II

PID Section

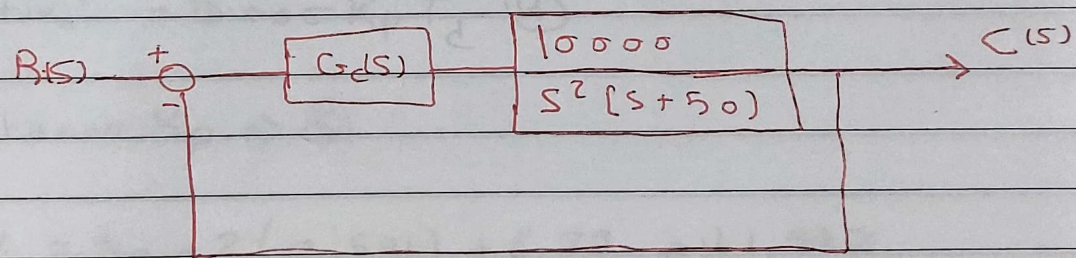
$$G_c(s) = \underbrace{K_p}_{\text{Proportional gain}} + \overbrace{K_i}^{\text{integral gain}} \frac{1}{s} + \underbrace{K_d}_{\text{Derivative gain}} s = K_p \left(1 + \frac{1}{\underbrace{T_i}_{\text{integral Time constant}} s} + \underbrace{T_d}_{\text{derivative Time constant}} s\right)$$

$$T_i = \frac{K_p}{K_i} \qquad \& \qquad T_d = \frac{K_d}{K_p}$$

Ex1) Design PD controller $G_c(s)$ that:

Maximum overshot $\leq 10\%$

settling Time < 1



$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \Rightarrow 0.1 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad \therefore \zeta = 0.5911$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow 1 = \frac{4}{0.59 * \omega_n} \Rightarrow \omega_n = 6.77 \text{ rad/s}$$

$\therefore$ PD controller

$$\therefore G_c(s) = K_p + K_d s = K_p (1 + T_d s)$$

$$G(s) = \frac{10000 \, K_p (1 + T_d s)}{s^2 (s+50)}, \quad H(s) = 1$$

Ch. eq = $1 + G(s)H(s) = 0$

$$s^3 + 50s^2 + 10000\,T_d K_p s + 10000 K_p = 0 \rightarrow ①$$

General Form:

$$(s^2 + 2\xi w_n s + w_n^2)(s + \alpha) = 0$$

$$s^3 + (2\xi w_n + \alpha)s^2 + (2\xi w_n \alpha + w_n^2)s + w_n^2 \alpha = 0 \rightarrow ②$$

From 1 & 2

$2\xi w_n \alpha = 50 \rightarrow ③$

$2\xi w_n \alpha + w_n^2 = 10000 K_p T_d$ ④

$w_n^2 \alpha = 10000 K_p \Rightarrow ⑤$

From ③ $\alpha = 50 - 2(0,591) * 6,77 = 41,997$

$\because \alpha \geq 5\xi w_n \therefore$ value accepted

From ⑤ $K_p = \dfrac{(6,77)^2 * 41,997}{10000} = 0,192$

$T_d = \dfrac{2 * 0,591 * 6,77 * 41,997 + (6,77)^2}{10000 * 0,192} = 0,1989$ sec

$\therefore G_c(s) = 0,192(1 + 0,1989s)$

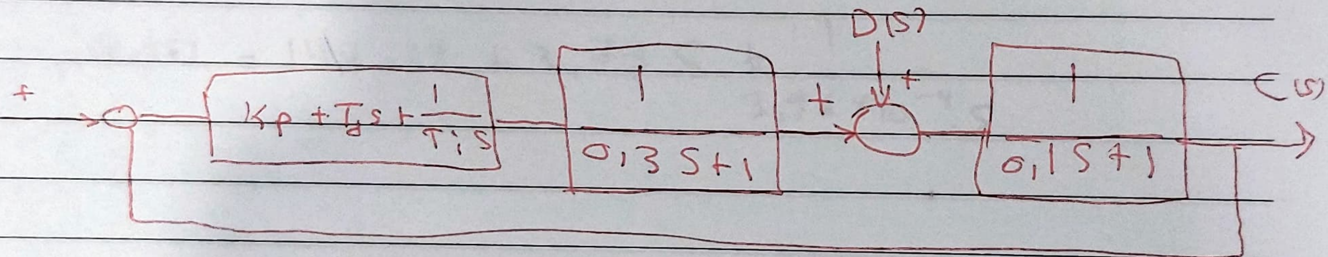
Design The Parameter of PID controller That following specifications

a) Settling Time $\leq 0,2$
b) Max overshot $\leq 5\%$
c) Steady State error due To unit Step = Zero

$\rightarrow$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \Rightarrow 0,05 = e^{-\pi\xi/\sqrt{1-\xi^2}} \Rightarrow \xi = 0,69$$

$$T_s = \frac{4}{\xi w_n} \Rightarrow 0,2 = \frac{4}{0,69 w_n} \Rightarrow w_n = 28,98$$



$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{K_p + T_d s + 1/T_i s}{(0,3s+1)(0,1s+1)} = 0$$

$$(0,3s+1)(0,1s+1) + K_p + T_d s + 1/T_i s = 0$$

$$0,03s^2 + 0,3s + 0,1s + 1 + K_p + T_d s + \frac{1}{T_i s} = 0$$

$$s^3 + \frac{(0,4+T_d)}{0,03}s^2 + \frac{(1+K_p)}{0,03}s + \frac{1}{0,03T_i} = 0 \rightarrow (1)$$

General Form: $s^2 + (2\xi w_n + \alpha)s^2 + (2\xi w_n\alpha + w_n^2)s + w_n^2\alpha = 0 \rightarrow (2)$

by comparison

$2\xi w_n + \alpha = (0,4+T_d)/0,03 \rightarrow (3)$

$2\xi w_n\alpha + w_n^2 = (1+K_p)/0,03 \rightarrow (4)$

$w_n^2\alpha = \frac{1}{0,03T_i} \rightarrow (5)$

$\therefore \alpha = 5\zeta w_n = 5(0,69)(28,98) = 99,981$ #

From 3 $T_d = (2 * 0,69 * 28,98 + 99,981) * 0,03 - \frac{0,4}{0,03}$

$= 3,79$

From 4 $K_p = 0,03(2\zeta w_n \alpha + w_n^2 - \frac{1}{0,03}) = 144,14$

From 5 $T_i = \frac{1}{0,03 w_n^2 \alpha} = 3,96 * 10^{-4}$

$\therefore G_c(s) = 144,14 + 3,79 s + \frac{1}{3,96 \times 10^{-4} . s}$