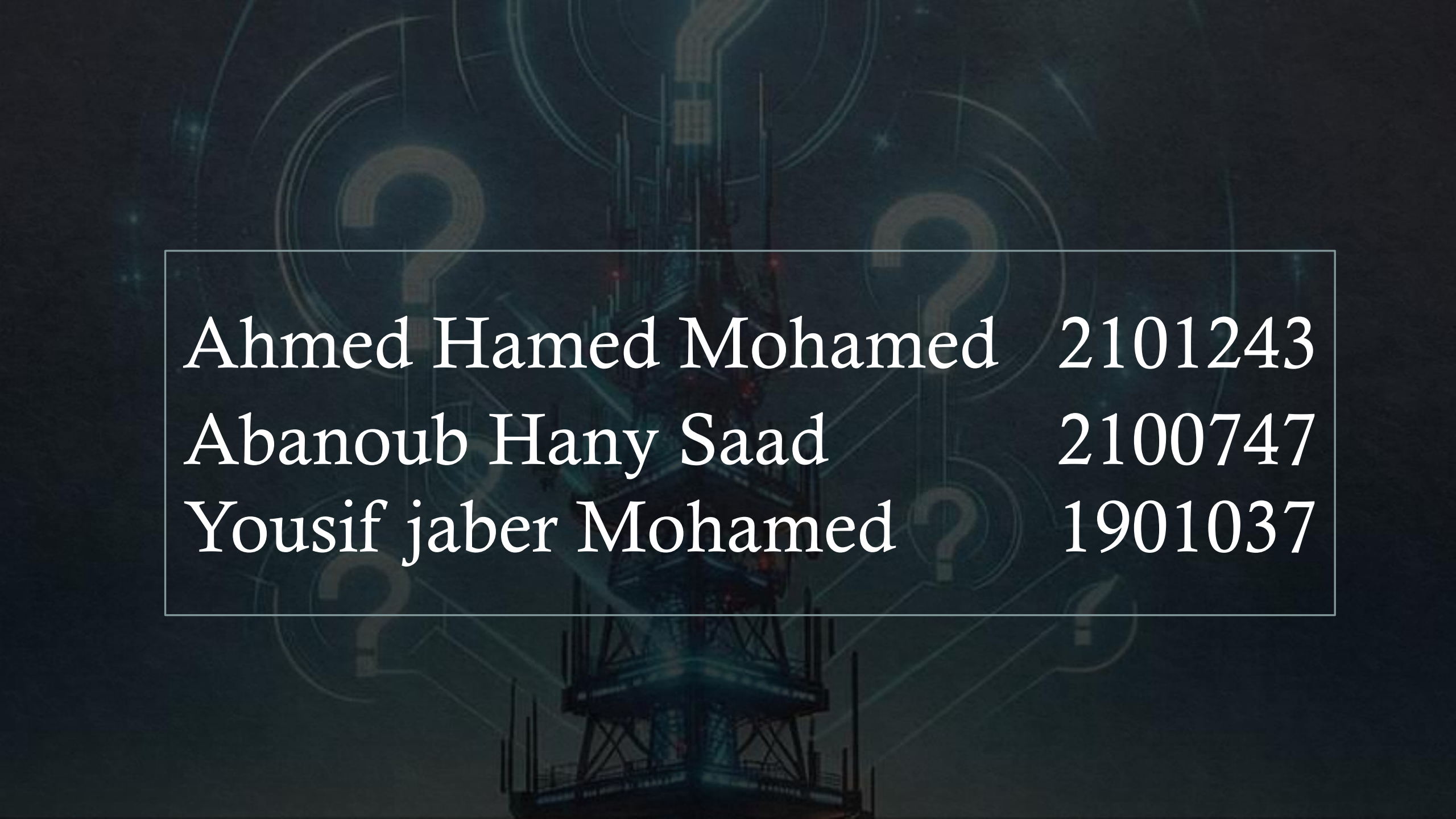
بسم الله الرحمن الرحيم

Signals & Systems Project



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1 Project specification

Fig. 1 shows ONE complete cycle of x(t).

A = 1 and T = 4 ms.

(a) Using Fourier Series analysis, determine

(i) The amplitude of the first harmonic a1.

(ii) The total power of x(t) PT.

(b) Using MATLAB determine PT and a1.

(c) Using MATLAB, plot the spectrum of x(t)

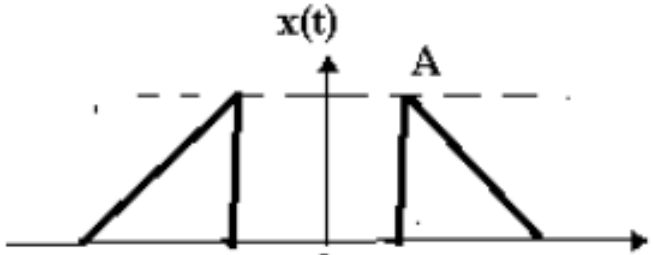
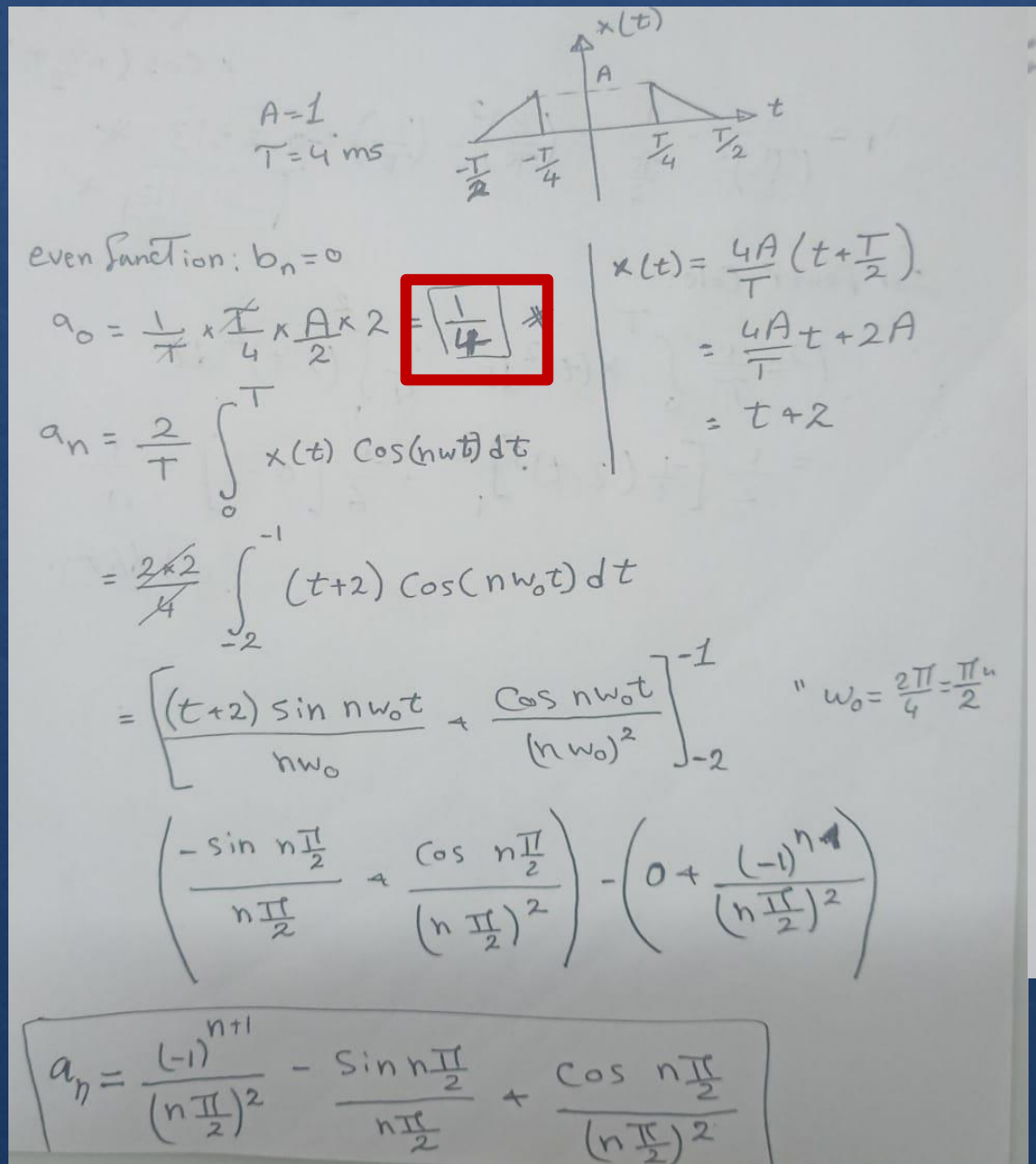


Fig. 1 One complete cycle

2 Manual Calculations



$$x(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{n^2 \omega_0^2} - \frac{\sin \frac{n\pi}{2}}{n\omega_0} - \frac{\cos \frac{n\pi}{2}}{n^2 \omega_0^2} \right) \times \cos(n\omega_0 t)$$

$$= \frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{(\frac{n\pi}{4})^2} - \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} + \frac{\cos \frac{n\pi}{2}}{(\frac{n\pi}{2})^2} \right) \times \cos(\tfrac{n\pi}{2} t)$$

$$a_1 = \frac{1}{(\frac{\pi}{2})^2} - \frac{1}{\frac{\pi}{2}} = \left(\frac{2}{\pi}\right)^2 - \left(\frac{2}{\pi}\right) = -0.2313$$

For Power Calc:

$$P = \frac{1}{T} \int_0^T x(t)^2 dt \overset{*}{=} \frac{2}{4} \int_1^2 (2-t)^2 dt$$

$$= \frac{1}{2} \left[\frac{-1}{3} (2-t)^3 \right]_1^2 = \frac{-1}{6} [0 - 1] = \frac{1}{6}$$

$$= 0.1667$$

3 Matlab Code

1

```
clear
clc
syms t n;
T = 4;
A = 1;
N = 100;

% Define the function x1(t)
x1 = piecewise(t >= 1 & t < 2, (-4*A/T)*(t-T/2),...
               t >= -T/4 & t <= T/4, 0,...
               t >= -2 & t < -T/4, (4*A/T)*(t+T/2));
%calculate an
an = (2/T)*int(x1*cos((n*pi/2)*t), t, -T/2, T/2);
%calculate a1
a1 = subs(an, n, 1);
% Calculate a0
a0 = (2/T)*int(x1, t, T/4, T/2);
% Display a0
fprintf('==============\n');
fprintf('a0= %f\n', a0);
% Display a1
fprintf('a1 = %f\n', a1);
annotation('textbox', [0.13, 0.72, 0.1, 0.1], 'String', ['a1: ', num2str(double(a1))]);
```

3 Matlab Code

2

```
% calculate x(t) by fourier series
%initialization value
x=a0;
for i = 1:N
    x = x + subs(an, n, i)*cos((i*pi/2)*t);
end

% Initialize power
p = (a0)^2; % Initial value of power
 % calculate total power
for i = 1:N
    p = p + (subs(an, n, i))^2/2;
end
% Display power
fprintf('Toal Power = %f\n', p);
annotation('textbox', [0.13, 0.82, 0.1, 0.1], 'String', ['Total Power: ',
num2str(double(p))]);
% Plot the result
subplot(2,1,1);
fplot(x, [-2, 2]); % Plot x(t) over the periodic interval [-2, 2]
xlabel('t');
ylabel('x(t)');
title('x(t)');
```

3 Matlab Code

3

```
% Initialize array to store harmonic coefficients
coefficients = zeros(1, N);

% Calculate coefficients up to N

for i = 1:N
    coefficients(i) = subs(an, n, i);
end

% Frequencies for each harmonic
frequencies = (1:N) * (1/T);

% Plot spectrum
subplot(2,1,2);
stem(frequencies, abs(coefficients), 'Marker', 'o');
% Set limits for x-axis
xlim([0, 15]);
xlabel('Frequency (Hz)');
ylabel('Amplitude');
title('Frequency Spectrum of x(t)');
fprintf('==============\n');
```

4 Simulation

N=100

Command Window

```
==============
a0= 0.250000
a1 = -0.231335
Toal Power = 0.165654
==============
fx >>
```

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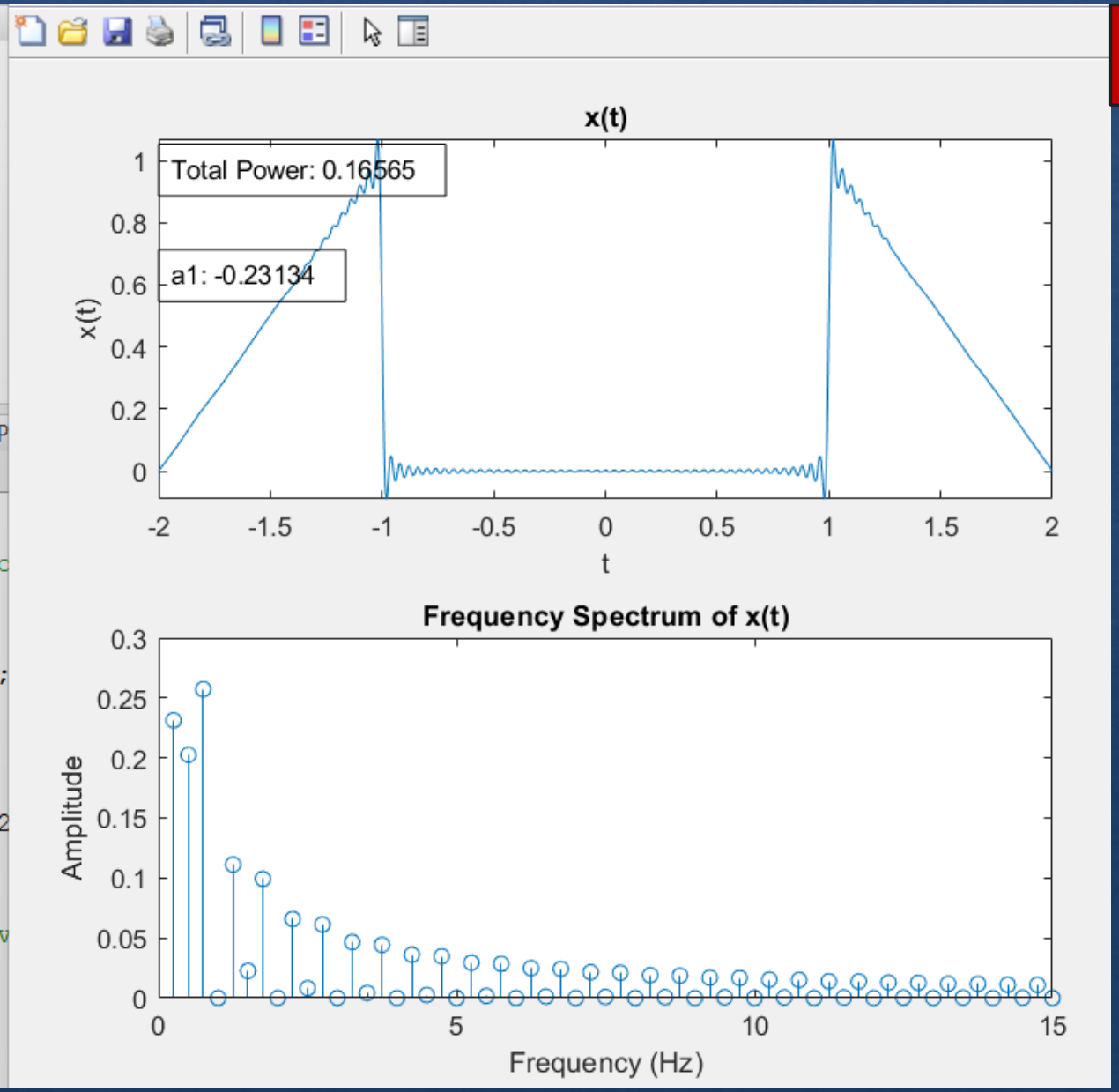
Project_of_singnal_fourier_series.m

```
32      % Initialize power
33 -    p = (a0)^2; % Initial value of po
34       % calculate total power
35 -    for i = 1:N
36 -        p = p + (subs(an, n, i))^2/2;
37 -    end
38      % Display power
39 -    fprintf('Toal Power = %f\n', p);
40 -    annotation('textbox', [0.13, 0.82
41      % Plot the result
42 -    subplot(2,1,1);
43 -    fplot(x, [-2, 2]); % Plot x(t) ov
44 -    xlabel('t');
45 -    ylabel('x(t)');
46 -    title('x(t)');
47
```



4 Simulation of x(t)

N=500

```
==============
a0= 0.250000
a1 = -0.231335
P1 = 0.089258


==============
>>
```

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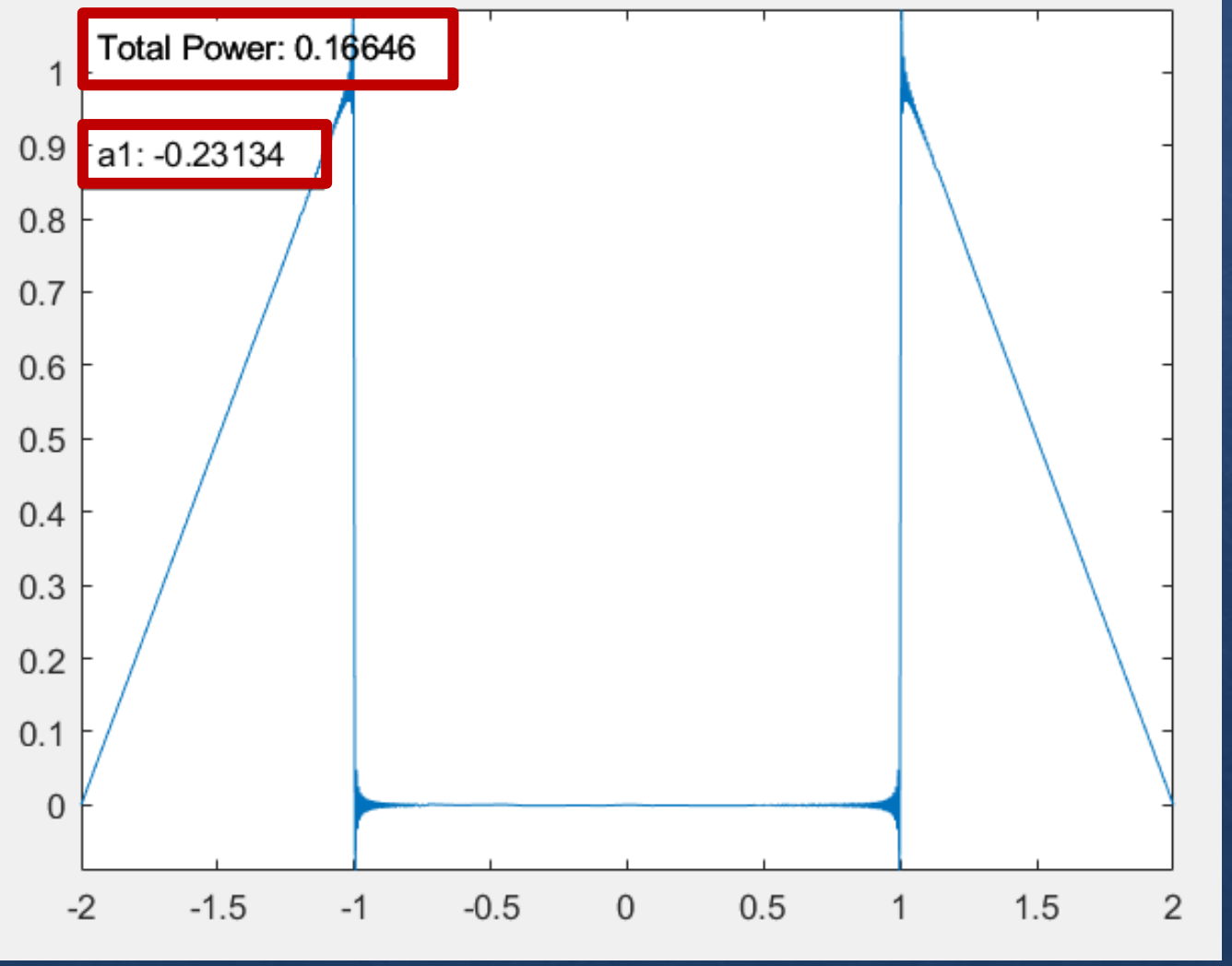
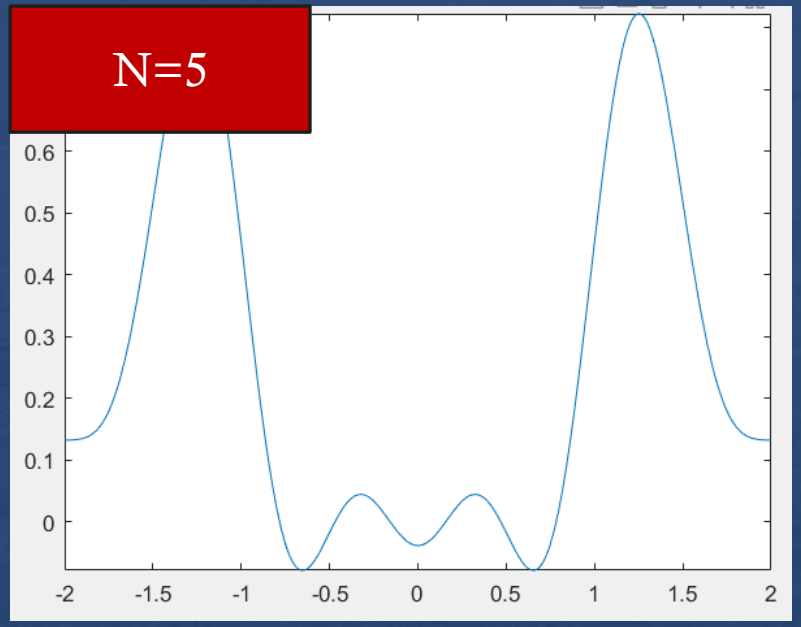
Project_of_singnal_fourier_series.m

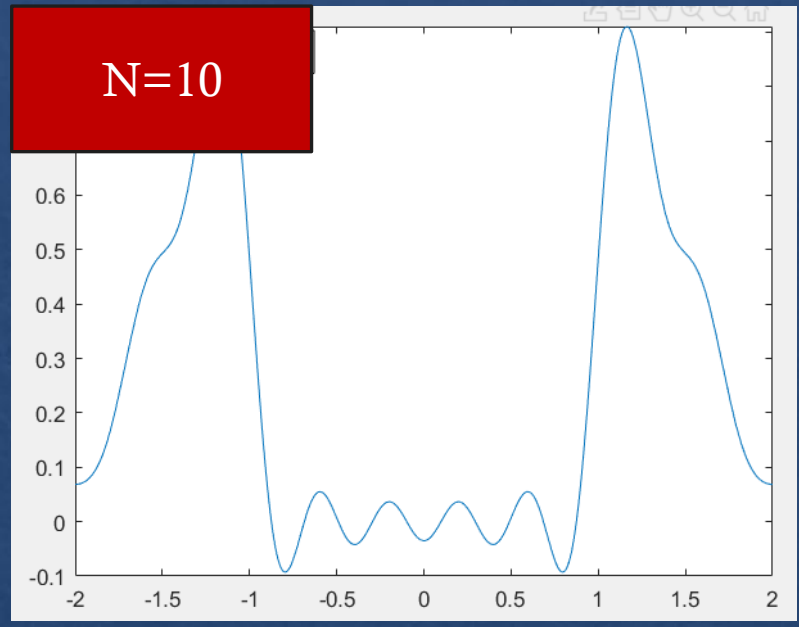
```
    clear
    clc
    syms t n;
    T = 4;
    A = 1;
    N = 500;

    % Define the function x1(t)
    x1 = piecewise(t >= 1 & t < 2, (
```



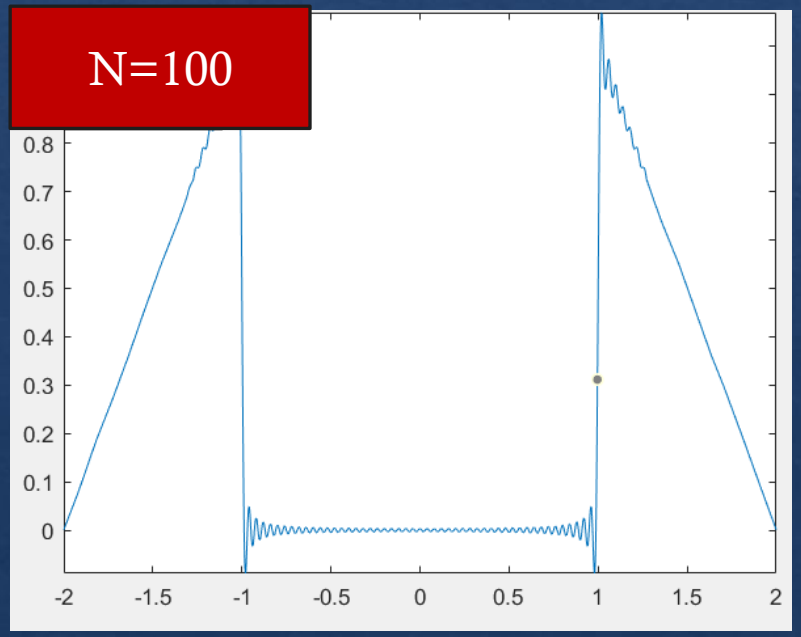
N=5



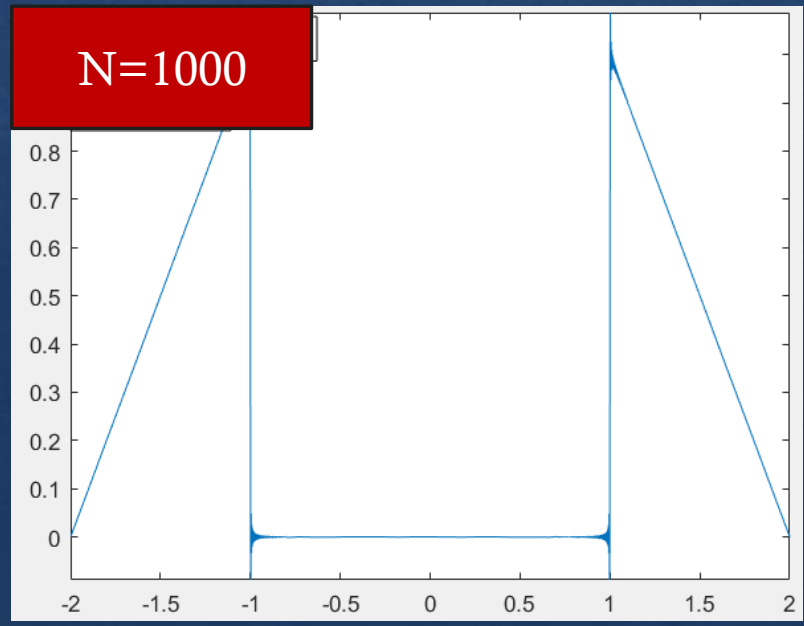
N=10



N=100



N=1000



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