

CAIRO UNIVERSITY
FACULTY OF SCIENCE
MATHEMATICS DEPARTMENT



Exam Midterm Math 493 (Operations research) 4th year

Time: 45 Mints

Date: 10-11-2022

Answer the following questions:

Question (1)

a) Solve the following (LP) problem by using two-phase technique

$$\begin{aligned} &\text{Maximize } Z = x_1 + x_2 + 4 \\ &\text{Subject to } -x_1 + 3x_2 \leq 9 ; x_1 + x_2 \geq 5 ; 3x_1 - x_2 \leq 9 \\ &\quad x_1, x_2 \geq 0 \end{aligned}$$

b) The following table represents a minimize (LP) in standard form:

	x_1	x_2	x_3	x_4	x_5	solution
z-eqn.	0	B	E	0	0	3
	1	C	1	0	0	A
	0	D	-1	1	0	2
	0	-1	1	0	1	4

Give conditions on the parameters A, B, C, D, and E so that:

- The table is in optimal form
- The table is in unbounded form
- The table is in feasible form (2)

$$\begin{aligned} b_1 &\leq 0 \\ b &< 0 \\ c_1, d &\leq 0 \\ c &\geq 0 \\ a &< 0 \end{aligned}$$

Question (2)

a) Prove that a (LP) problem in standard form has a finite optimal solution it has an extreme optimal point.

b) Consider the following (LP) problem:

$$\begin{aligned} &\text{Minimize } Z = -x_1 - x_2 \\ &\text{Subject to } -x_1 + x_2 \leq 2 ; -x_1 + x_2 \geq -2 ; x_1(-) \\ &\quad 0 \leq x_1 \leq 4 ; 0 \leq x_2 \leq 4. \end{aligned}$$

Solve the (LP) problem graphically and find its dual problem.

Answer the following questions:

Question (1)

- a) Solve the following (ILP) problem by apply Gomory's cutting plane method

$$\begin{aligned} \text{Minimize } & Z = x_1 - 2x_2 \\ \text{Subject to } & 2x_1 + x_2 \leq 5 \\ & -4x_1 + 4x_2 \leq 5 \\ & x_1, x_2 \geq 0 \text{ And } x_2 \text{ integer} \end{aligned}$$

$x_1, x_2 \geq 0$

The optimal continuous solution for given (ILP) problem is given by the table:

Basis	X_1	X_2	X_3	X_4	Solution
Z	0	0	-1/3	-5/12	-15/4
X_1	1	0	1/3	-1/12	5/4
X_2	0	1	1/3	1/6	5/2

$2 \times \frac{1}{2}$

- b) Find the dual problem to the problem in (a) after change $x_1, x_2 \geq 0$ and x_2 integer to $x_1, x_2 \geq 0$
- c) Solve the problem in (b) graphically and specify $(S, \theta, \theta_x, \theta_\mu, z^*, x^*)$

Question (2)

Select (True) or (False) and correct the wrong answer for the following statements:

- a. The dual simplex method solves any (LP) problem. \times
- b. The feasible region of (ILP) problem is convex set. \times discrete
- c. The variables in (ILP) problem are non-negative and integers. \checkmark
- d. A point $x \in S$ is degenerate if the number of constraints active at x is equal the dimension of the feasible region S . \checkmark
- e. To solve maximize (LP) problem by M- technique we add $M \sum_{i=1}^k R_i$ to objective function. \checkmark
- f. To solve (LP) problem by two-phase Method the objective function in phase (II) is a function of basic variable. \times