

CAIRO UNIVERSITY

FACULTY OF SCIENCE

MATHEMATICS DEPARTMENT

Exam Midterm Math 493 (Operations research)  
4<sup>th</sup> year



Time allowed: 45 minutes

Date: 10-12-2020

Answer the following questions:

1) Consider the following (LP) problem:

$$\text{Maximize } Z = 3x_1 + x_2 + 5$$

$$\text{Subject to: } x_1 + x_2 = 3$$

$$x_1 + 3x_2 \leq 8$$

$$2x_1 + x_2 \leq 3; x_1, x_2 \geq 0$$

a) Solve the above (LP) problem by:

i) Graphical method and specify  $S, \theta, \theta_x$  and  $\theta_\mu$ .

ii) Two-Phase technique.

b) Find redundant constraints and degenerate points of this (LP) problem.

c) Find the Dual problem of the (LP) problem after change  $x_2$  by  $x_2$  free.

2) a) prove that if a maximize (LP) problem in standard form has finite optimal solution then it has an extreme optimal point.

b) The following tableau represents a Minimize (LP) problem in standard form:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	Solution
Z-eqn.	0	b	e	0	0	-9
	1	c	1	0	0	a
	0	d	-1	1	0	2
	0	-1	1	0	1	4

Give conditions on the parameters a, b, c, d, and e so that:

$b, e \leq 0$   $0, -ve \leftarrow \frac{e}{c}$

i) The table is in optimal form. ii) The table is in unbounded form.

iii) The table is in infeasible form (2).

$\rightarrow -ve$  sol.  $\rightarrow$   $\frac{b}{c} < 0$   $\frac{e}{c} > 0$

$c > 0$   
 $a < 0$

Best Wishes

infeasible form (1)

$0 \mid 0 \mid 0 \mid \frac{b}{c} > 0$

$\exists b_i \neq 0, a_{ik} = 0 \forall k$

$-ve$   
 $-ve, 0$   
 $-ve, 0$   
 $-ve, 0$

$b < 0$   
 $c, d \leq 0$

$\exists c_i < 0, a_{ik} < 0$

$\exists b_i < 0, a_{ik} > 0, \forall k$



Answer the following questions:

- (1) Solve the following (ILP) problem by apply Gomory's cutting plane method

$$\begin{aligned} \text{Minimize } & Z = x_1 - 2x_2 \\ \text{Subject to } & 2x_1 + x_2 \leq 5 \\ & -4x_1 + 4x_2 \leq 5 \end{aligned}$$

$$x_1, x_2 \geq 0 \text{ And integers}$$

The optimal continuous solution for given (ILP) problem is given by the table:

Basis	$x_1$	$x_2$	$x_3$	$x_4$	Solution
Z	0	0	-1/3	-5/12	-15/4
$x_1$	1	0	1/3	-1/12	5/4
$x_2$	0	1	1/3	1/6	5/2

- (2) Solve the following (FLP) problem

$$\begin{aligned} \text{Max } & \frac{x_1 + 2x_2}{4x_1 + 3x_2 + 3} \\ \text{Subject to } & x_1 + x_2 \leq 2 ; \quad -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- (3) Select (True) or (False) and correct the wrong answer for the following statements:

- The dual simplex method solves any (LP) problem. ✗
- The feasible region of (ILP) problem is convex set. *discrete*
- In (FLP) problem the objective function

$$w = \frac{p^t x + a}{q^t x + b} ;$$

$$(q^t x + b) > 0 \text{ or } (q^t x + b) < 0 \\ \forall x \in S$$

- The feasible region of (FLP) problem is discrete set. *Linear*
- The variables in (ILP) problem are non-negative and integers. ✓

*Best Wishes*