MDDELING OF ELECTRICAL SYSTEM

Section 3

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Resistance

Time Domain

$$v_R(t) = i_R(t)\, R$$

s Domain

$$V_R(s) = R\, I(s)$$

Inductance

Time Domain

$$v_L(t) = L \frac{di_L(t)}{dt}$$

s Domain

$$V_L(s) = LS\, I(s)$$

Capactance

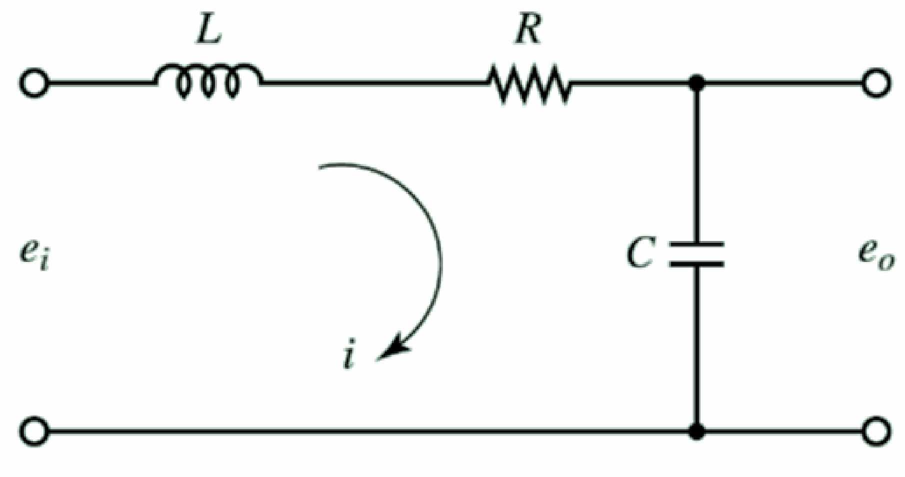
Time Domain

$$v_c(t) = \frac{1}{C} \int i_c(t)\, dt$$

s Domain

$$V_c(s) = \frac{1}{Cs} I(s)$$

Find the transfer function of the electrical circuit shown in the following figure



KVL

$\sum V = 0$

$v_{L(t)} + v_{R(t)} + v_{C(t)} = e_{i(t)}$

$$L\frac{di\,(t)}{dt} + Ri(t) + \frac{1}{C}\int i\,dt = e_i(t)$$

$$LS\,I(s) + R\,I(s) + \frac{1}{Cs}\,I(s) = E_{i(s)}$$

$$\boxed{I(s)[LS + R + \frac{1}{Cs}] = E_i(s) \longrightarrow \{1\}}$$

$$e_0(t) = v_C(t) = \frac{1}{c}\int i\,dt$$

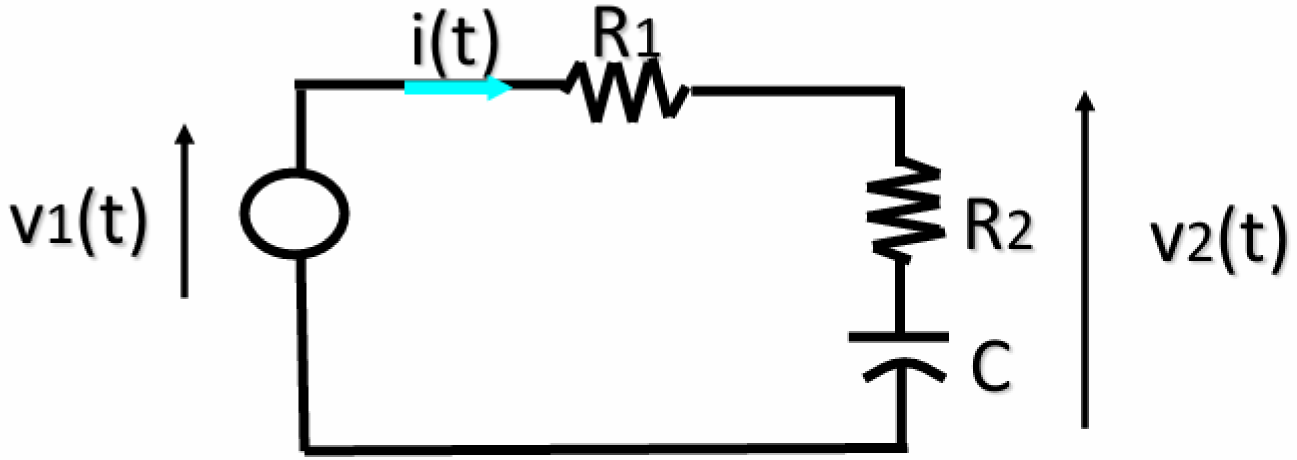
$$\boxed{E_0(s) = \frac{1}{Cs}\,I(s) \longrightarrow \{2\}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{LS + R + \frac{1}{Cs}} = \frac{1}{CL\,S^2 + CR\,S + 1}$$

KVL

$\sum V = 0$

$v_{R1(t)} + v_{R2(t)} + v_{C(t)} = v_{1(t)}$



$$R_1\, i(t) + R_2\, i(t) + \frac{1}{C}\int i\, dt = v_1(t)$$

$$R_1\ I(s) + R_2\ I(s) + \frac{1}{Cs}\ I(s) = V_{1\,(s)}$$

$$\boxed{I(s)[R_1 + R_2 + \frac{1}{Cs}] = V_1(s) \longrightarrow \quad \{1\}}$$

$$v_2(t) = R_2\, i(t) + \frac{1}{C}\int i\, dt$$

$$\boxed{V_2(s) = R_2\ I(s) + \frac{1}{Cs}\ I(s) = I(s)[R_2 + \frac{1}{Cs}] \longrightarrow \{2\}}$$

$$E_0(s) = \frac{1}{Cs}\ I(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2\, Cs + 1}{R_1\, Cs + R_2\, Cs + 1}$$

KVL

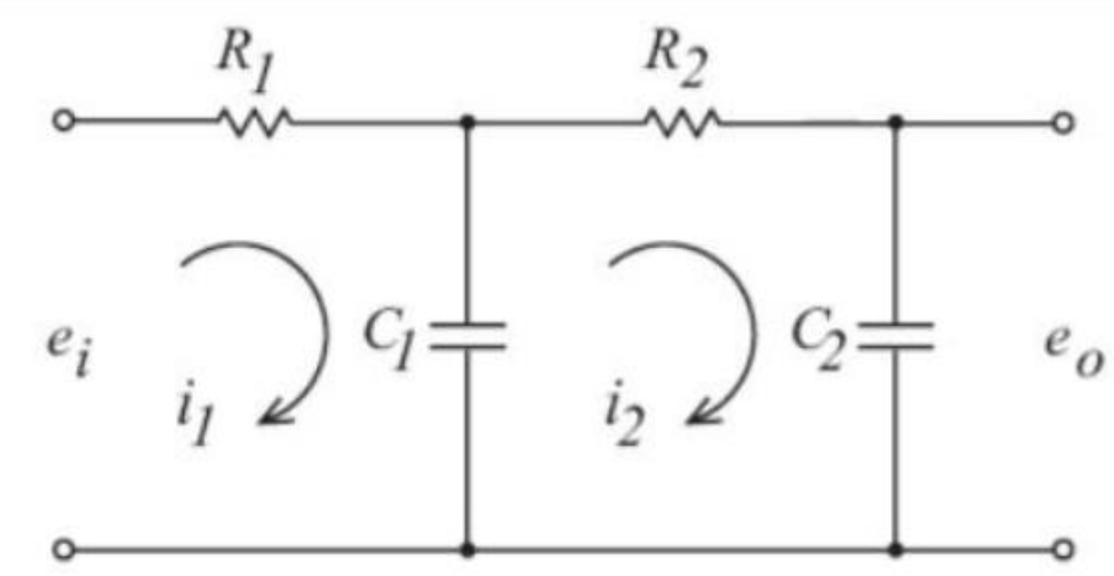
$\sum V = 0$

at i_1

$$R_1\, i_1(t) + \frac{1}{C_1}\int (i_1 - i_2)\, dt = e_i(t)$$

$$R_1\, I_1(s) + \frac{1}{C_1\, s}\,(I_1(s) - I_2(s)\,) = E_i(s)$$

$$\boxed{I_{1(s)}\left[R_1 + \frac{1}{C_1\, s}\right] - I_2(s)\frac{1}{C_1\, s} = E_{i(s)} \longrightarrow \{1\}}$$



at i_2

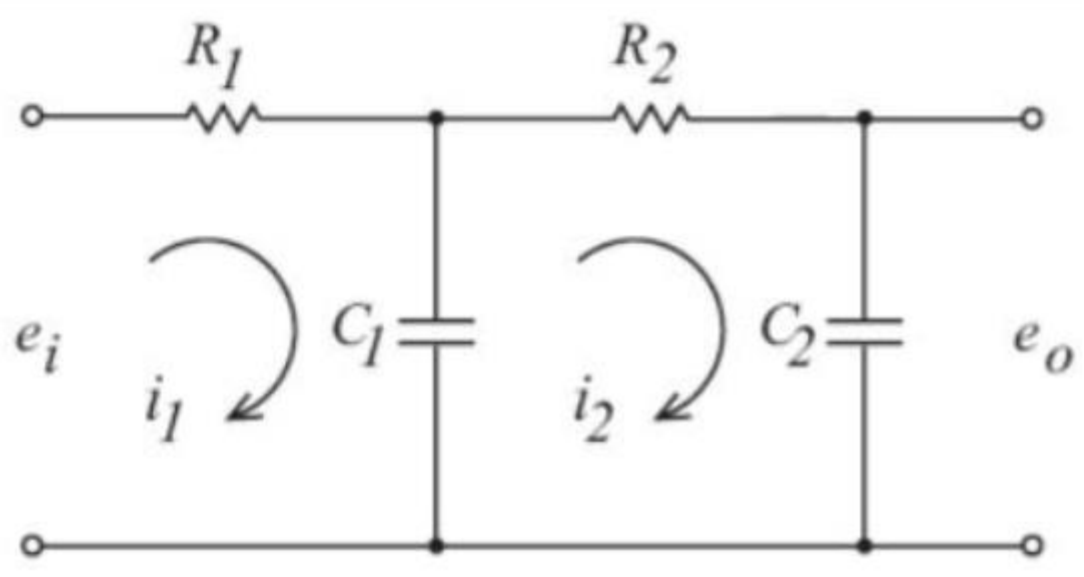
$$R_2 i_2(t) + \frac{1}{C_1}\int (i_2 - i_1)\, dt + \frac{1}{C_2}\int i_2\, dt = 0$$

$$R_2\, I_2(s) + \frac{1}{C_1 s}\,(I_2(s) - I_1(s)\,) + \frac{1}{C_2 s} I_2(s) = 0$$

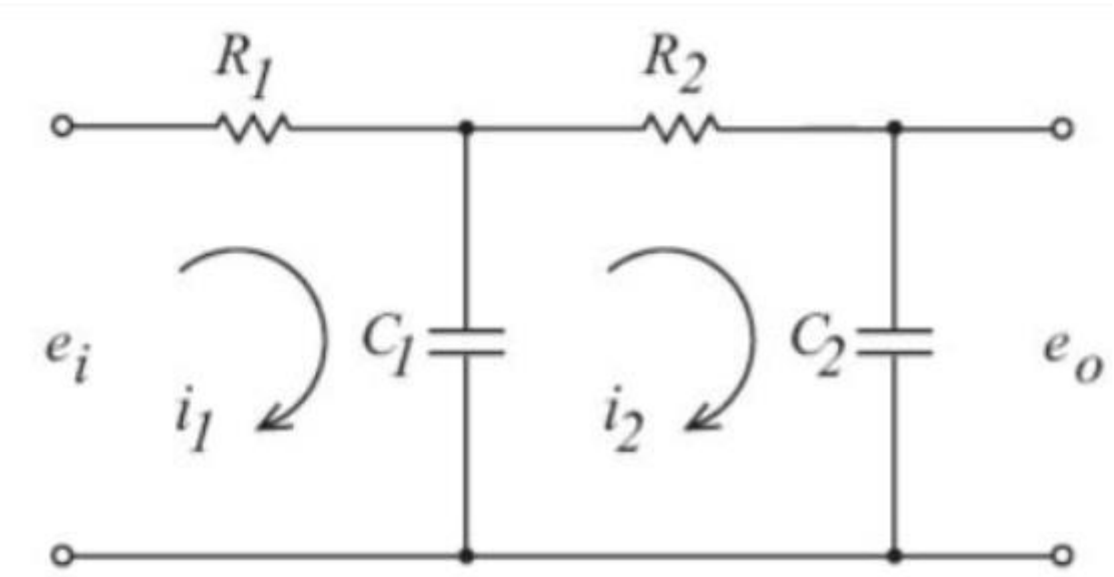
$$\boxed{I_{2(s)}\left[R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] - I_1(s)\frac{1}{C_1 s} = 0 \longrightarrow \{2\}}$$

$$e_0(t) = v_{C2}(t) = \frac{1}{C_2}\int i_2\, dt$$

$$E_0(s) = \frac{1}{C_2 s}\, I_2(s) \longrightarrow \boxed{I_2(s) = C_2\, s\, E_0(s) \longrightarrow \{3\}}$$



{3} *in* {2}



$$C_2 s\, E_0(s)\left[R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] - I_1(s)\frac{1}{C_1 s} = 0$$

$$\boxed{I_1(s) = C_1 C_2 s^2 E_0(s)\left[R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] \longrightarrow \{4\}}$$

{3} *and* {4} *in* {2}

$$C_1 C_2 s^2 E_0(s)\left[R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right]\left[R_1 + \frac{1}{C_1 s}\right] - C_2 s\, E_0(s)\frac{1}{C_1 s} = E_{i\,(s)}$$

Simplify

$$C_1 C_2 s^2 E_0(s)\left[R_1 R_2 + \frac{R_1}{C_1 s} + \frac{R_1}{C_2 s} + \frac{R_2}{C_1 s} + \frac{1}{C_1^2 s^2} + \frac{1}{C_1 C_2 s^2}\right] - C_2 s\, E_0(s)\frac{1}{C_1 s} = Ei_{(s)}$$

$$E_0(s)\left[R_1 R_2 C_1 C_2 s^2 + R_1 C_2 s + R_1 C_1 s + R_2 C_2 s + \frac{C_2}{C_1} + 1\right] - E_0(s)\frac{C_2}{C_1} = Ei_{(s)}$$

$$E_0(s)\left[R_1R_2C_1C_2s^2 + R_1C_2s + R_1C_1s + R_2C_2s + \frac{C_2}{C_1} + 1\right] - E_0(s)\frac{C_2}{C_1} = E_i(s)$$

$$E_0(s)\left[R_1R_2C_1C_2s^2 + R_1C_2s + R_1C_1s + R_2C_2s + \frac{C_2}{C_1} - \frac{C_2}{C_1} + 1\right] = E_i(s)$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{R_1R_2C_1C_2s^2 + R_1C_2s + R_1C_1s + R_2C_2s + 1}$$