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LAFDS Session 1 Homework

• Questions :

① . angle between: $r = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 3 \end{bmatrix}$, $S = \begin{bmatrix} 1 \\ \sqrt{3} \\ 3 \\ -3 \end{bmatrix}$

$\because$

$$r \cdot S = |r| * |S| * \cos(\theta)$$

$$\therefore \cos(\theta) = \frac{r \cdot S}{|r|\,|S|} = \frac{((1*1) + (0*\sqrt{3}) + (-1*3) + (3*-3))}{\sqrt{1+1+9} \quad * \sqrt{1+3+9+9}}$$

$$= \frac{-\cancel{11}}{\cancel{11}\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = 135^\circ$$

$\therefore$ The angle $(\theta) = 135^\circ$

② The angle would be a right angle if the two vectors are normal to each other; and so their dot-product would be "Zero"

$\therefore$ $A = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $B = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $C = \begin{pmatrix} 6 \\ -1 \\ -3 \end{pmatrix}$

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So:

$A.B = (2-2+0) = 0.0$

$A.C = (6+3+0) = 9$

$B.C = (12-1+6) = 17$

So, the right angle is at vertex between A, B

③

a)
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 20 \\ 30 & 40 \\ 50 & 60 \end{bmatrix} = \begin{bmatrix} 11 & 22 \\ 33 & 44 \\ 55 & 66 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (1*3) + (2*4) = 11$$

c)
$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 2 \end{bmatrix}$$

d)
$$\begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ 110 & 20 \end{bmatrix}$$

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e)

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

- There is no answer; as the condition of matrix multiplication $n \times m, m * k$ and this is $1 \times \cancel{3}, \cancel{2} * 1$ Not valid.

f)

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 7 \end{bmatrix}$$

- The same as (e); dimensions can't be multiplied.

g)

$$\begin{bmatrix} 0 & 1 & 2 \\ 10 & -10 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 21 & 24 \\ 5 & 10 & 15 \end{bmatrix}$$

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(4) . We Can represent this problem as a system of Linear equations:

Suppose sides are : A, B, C

. as $A + 11 = B - 11 = C$, $4 * A = 3 * C + 10$

=

$A - C = -11$ — (1)

$B - C = 11$ — (2)

$4A - 3C = 10$ — (3)

From (1),(2),(3) we represent a matrix - vector form

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 4 & 0 & -3 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -11 \\ 11 \\ 10 \end{bmatrix}$$

using Gaus elamination:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -11 \\ 0 & 1 & -1 & 11 \\ 4 & 0 & -3 & 10 \end{array}\right]$$ row-3 - 4 (row-1)

=

$$= \begin{bmatrix} 1 & 0 & -1 & -11 \\ 0 & 1 & -1 & 11 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$ row 1 + row 3 , row 2 + row 3

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 43 \\ 0 & 1 & 0 & 65 \\ 0 & 0 & 1 & 54 \end{array}\right]$$, so $\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 43 \\ 65 \\ 54 \end{bmatrix}$ #

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⑤ Suppose the containers are (x, y)

So:

$$240x + 260y = 52° * (240+260)$$
$$180x + 120y = 46\,(180+120)$$

where (x, y) here are the temperatures factrs.

So;

$$\begin{bmatrix} 240 & 260 \\ 180 & 120 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 52*500 \\ 46*300 \end{bmatrix}$$

using Gauss elimination:

$$\left[\begin{array}{cc|c} 240 & 260 & 52*500 \\ 180 & 120 & 46*300 \end{array}\right] \begin{array}{l} row1/20 \\ row2/20 \end{array}$$

$$\left[\begin{array}{cc|c} 12 & 13 & 1300 \\ 9 & 6 & 690 \end{array}\right] \quad row2 - \frac{3}{4}\,row1$$

$$\left[\begin{array}{cc|c} 12 & 13 & 1300 \\ 0 & -3.75 & -285 \end{array}\right] \xrightarrow{\times\frac{-4}{15}} \left[\begin{array}{cc|c} 12 & 13 & 1300 \\ 0 & 1 & 76 \end{array}\right] \quad row1 - 13\,row2$$

$$= \left[\begin{array}{cc|c} 12 & 0 & 312 \\ 0 & 1 & 76 \end{array}\right] \div 12 = \left[\begin{array}{cc|c} 1 & 0 & 26 \\ 0 & 1 & 76 \end{array}\right]$$

So, temperatures are: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 76 \end{bmatrix}$

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6 let x, y, z be the Cuboid dimensions:

$\therefore$ area $= 2xz + 2xy + 2zy$

So;

at: $(x+1)$: $2(x+1)y + 2(x+1)z + 2zy =$ area $+54$

$\therefore$ $\cancel{2xy + 2xz + 2yz} + 2y + 2z = \cancel{area} + 54$

$\therefore$ $2y + 2z = 54$ — ①

at: $(y+2)$: $2x(y+2) + 2xz + 2(y+2)z =$ area $+ 96$

$\therefore$ $\cancel{2xy + 2xz + 2yz} + 4x + 4z = \cancel{area} + 96$

$\therefore$ $4x + 4z = 96$ — ②

at: $(z+3)$: $2xy + 2x(z+3) + 2y(z+3) =$ area $+ 126$

$\therefore$ $\cancel{2xy + 2xz + 2yz} + 6x + 6y = 126 + \cancel{area}$

$\therefore$ $6x + 6y = 126$ — ③

from ①, ②, ③

in matrix vector form:

$$\begin{bmatrix} 0 & 2 & 2 \\ 4 & 0 & 4 \\ 6 & 6 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 54 \\ 96 \\ 126 \end{bmatrix}$$

$$\therefore \left[\begin{array}{ccc|c} 0 & 2 & 2 & 54 \\ 4 & 0 & 4 & 96 \\ 6 & 6 & 0 & 126 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & 4 & 54 \\ 6 & 6 & 0 & 96 \\ 0 & 2 & 2 & 126 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & -198 \\ 0 & 6 & -6 & 15 \\ 0 & 1 & 1 & 99 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 198 \\ 0 & 1 & -1 & 5/2 \\ & 1 & 1 & 99 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 198 \\ 0 & 1 & -1 & 5/2 \\ 0 & 0 & 2 & 96.5 \end{array}\right] \Rightarrow \therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$

using Numpy

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LAFDS Session 2 Homework

1. To express $\begin{pmatrix}9\\6\end{pmatrix}$ as a Linear Combination of $\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}1\\-4\end{pmatrix}$

Then:

$$\begin{pmatrix}1\\2\end{pmatrix}x + \begin{pmatrix}1\\-4\end{pmatrix}y = \begin{pmatrix}9\\6\end{pmatrix}$$

or

$$\begin{bmatrix}1 & 1\\2 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}9\\6\end{bmatrix}$$

Using Gauss elimination to solve for x, y :

$$\left[\begin{array}{cc:c}1 & 1 & 9\\2 & -4 & 6\end{array}\right] \quad row2 - 2*row1 \quad \therefore = \begin{bmatrix}1 & 1 & 9\\0 & -6 & -12\end{bmatrix} \div -6$$

$$= \begin{bmatrix}1 & 1 & 9\\0 & 1 & 2\end{bmatrix} \quad row(1) - row(2) \quad \therefore = \begin{bmatrix}1 & 0 & 7\\0 & 1 & 2\end{bmatrix}$$

So, $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}7\\2\end{pmatrix}$

Then,

$$\begin{pmatrix}1\\2\end{pmatrix}*7 + \begin{pmatrix}1\\-4\end{pmatrix}*2 = \begin{pmatrix}9\\6\end{pmatrix}$$

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② If the vector lies in a span; it should be a linear Combination of the basis vectors.

but for

$$X_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ and, } X_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad X_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

vectors X_2, X_3 represent a $\mathbb{R}^2$ span as

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$$ row 2 = 2 * row 1 So using Gauss elimination

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 3 \end{array}\right] = \left[\begin{array}{ccc} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 3 & 1 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 3 & 1 & 3 \end{array}\right]$$

row 2 − 3 * row 1

$$\therefore \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -5 & -3 \end{array}\right] \div -5 = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 3/5 \end{array}\right]$$ row 1 − 2 row 2

row
$$\left[\begin{array}{cc|c} 1 & 0 & 4/5 \\ 0 & 1 & 3/5 \end{array}\right] \quad \text{So, } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$$

So,

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} * 4/5 + \begin{pmatrix} 2 \\ 1 \end{pmatrix} * 3/5 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

So, the vector lies in the span.

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③

a) $\because\ V_1 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $W = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$

$\therefore \begin{pmatrix} -2 \\ 3 \end{pmatrix} X = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$ $\therefore\ -2X_1 = -8$ $\therefore X_1 = 4$
$3X_2 = 12$ $\therefore X_2 = 4$ } both equal

Then:

$V_1 * 4 = W$

b) $V_1, V_2 = \left(\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & 2 & -6 \\ 5 & 0 & 10 \end{array}\right)$, $\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} X + \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} y = \begin{pmatrix} 4 \\ -6 \\ 10 \end{pmatrix}$

$\therefore\ 2X + 0y = 4$ $\therefore X = 2$
$0X + 2y = -6$ $y = -3$ So,
$5X + 0y = 10$ $X = 2$

$\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} * 2 - 3 \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 10 \end{pmatrix}$

or

$2V_1 - 3V_2 = W$

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④ Matching:

1. e 4. f
2. d 5. c
3. b 6. a

⑤ as $S = \{V_1, V_2, V_3, V_4, V_5\}$

Then to find the basis of the Span:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 1 & 7 \\ 2 & 1 & -1 & 4 & 0 \\ -1 & 1 & 5 & -1 & 2 \end{bmatrix} \begin{array}{l} \\ \text{Row2} - \text{row1} * 2 \\ \text{row3} - 2*\text{row1} \\ \text{row4} + \text{row1} \end{array}$$

Then:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ 0 & 2 & 6 & 0 & 4 \end{bmatrix} \begin{array}{l} \\ \\ \text{row3} + \text{row2} \\ \text{row4} - 2*\text{row2} \end{array}$$

Then:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ \cancel{0} & \cancel{1} & \cancel{3} & \cancel{-1} & \cancel{3} \\ \cancel{0} & \cancel{-1} & \cancel{-3} & \cancel{2} & \cancel{-2} \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 & -2 \end{bmatrix} \quad \text{row 4} - 2*\text{row 3}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & d \end{bmatrix} \quad \text{row 1} - \text{row 2}$$

$$\begin{bmatrix} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & d \end{bmatrix} \quad \begin{matrix} \text{row 1} - \text{row } 3*2 \\ \text{row 2} + \text{row 3} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & d \end{bmatrix}$$

So, as V_1, V_2, V_4 Contains the basis diagonals. then V_3, V_5 Can be Linearly Combiened out of them

Then, the span of (S) is V_1, V_2, V_4

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 4 \\ -1 & 1 & -1 \end{bmatrix} \longleftarrow \text{This is a Combination and Can be removed.}$$

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LAFDS Session 3&4 Homework

①

a) $M = \begin{bmatrix} 17 & -11 \\ 6 & -3 \end{bmatrix}$

$\therefore |m| = (17 * -3) - (6 * -11) = 15$ #

b)
$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{bmatrix}$$

$\therefore |m| = 1((3*-5)-(4*1)) - 1((2*-5)-(3*1)) + 2((2*4)-(3*3))$
$= -19 + 13 - 2 = -8$ #

②

a)
$$A = \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$$

& $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore |A| = (-3*3) - (3*-2) = -3$

$\therefore A^{-1} = \frac{-1}{3} \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix}$ #

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b)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1) $|A| = 1(0) - 0(1) + (-1) = -1$

&

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & +1 & 1 \end{bmatrix}$$

$$\text{Cof}(A) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

3

a) The rank = 2 ; as the two rows are independent

b) The matrix is full rank (3) as the three columns and rows are independent of each other.

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(4)

a) $\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 5 \\ 1 & -1 & 1 & 2 & 6 \\ 4 & 1 & 6 & 5 & 9 \end{array}\right]$

- rank of matrix: full rank; (3)

Since we have rank of 3;

and Column (4) = Column (3) + Column (2)

assume $x_4 = 0.0$

$\therefore \begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 1 & -1 & 1 & 2 & 6 \\ 4 & 1 & 6 & 5 & 9 \end{bmatrix} \xrightarrow[R_3-4R_1]{R_2-1R_1} \begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 0 & -5 & -2 & 3 & 1 \\ 0 & -15 & -6 & 9 & -11 \end{bmatrix}$ Row3 + 3 Row

$\begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 0 & 1 & 2/5 & 3/5 & 1/5 \\ 0 & 0 & 0 & 0 & -12 \end{bmatrix}$

From this system; equations has no Solution.

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b) $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 2 & -4 & 1 & 1 & 2 \\ 1 & -2 & -2 & 3 & 1 \end{array}\right]$

$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 3 & -4 \\ 0 & 0 & -3 & 6 & -2 \end{array}\right]$ row3 − 3row2 $= \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 3 & -4 \\ 0 & 0 & 0 & -3 & 10 \end{array}\right]$

row2 + row3 $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 0 & 6 \\ 0 & 0 & 0 & -3 & 10 \end{array}\right]$

∴ from system:

$X_4 = \frac{-10}{3}$, $X_3 = -6$

$X_1 - 2X_2 - \frac{10}{3} + 6 = 3$

$\therefore X_1 - 2X_2 = \frac{1}{3}$

$\therefore X_1 - 2X_2 = \frac{1}{3}$

∴ (X_1, X_2) may have any Solution. but we can't determine because of lack of equations.

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c)
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & 1 & 4 \\ 0 & 5 & 2 & 1 \end{array}\right] \begin{array}{l} \\ row2 - 2\,row1 \\ row3 - 3\,row1 \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -5 & -5 & -4 \\ 0 & -5 & -8 & 1 \\ 0 & 5 & 2 & 1 \end{array}\right] \begin{array}{l} \\ row2 / -5 \\ row3 - row2 \\ row4 + row2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 4/5 \\ 0 & 0 & -3 & 5 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

bot row-4 , row-3 gives ;

$-3X_3 = -3 \quad \therefore X_3 = 1$

$-3X_3 = 5 \quad X_3 = 5/-3$ } impossible.

&

$X_2 + X_3 = 4/5$

and; $X_1 + 2X_2 + 3X_3 = 1$

This system has infinite number of solutions.

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5

a)

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \& \quad AV - \lambda V = 0.0$$

$\therefore (A - \lambda I) V = 0.0$

So,

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 1 = 4 - 4\lambda + \lambda^2 - 1 = 0.0$$

$\therefore \lambda^2 - 4\lambda + 3 = 0.0$

$(\lambda - 3)(\lambda - 1) = 0.0$

So,

$\lambda_1 = 3 \quad , \quad \lambda_2 = 1$

@ $\lambda_1 = 3$

$$\therefore \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore$

$2x - y = 3x \quad , \quad -x + 2y = 3y$

$\therefore$ both reveals that : $x = -y$,

So : eigen-vector is any vecter satisfy $x = -y$

for $\lambda_1 = 3$ then $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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@ $\lambda_2 = 1$ $\therefore$ $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore$ $2x - y = x$,
$-x + 2y = y$ both gives $x = y$

$\therefore$ at $\lambda_1 = 3$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

at $\lambda_2 = 1$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ //

Trace $= 2 + 2 = 4$
& $\lambda_1 + \lambda_2 = 3 + 1 = 4$ $\therefore$ Trace $= \lambda_1 + \lambda_2$
&

$|A| = (2*2) - (-1*-1) = 3$
$\lambda_1 * \lambda_2 = 3*1 = 3$ $\therefore$ determinant $= \lambda_1 \lambda_2$

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b)

$$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} , (A^2 - \lambda I) v = 0.0$$

$$\begin{bmatrix} 5-\lambda & -4 \\ -4 & 5-\lambda \end{bmatrix} = 25 - 10\lambda + \lambda^2 - 16 = 0.0$$

$$\therefore \lambda^2 - 10\lambda + 9 = (\lambda - 9)(\lambda - 1) = 0.0$$

$$\therefore \lambda_1 = 1 , \lambda_2 = 9$$

@ $\lambda_1 = 1$

$$\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore$

$$5x - 4y = x , -4x + 5y = y$$

both gives that $x = y$

@ $\lambda_2 = 9$

$$\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 9 \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore$ ~~5~~

$$5x - 4y = 9x , -4x + 5y = 9y$$

both gives that $x = -y$

$\therefore$ ~~at~~

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So, at $\lambda_1 = 1$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

at $\lambda_2 = 9$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Trace $= 5 + 5 = 10$

$\lambda_1 + \lambda_2 = 1 + 9 = 10$ So; Trace $= \lambda_1 + \lambda_2$

&

$|A^2| = 25 - 16 = 9$

$\lambda_1 \cdot \lambda_2 = 1 * 9 = 9$ so; determinant $= \lambda_1 \lambda_2$

c)

$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\therefore$

$\frac{1}{3}\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = \frac{1}{3}(\lambda^2 - 4\lambda + 3) = 0.0$

$\therefore$ the same as before:

$\lambda_1 = 1$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ +1 \end{bmatrix}$

$\lambda_2 = 3$, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Trace $= \lambda_1 + \lambda_2 = 4$

determinant $= \lambda_1 \cdot \lambda_2 = 3$

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d)

$$A+4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} = 36 - 12\lambda + \lambda^2 - 1 = 0.0$$

$$\lambda^2 - 12\lambda + 35 = (\lambda - 7)(\lambda - 5) = 0.0$$

$$\therefore \lambda_1 = 7 \quad , \quad \lambda_2 = 5$$

$\therefore$ ~~$+\lambda$~~

@ $\lambda_1 = 7$ $\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 7\begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore 6x - y = 7x$

$, -x + 6y = 7y$ both gives $x = -y$

@ $\lambda_2 = 5$ $\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 5\begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore$ $6x - y = 5x$

$-x + 6y = 5y$ both gives $x = y$

$\therefore$ at $\lambda_1 = 7$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

at $\lambda_2 = 5$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ #

Trace $= \lambda_1 + \lambda_2 = 12$

determinant $= \lambda_1 \cdot \lambda_2 = 35$