

MTE 408 | ROBOTICS



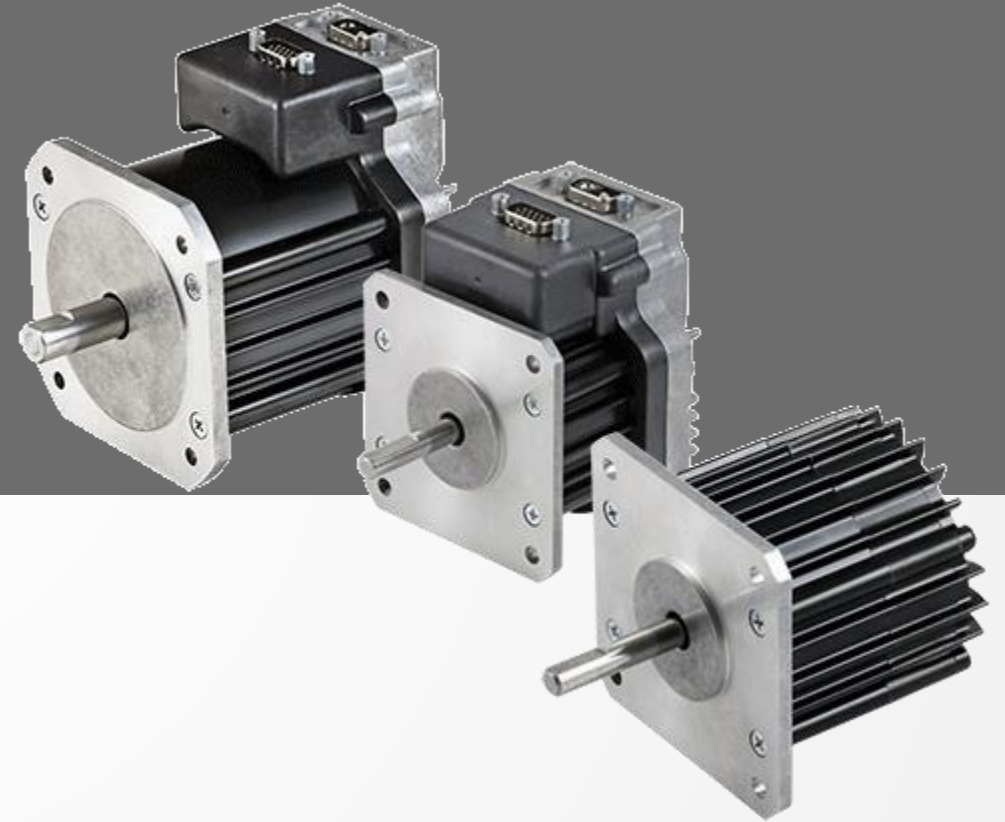
MOTOR SIZING PART I: MOTOR SELECTION

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WALEED ELBADRY

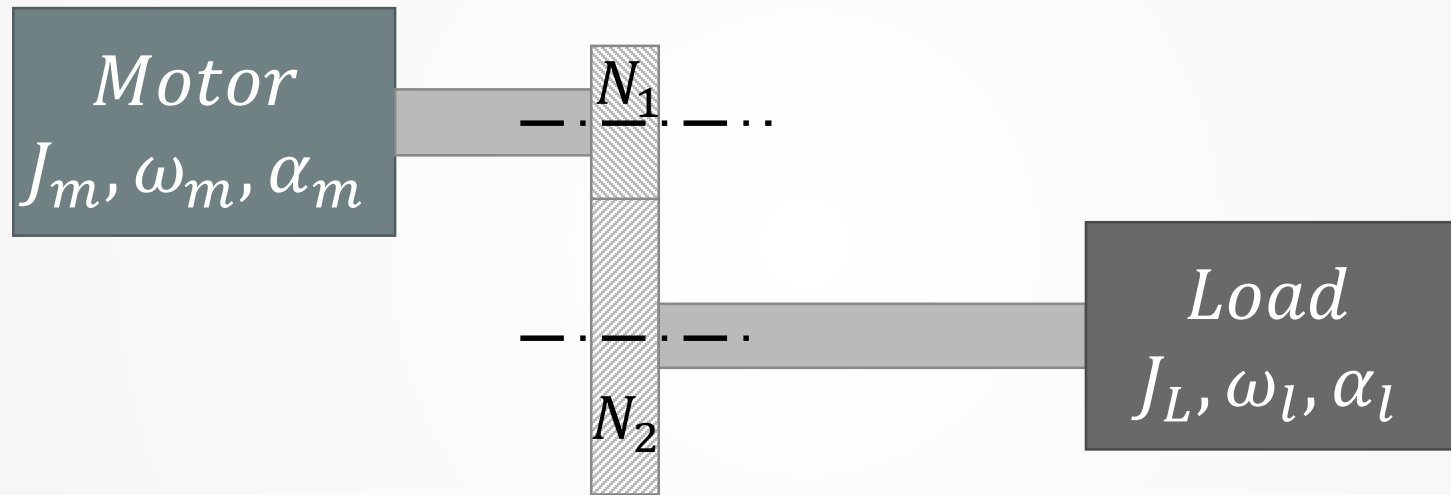
DECEMBER 2022

Motor Sizing

And its relationship to trajectory generation

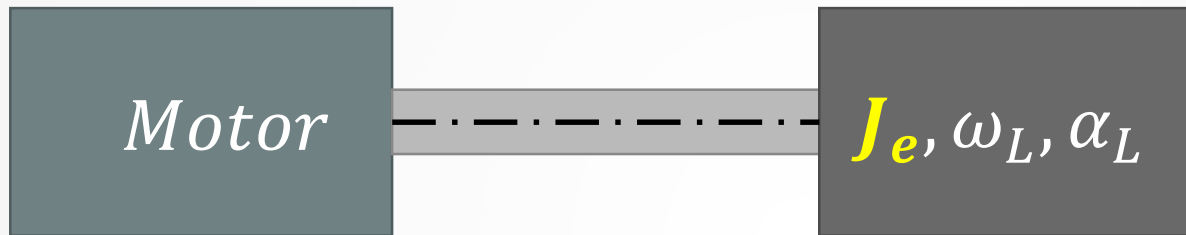


Reflected Inertia (Review)



*The mechanical load needs to rotate at **specific angular velocity and acceleration***
From (trajectory generation)

Reflected Inertia (Review)



The *load inertia* must be *reflected to motor side* to pick the suitable motor to maintain the *desired velocity* and *acceleration*

We need to find the *equivalent inertia* J_e

Notation

ω_L ... Load angular velocity ($\frac{\text{rad}}{\text{s}}$)

ω_m ... Motor angular velocity ($\frac{\text{rad}}{\text{s}}$)

α_L ... Load angular acceleration ($\frac{\text{rad}}{\text{s}^2}$)

α_m ... Motor angular acceleration ($\frac{\text{rad}}{\text{s}^2}$)

T_L ... Load resistive torque (N.m)

T_m ... Motor driving torque (N.m)

J_L ... Load rotational moment of inertia (kg.m^2)

J_m ... Motor shaft moment of inertia (kg.m^2)

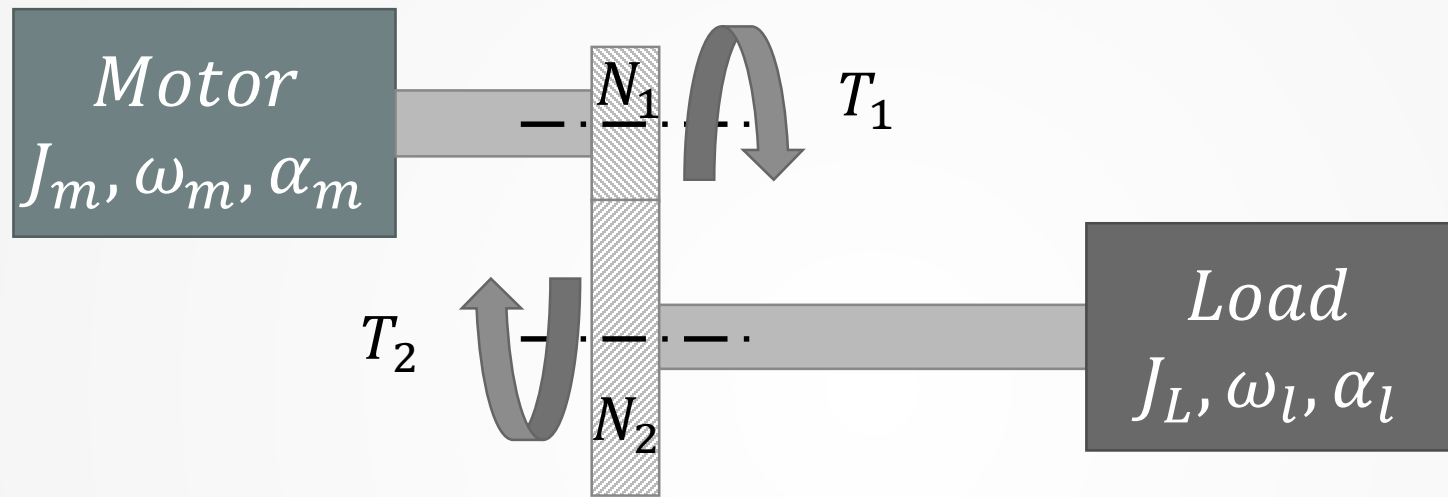
J_{gb} ... Gearbox moment of inertia (kg.m^2)

i ... Gearbox ratio > 1 (**boosting torque**)

$\omega_m > \omega_L$ and $T_m < T_L$

N ... Number of teeth of gear

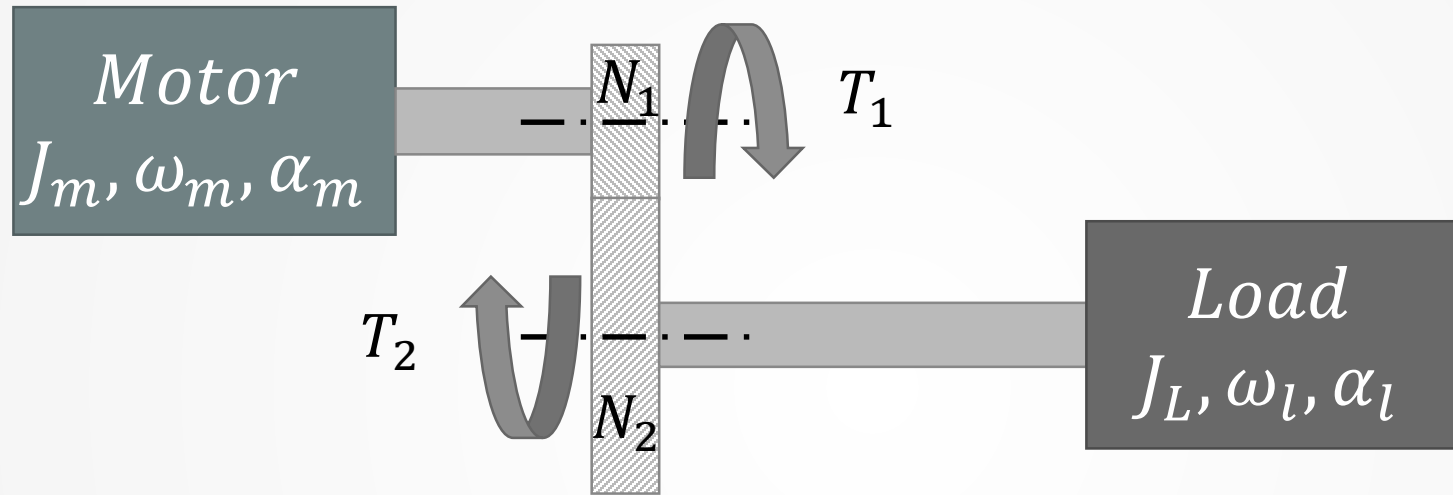
Reflected Inertia (Review)



For power transmission (Gears) $P = T \cdot \omega$

$$\begin{array}{c}
 V_1 = V_2 \\
 \xrightarrow{r_1} \boxed{\frac{\omega_1}{\omega_2}} \xrightarrow{r_2}
 \end{array}
 \quad
 \begin{array}{c}
 P_1 = P_2 \\
 \xrightarrow{T_1} \boxed{\frac{\omega_1}{\omega_2}} \xrightarrow{T_2}
 \end{array}
 \quad
 \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1}$$

Reflected Inertia (Review)

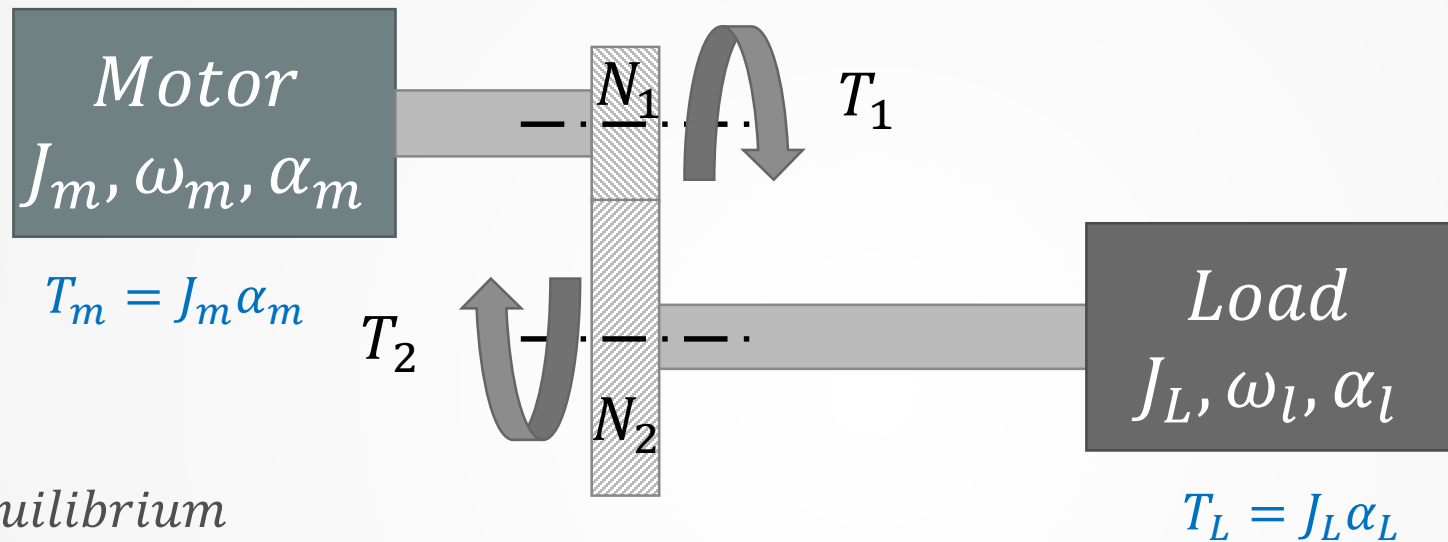


$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1} \rightarrow T_2 = T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1} \rightarrow \omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

Assuming Gearbox inertia is ignored

Reflected Inertia (Review)



$$T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

$$\omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

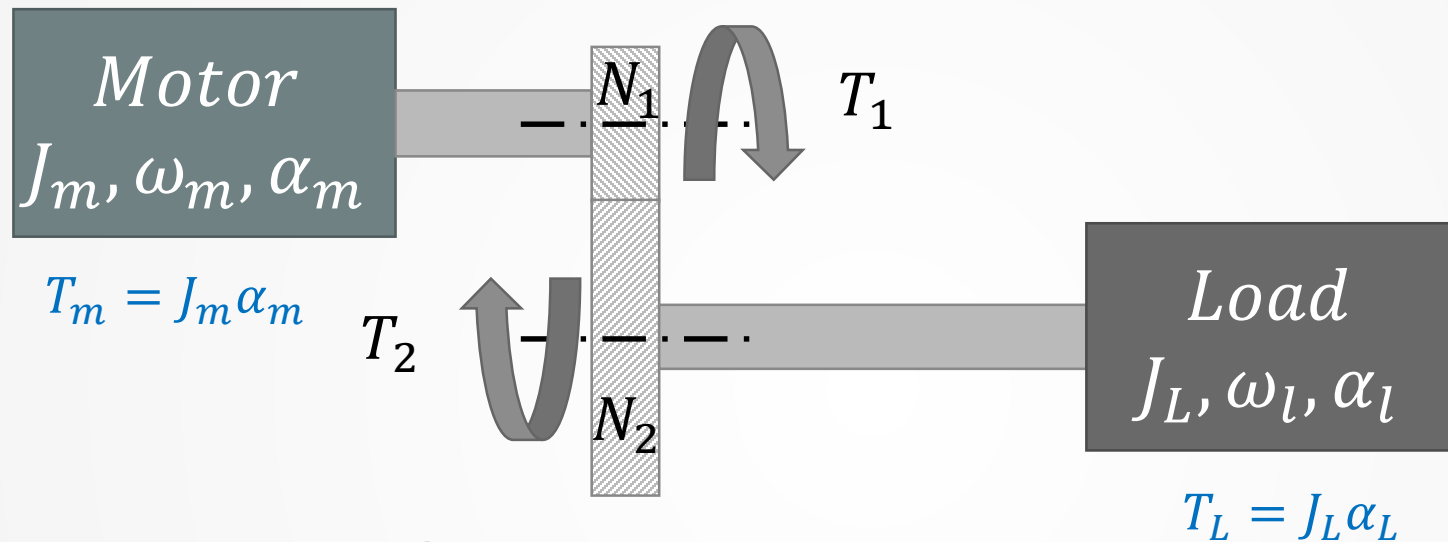
Rotational equilibrium

$$\sum T = 0 \rightarrow T_m + T_L = J_m \alpha_m + J_L \alpha_L = 0$$

Reflection using equations 1 and 2

$$J_L \alpha_L = T_L \rightarrow J_L \alpha_m \frac{N_1}{N_2} = T_m \frac{N_2}{N_1} \rightarrow J_L \left(\frac{N_1}{N_2} \right)^2 \alpha_m = T_m, \text{ but we know that } T_m = J_m \alpha_m$$

Reflected Inertia (Review)



$$T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

$$\omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

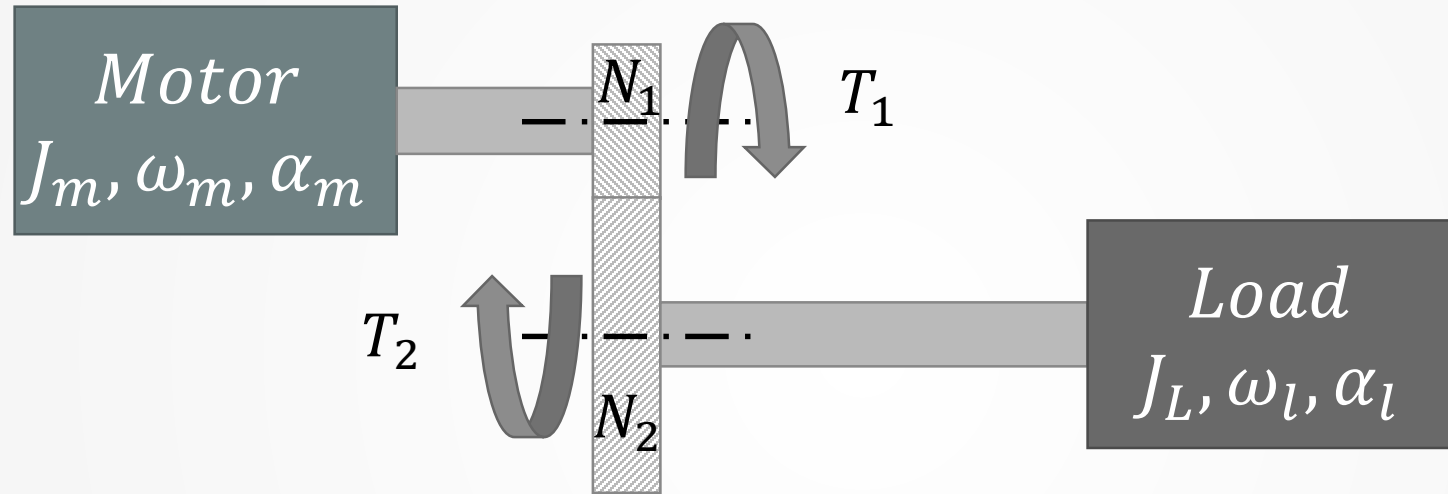
Reflection using equations 1 and 2

$$\because J_L \alpha_L = T_L \rightarrow J_L \alpha_m \frac{N_1}{N_2} = T_m \frac{N_2}{N_1} \rightarrow J_L \left(\frac{N_1}{N_2} \right)^2 \alpha_m = T_m$$

Rotational equilibrium

$$\because T_m + T_{m(\text{reflected})} = J_m \alpha_m + J_L \alpha_L = J_m \alpha_m + J_L \left(\frac{N_1}{N_2} \right)^2 \alpha_m = \left(J_m + J_L \left(\frac{N_1}{N_2} \right)^2 \right) \alpha_m = T_{m(\text{overall})}$$

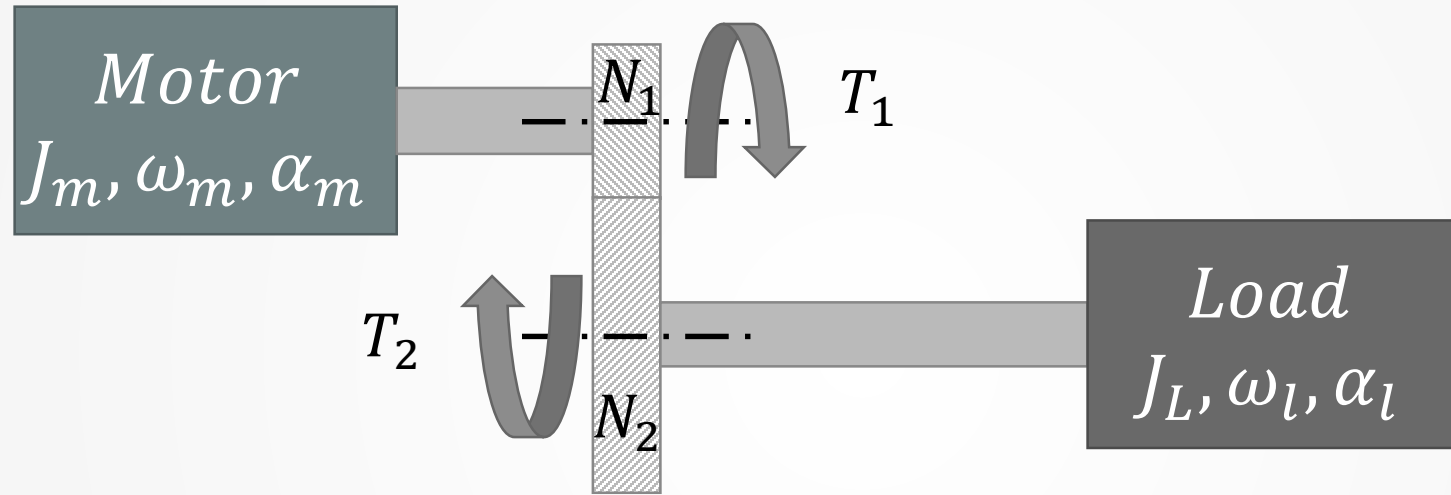
Reflected Inertia (Review)



$$\therefore J_e = J_m + J_L \left(\frac{N_1}{N_2} \right)^2 \text{ kg.m}^2 \text{ (Equivalent Reflected Inertia)}$$

$$\therefore J_e = J_m + \frac{J_L}{i^2}, \quad i = \frac{N_2}{N_1}, i \dots \text{Gear Ratio} = \frac{\text{Driven No. of teeth}}{\text{Driver No. of teeth}}$$

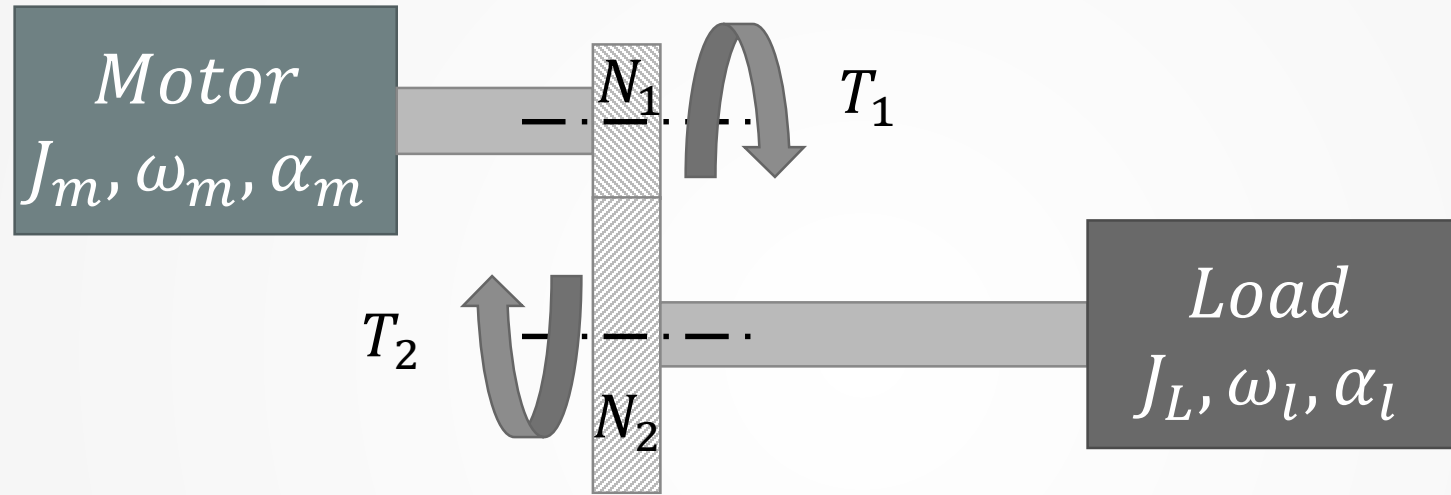
Motor Torque



$$\therefore T_m = \left(J_m + \frac{J_L}{i^2} \right) \cdot \alpha_m^{max} = \left(J_m + \frac{J_L}{i^2} \right) \cdot (i \cdot \alpha_L^{max}),$$

$$\text{since } \alpha_m^{max} = \frac{N_2}{N_1} \alpha_L^{max} = i \cdot \alpha_L^{max}, \alpha_m^{max} > \alpha_L^{max}$$

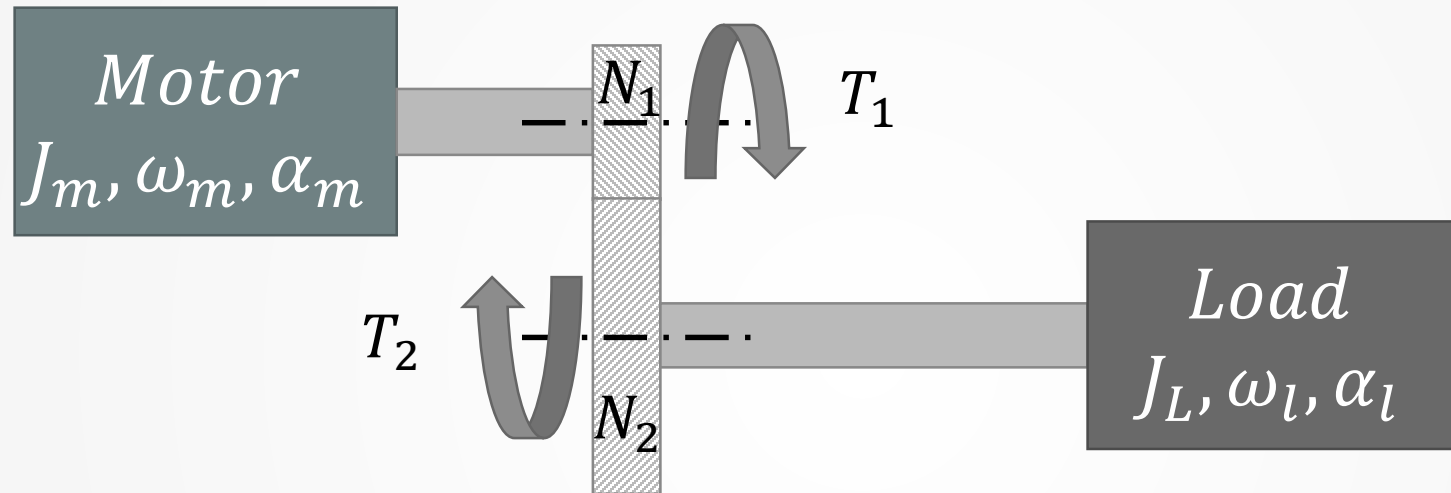
Maximum Motor Torque



$$\therefore T_m = \left(J_m + \frac{J_L}{i^2} \right) \cdot (i \cdot \alpha_L^{max}) = J_m \cdot i \cdot \alpha_L^{max} + \frac{J_L}{i} \cdot \alpha_L^{max}$$

$$\text{To find } T_m^{max} = \frac{\partial T_m}{\partial i} = 0 \rightarrow J_m \cdot \alpha_L^{max} - \frac{J_L}{i^2} \cdot \alpha_L^{max} = 0$$

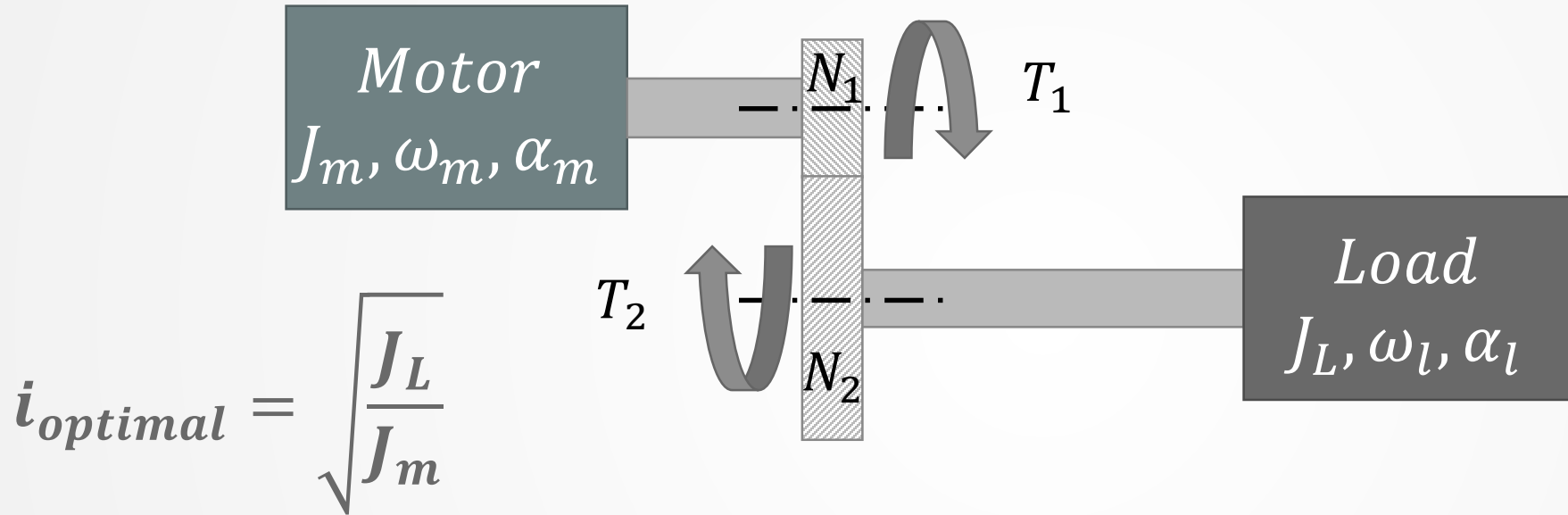
Optimal Gear Ratio



$$T_m^{max} = (J_m - \frac{J_L}{i^2}) \cdot \alpha_L^{max} = 0$$

$$\text{Either } \alpha_L^{max} = 0 \text{ (rejected) or } \left(J_m - \frac{J_L}{i^2} \right) = 0 \rightarrow i_{optimal} = \sqrt{\frac{J_L}{J_m}}$$

Inertia Mismatch



Let $\frac{J_L}{i^2} = J_L^{eq} \rightarrow i_{opt}^2 = \frac{J_L}{J_m} \rightarrow \frac{\frac{J_L}{i_{opt}^2}}{J_m} = 1 \rightarrow \frac{J_L^{eq}}{J_m} = 1$ (*Inertia Mismatch*)

Inertia Mismatch

$$\frac{J_L^{eq}}{J_m} = 1 - 10 \text{ (*Inertia Mismatch*)}$$

1 – 3 → *Precise servo systems*

3 – 6 → *moderate accuracy (industrial systems)*

6 – 10 → *rough handling systems*

Required Motor Power

$$\therefore T_m = \left(J_m + \frac{J_L}{i^2} \right) \cdot i \cdot \alpha_L^{max} \text{ and } P_m = T_m \cdot \omega_m^{max}$$

$$\therefore P_m = \left(J_m + \frac{J_L}{i^2} \right) \cdot i \cdot \alpha_L^{max} \cdot i \cdot \omega_L^{max}$$

$$\therefore P_m = \left(J_m \cdot i^2 + i^2 \cdot \frac{J_L}{i^2} \right) \cdot \alpha_L^{max} \cdot \omega_L^{max}$$

$$\therefore P_m = \left(J_m \cdot \frac{J_L}{J_m} + J_L \right) \cdot \alpha_L^{max} \cdot \omega_L^{max} = 2 \cdot J_L \cdot \alpha_L^{max} \cdot \omega_L^{max} = \frac{2}{\eta} \cdot J_L \cdot \alpha_L^{max} \cdot \omega_L^{max}$$

$$\omega_m^{max} = i \cdot \omega_L^{max}$$

$$\alpha_m^{max} = i \cdot \alpha_L^{max}$$

$$i_{opt}^2 = \frac{J_L}{J_m}$$

$\eta(\text{efficiency}) = 80\% - 90\%$, if not given, then take it as $72\% = 0.72$

Design Procedure

1. *Design the mechanical mechanism*
2. *Motion planning and trajectory generation.*
3. *Obtain J_L , ω_L^{\max} and α_L^{\max} (From motion study on SolidWORKS)*
4. *Select the desired mismatch (with respect to application)*
5. *calculate the desired power $\rightarrow P_m = \frac{2}{\eta} \cdot J_L \cdot \alpha_L^{\max} \cdot \omega_L^{\max}$*
6. *Select the motor from the catalog (vendor catalog)*

Next is *parametric check*

Design Procedure

Parametric Check :

I. Rated speed factor $\rightarrow \frac{i \cdot \omega_L^{max}}{\omega_m^{max}} \leq 0.8 - 1$

II. Torque factor $\rightarrow \frac{RMS \text{ Loading Torque } (T_L^{RMS})}{Motor \text{ Rated Torque } (T_m^{Rated})} \leq 0.8 - 1$

III. Maximum Torque factor $\rightarrow \frac{\frac{T_L^{max}}{i}}{T_m^{max}} \leq 0.8 - 1$

Example

Choose the servo motor operating on a robot joint with the following parameters:

$$J_L = 200 \text{ Kg.m}^2, T_f = 120 \text{ N.m}, \omega_L^{max} = 0.15 \frac{\text{rad}}{\text{s}}, \alpha_L^{max} = 0.44 \frac{\text{rad}}{\text{s}^2}$$

$$\eta = 0.72$$

Solution

$$\therefore P_m = \frac{2}{\eta} (T_f + J_L \cdot \alpha_L^{max}) \cdot \omega_L^{max} = \frac{2}{0.72} (120 + 200 * 0.44) * 0.15 = 404.44 \text{ W (rated power)}$$

Selection of fitting motor from catalog

$$\therefore P_m = \frac{2}{\eta} (T_f + J_L \cdot \alpha_L^{max}) \cdot \omega_L^{max} = \frac{2}{0.72} (120 + 200 * 0.44) * 0.15 = 404.44 \text{ W (rated power)}$$

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$$\eta = 0.72$$

Solution

$$\therefore P_m = \frac{2}{\eta} (T_f + J_L \cdot \alpha_L^{max}) \cdot \omega_L^{max} = \frac{2}{0.72} (120 + 200 * 0.44) * 0.7 = 404.44 \text{ W (rated power)}$$

Selection of fitting motor from **catalog**

Model	Speed n^{rated} (min^{-1})	Torque T^{rated} (N.m)	Power P^{rated} (KW)	Power V^{rated} (V)	Current I^{rated} (A)	Maximum Torque T^{max} (N.m)	Frequency f^{rated} (Hz)	Inertia J_m (kg.cm^2)
MDSKS 036-23,200	4000	1.3	0.54	120	0.9	7.2	200	0.36

Example

Choose the servo motor operating on a robot joint with the following parameters:

$$J_L = 200 \text{ Kg.m}^2, T_f = 120 \text{ N.m}, \omega_L^{max} = 0.15 \frac{\text{rad}}{\text{s}}, \alpha_L^{max} = 0.44 \frac{\text{rad}}{\text{s}^2}$$

$$\eta = 0.72$$

Solution

$$P_{m(\text{required})}^{rated} = 404.44 \text{ W} \rightarrow P_{m(\text{catalog})}^{rated} = 540 \text{ W}$$

$$n_{m(\text{catalog})}^{rated} = 4000 \text{ RPM} \rightarrow \omega_{m(\text{catalog})}^{rated} = \frac{2 * \pi * 4000}{60} = 418 \frac{\text{rad}}{\text{s}}$$

$$i_{opt} = \sqrt{\frac{J_L}{J_m}} = \sqrt{\frac{200 * 10^4}{0.36}} = 2357 \text{ (gear train)}$$

Example

Choose the servo motor operating on a robot joint with the following parameters:

$$J_L = 200 \text{ Kg.m}^2, T_f = 120 \text{ N.m}, \omega_L^{\max} = 0.15 \frac{\text{rad}}{\text{s}}, \alpha_L^{\text{rated}} = 0.44 \frac{\text{rad}}{\text{s}^2}$$

$$\eta = 0.72$$

Solution

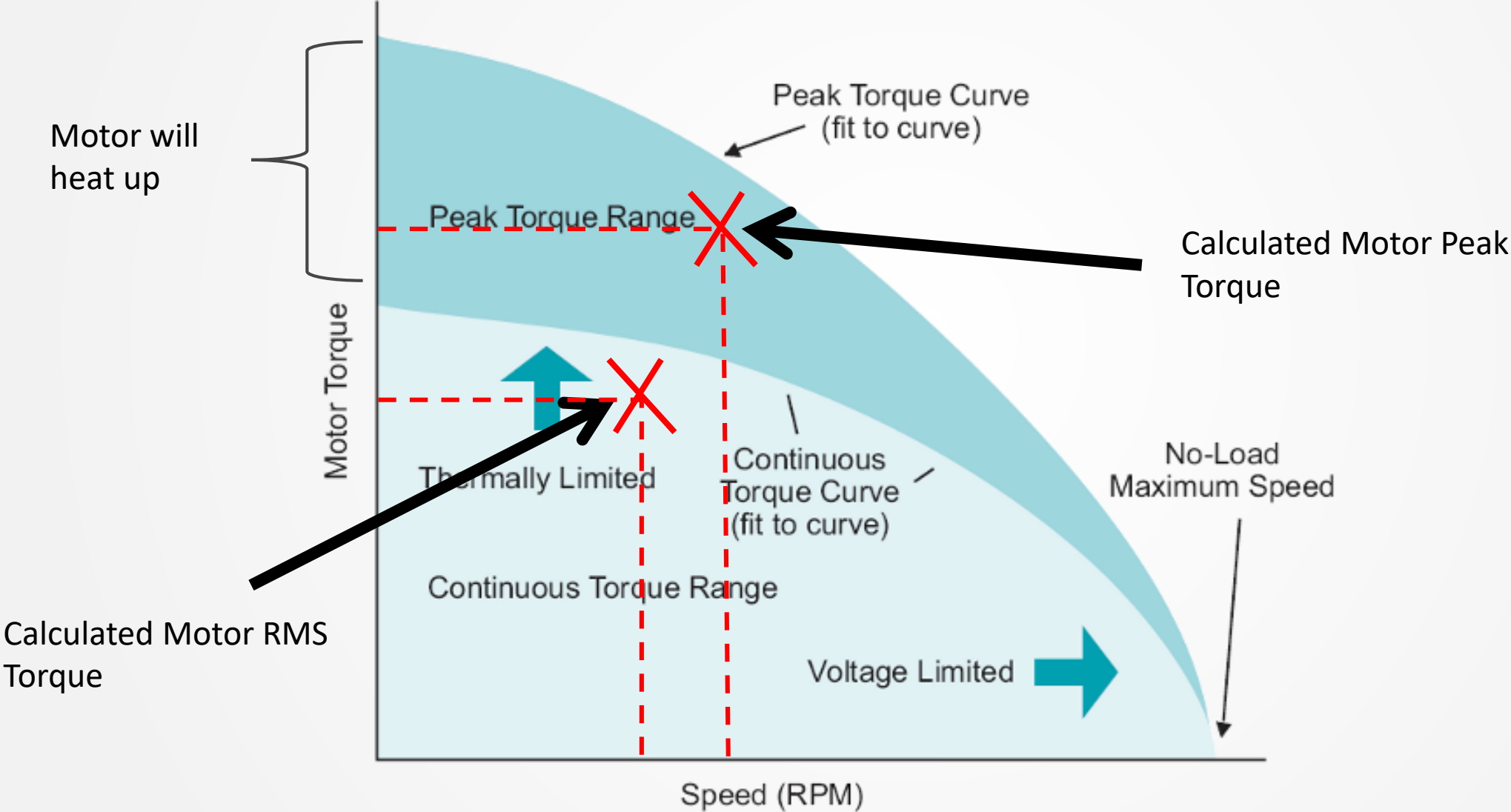
$$P_{m(\text{catalog})}^{\text{rated}} = 540 \text{ W} \quad \omega_{m(\text{catalog})}^{\text{rated}} = 418 \frac{\text{rad}}{\text{s}} \quad i_{\text{opt}} = 373$$

Checking:

$$\text{Rated speed factor} \rightarrow \frac{i \cdot \omega_L^{\max}}{\omega_m^{\max}} = \frac{2357 * 0.15}{418} = 3.94 \leq 0.8 - 1 \text{ (OK)}$$

$$\text{Maximum Torque factor} \rightarrow \frac{\frac{T_L^{\max}}{i}}{T_m^{\text{rated}}} = \frac{\frac{200 * 0.44}{2357}}{1.3} = 0.029 \leq 0.8 - 1 \text{ (OK)}$$

Motor Torque-Speed Chart

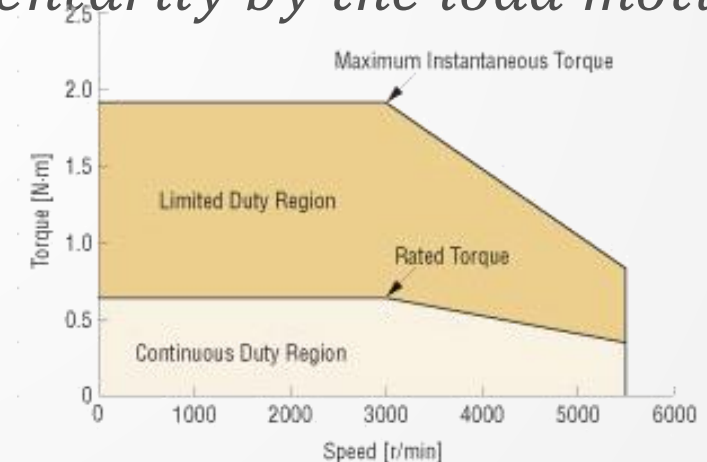
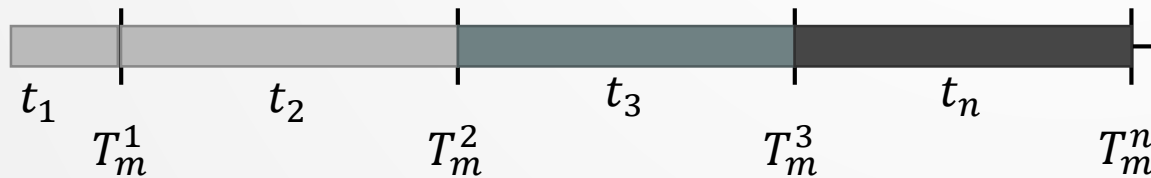


Notes

RMS Torque : Represents the average torque of resistive load during trajectory. It is known as “continuous or rated torque”

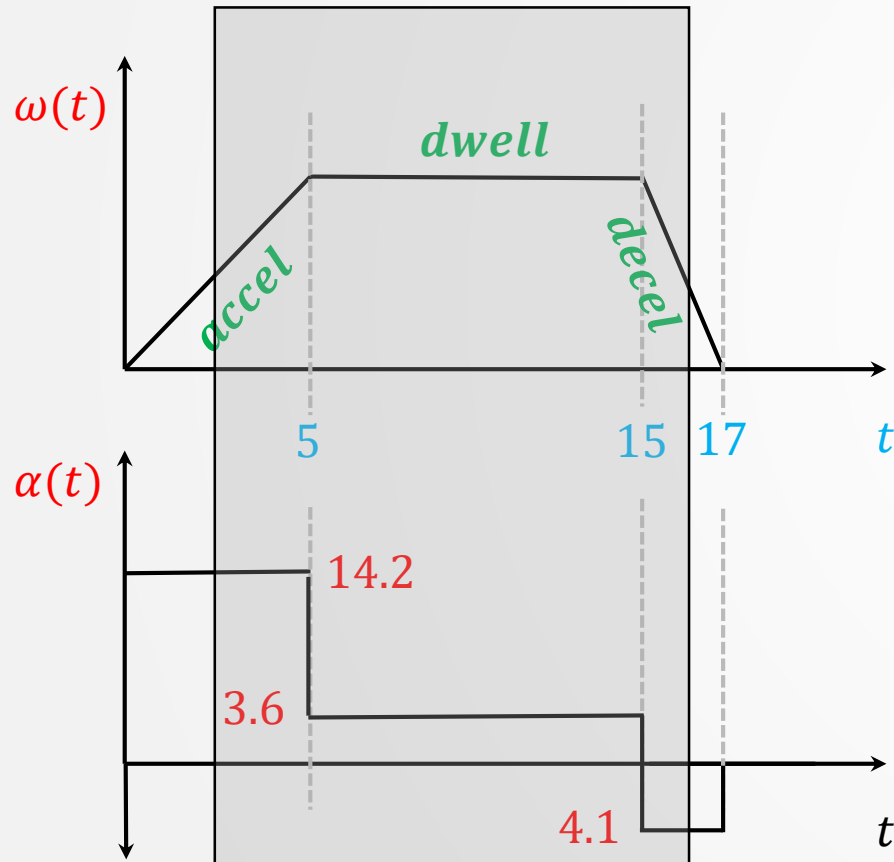
$$T_m^{RMS} = \sqrt{\frac{\sum_{k=0}^n T_m^k t_k}{\sum_{k=1}^n t_k}} = \sqrt{\frac{(T_m^1)^2 t_1 + (T_m^2)^2 t_2 + (T_m^3)^2 t_3 + \dots (T_m^n)^2 t_n}{t_1 + t_2 + t_3 + \dots t_n}} \quad (N.m)$$

Peak Torque : Is the worst torque reached (momentarily by the load motion). Also referred to as "intermittent torque"



Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2 \text{ Kg.cm}^2$



SOLUTION

$$J_m = 0.2 * 10^{-4} \text{ Kg.m}^2$$

Computing Torque for each motion segment

$$T_{\text{accel}} = (J_m)(\alpha_{\text{accel}}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} \text{ N.m}$$

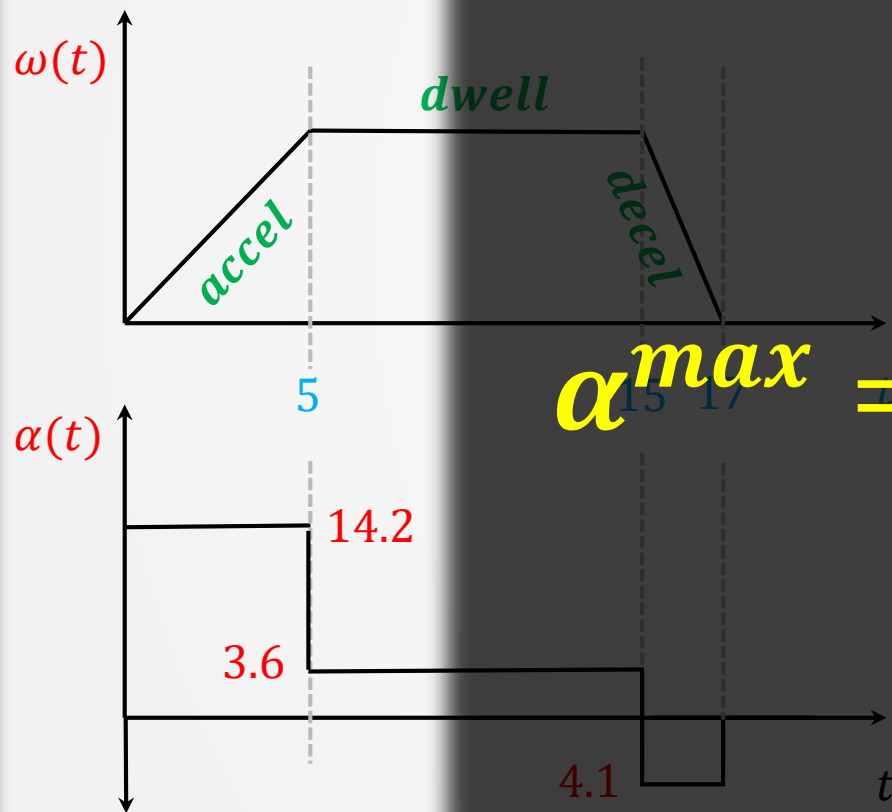
$$T_{\text{dwell}} = (J_m)(\alpha_{\text{dwell}}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} \text{ N.m}$$

$$T_{\text{decel}} = (J_m)(\alpha_{\text{decel}}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} \text{ N.m}$$

Show that $T_{\text{RMS}} = 1.66 * 10^{-4} \text{ N.m}$

Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2 \text{ Kg.cm}^2$



SOLUTION

$$J_m = 0.2 * 10^{-4} \text{ Kg.m}^2$$

Computing Torque for each motion segment

$$\alpha^{max} = \frac{\omega^{max}}{t} \rightarrow \omega^{max} = 71 \frac{\text{rad}}{\text{s}}$$

$$T_{\text{accel}} = (J_m)(\alpha_{\text{accel}}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} \text{ N.m}$$

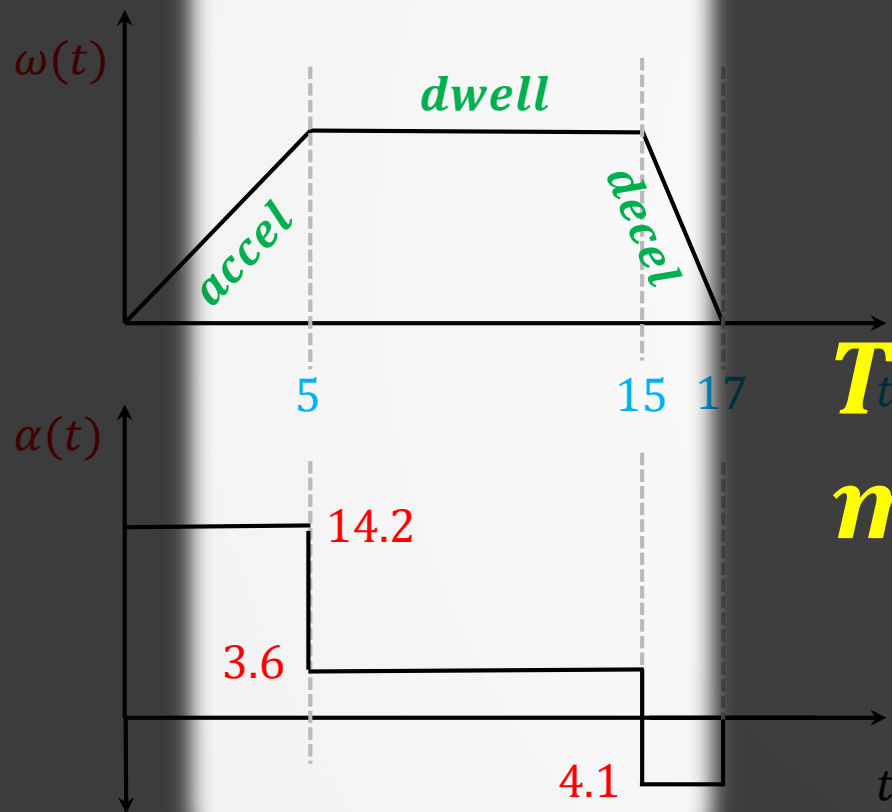
$$T_{\text{dwell}} = (J_m)(\alpha_{\text{dwell}}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} \text{ N.m}$$

$$T_{\text{decel}} = (J_m)(\alpha_{\text{decel}}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} \text{ N.m}$$

Show that $T_{\text{RMS}} = 1.66 * 10^{-4} \text{ N.m}$

Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2 \text{ Kg.cm}^2$



SOLUTION

$$J_m = 0.2 * 10^{-4} \text{ Kg.m}^2$$

Computing Torque for each motion segment

Torque needed to overcome motion friction

$$T_{\text{accel}} = (J_m)(\alpha_{\text{accel}}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} \text{ N.m}$$

$$T_{\text{dwell}} = (J_m)(\alpha_{\text{dwell}}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} \text{ N.m}$$

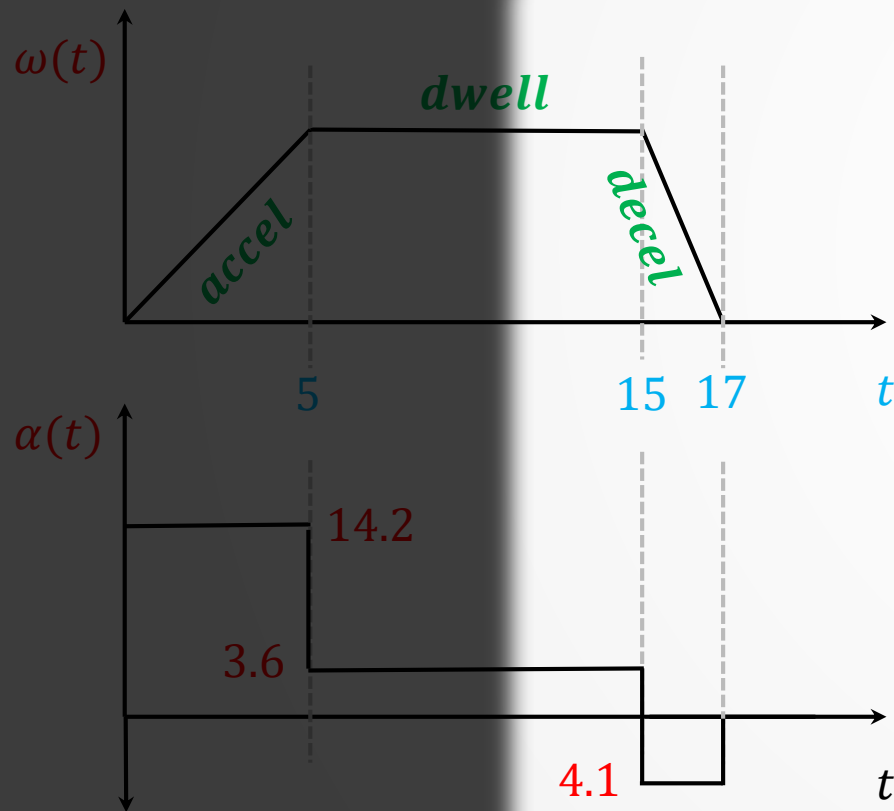
$$T_{\text{decel}} = (J_m)(\alpha_{\text{decel}}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} \text{ N.m}$$

Show that $T_{RMS} = 1.66 * 10^{-4} \text{ N.m}$

Example

$$\alpha^{decel} = 4.1 \rightarrow \omega^{decel} = ? \frac{rad}{s}$$

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2 \text{ Kg cm}^2$



SOLUTION

$$J_m = 0.2 * 10^{-4} \text{ Kg.m}^2$$

Why deceleration torque is lower than acceleration torque?

Computing Torque for each motion segment

$$T_{accel} = (J_m)(\alpha_{accel}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} \text{ N.m}$$

$$T_{dwell} = (J_m)(\alpha_{dwell}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} \text{ N.m}$$

$$T_{decel} = (J_m)(\alpha_{decel}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} \text{ N.m}$$

Show that $T_{RMS} = 1.66 * 10^{-4} \text{ N.m}$

VISUAL SIZER (Axis Design)

VisualSizer Pro - [Axis Design 1]

File Axis Units Window Help

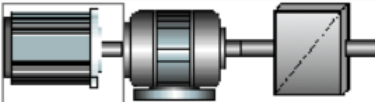
New Open Save Help

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Motor Sizing.Dat

- Axis # 1
 - Axis Design
 - Velocity Profile
 - Motor Selection
 - Report Generator
 - Performance Curves

Analyze Clear Motor Motor Database Append Replace Insert Cut Copy Help



Loads Reduction Misc. Special

Clipboard Conveyor Generic

Linear Actuator Leadscrew Nip Roll

Rack-Pinion Rotary Table Rotary Table w/ Gear

Winder - Center Driven Winder - Surface Driven

Load Parameters None

Parameter	Data	Unit

DC Servo Brushless Motor

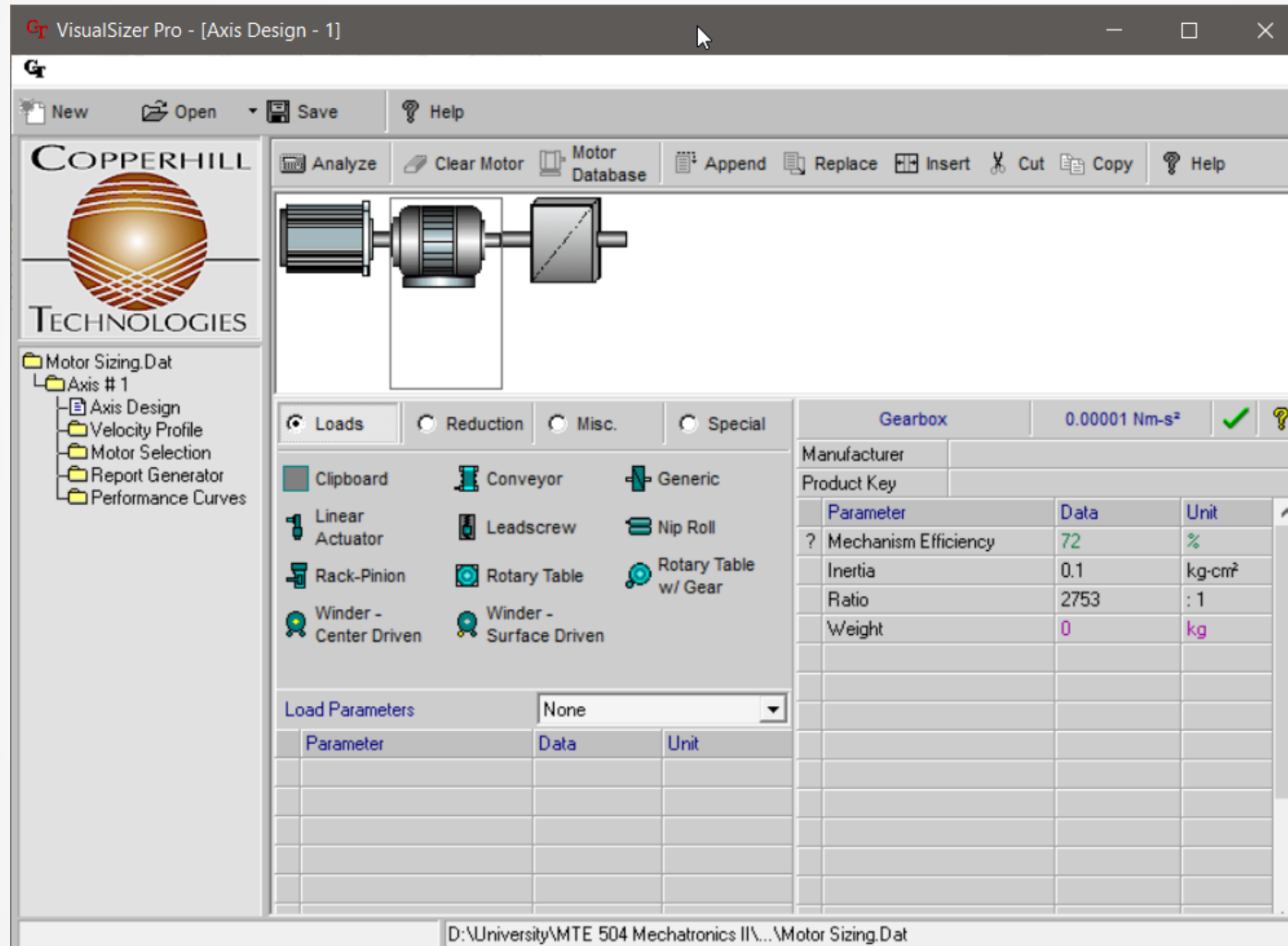
Manufacturer AutomationDirect.Com

Product Key SVL-207

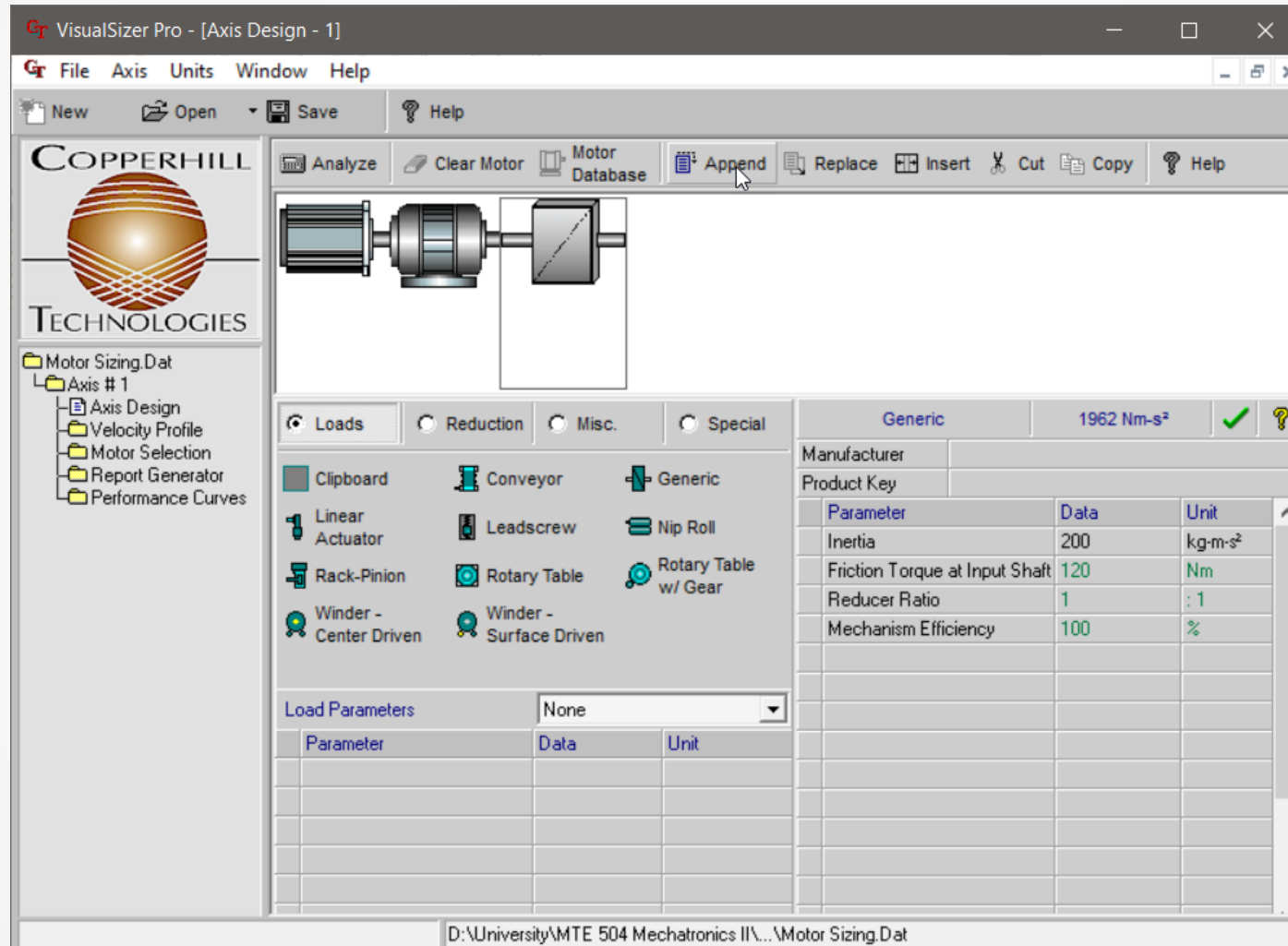
Parameter	Data	Unit
Rated Speed	4000	rpm
Rated Torque	1.3	Nm
Max. (Peak) Torque	7.6	Nm
Rotor Inertia	0.36	kg-cm ²
Kt	0	Nm/A
Weight	0	kg
Brake Inertia	0	Nm-s ²
Gear Ratio	1	: 1
Gear Inertia	0	Nm-s ²

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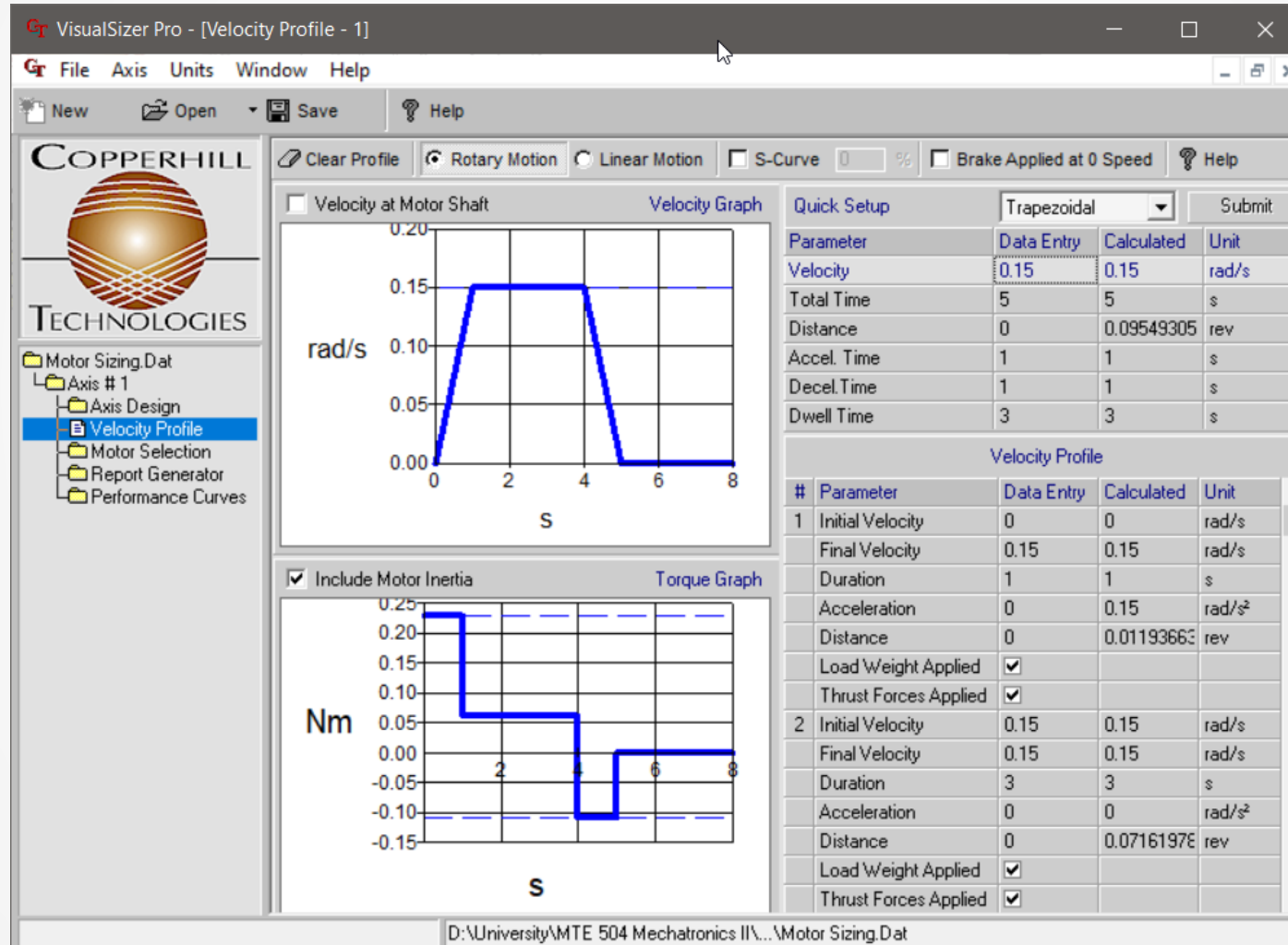
VISUAL SIZER (Axis Design)



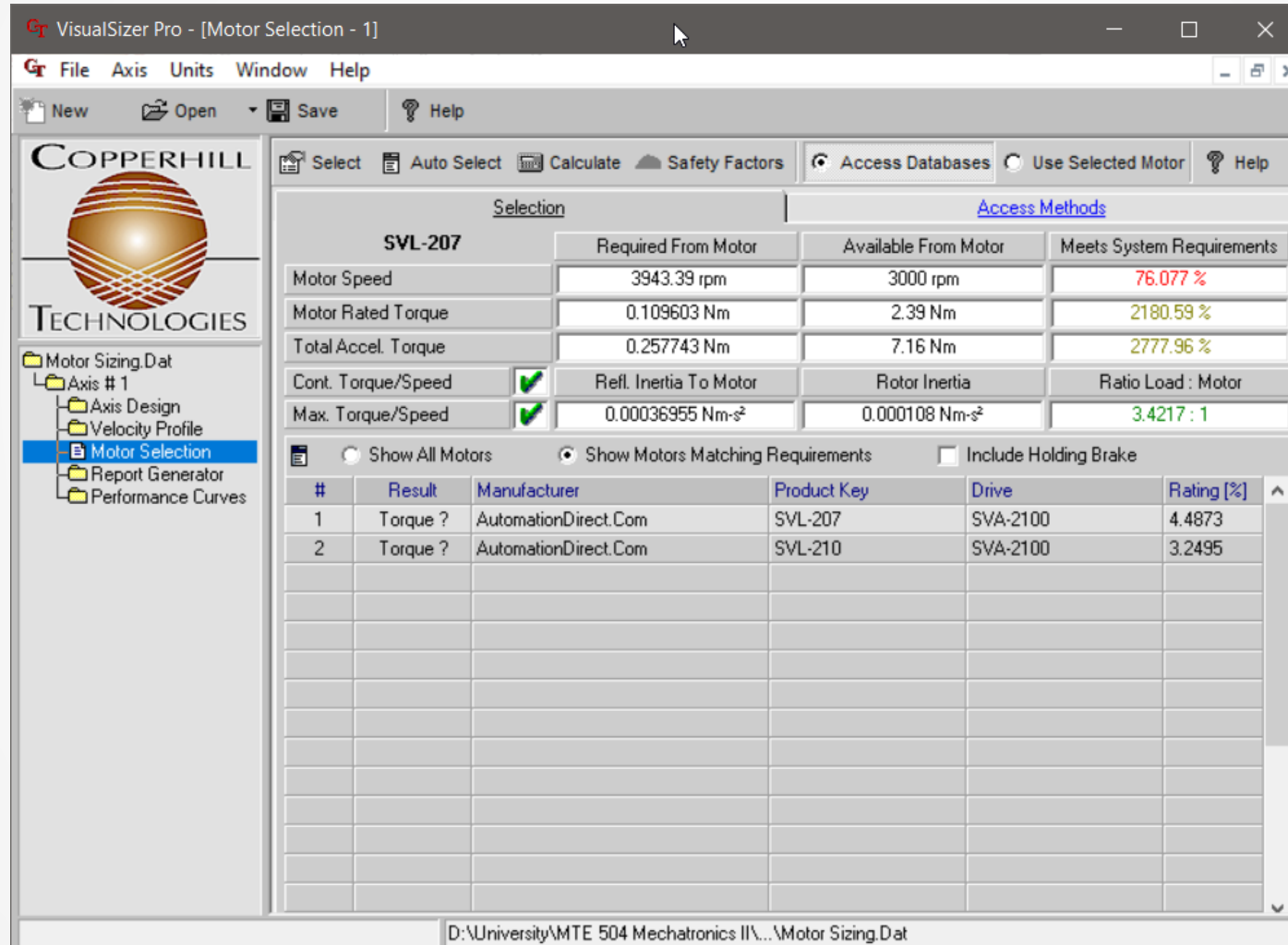
VISUAL SIZER (Axis Design)



VISUAL SIZER (Trajectory)



VISUAL SIZER (Motor Selection and Check)



VISUAL SIZER (Selection Report)

The screenshot displays the VisualSizer Pro software interface, specifically the Report Generator window. The window title is "VisualSizer Pro - [Report Generator - 1]". The menu bar includes "File", "Axis", "Units", "Window", and "Help". The toolbar contains "New", "Open", "Save", and "Help". The left sidebar shows the project structure: "Motor Sizing.Dat" > "Axis # 1" > "Axis Design", "Velocity Profile", "Motor Selection", "Report Generator" (selected), and "Performance Curves". The main report area displays the following information:

VisualSizer Report Generator - Version 5.80.070 - Professional

COPPERHILL TECHNOLOGIES Visual

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Date : 11-23-2019
File : D:\University\MTE 504 Mechatronics II\Labs\Lab 5\...\Moto

Axis : 1

Selected Motor :

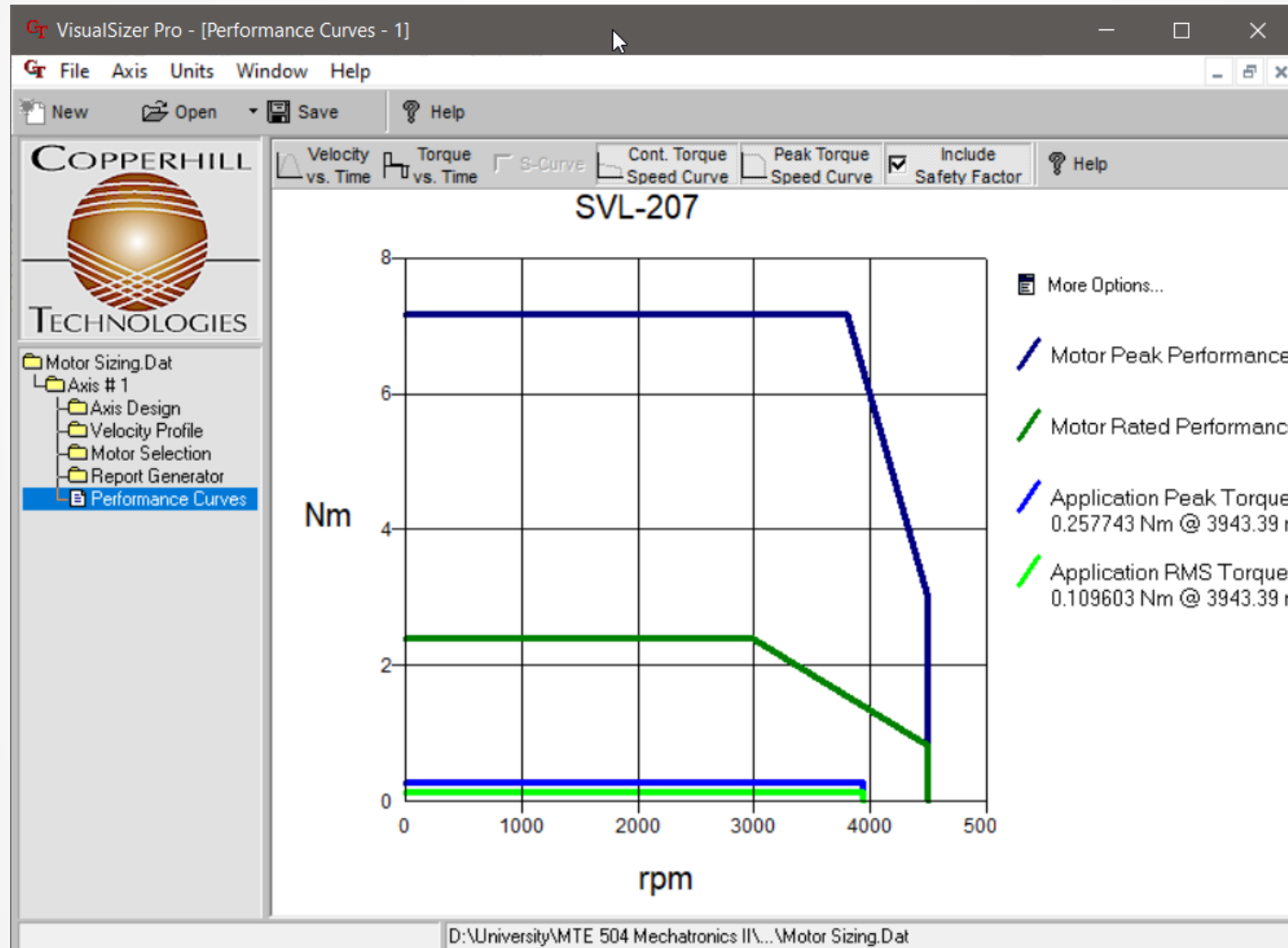
Manufacturer	AutomationDirect.C
Product Family	SVL - Low Inertia
Product Key	SVL-207
Drive Module	SVA-2100

Rated Speed	3000	rpm
Rated Torque	2.39	Nm
Max. (Peak) Torque	7.16	Nm
Rotor Inertia	0.000108	Nm-s ²
Kt	0	Nm/A

Motor Performance :
Rated speed is lower than max. velocity - Check motor performance

D:\University\MTE 504 Mechatronics II\...\Motor Sizing.Dat

VISUAL SIZER (Torque-Speed Curve)



NEXT LECTURE

Motor Sizing with Gearbox Inertia

Motion Study with SolidWORKS

Custom Trajectory with LabVIEW and SolidWORKS



End of Lecture

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