State Space Representation

System representation methodes

Transfore Funtion representation — State space representation

- we can represent a nonlinear systems
- Applicable for Multi input Multi output systems (MIMO)

Steps:

1. Choose state variables $X_1, X_2, X_3 \ldots$
2. Get the first derivative of each variable interms of state variable $\dot{X}_1, \dot{X}_2 \ldots$
3. write state Equation $\dot{X}(T) = AX + BU$
   - A: state matrix
   - X: state vector
   - B: input matrix
   - U: input vector
4. write the output equation $Y(T) = CX + DU$
   - C: output matrix
   - D: feed forward matrix

Find state space modeling

$$\ddot{y} + 4\dot{y} + y = 5r$$

because biggest order here is 2 I will put 2 states

1 $x_1 = y$ lowest order $\rightarrow \dot{x}_1 = \dot{y} = x_2 \rightarrow (1)$

$x_2 = \dot{y} \rightarrow \dot{x}_2 = \ddot{y} = 5r - 4\dot{y} - y$

$= 5r - 4x_2 - x_1 \rightarrow (2)$

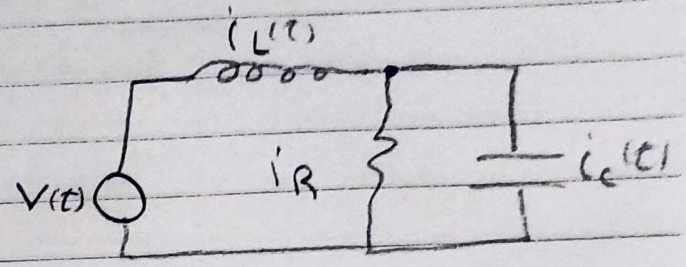
$\dot{X} = AX + Bu$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} r$$

$y = CX + Du$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ex 1: For the electrical system shown find the state space modeling if the output is i_R



apply KCL

$$i_L(t) = i_R(t) + i_c(t)$$
$$= \frac{V_c}{R} + C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = -\frac{V_c}{RC} + \frac{i_L}{C} \rightarrow (1)$$

apply KVL:

Note: in Electrical systems the states current in the coil and volt in capacitor

$$V_t = V_L(t) + V_c(t)$$

$$V(t) = L\frac{di(t)}{dt} + V_c$$

$$\therefore \frac{di}{dt} = \frac{V(t)}{L} - \frac{V_c(t)}{L} \rightarrow (2)$$

$\boxed{1}$ $X_1 = V_c \longrightarrow \dot{X}_1 = \frac{dV_c}{dt}$

$X_2 = i_L \longrightarrow \dot{X}_2 = \frac{di_L(t)}{dt}$

$$\dot{X} = A \quad X + B \quad u$$

From (1): $\dot{X}_1 = -\frac{X_1}{R_c} + \frac{X_2}{C}$

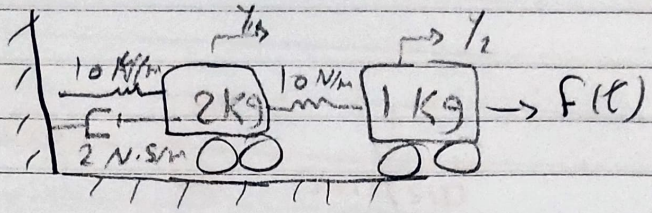
From (2) $\dot{X}_2 = -\frac{X_1}{L} + \frac{V_t}{L}$

$$\Rightarrow \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V(t)$$

$$y = C \quad X \quad + D \quad u$$

output is: $i_R = \frac{V_c}{R} = \frac{X_1}{R} \rightarrow i_R = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + 0$

Find The State Space Modeling



Sol

For m = 2 kg

$$2\ddot{y}_1 + 10y_1 + 10(y_1 - y_2) + 2\dot{y}_1 = 0 \rightarrow ①$$

For m = 1 kg

$$\ddot{y}_2 + 10y_2 - 10y_1 = F(t) \rightarrow ②$$

$x_1 = y_1 \rightarrow \dot{x}_1 = x_2$

$x_2 = \dot{y}_1 \rightarrow \dot{x}_2 = -\dot{y}_1 - 10y_1 + 5y_2 =$

$x_3 = y_2 \rightarrow \dot{x}_3 = x_4$

$x_4 = \dot{y}_2 \quad \dot{x}_4 = F(t) - 10y_2 + 10y_1 = 10x_1 - 10x_3 + f(t)$

$\therefore \dot{X} = AX + BU$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -10 & -1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 10 & 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

$y = CX + DU$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Find S-S representation

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+3)}$$

Use ① Phase methode ② diagonal methode

① Phase methode:

$$\frac{Y(s)}{U(s)} = \frac{1}{\underbrace{(s+1)(s+3)}_{s^2+4s+3}}$$

$$s^2 Y(s) + 4s\, Y(s) + 3\, Y(s) = U(s)$$

$$\ddot{y} + 4\dot{y} + 3y = u$$

$x_1(t) = y$ $\quad \dot{x}_1(t) = x_2$

$x_2(t) = \dot{y}$ $\quad \dot{x}_2(t) = u - 3x_1(t) - 4x_2(t)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

b) diagonal methode

$$G(S) = \frac{1}{(S+1)(S+3)} = \frac{A}{S+1} + \frac{B}{S+3}$$

$$1 = A(S+3) + B(S+1)$$

at $S = -1$

$1 = 2A \quad \therefore A = \frac{1}{2}$

at $S = -3$

$1 = -2B \quad \therefore B = -\frac{1}{2}$

$$\therefore G(S) = \frac{1/2}{S+1} + \frac{-1/2}{S+3}$$

$$Y(S) = \frac{1/2\, U(S)}{S+1} + \frac{-1/2\, U(S)}{S+3}$$

$$X_1 = \frac{1/2\, U(S)}{S+1} \Rightarrow S X_1 + X_1 = 1/2\, U(S)$$

$$\dot{X}_1(t) + X_1(t) = 1/2\, U(t)$$

$$\dot{X}_1 = 1/2\, U(t) - X_1(t) \rightarrow (1)$$

$$X_2 = \frac{-1/2\, U(S)}{S+3} \Rightarrow S X_2 + 3X_2 = -1/2\, U(S)$$

$$\dot{X}_2(t) + 3X_2(t) = -1/2\, U(t)$$

$$\dot{X}_2(t) = -1/2\, U(t) - 3X_2(t) \rightarrow (2)$$

$$Y(t) = X_1(t) + X_2(t)$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} u(t)$$

$$Y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

For Following system obtain $Y(s)/U(s)$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$Y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} X(t)$$

$$\frac{Y(s)}{U(s)} = C(SI - A)^{-1} B + D$$

$$SI - A = \underbrace{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}}_{SI} - \underbrace{\begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}}_{A} = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{|SI - A|} \cdot \text{adj}^T (SI - A)$$

$$|SI - A| = (s+5)(s+1) + 3 = s^2 + 6s + 8$$

$$\text{adj}^T = \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{s^2+6s+8}\begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+1}{s^2+6s+8} & \frac{-1}{s^2+6s+8} \\ \frac{3}{s^2+6s+8} & \frac{s+5}{s^2+6s+8} \end{bmatrix}$$

$$C(SI - A)^{-1}B = \underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times 8}{\begin{bmatrix} \frac{s+1}{s^2+6s+8} & \frac{-1}{s^2+6s+8} \\ \frac{3}{s^2+6s+8} & \frac{s+5}{s^2+6s+8} \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$= \underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \begin{bmatrix} \frac{s+1}{s^2+6s+8} - \frac{1}{s^2+6s+8} \\ \frac{3}{s^2+6s+8} + \frac{s+5}{s^2+6s+8} \end{bmatrix}$$

$$= \frac{s+1}{s^2+6s+8} - \frac{1}{s^2+6s+8} + \frac{6}{s^2+6s+8} + \frac{2(s+5)}{s^2+6s+8}$$

$$= \frac{s+1-1+6+2s+10}{s^2+6s+8}$$

$$= \frac{3s+16}{s^2+6s+8}$$