Section 6

Routh stability

$$Ch.eq = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots + a_n = 0$$

Conditions For stability:-

1- All Coefficient of the Polynomial must have same Sign

2 All The power of 's' must be Present in Ch.eq

These Two Conditions are necessary for a system To be stable

but not sufficient

Steps:

1- arrange The Ch.eq according To The Power

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots + a_n$$

Ex: $s^3 + 6s^2 + 11s + 6$

2- Table:

$$\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \\
s^{n-1} & a_1 & a_3 & a_5 \\
s^{n-2} & b_1 & b_2 & \\
 & c_1 & & \\
 & a_n & &
\end{array}$$

All The first Two rows written from the Ch.eq

Third row: $b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$

$$b_2 = \frac{a_1 a_4 - a_0 \cdot a_5}{a_1}$$

3- No. of sign changes in first column equal to No of poles

Ex 1: is the closed loop system stable:

$$G(s) = \frac{12(s+4)}{s(s+1)(s+3)(s^2+2s+10)}$$

Ch. eq $= 1 + G(s)H(s) = 0$ $\quad$ $H(s) = 1$ (because closed loop)

$$= s^5 + 6s^4 + 21s^3 + 46s^2 + 42s + 48 = 0$$

s^5	1	21	42	
s^4	6	46	48	
s^3	$\frac{6\times21-1\times46}{6}$	$\frac{6\times42-48}{6}$	0	
s^2	$30\frac{7}{10}$	48,1	0	
s^1	13,15	0		
s^0	48			

$\therefore$ No of sign changes $= 0$

$\therefore$ System is stable

Ex2: Check The stability for of The system for value K_v and comment on your result

$$G(s) = \frac{K_v}{s(4s+1)(s+1)}$$

Ch. eq $\rightarrow 1 + G(s)H(s) = 0$

$$4s^3 + 5s^2 + s + K_v = 0$$

s^3	4	1
s^2	5	K_v
s^1	b_1	0
s^0	c_1	

$c_1 = K_v$

$$b_1 = \frac{5 - 4K_v}{5}$$

for stability $b_1 > 0$ & $c_1 > 0$

$$\therefore \frac{5-4K_v}{5} > 0 \quad \& \quad \boxed{K_v > 0} \; (1)$$

$$5 - 4K_v > 0$$

$$\boxed{K_v < \frac{5}{4}} \; (2)$$

From (1) & (2) $0 < K_v < \frac{5}{4}$ system is stable

Special Case 1: First Element of any of The rows is zero and remaining rows contains at least one non zero

Effect: The term of The next row becomes infinite → routh test fail

Ex: $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$

s^5	1	3	2
s^4	2	6	1
s^3	$0 \to \varepsilon$	1.5	
s^2	∞ $c_1 = \frac{6\varepsilon - 3}{\varepsilon}$	1	
s^1	d_1	0	
s^0			

Solution of Problem

[1] Put ε in place of zero
small positive num

[2] Complete routh Test with ε

[3] Take $\lim_{\varepsilon \to 0}$

$$d_1 = \frac{\left(\frac{6\varepsilon - 3}{\varepsilon}\right) * 1.5 - \varepsilon}{\frac{6\varepsilon - 3}{\varepsilon}}$$

[3] Take $\lim_{\varepsilon \to 0}$ for c_1 & d_1

$$\lim_{\varepsilon \to 0} \left(\frac{6\varepsilon - 3}{\varepsilon}\right) = 6 - \lim_{\varepsilon \to 0} \frac{3}{\varepsilon} = -\infty$$

$$\lim_{\varepsilon \to 0} d_1 = 1.5$$

System is unstable

No of sign changes = 2

Special Case 2

s^5	a	b	c
s^4	d	e	f
s^3	0	0	0
s^2			

→ Row of zeros

Effect: The terms of next row cannot be determined and the Routh test fails

To solve this use coefficient of a row which is just above the row of zeros Auxiliary Polynomial

$$A(s) = d.s^4 + es^2 + f$$

$$\frac{dA(s)}{d(s)} = 4ds^3 + 2es$$

Ex: Ch. eq: $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	
s^3	$0 \to 8$	$0 \to 24$	$0 \to 0$	→ we find row of zeros
s^2	6	16		
s^1	8/3			
s^0	16			

$$A(s) = 2s^4 + 12s^2 + 16$$

$$\frac{dA(s)}{ds} = 8.s^3 + 24.s$$