Aerospace Engineering Department

Faculty of Engineering

Cairo University

4^{th} year

AER4420

Design of control systems for aeronautical and space vehicles

Task (4)

LINEARIZATION AND APPROXIMATIONS

In this task we will be linearizing the airplane equations of motion to be able to use the tools we studied in Linear control systems theory (e.g. Transfer Functions, Root locus, bode plot, etc.); in order to analyze the airplane dynamics and hence, design a suitable control system as per design requirements.

Task statement

Part I:

Step 1: "Linearize the complete set of the equations of motion of fixed wing airplane at a symmetric flight reference condition at which reference angle of attack ($\alpha o \neq 0$). Decouple the equations into Longitudinal & Lateral dynamics and write them in the state space form."

Note: You may refer to Etkin for a good derivation of the linearized equations of motion.

The equations in many references are derived neglecting the $\dot{w}$ related derivatives, please don't neglect them.

Step 2: "State the short period & Long period approximations of the Longitudinal dynamics"

Step 3: "Find the step response of your airplane using all the models of the longitudinal dynamics you have obtained (Non-linear simulation, Linear full model, Short period approximation, Long period approximation). Compare & discuss the results introduced by the different models at each of the following set of inputs respectively (each one at a time)

• Elevator: $\delta_{elevator} = 1^{\circ}, 5^{\circ}, 25^{\circ}$
• Thrust: $\delta_{thrust} = 2000, 10000\ lbs$

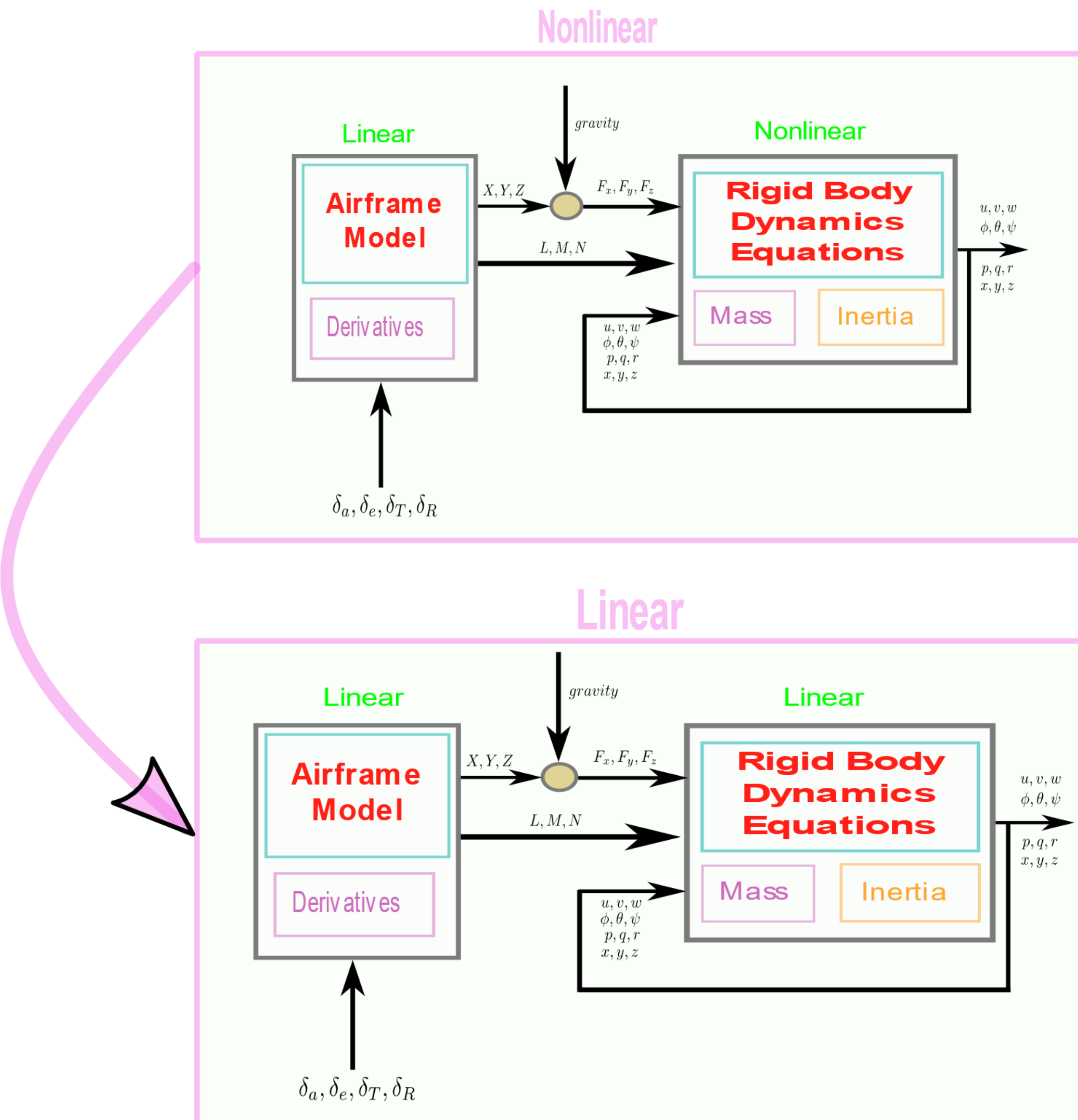
Step 4: "Extract all the transfer functions of your airplane using all the models of the longitudinal dynamics you have obtained (Linear full model, Short period approximation, Long period approximation). Plot & Compare the Root locus & Bode plot of the different representations of each transfer function.

Part II:

Step 1: "State the 3-DOF, 2-DOF, 1-DOF approximations of the Lateral dynamics"

Step 2: "Find the step response of your airplane using all the models of the lateral dynamics you have obtained (Non-linear simulation, Linear full model, 3-DOF, 2-DOF, 1-DOF). Compare & discuss the results introduced by the different models at each of the following set of inputs respectively (each one at a time)
• Aileron: $\delta_{aileron} = 1^{\circ}, 5^{\circ}, 10^{\circ}, 25^{\circ}$
• Rudder: $\delta_{rudder} = 1^{\circ}, 5^{\circ}, 10^{\circ}, 25^{\circ}$

Step 3: "Extract all the transfer functions of your airplane using all the models of the lateral dynamics you have obtained (Linear full model, 3-DOF, 2-DOF, 1-DOF). Plot & Compare the Root locus & Bode plot of the different representations of each transfer function.



Nonlinear
Linear
Airframe Model
Derivatives
X, Y, Z
gravity
F_x, F_y, F_z
L, M, N
Nonlinear
Rigid Body Dynamics Equations
Mass
Inertia
u, v, w
φ, θ, ψ
p, q, r
x, y, z
δ_a, δ_e, δ_T, δ_R
Linear
Linear
Airframe Model
Derivatives
X, Y, Z
gravity
F_x, F_y, F_z
L, M, N
Linear
Rigid Body Dynamics Equations
Mass
Inertia
u, v, w
φ, θ, ψ
p, q, r
x, y, z
δ_a, δ_e, δ_T, δ_R

General Hints and Notes

- When comparing root locus plots, having several plots in the same figure may cause the figure to be overcrowded and not very useful for extracting meaningful information. In this case, it might be better to compare them side by side.
- When comparing bode plots, it is often better to have several plots on the same figure as they will show when certain approximations are valid to represent the full linear model and when they are not.
- Make sure to use “minreal” function to get the minimum realization of the obtained transfer function.
- If you plot two lines that are coinciding with each other, it is better to make one of them dashed to get better display.
- Always comment on the obtained figures, figures are plotted to deduce useful information, not just fill the report.

Note on magnitude of the response from the linear simulation

The response obtained from the transfer function is not the magnitude of the response, but it is the change in the states value from their initial values, i.e. (Δu) not (u), so do not forget to add this response to the initial values of the states to obtain the magnitude of the response for this input, for example $u = u_o + \Delta u$.

Note on the step function in Matlab

When you use the function (step) in Matlab, it gives you the response for a unit step input, i.e. the magnitude of the input is 1

For example, if you are getting the response of the transfer function (u/δ_e) you are getting the response of (Δu) to a step input in the elevator of magnitude 1, and since the control actions are in (rad) this means that the elevator's deflection is $1\, rad = 57.32^o$!! This is very large.

So, when you use the function step do not forget to multiply the response by the amplitude of the control action which you write in the excel sheet, and which you get the response of the non-linear simulation code for.

Simulink Bonus

Use Model Linearizer to extract the state space model and the transfer functions from the Nonlinear Simulink model you built in the previous task. Compare them to what you have from manual linearization.

References

1) John H. Blakelock - Automatic Control of Aircraft and Missiles-Wiley-Interscience (1991)
2) Flight Stability and Automatic Control - Robert C. Nelson
3) Etkin B., Reid L.D. - Dynamics of flight_ Stability and control-Wiley (1996)
4) NASA CR-2144--Heffley--Aircraft Handling Qualities Data