Frist order system

General Form:

$$T.F = \frac{K}{\tau S+1}$$

$K \rightarrow$ DC gain: steady state gain of the system when input is constant

if $K = 2$ and input is constant 5 output will eventually settle at $2 \times 5 = 10$

τ Time constant: indecates to how fast or slow the response of the system

- also defines time for system output to reach 63% of its final value
- smaller time constant mean faster respone

second order system

$$T.F = \frac{K\,w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

w_n (natural frequency): it defines how fast the system oscillates when undamped

ζ (damping ratio): it influences the rate of decay of oscillations and system stability

Characteristic Equ: $s^2 + 2\zeta w_n s + w_n^2$

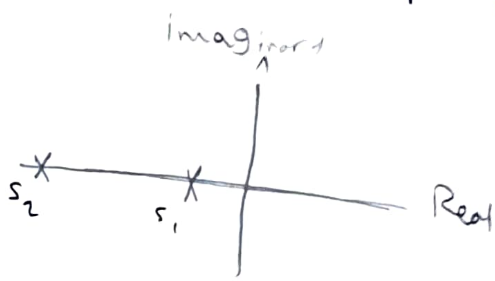
From general role

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\zeta w_n \pm w_n\sqrt{\zeta^2 - 1}$$

Poles of the system

when $\zeta > 1 \rightarrow s_1 \neq s_2$ and values s_1 & s_2 will be real negative

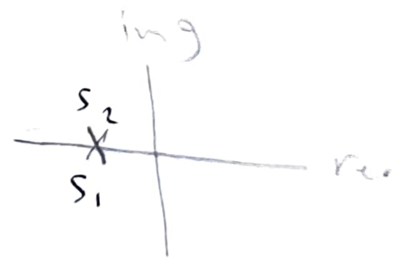
over damped



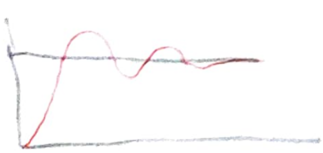
system is stable

when $\zeta = 1 \rightarrow s_1 = s_2$ and values will be real

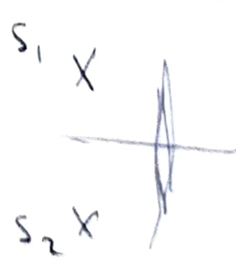
critical damped



when $0 < \zeta < 1$ $s_1 \neq s_2$ but complex number



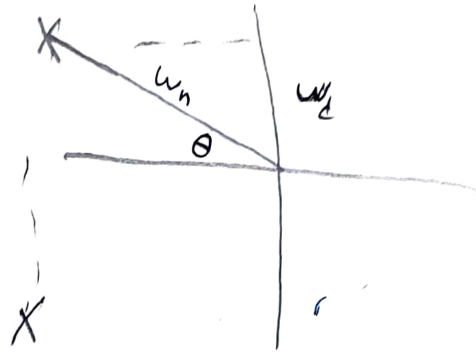
under damped



ossilation and then stable

$T_r = \dfrac{\pi - \theta}{W_d}$

damped frequency



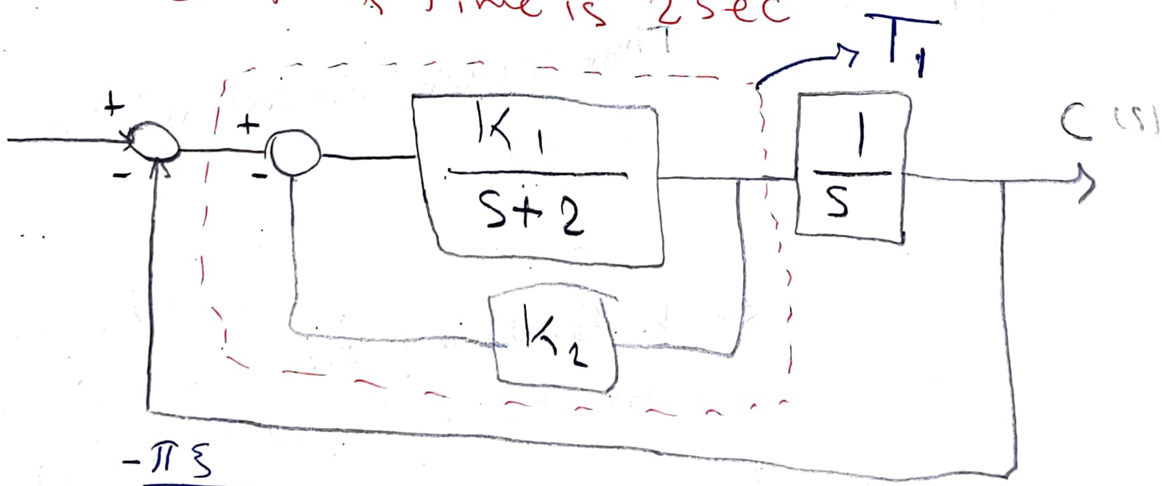
$\theta = \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)$

$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} * 100$

$T_p = \dfrac{\pi}{w_d}$, $w_d = w_n\sqrt{1-\xi^2}$

$T_s = 4T = \dfrac{4}{\xi W_n}$, $T = \dfrac{1}{\xi W_n}$ → when error 2%

determine The Value of K_1 , K_2 when system has maximum overshoot in unit step response is 25% and Peak time is 2 sec



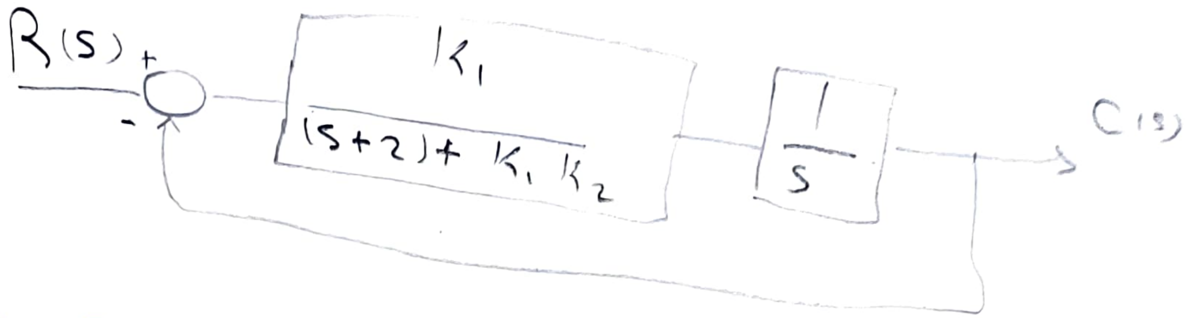
$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} * 100 \Rightarrow 0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$\therefore \xi = 0.403$

$T_p = \dfrac{\pi}{w_d} = \dfrac{\pi}{w_n\sqrt{1-\xi^2}} \Rightarrow 2 = \dfrac{\pi}{w_n\sqrt{1-(0.403)^2}}$

$\therefore W_n = 1.716$ rad/s

$$T_1 = \frac{G(s)}{1 - G(s)\,H(s)} = \frac{K_1}{(s+2) + K_1 K_2}$$



$$T_2 \Rightarrow G(s) = \frac{K_1}{s[(s+2)+K_1K_2]} \quad , \quad H(s) = 1$$

$$Ch.eq = 1 + G(s)\,H(s) = 1 + \frac{K_1}{s[(s+2)+K_1K_2]} * 1 = 0$$

$$= \frac{s[(s+2)+K_1K_2] + K_1}{\text{- - -}} \cancel{= \frac{0}{1}}$$

$$= s[(s+2)+K_1K_2] + K_1 = s^2 + 2s + K_1K_2\,s + K_1$$

$$= s^2 + (2+K_1K_2)\,s + K_1 = 0 \longrightarrow (1)$$

General form for 2nd order system

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

by comparison:

$$K_1 = \omega_n^2 = (1{,}716)^2 = 2{,}94 \quad \#$$

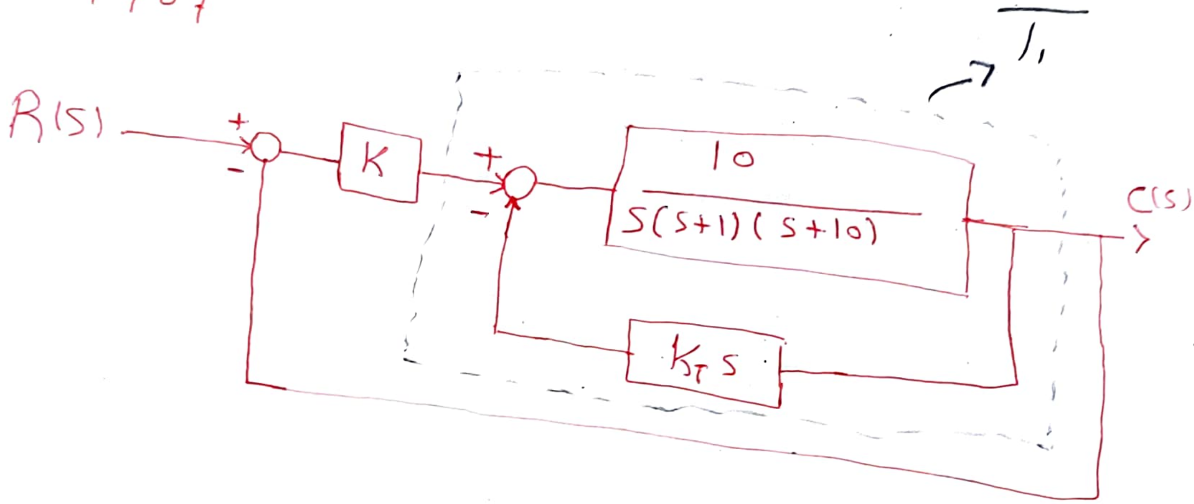
$$2 + K_1K_2 = 2\zeta\omega_n \Rightarrow 2 + K_2(2{,}94) = 2(0{,}403)(1{,}716)$$

$$\therefore K_2 = -0{,}209 \quad \#$$

Find The value of K_t, K_t So That following Specifications are satisfied:

$K_v = 1$

$\xi = 0.707$



$$T_1 = \frac{G(S)}{1 + G(S)\,H(S)} = \frac{10}{S(S+1)(S+10) + 10 K_T S}$$

$$G(S) = \frac{10 \cdot K}{S(S+1)(S+10) + 10 K_T S} \quad , \quad H(S) = 1$$

Ch.eq $\Rightarrow$ $1 + G(S)\,H(S) = S(S+1)(S+10) + 10 K_T S + 10K = 0$

$= S^3 + 10S^2 + S^2 + 10S + 10K_T S + 10K = 0$

$= S^3 + 11S^2 + (10 + 10K_T)\,S + 10K = 0 \quad \rightarrow (1)$

General form for 3rd order Ch.eq system

$(S + \alpha)(S^2 + 2\xi W_n S + W_n^2) = 0$

$S^3 + 2\xi W_n S^2 + W_n^2 S + S^2\alpha + 2\xi W_n \alpha S + W_n^2 \alpha$

$S^3 + (2\xi W_n + \alpha)\,S^2 + (W_n^2 + 2\xi W_n \alpha)\,S + W_n^2 \alpha = 0 \quad \rightarrow (2)$

$$K_v = \lim_{s\to 0} s\,G(s)\,H(s) = \lim_{s\to 0} s \cdot \frac{10K}{s(s+1)(s+10) + 10K_T s}$$

$$= \lim_{s\to 0} \frac{10K}{(s+1)(s+10) + 10K_T s} = \frac{10K}{10 + 10K_T} = \frac{K}{1+K_t}$$

$$K_v = \frac{K}{1+K_T} \quad \therefore K = 1 + K_T \rightarrow (3)$$

by comparison between (1), (2):

$\rightarrow 2\zeta w_n + \alpha = 11 \rightarrow (4)$

$\rightarrow w_n^2 + 2\zeta w_n \alpha = 10 + 10K_T \rightarrow (5)$

$\rightarrow w_n^2 \alpha = 10K \rightarrow (6)$

(3) in (5) $\Rightarrow w_n^2 + 2\zeta w_n \alpha = 10K \rightarrow (7)$

(6) in (7) $\Rightarrow w_n^2 + 2\zeta w_n \alpha = w_n^2 \alpha \rightarrow (8)$

From (4) $\alpha = 11 - 2\zeta w_n \rightarrow (9)$

(9) in (8) $w_n^2 + 2\zeta w_n (11 - 2\zeta w_n) = w_n^2 (11 - 2\zeta w_n)$

When $\zeta = 0{,}707$

$1{,}414\, w_n^3 - 11{,}99\, w_n^2 + 15{,}554\, w_n = 0$

rejected $\leftarrow$ $\therefore w_n = 0$, $w_{n_1} = 6{,}88$, $w_{n_2} = 1{,}5986$

From eq (9) $\alpha_1 = 1{,}27$, $\alpha_2 = 8{,}739$

shifting $\leftarrow$

Role $\alpha \geq 5\zeta w_n$

we will find ~~that~~ $w_{n1} \Rightarrow$ rejected

$\therefore$ in (6) $K = 2{,}2$; in (3) $K_T = 1{,}2$ | $w_{n2} \Rightarrow$ accepted $\rightarrow \alpha = 8{,}739$