

```
iteration number (7)
Xn-1 = -1.37936841765970, Xn = -1.37936459422430
f(-1.379)= 0.00005 & f(-1.379)= 0.0
```

```
-----
-----
```

Solution

The root of the Equation= (-1.37936459422430)
with guess (25) and reached after (7 iterations)

e)

```
In [9]: EX1e = newton_raphson(ex1, 50)
print('\nSolution \n', EX1e)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = 50, Xn = 24.4254856569177
f(50)= 2490.35034 & f(24.425)= 589.00287
```

```
iteration number (2)
Xn-1 = 24.4254856569177, Xn = 10.5186473541231
f(24.425)= 589.00287 & f(10.519)= 115.23245
```

```
iteration number (3)
Xn-1 = 10.5186473541231, Xn = 1.03698932094905
f(10.519)= 115.23245 & f(1.037)= -4.01280
```

```
iteration number (4)
Xn-1 = 1.03698932094905, Xn = 1.41262296149367
f(1.037)= -4.01280 & f(1.413)= 0.42036
```

```
iteration number (5)
Xn-1 = 1.41262296149367, Xn = 1.37952504038751
f(1.413)= 0.42036 & f(1.380)= 0.00202
```

```
iteration number (6)
Xn-1 = 1.37952504038751, Xn = 1.37936459821508
f(1.380)= 0.00202 & f(1.379)= 0.0
```

Solution

```
The root of the Equation= (1.37936459821508)
with guess (50) and reached after (6 iterations)
```

f)

```
In [10]: EX1f = newton_raphson(ex1, 100)
print('\nSolution \n', EX1f)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
Xn-1 = 100, Xn = 48.7454384988992
f(100)= 9991.37681 & f(48.745)= 2375.61047
```

```
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-----
```

```
iteration number (2)
Xn-1 = 48.7454384988992, Xn = 21.5967690939719
f(48.745)= 2375.61047 & f(21.597)= 475.65279
```

```
-----
-----
```

```
iteration number (3)
Xn-1 = 21.5967690939719, Xn = 11.4842195691050
f(21.597)= 475.65279 & f(11.484)= 127.19300
```

```
-----
-----
```

```
iteration number (4)
Xn-1 = 11.4842195691050, Xn = 2.48815834094555
f(11.484)= 127.19300 & f(2.488)= 14.13094
```

```
-----
-----
```

```
iteration number (5)
Xn-1 = 2.48815834094555, Xn = 1.20997479565541
f(2.488)= 14.13094 & f(1.210)= -2.06639
```

```
-----
-----
```

```
iteration number (6)
Xn-1 = 1.20997479565541, Xn = 1.38544925227961
f(1.210)= -2.06639 & f(1.385)= 0.07659
```

```
-----
-----
```

```
iteration number (7)
Xn-1 = 1.38544925227961, Xn = 1.37937026950762
f(1.385)= 0.07659 & f(1.379)= 0.00007
```

```
-----
-----
```

```
iteration number (8)
Xn-1 = 1.37937026950762, Xn = 1.37936459422703
f(1.379)= 0.00007 & f(1.379)= 0.0
```

```
-----
-----
```

Solution

```
The root of the Equation= (1.37936459422703)
with guess (100) and reached after (8 iterations)
```

Bisedtion

```
In [13]: def bisection(func, a, b):
    accuracy = float(input('input the senetivity: '))
    iterations = int(input('input max no. of iterations if it cant reach near: '))
    upper, lower = a, b
    if func(a) == 0:
        return a
    if func(b) == 0:
        return b
    mid = a
    iters = 0
    while (func(mid) > accuracy or func(mid) < -accuracy) and iters < iterations:
        iters += 1
        mid = (a+b)/2
        print(f'\niteration number ({iters})')
        print('a=',a,', b=',b,', mid=',mid)
        print(f'f({round(a,3)})= {round(func(a),5)} & f({round(b,3)})= {round(func(b),5)}')
        print('-----')
        if (func(a) * func(mid)) < 0:
            b = mid
        elif (func(b) * func(mid)) < 0:
            a = mid
        else:
            break
    return f''The root of the Equation= ({mid})
    between ({upper}, {lower}) and reached after ({iters} iterations)''
```

a)


```
In [14]: def ex2a(x):
          return x - 2**(-x)

EX2a = bisection(ex2a,0,1)
```

```
input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20
```

```
iteration number (1)
a= 0 , b= 1 , mid= 0.5
f(0)= -1 & f(1)= 0.5 & f(0.5)= -0.20711
```

```
-----
-----
```

```
iteration number (2)
a= 0.5 , b= 1 , mid= 0.75
f(0.5)= -0.20711 & f(1)= 0.5 & f(0.75)= 0.1554
```

```
-----
-----
```

```
iteration number (3)
a= 0.5 , b= 0.75 , mid= 0.625
f(0.5)= -0.20711 & f(0.75)= 0.1554 & f(0.625)= -0.02342
```

```
-----
-----
```

```
iteration number (4)
a= 0.625 , b= 0.75 , mid= 0.6875
f(0.625)= -0.02342 & f(0.75)= 0.1554 & f(0.688)= 0.06657
```

```
-----
-----
```

```
iteration number (5)
a= 0.625 , b= 0.6875 , mid= 0.65625
f(0.625)= -0.02342 & f(0.688)= 0.06657 & f(0.656)= 0.02172
```

```
-----
-----
```

```
iteration number (6)
a= 0.625 , b= 0.65625 , mid= 0.640625
f(0.625)= -0.02342 & f(0.656)= 0.02172 & f(0.641)= -0.00081
```

```
-----
-----
```

```
iteration number (7)
a= 0.640625 , b= 0.65625 , mid= 0.6484375
f(0.641)= -0.00081 & f(0.656)= 0.02172 & f(0.648)= 0.01047
```

```
-----
-----
```

```
iteration number (8)
a= 0.640625 , b= 0.6484375 , mid= 0.64453125
f(0.641)= -0.00081 & f(0.648)= 0.01047 & f(0.645)= 0.00483
```

```
-----
-----
```

```
iteration number (9)
```

```
a= 0.640625 , b= 0.64453125 , mid= 0.642578125
f(0.641)= -0.00081 & f(0.645)= 0.00483 & f(0.643)= 0.00201
```

```
iteration number (10)
a= 0.640625 , b= 0.642578125 , mid= 0.6416015625
f(0.641)= -0.00081 & f(0.643)= 0.00201 & f(0.642)= 0.0006
```

```
iteration number (11)
a= 0.640625 , b= 0.6416015625 , mid= 0.64111328125
f(0.641)= -0.00081 & f(0.642)= 0.0006 & f(0.641)= -0.0001
```

```
iteration number (12)
a= 0.64111328125 , b= 0.6416015625 , mid= 0.641357421875
f(0.641)= -0.0001 & f(0.642)= 0.0006 & f(0.641)= 0.00025
```

```
iteration number (13)
a= 0.64111328125 , b= 0.641357421875 , mid= 0.6412353515625
f(0.641)= -0.0001 & f(0.641)= 0.00025 & f(0.641)= 7e-05
```

```
iteration number (14)
a= 0.64111328125 , b= 0.6412353515625 , mid= 0.64117431640625
f(0.641)= -0.0001 & f(0.641)= 7e-05 & f(0.641)= -2e-05
```

```
iteration number (15)
a= 0.64117431640625 , b= 0.6412353515625 , mid= 0.641204833984375
f(0.641)= -2e-05 & f(0.641)= 7e-05 & f(0.641)= 3e-05
```

```
iteration number (16)
a= 0.64117431640625 , b= 0.641204833984375 , mid= 0.6411895751953125
f(0.641)= -2e-05 & f(0.641)= 3e-05 & f(0.641)= 1e-05
```

```
In [15]: print(EX2a)
```

```
The root of the Equation= (0.6411895751953125)
    between (0, 1) and reached after (16 iterations)
```

b)

```
In [16]: def ex2b(x):
import math
return math.exp(x)-x**2+3*x-2
EX2b = bisection(ex2b, 0, 1)
```

input the senetivity: 0.00001

input max no. of iterations if it cant reach near: 20

iteration number (1)

a= 0 , b= 1 , mid= 0.5

f(0)= -1.0 & f(1)= 2.71828 & f(0.5)= 0.89872

iteration number (2)

a= 0 , b= 0.5 , mid= 0.25

f(0)= -1.0 & f(0.5)= 0.89872 & f(0.25)= -0.02847

iteration number (3)

a= 0.25 , b= 0.5 , mid= 0.375

f(0.25)= -0.02847 & f(0.5)= 0.89872 & f(0.375)= 0.43937

iteration number (4)

a= 0.25 , b= 0.375 , mid= 0.3125

f(0.25)= -0.02847 & f(0.375)= 0.43937 & f(0.312)= 0.20668

iteration number (5)

a= 0.25 , b= 0.3125 , mid= 0.28125

f(0.25)= -0.02847 & f(0.312)= 0.20668 & f(0.281)= 0.08943

iteration number (6)

a= 0.25 , b= 0.28125 , mid= 0.265625

f(0.25)= -0.02847 & f(0.281)= 0.08943 & f(0.266)= 0.03056

iteration number (7)

a= 0.25 , b= 0.265625 , mid= 0.2578125

f(0.25)= -0.02847 & f(0.266)= 0.03056 & f(0.258)= 0.00107

iteration number (8)

a= 0.25 , b= 0.2578125 , mid= 0.25390625

f(0.25)= -0.02847 & f(0.258)= 0.00107 & f(0.254)= -0.0137


```
iteration number (9)
a= 0.25390625 , b= 0.2578125 , mid= 0.255859375
f(0.254)= -0.0137 & f(0.258)= 0.00107 & f(0.256)= -0.00631
-----
-----
```

```
iteration number (10)
a= 0.255859375 , b= 0.2578125 , mid= 0.2568359375
f(0.256)= -0.00631 & f(0.258)= 0.00107 & f(0.257)= -0.00262
-----
-----
```

```
iteration number (11)
a= 0.2568359375 , b= 0.2578125 , mid= 0.25732421875
f(0.257)= -0.00262 & f(0.258)= 0.00107 & f(0.257)= -0.00078
-----
-----
```

```
iteration number (12)
a= 0.25732421875 , b= 0.2578125 , mid= 0.257568359375
f(0.257)= -0.00078 & f(0.258)= 0.00107 & f(0.258)= 0.00014
-----
-----
```

```
iteration number (13)
a= 0.25732421875 , b= 0.257568359375 , mid= 0.2574462890625
f(0.257)= -0.00078 & f(0.258)= 0.00014 & f(0.257)= -0.00032
-----
-----
```

```
iteration number (14)
a= 0.2574462890625 , b= 0.257568359375 , mid= 0.25750732421875
f(0.257)= -0.00032 & f(0.258)= 0.00014 & f(0.258)= -9e-05
-----
-----
```

```
iteration number (15)
a= 0.25750732421875 , b= 0.257568359375 , mid= 0.257537841796875
f(0.258)= -9e-05 & f(0.258)= 0.00014 & f(0.258)= 3e-05
-----
-----
```

```
iteration number (16)
a= 0.25750732421875 , b= 0.257537841796875 , mid= 0.2575225830078125
f(0.258)= -9e-05 & f(0.258)= 3e-05 & f(0.258)= -3e-05
-----
-----
```

```
iteration number (17)
a= 0.2575225830078125 , b= 0.257537841796875 , mid= 0.25753021240234375
f(0.258)= -3e-05 & f(0.258)= 3e-05 & f(0.258)= -0.0
-----
-----
```

In [17]: `print(EX2b)`

The root of the Equation= (0.25753021240234375)
between (0, 1) and reached after (17 iterations)

c)

In [22]: `def ex2c(x):
 import math
 return 2*x*math.cos(2*x)-(x+1)**2`

`EX2c1 = bisection(ex2c, -3, -2)
print('\nSecond \n')
EX2c2 = bisection(ex2c, -1, 0)`

```
-----
iteration number (2)
a= -1 , b= -0.5 , mid= -0.75
f(-1)= 0.83229 & f(-0.5)= -0.7903 & f(-0.75)= -0.16861
-----

iteration number (3)
a= -1 , b= -0.75 , mid= -0.875
f(-1)= 0.83229 & f(-0.75)= -0.16861 & f(-0.875)= 0.29631
-----

iteration number (4)
a= -0.875 , b= -0.75 , mid= -0.8125
f(-0.875)= 0.29631 & f(-0.75)= -0.16861 & f(-0.812)= 0.05288
-----
```

In [23]: `print(EX2c1)
print('\nSecond \n')
print(EX2c2)`

The root of the Equation= (-2.1913070678710938)
between (-3, -2) and reached after (17 iterations)

Second

The root of the Equation= (-0.7981605529785156)
between (-1, 0) and reached after (18 iterations)

```
In [20]: def ex2d(x):
import math
return x*math.cos(x)-2*(x**2)+3*x-1

EX2d1 = bisection(ex2d, 0.2, 0.3)
print('\nSecond \n')
EX2d2 = bisection(ex2d, 1.2, 1.3)
```

input the senetivity: 0.00001
input max no. of iterations if it cant reach near: 20

iteration number (1)
a= 0.2 , b= 0.3 , mid= 0.25
f(0.2)= -0.28399 & f(0.3)= 0.0066 & f(0.25)= -0.13277

iteration number (2)
a= 0.25 , b= 0.3 , mid= 0.275
f(0.25)= -0.13277 & f(0.3)= 0.0066 & f(0.275)= -0.06158

iteration number (3)
a= 0.275 , b= 0.3 , mid= 0.2875
f(0.275)= -0.06158 & f(0.3)= 0.0066 & f(0.287)= -0.02711


```
In [21]: print(EX2d1)
print('\nSecond \n')
print(EX2d2)
```

The root of the Equation= (0.29752807617187504)
between (0.2, 0.3) and reached after (14 iterations)

Second

The root of the Equation= (1.256622314453125)
between (1.2, 1.3) and reached after (14 iterations)

Fixed_Point Iterations:

```

In [24]: '''
As  $f(x) = x^3 - x - 1$ 
Then we let  $g(x) = (x+1)^{1/3}$ , as  $|g'(x)| < 1$  in the interval  $[1,2]$ 
'''
def g(x):
    return (x+1)**(1/3)

def fixed_point(func, x=0):
    accuracy = float(input('input the senetivity: '))
    iterations = int(input('input max no. of iterations if it cant reach near: '))
    guess = x

    iters = 0
    while abs(x-func(x)) > accuracy and iters<iterations:
        iters += 1
        print(f'\niteration number ({iters})')
        print(f'X = {x}, g(x) = {g(x)}, |x-func(x)|= {abs(x-func(x))}')
        print('-----')

        x = func(x)
    return f'''There is a fixed point solution = {x}
    using (P= {guess}) and after ({iters}) iterations'''

```

```

In [25]: EX3 = fixed_point(g, 1)

input the senetivity: 0.01
input max no. of iterations if it cant reach near: 20

iteration number (1)
X = 1, g(x) = 1.2599210498948732, |x-func(x)|= 0.2599210498948732
-----
-----

iteration number (2)
X = 1.2599210498948732, g(x) = 1.3122938366832888, |x-func(x)|= 0.0523727867884
15615
-----
-----

iteration number (3)
X = 1.3122938366832888, g(x) = 1.3223538191388249, |x-func(x)|= 0.0100599824555
36076
-----
-----

```

```

In [26]: print(EX3)

There is a fixed point solution = 1.3223538191388249
        using (P= 1) and after (3) iterations

```

```

In [ ]:

```

