Operations Research
Week 6 Lecture Notes

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Shadow Price

Contents

1. Introduction

In linear programming problems the shadow price of a constraint is the difference between the optimized value of the objective function and the value of the objective function, evaluated at the optional basis, when the right hand side (RHS) of a constraint is increased by one unit.

OR

A shadow price of a resource constraint in linear programming is usually defined as the maximum price which should be paid to obtain an additional unit of resource.

OR

The rate of change in the optimal objective value of a linear programming model obtained by changing right hand side of the constraint by one unit.

2. Working Rule

Following are the working rule for the calculation of shadow price:

a. Find the graphical solution of LPP

b. Calculate the shadow price

c. Compute the range of applicability of the shadow price calculated in also known as "Feasibility Ranges"

3. Explanation with Example

A company produces two types of products on two machines. A unit of product – I requires 2 hours on machine – I and 1 hour on machine – II. A unit of product – II requires 1 hour on machine – I and 2 hours on machine – II. The revenue per unit of product – I and II are \$300 and \$200 respectively. The total processing time available for each machine is 8 hours.

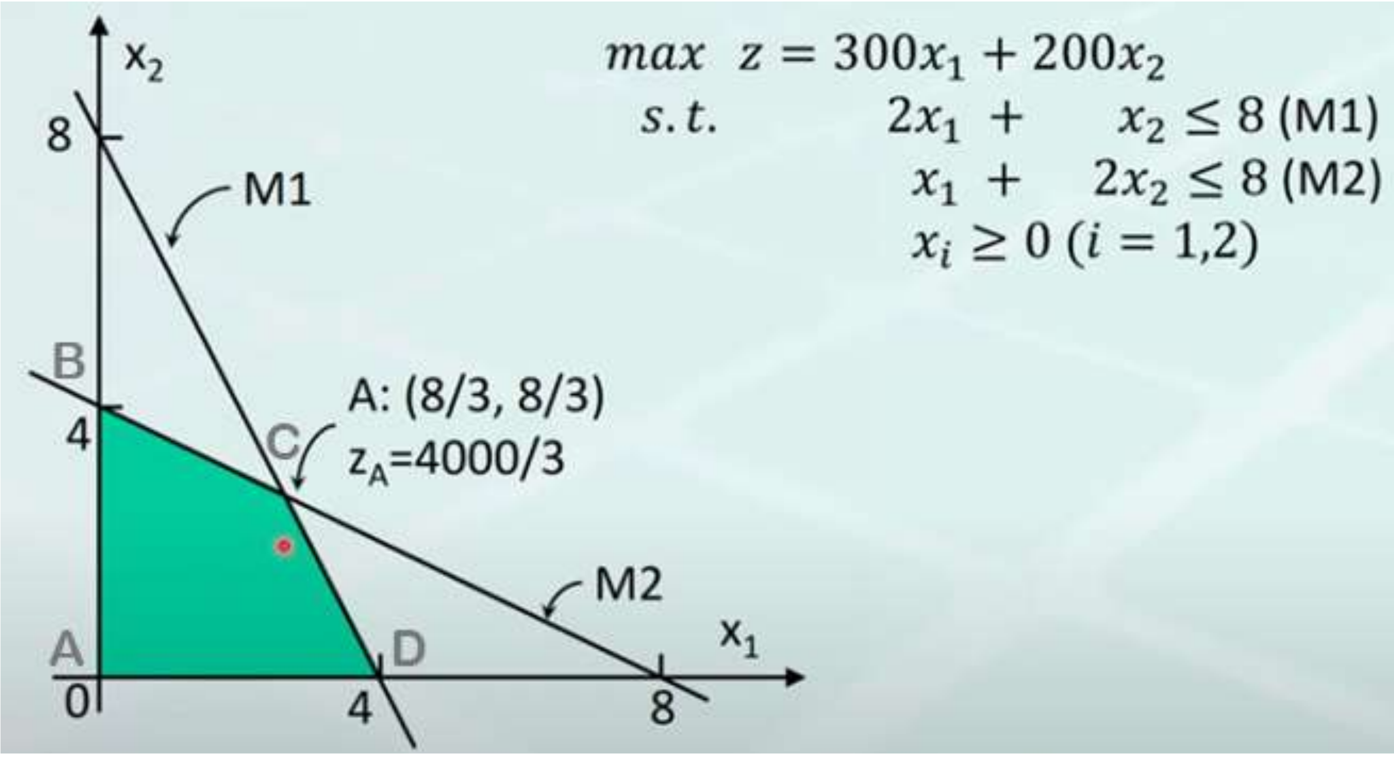
a. Find the optimal solution.
b. Determine the shadow prices and their feasibility ranges.
c. If the company wants to increase the capacity of both machines, which machine will receive higher priority?
d. Suggestion is made to increase the capacities of both the machines by \$50, is this advisable?

Solution: Part a. & b.

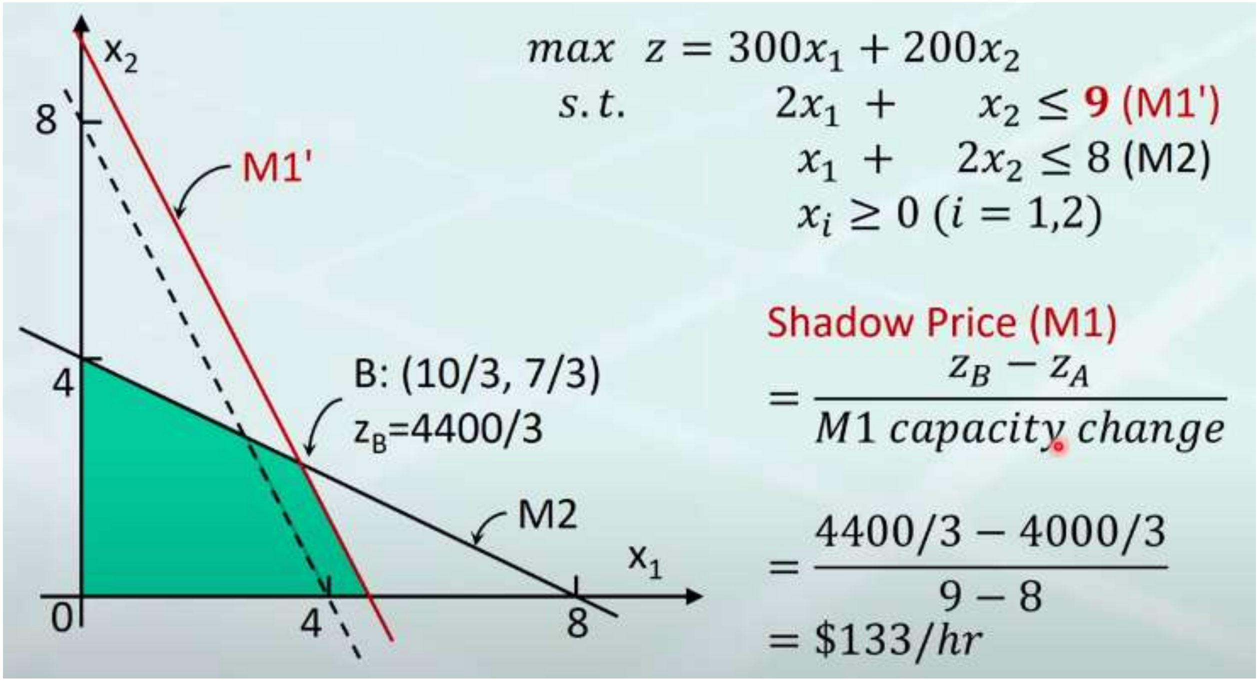
Step 1:

	Product 1	Product 2	Daily Availability
Machine 1	2 hours	1 hour	8 hours
Machine 2	1 hour	2 hours	8 hours
Unit Profit	\$300	\$200	

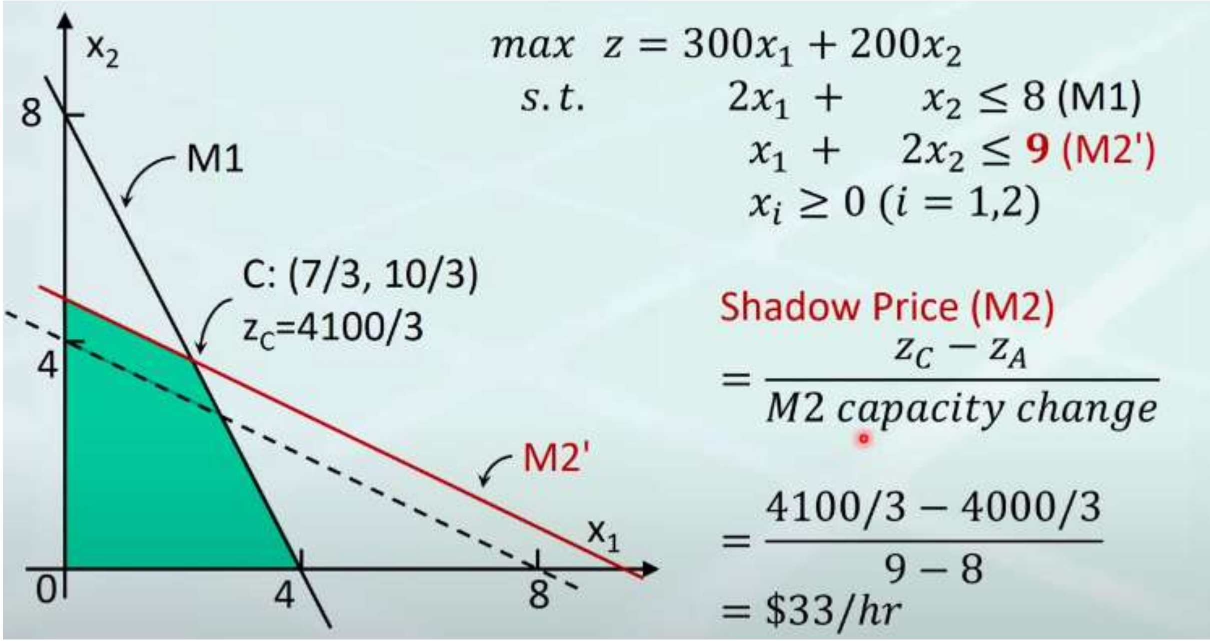
$$\begin{aligned} \max\ z &= 300x_1 + 200x_2 \\ s.t.\quad & 2x_1 + x_2 \le 8 \text{ (M1)} \\ & x_1 + 2x_2 \le 8 \text{ (M2)} \\ & x_i \ge 0\ (i = 1,2) \end{aligned}$$



Step II:



Calculation of Shadow price after increasing the constraint – I by 1.



Calculation of Shadow price after increasing the constraint – I by 1.

Final Conclusion

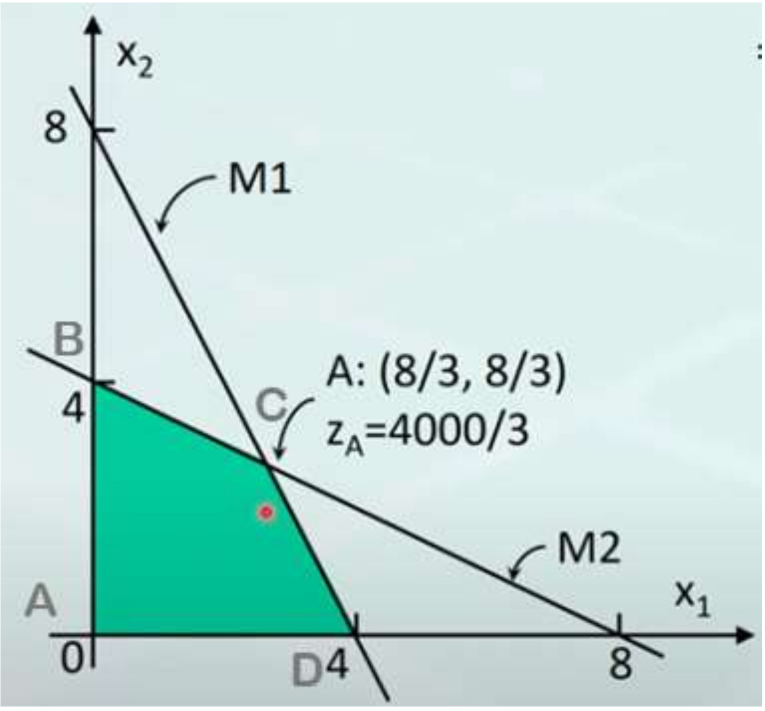
1. The revenue from Machine – I will be \$133 per hour.
2. The revenue from Machine – II will be \$33 per hour.

Step III:

Let the expressions for the capacity of Machine – I and Machine – II are given by

$M_I = 2x_1 + x_2$
$M_{II} = x_1 + 2x_2$
From figure we can see that the Minimum capacity of Machine – I is at B(0,4) that is Given by
$M_I = 2(0) + (4) = 4$ hrs

Similarly, from figure we can see that the Maximum capacity of Machine – I is at D(4,0) that is Given by
$M_I = 2(4) + (0) = 8$ hrs



From figure we can see that the Minimum capacity of Machine – II is at D(4,0) that is Given by
$M_{II} = 4 + 2(0) = 4$ hrs

Similarly, from figure we can see that the Maximum capacity of Machine – II is at B(4,0) that is Given by
$M_{II} = 2(4) + (0) = 8$ hrs

So the feasibility ranges for both the machines are given by

$$4 \leq M_I \leq 8$$
$$4 \leq M_{II} \leq 8$$

Which shows that the shadow prices will remain applicable for the above ranges.

Part c.
The shadow price for Machine – I and Machine – II are given as follows:

- Shadow price for Machine – I will be \$133 per hour.
- Shadow price for Machine – II will be \$33 per hour.

Since Machine – I is given higher revenue of \$133 per hour so that is why Machine – I will be given higher priority.

Part d.

- The additional net revenue per hour for Machine – I is given by
- Additional net revenue per hour $M_I = 133 - 50 = \$83$
- Similarly, the additional net revenue per hour for Machine – II is given by
- Additional net revenue per hour $M_{II} = 33 - 50 = -\$20$

This shows that this increase is advisable only for Machine – I

4. Practice Questions

1. A company produces two types of products on two machines. A unit of product – I requires 3 hours on machine – I and 1 hour on machine – II. A unit of product – II requires 1 hour on machine – I and 3 hours on machine – II. The revenue per unit of product – I and II are $30 and $20 respectively. The total processing time available for each machine is 6 hours.

a. Find the optimal solution.
b. Determine the shadow prices and their feasibility ranges.
c. If the company wants to increase the capacity of both machines, which machine will receive higher priority?

2. A company produces two types of products on two machines. A unit of product – I requires 3 hours on machine – I and 1 hour on machine – II. A unit of product – II requires 1 hour on machine – I and 3 hours on machine – II. The revenue per unit of product – I and II are $30 and $20 respectively. The total processing time available for each machine is 8 hours.

d. Find the optimal solution.
e. Determine the shadow prices and their feasibility ranges.
f. If the company wants to increase the capacity of both machines, which machine will receive higher priority?