lec

1 How to Convert diffrential equation into difference equation

$$\dot{y}(t) + 2y(t) = u(t)$$

$$\dot{y}(t) = \frac{dy}{dt} = \frac{\Delta y}{\Delta t} = \frac{y(k+1) - y(k)}{T}$$

let $T = 1$ sec

$$y(k+1) - y(k) + 2y(k) = u(k)$$

$$\boxed{y(k+1) + y(k) = u(k)}$$

2 Relation between s domain and Z-domain

$$s \longleftrightarrow Z$$

laplace $\Rightarrow$ $Y(s) = L\{y(t)\} = \int_0^\infty y(t)\, e^{-st}\, dt$

Z-Transform $\Rightarrow$ $Y(Z) = Z\{y(k)\} = \sum_{k=0}^{\infty} y(k)\, Z^{-k}$

discretization

$$\sum_{k=0}^{\infty} y(kT)\, e^{-skT} = \sum_{k=0}^{\infty} y(kT) \left(e^{sT}\right)^{-k}$$

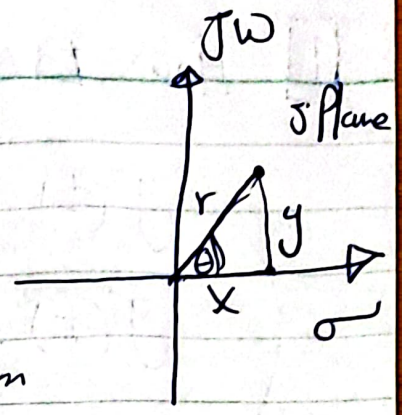
$$\Rightarrow \boxed{Z = e^{Ts}} = e^{T(\sigma + j\omega)} = e^{T\sigma} * e^{j\omega T} = r e^{j\theta}$$

$$s = \underset{\text{real}}{\sigma} + \underset{\text{imaginary}}{j\omega} \qquad \theta = \omega T, \quad r = e^{T\sigma}$$

$Z = r e^{j\theta}$ exponential form

$Z = r \angle \theta$ Polar form

$Z = X + jy$ Cartesian form

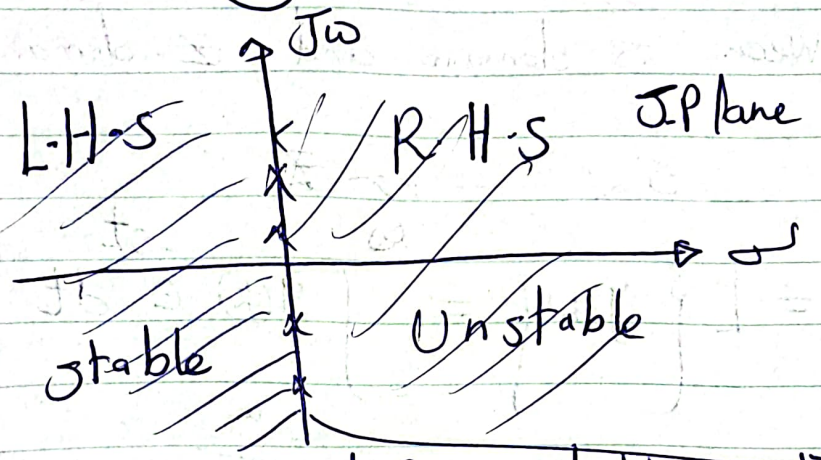


$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1} \frac{y}{x}$

$r = e^{T\sigma}$

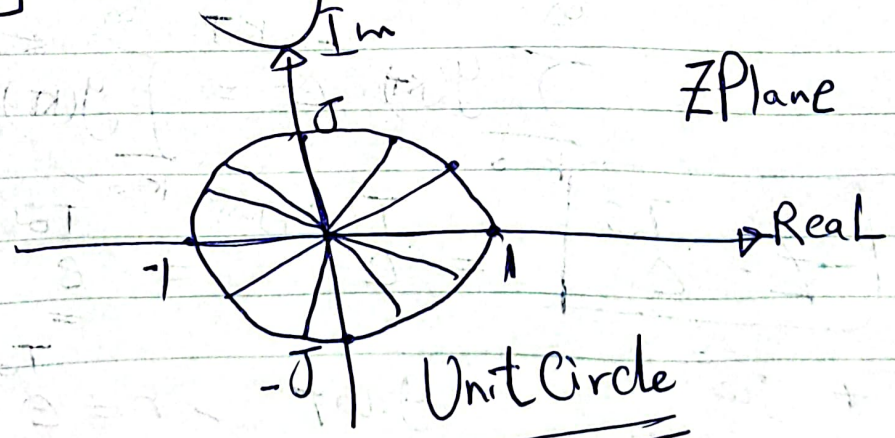
3 Stability in s-plane



Critically Stable

$\sigma = 0 \Rightarrow r = e^{T\sigma} = e^{0} = 1$

4 Stability in Z-Plane or Z-domain



Unit Circle

* Point lies on the Imaginary axis $\Rightarrow \sigma = 0 \Rightarrow r = e^{\sigma T} = 1$

* Point lies on the L.H.S $\Rightarrow \sigma = -ve \Rightarrow r = e^{-ve} < 1 \Rightarrow$ inside the Circle

* Points lies on the R.H.S $\Rightarrow \sigma = +ve \Rightarrow r = e^{+ve} > 1$

$\Rightarrow$ outside the Circle

$\Downarrow$

* if Poles lies inside the Unit Circle ($r<1$) $\Rightarrow$ system is stable.

* if Poles lies outside the Unit Circle ($r>1$) $\Rightarrow$ system is Unstable

* if the poles lies on the Unit Circle ($r=1$) $\Rightarrow$ system is Critically stable.

$\boxed{4}$ Solution of difference equation

- ① using Recursive Method $\Rightarrow$ Produce Samples
- ② Using Z-Transform $\Rightarrow$ Produce closed form

$y(0) = y(1) = 0$, $u(k) = \begin{cases} 1 & k \geq 1 \\ 0 & k = 0 \end{cases}$

Example $y(k+2) - 0.75\,y(k+1) + 0.125\,y(k) = u(k)$

Recursive Method $y(k+2) = 0.75\,y(k+1) - 0.125\,y(k) + u(k)$

$k=0 \Rightarrow y(2) = 0.75\,\cancel{y(1)}^{0} - 0.125\,\cancel{y(0)}^{0} + \cancel{u(0)}^{0} = 0$

$k=1 \Rightarrow y(3) = 0.75\,\cancel{y(2)}^{0} - 0.125\,\cancel{y(1)}^{0} + \cancel{u(1)}^{1} = 1$

$k=2 \Rightarrow y(4) = 0.75\,\cancel{y(3)}^{1} - 0.125\,\cancel{y(2)}^{0} + \cancel{u(2)}^{1} = 1.75$

$\vdots$

Using Z-Transform $y(k+2) - 0.75\,y(k+1) + 0.125\,y(k) = u(k)$

$$\left(Z^2 - 0.75Z + 0.125\right) Y(Z) = U(Z)$$

$$U(Z) = Z^{-1} * \frac{Z}{Z-1} = \frac{1}{Z-1}$$

$$\frac{Y(Z)}{Z} = \frac{1}{Z(Z-1)} * \frac{1}{Z^2 - 0.75Z + 0.125}$$

$$\frac{Y(Z)}{Z} = \frac{1}{Z(Z-1)(Z-0.5)(Z-0.25)}$$

$$\frac{Y(Z)}{Z} = \frac{A}{Z} + \frac{B}{Z-1} + \frac{C}{Z-0.5} + \frac{D}{Z-0.25}$$

$$A = -8, \quad B = \frac{8}{3}, \quad C = -16, \quad D = \frac{64}{3}$$

$$Y(z) = -8 + \frac{8/3\, z}{z-1} - \frac{16z}{z-0.5} + \frac{64/3\, z}{z-0.25}$$

$$y(k) = -8\delta(k) + \frac{8}{3} - 16\,(0.5)^k + \frac{64}{3}\,(0.25)^k$$

$$k=0 \Rightarrow y(0) = -8 + \frac{8}{3} - 16 + \frac{64}{3} = 0$$

$$k \geq 1 \Rightarrow y(k) = \frac{8}{3} - 16\,(0.5)^k + \frac{64}{3}\,(0.25)^k$$

$$y(k) = \begin{cases} 0 & k=0 \\ \frac{8}{3} - 16\,(0.5)^k + \frac{64}{3}\,(0.25)^k & k \geq 1 \end{cases}$$

$$k=1 \Rightarrow y(1) = \frac{8}{3} - 16 * 0.5 + \frac{64}{3} * 0.25 = 0$$

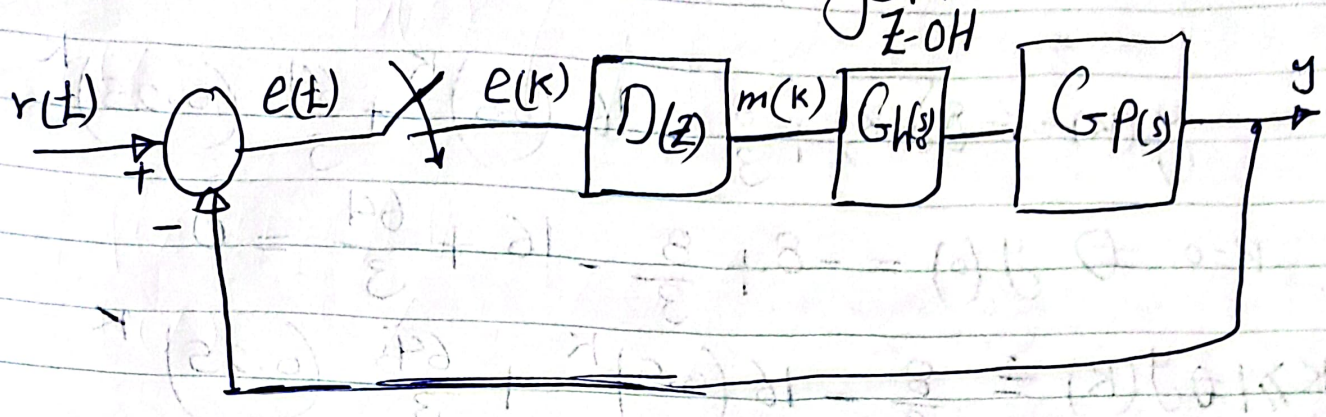
$$k=2 \Rightarrow y(2) = \frac{8}{3} - 16 * (0.5)^2 + \frac{64}{3} * (0.25)^2 = 0$$

$$k=3 \Rightarrow y(3) = \frac{8}{3} - 16\,(0.5)^3 + \frac{64}{3} * (0.25)^3 =$$

$$y(3) = \frac{8}{3} - \frac{\cancel{16}^{\,2}}{\cancel{8}_{\,1}} + \frac{\cancel{64}}{3} * \frac{1}{\cancel{4}} * \frac{1}{\cancel{4}} * \frac{1}{\cancel{4}}$$

$$y(3) = \frac{8}{3} - 2 + \frac{1}{3} = \frac{9}{3} - 2 = 3 - 2 = \underline{\underline{1}}$$

5 Open Loop discrete time system



$$D(z) = \frac{M(z)}{E(z)}$$

Mathmatical Model

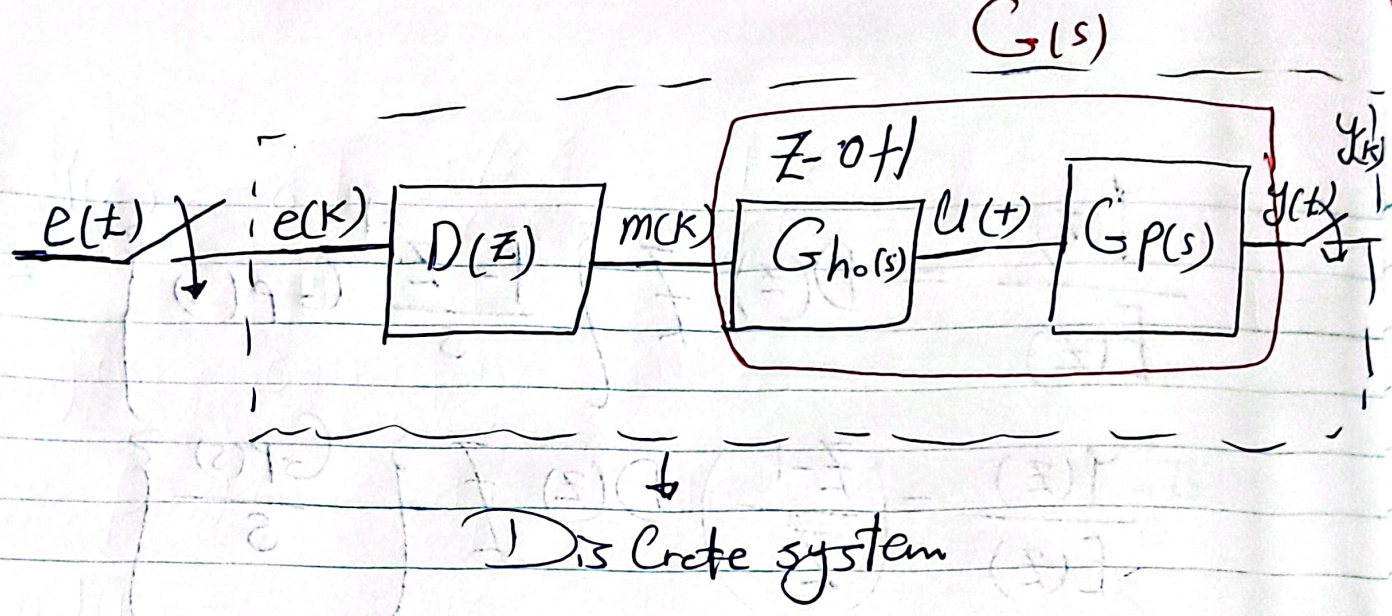
- Difference equation
- Transfer function

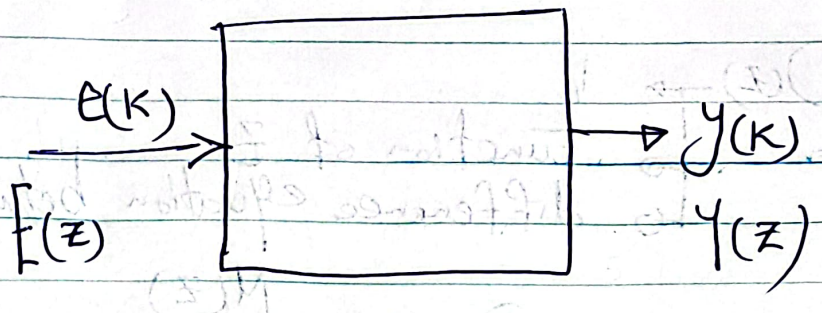
Analysis of the system

- TF or difference equation
- Time response
- Stability
- Root locus

systems $\Rightarrow$ divide the Problem

- open loop system
- closed loop system





$$\frac{Y(Z)}{E(Z)} = Z\left\{ D(Z)\, G(s) \right\} = D(Z)\, Z\left\{ G(s) \right\}$$

$$\frac{Y(Z)}{E(Z)} = D(Z)\, Z\left\{ G_{ho}(s)\, G_p(s) \right\}$$

$$G_{sho}(s) = \frac{1 - e^{-Ts}}{s}$$

$$\frac{Y(Z)}{E(Z)} = D(Z)\, Z\left\{ \frac{1 - e^{-Ts}}{s}\, G_p(s) \right\}$$

$$\boxed{Z = e^{Ts}}$$

$$\frac{Y(z)}{E(z)} = D(z)\, \mathcal{Z}\left\{ \frac{1-z^{-1}}{s} G_p(s) \right\} * \frac{z}{z}$$

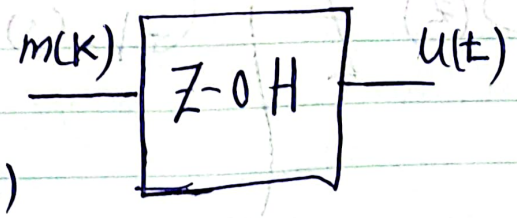
$$\frac{Y(z)}{E(z)} = \left(\frac{z-1}{z}\right) D(z)\, \mathcal{Z}\left\{ \frac{G_p(s)}{s} \right\}$$

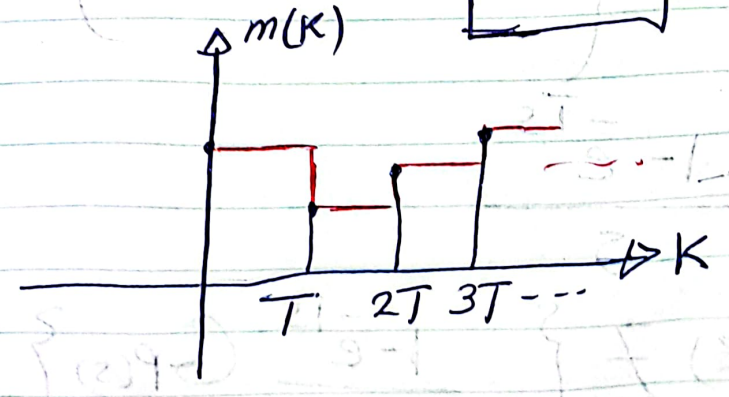
$D(z) \rightarrow$ 1,

- Function of z
- difference equation between $e(k), m(k)$

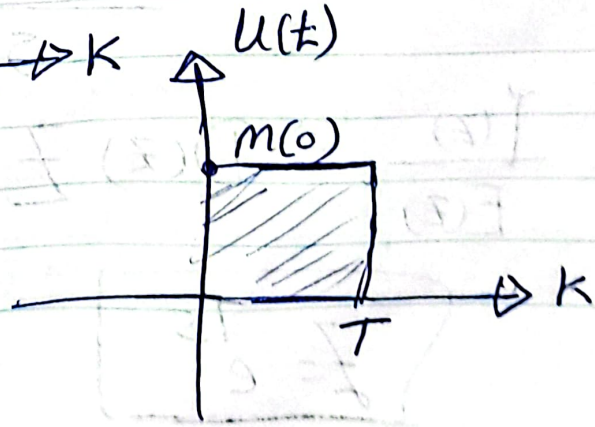
$$D(z) = \frac{M(z)}{E(z)}$$

$\boxed{6}$ Z-OH $\Rightarrow$ $TF = \dfrac{1-e^{-Ts}}{s} = G_{ho}(s)$





Take One Sample $\Rightarrow$



Unit step

$$u(t) = m(0)\left[u_s(t) - u_s(t-T)\right]$$

output input

$$U(s) = m(0)\left[\frac{1}{s} - \frac{1}{s}e^{-Ts}\right] = m(0)\,\frac{1-e^{-Ts}}{s}$$

$$\frac{\text{Output}}{\text{input}} = TF = \frac{U(s)}{m(0)} = G_{Ho}(s) = \frac{1-e^{-Ts}}{s}$$

$\boxed{T}$ Time Response $\Rightarrow y(k)$

$$\frac{Y(z)}{E(z)} = \left(\frac{z-1}{z}\right)\left(D(z)\right) Z\left\{\frac{G_P(s)}{s}\right\}$$

$e(k)$ → Unit step $\Rightarrow E(z) = \frac{z}{z-1}$

→ Unit impulse $\Rightarrow E(z) = 1$

~~Substitute~~ Then take inverse Z-Transform to get $y(k)$

Example ① given $G_P(s) = \frac{1}{s+1}$, $T = 1\,sec$

$m(K+1) = 2e(K+1) - e(K)$

Find ① total TF $\frac{Y(z)}{E(z)}$

② Unit step Response

③ Unit Impulse Response

④ Discuss the stability