Aerospace Engineering Department

Faculty of Engineering

Cairo University

4th year

AER4420

Design of control systems for aeronautical and space vehicles

Task (3)

AIRFRAME MODEL
AIRPLANE SIMULATOR PART II

Note: This will be the last part of the simulator you will be using from now on in the course, you have to verify and validate your results in one of two ways:

- Compare results with the simulink simulator you will build.
- Compare results with one of your colleagues.

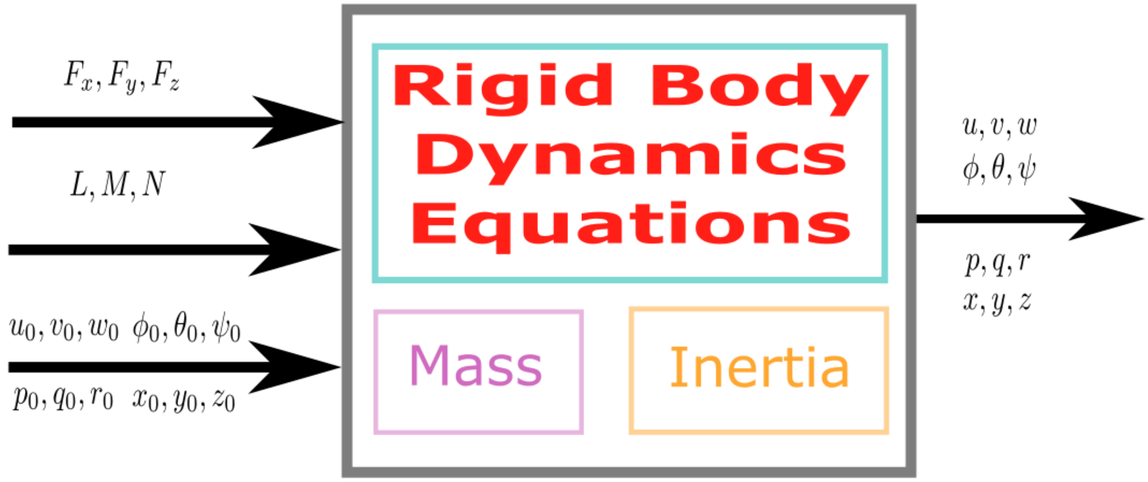
Task statement

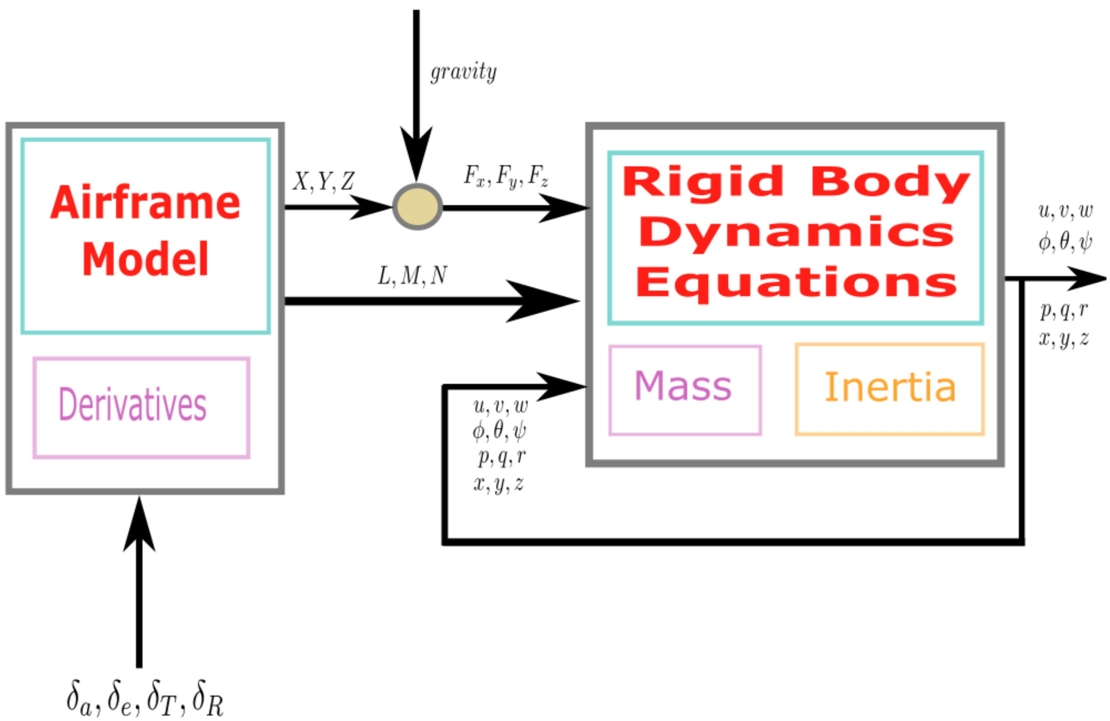
Step 1: Write a code that calculates the (Aerodynamic & Thrust) (Forces & Moments) acting on an Airplane due to pilots input signals ($\delta_{aileron}$, δ_{rudder}, $\delta_{elevator}$, δ_{thrust}) knowing its stability & control derivatives at nominal flight condition.

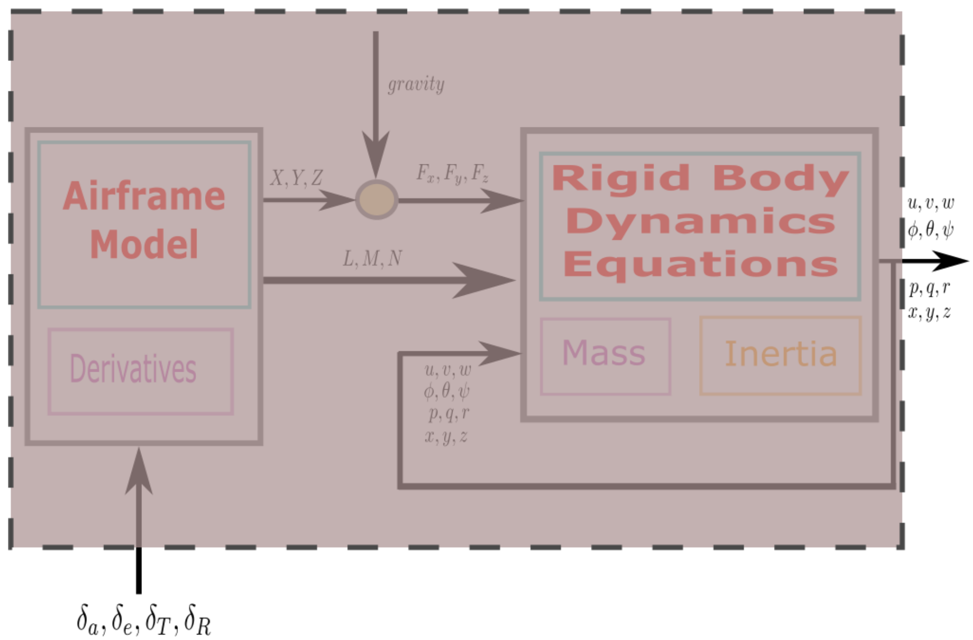
Consult the following document (NASA CR-2144 AIRCRAFT HANDLING QUALITIES DATA) as a reference for the Airplanes' parameters including the (Stability derivatives, Mass, Inertias) at the Reference Flight conditions. Each team will be assigned an Airplane and a Flight condition.

Step 2: "Combine the (Airframe Model) code with the (RBD solver) you built in the previous task in order to build the complete (Airplane Non-linear Flight Simulator)". The idea is to use the (Pilots inputs) to calculate the Aerodynamic & Thrust (Forces & Moments) acting on the airplane, and then use these calculated Forces & Moments to solve the (RBD) equations to calculate the new states of the airplane in the next time step, and repeat this procedure at each time step.

Note: do not forget to add the (Gravitational Forces) to the (Aerodynamic & Thrust Forces) before using them in the (RBD solver).







gravity
Airframe Model
Derivatives
X, Y, Z
F_x, F_y, F_z
L, M, N
Rigid Body Dynamics Equations
Mass
Inertia
u, v, w
ϕ, θ, ψ
p, q, r
x, y, z
$\delta_a, \delta_e, \delta_T, \delta_R$

Forces:

The following set of **linear** equations represents the **change** in the Aerodynamic & thrust forces & moments, they are function of:

- Stability Derivatives.
- Control Derivatives.
- The perturbation change in the states and the control surfaces deflections from their values at the trim condition.

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T$$
$$\Delta Y = \frac{\partial Y}{\partial v}\Delta v + \frac{\partial Y}{\partial p}\Delta p + \frac{\partial Y}{\partial r}\Delta r + \frac{\partial Y}{\partial \delta_r}\Delta\delta_r$$
$$\Delta Z = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial \dot{w}}\Delta\dot{w} + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_e}\Delta\delta_e + \frac{\partial Z}{\partial \delta_T}\Delta\delta_T$$
$$\Delta L = \frac{\partial L}{\partial v}\Delta v + \frac{\partial L}{\partial p}\Delta p + \frac{\partial L}{\partial r}\Delta r + \frac{\partial L}{\partial \delta_r}\Delta\delta_r + \frac{\partial L}{\partial \delta_a}\Delta\delta_a$$
$$\Delta M = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial \dot{w}}\Delta\dot{w} + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_e}\Delta\delta_e + \frac{\partial M}{\partial \delta_T}\Delta\delta_T$$
$$\Delta N = \frac{\partial N}{\partial v}\Delta v + \frac{\partial N}{\partial p}\Delta p + \frac{\partial N}{\partial r}\Delta r + \frac{\partial N}{\partial \delta_r}\Delta\delta_r + \frac{\partial N}{\partial \delta_a}\Delta\delta_a$$

Note: $\Delta X, \Delta Y, \Delta Z, \Delta L, \Delta M, \Delta N$ are the **changes** in the forces & moments, i.e. these are not the absolute values of the forces and moments. They should be added to the reference values at the trim condition $X_0, Y_0, Z_0, L_0, M_0, N_0$ to calculate the absolute values X, Y, Z, L, M, N.

Similarly, $\Delta u, \Delta v, \Delta w$, ... are the changes in the states values from their values at the reference condition $\Delta u = u - u_0, \Delta v = v - v_0, \Delta w = w - w_0$, ...

Hence:

Inputs and outputs of the (Airframe Model) are perturbations from the reference values
Inputs and outputs of the RBD are absolute values

So that: "Perturbation values resulting from the (Airframe Model) should be added to the reference values before passing them to the (RBD) and the absolute values resulting from the (RBD) should be converted to perturbation values by subtracting the reference values from them.”

The total forces acting on an airplane are:

- Aerodynamic forces.
- Thrust force.
- Gravity force.

$$X - \mathrm{mgsin}\,\theta = m(\dot{u}^E + qw^E - rv^E)$$

$$Y + \mathrm{mgcos}\,\theta\,\sin\phi = m(\dot{v}^E + ru^E - pw^E)$$

$$Z + \mathrm{mgcos}\,\theta\,\cos\phi = m(\dot{w}^E + pv^E - qu^E)$$

Equilibrium state

Initially at the reference flight condition the airplane is in an equilibrium state, which means:

$$\sum Forces = 0 \qquad \& \qquad \sum Moments = 0$$

$$X_0 - mg\sin\,\theta_0 = 0 \rightarrow X_0 = mg\sin\,\theta_0$$
$$Y_0 - mg\cos\,\theta_0\sin\,\emptyset_0 = 0 \rightarrow Y_0 = -mg\cos\,\theta_0\sin\,\emptyset_0$$
$$Z_0 - \mathrm{mgcos}\,\theta_0\cos\emptyset_0 = 0 \rightarrow Z_0 = -\,\mathrm{mgcos}\,\theta_0\cos\emptyset_0$$
$$\therefore X = X_0 + \Delta X = \Delta X + mg\sin\,\theta_0$$
$$Y = Y_0 + \Delta Y = \Delta Y - mg\cos\,\theta_0\sin\,\emptyset_0$$
$$Z = Z_0 + \Delta Z = \Delta Z - mg\cos\,\theta_0\cos\,\emptyset_0$$

And the total force acting on the airplane (this value is the input which you will give to the RBD)

$$F_x \;= X - mg\sin\,\theta = \Delta X + mg\sin\,\theta_0 - mg\sin\,\theta$$
$$F_y \;= Y + mg\cos\,\theta\sin\,\emptyset = \Delta Y + mg\cos\,\theta\sin\,\emptyset$$
$$F_z = Z + mg\cos\,\theta\cos\,\emptyset = \Delta Z - mg\cos\,\theta_0\cos\,\emptyset_0 + mg\cos\,\theta\sin\,\emptyset$$

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ L \\ M \\ N \end{Bmatrix} = \begin{Bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta L \\ \Delta M \\ \Delta N \end{Bmatrix} + \begin{Bmatrix} \mathrm{X}_0 \\ \mathrm{Y}_0 \\ \mathrm{Z}_0 \\ L_0 \\ M_0 \\ N_0 \end{Bmatrix} + \begin{Bmatrix} -mg\sin\,\theta \\ mg\cos\,\theta\sin\,\emptyset \\ mg\cos\,\theta\cos\,\emptyset \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} mg\sin\,\theta_0 \\ -mg\cos\,\theta_0\sin\,\emptyset_0 \\ -mg\cos\,\theta_0\cos\,\emptyset_0 \end{Bmatrix} \ \& \ \begin{Bmatrix} L_0 \\ M_0 \\ N_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Types of Body axes

Consult “Dynamics of Flight, Bernard Etkin” pages 101-103 to review the concept of the **Body axes** of the airplane and its types (principal axes, stability axes, body axes).

You should note that the stability derivatives & Inertias of an airplane have different values and symbols according to the type of the body axes they are represented in.

Very important: Study the symbols and definitions stated in (NASA CR-2144) **appendices A&B**, then use the tables of the derivatives represented in the (**Body axes**) to extract the derivatives according to your flight condition.

LONGITUDINAL DIMENSIONAL DERIVATIVES

(BODY AXIS SYSTEM)

F/C #	1	2	3	4	5	6	7	8	9	10
H	SL	SL	SL	SL	20 K	20 K	20 K	40 K	40 K	40 K
M	.198	.249	.450	.650	.500	.650	.800	.700	.800	.900
XU *	-.0209	-.0108	-.00499	-.00777	-.00247	-.00280	-.00643	.00187	-.00276	-.0200
ZU *	-.202	-.150	-.0807	-.126	-.0679	-.0832	-.0941	-.0696	-.0650	-.0424
MU *	.000117	.000181	.000146	-.000199	.000247	.885E-4	-.000222	.000259	.000193	-.623E-4
XW	.122	.106	.0743	.0345	.0782	.0482	.0253	.0263	.0389	.0159
ZW	-.512	-.613	-.736	-.963	-.433	-.539	-.624	-.292	-.317	-.401
MW	-.00177	-.00193	-.00262	-.00239	-.00170	-.00190	-.00153	-.00101	-.00105	-.00190
ZWD	.0334	.0338	.0297	.0293	.0157	.0156	.0144	.00704	.00556	.00614
ZQ	-6.22	-7.58	-10.4	-12.8	-6.39	-8.09	-9.98	-4.32	-5.16	-6.71
MWD	-.000246	-.000240	-.000221	-.000228	-.000125	-.000155	-.000212	-.905E-4	-.000116	-.000160
MQ	-.357	-.437	-.699	-.925	-.421	-.535	-.659	-.284	-.339	-.401
XDE	.959	.971	1.18	0.	2.02	1.15	0.	1.93	1.44	.781
ZDE	-6.42	-9.73	-21.8	-32.4	-16.9	-26.4	-32.7	-15.1	-17.9	-18.6
MDE	-.378	-.574	-1.40	-2.07	-1.09	-1.69	-2.08	-.970	-1.16	-1.22
XDTH	.570E-4	.570E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4
ZDTH	-.249E-5	-.249E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5
MDTH	.310E-6	.310E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6

LATERAL-DIRECTIONAL DIMENSIONAL DERIVATIVES

(BODY AXIS SYSTEM)

F/C #	1	2	3	4	5	6	7	8	9	10
H	SL	SL	SL	SL	20 K	20 K	20 K	40 K	40 K	40 K
M	.198	.249	.450	.650	.500	.650	.800	.700	.800	.900
YV	-.0890	-.0997	-.143	-.197	-.0822	-.104	-.120	-.0488	-.0558	-.0606
YB	-19.7	-27.8	-71.7	-143.	-42.6	-70.4	-99.4	-33.1	-43.2	-52.8
LB'	-1.33	-1.63	-3.19	-5.45	-2.05	-2.96	-4.12	-1.45	-3.05	-1.32
NB'	.168	.247	.810	1.82	.419	.923	1.62	.404	.598	.971
LP'	-.975	-1.10	-1.12	-1.47	-.652	-.804	-.974	-.404	-.465	-.459
NP'	-.166	-.125	-.0706	-.0214	-.0701	-.0531	-.0157	-.0366	-.0316	.00284
LR'	.327	.198	.379	.256	.376	.317	.292	.312	.388	.280
NR'	-.217	-.229	-.246	-.344	-.140	-.193	-.232	-.0963	-.115	-.141
Y*CA	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
L'CA	.227	.318	.229	.372	.128	.210	.310	.0964	.143	.186
N'CA	.0264	.0300	.0285	.0371	.0177	.0199	.0127	.00875	.00775	-.00611
Y*CR	.0148	.0182	.0226	.0213	.0131	.0142	.0124	.00777	.00729	.00464
L'CR	.0636	.110	.254	.318	.148	.211	.183	.115	.153	.100
N'CR	-.151	-.233	-.614	-.970	-.391	-.616	-.922	-.331	-.475	-.442

Note on Lateral-Directional derivatives:

The Lateral-Directional derivatives given in the table are **(dashed)**, these are not the values to be used in the (forces & moments equations), check **appendix B** in the report to find the relation between the (dashed & undashed derivatives) to calculate the (undashed ones).

$$L'_\beta = (L_\beta + I_{xz}N_\beta/I_x)G \qquad 1/\text{sec}^2$$

$$L'_p = (L_p + I_{xz}N_p/I_x)G \qquad 1/\text{sec}$$

$$L'_r = (L_r + I_{xz}N_r/I_x)G \qquad 1/\text{sec}$$

$$L'_{\delta_r} = (L_{\delta_r} + I_{xz}N_{\delta_r}/I_x)G \qquad 1/\text{sec}^2$$

$$L'_{\delta_a} = (L_{\delta_a} + I_{xz}N_{\delta_a}/I_x)G \qquad 1/\text{sec}^2$$

$$N'_\beta = (N_\beta + I_{xz}L_\beta/I_z)G \qquad 1/\text{sec}^2$$

$$N'_p = (N_p + I_{xz}L_p/I_z)G \qquad 1/\text{sec}$$

$$N'_r = (N_r + I_{xz}L_r/I_z)G \qquad 1/\text{sec}$$

$$N'_{\delta_r} = (N_{\delta_r} + I_{xz}L_{\delta_r}/I_z)G \qquad 1/\text{sec}^2$$

$$N'_{\delta_a} = (N_{\delta_a} + I_{xz}L_{\delta_a}/I_z)G \qquad 1/\text{sec}^2$$

$$G = \frac{1}{1 - \frac{I_{xz}^2}{I_x I_z}}$$

Note: writing these equations in matrix form may help you a lot.

Note on pitch angle at equilibrium.

Please do not put the value of $\Theta_0 = 0$, as the pitch angle is the summation of the angle of attack and the climb angle or flight path angle γ.

$$\theta = \alpha + \gamma$$
$$\theta_0 = \alpha_0 + \gamma_0$$

You will find the values of α_0 & γ_0 in the tables in NASA report in the flight condition table. Also, the initial values of the velocities (u_0, v_0, w_0) should be calculated from the value of the total speed along with the angle of attack and the side slip angle.

F/C #

H(FT)

M(-)

VTO(FPS)

VTO(KTAS)

VTO(KCAS)

W(LBS)

C.G.(MGC)

IX (SLUG-FT SQ)

IY (SLUG-FT SQ)

IZ (SLUG-FT SQ)

IXZ(SLUG-FT SQ)

EPSILCN(DEG)

Q(PSF)

QC(PSF)

ALPHA(DEG)

GAMMA(DEG)

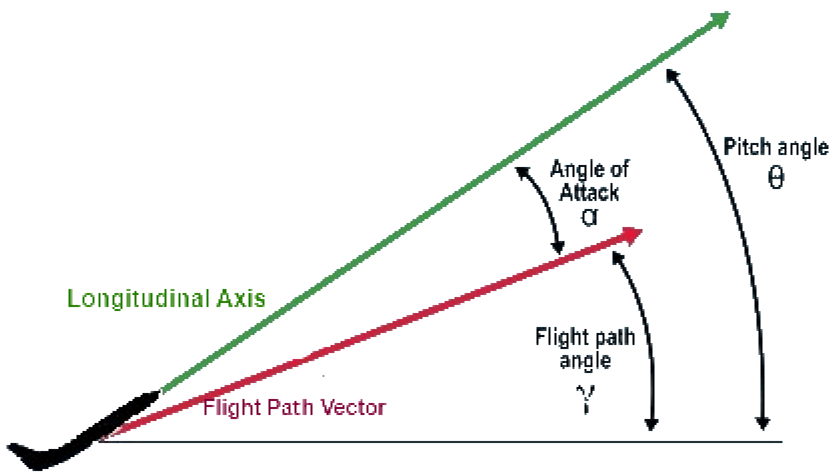
LXP(FT)

LZP(FT)

ITH(DEG)

XI (DEG)

LTH(FT)

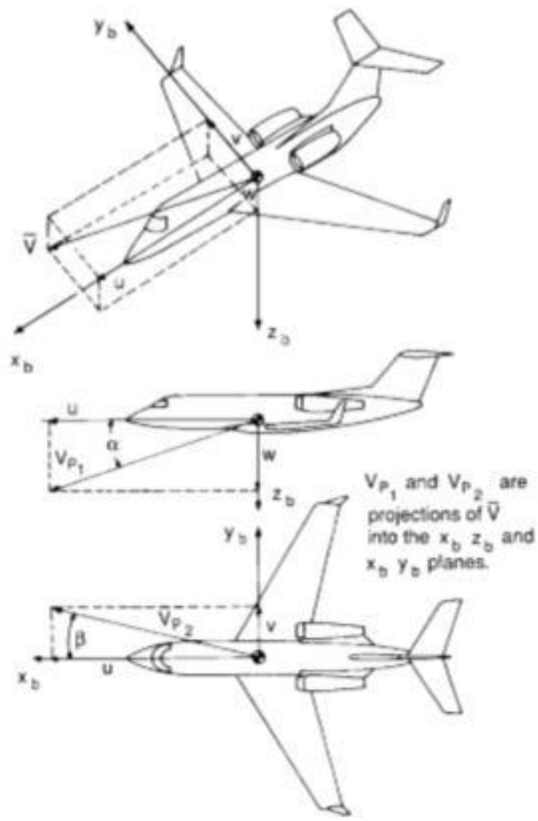


Angle of attack and sideslip angle

$$\alpha = \tan^{-1}\frac{w}{u}$$

and

$$\beta = \sin^{-1}\frac{v}{V}$$



Validation for B747 FC5

Use the validation_helper.m script to verify the results for B747 FC5 and replicate the document output.

Validation for your plane

Use the Excel_Validation_App.p script to verify the results for your assigned plane and flight condition. **Then generate the plots for your A/C.**

Simulink simulator

Use the 6 DOF Euler angles block in Simulink to solve the above problem and compare the results using plots.

Reminder: Adjust the solver settings for Simulink to match your Matlab solver.

References

1) John H. Blakelock - Automatic Control of Aircraft and Missiles-Wiley-Interscience (1991)
2) Flight Stability and Automatic Control - Robert C. Nelson
3) Etkin B., Reid L.D. - Dynamics of flight_ Stability and control-Wiley (1996)
4) NASA CR-2144--Heffley--Aircraft Handling Qualities Data