MTE 408 | ROBOTICS

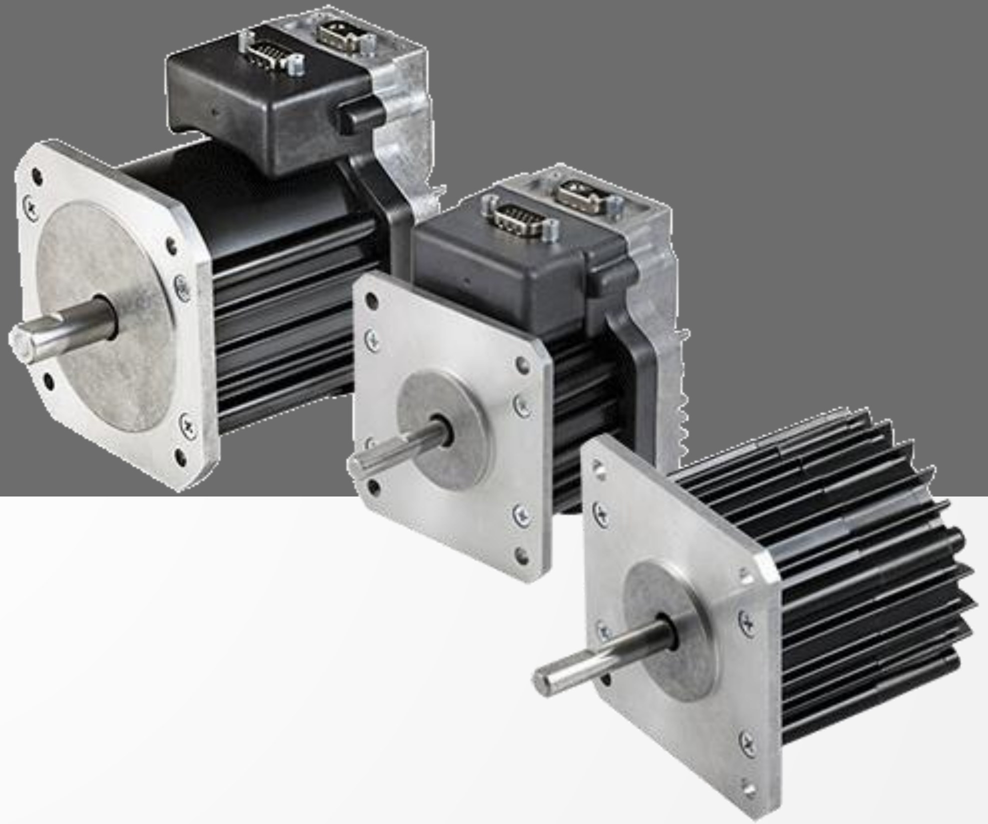
MOTOR SIZING
PART I: MOTOR SELECTION

PROF. FARID TOLBA (رحمه الله)
WALEED ELBADRY

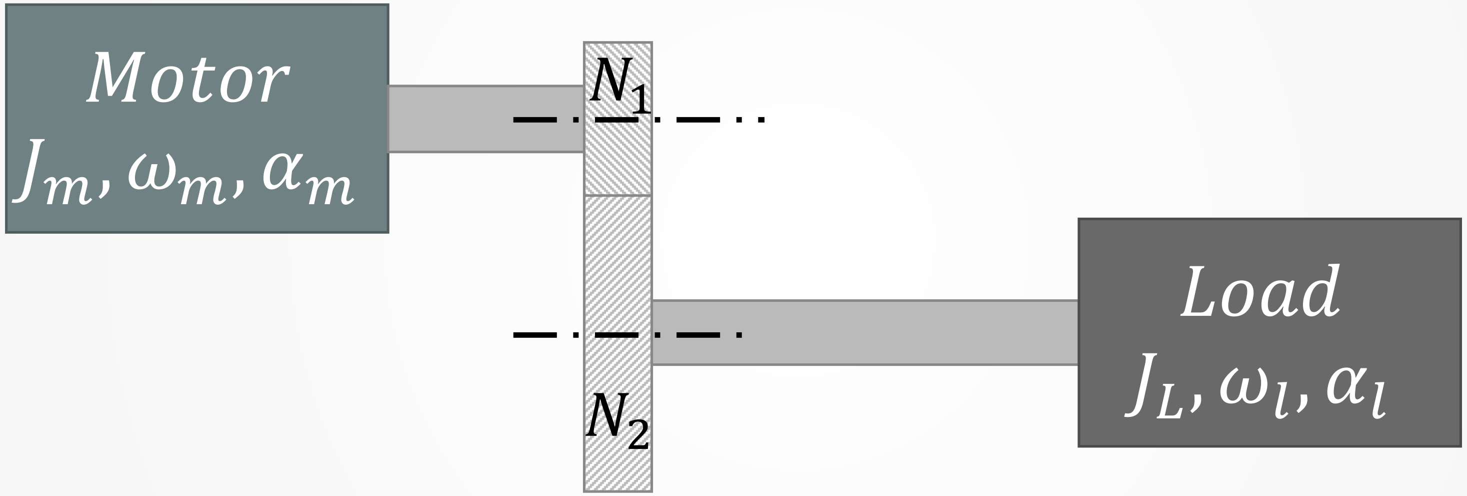
DECEMBER 2022

Motor Sizing

And its relationship to trajectory generation

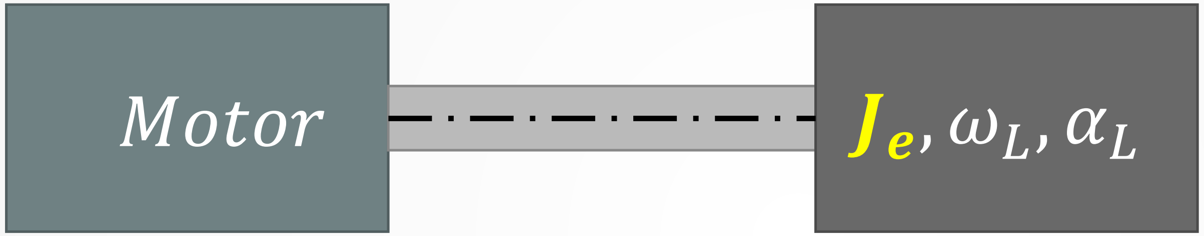


Reflected Inertia (Review)



The mechanical load needs to rotate at specific angular velocity and acceleration
From (trajectory generation)

Reflected Inertia (Review)



$The\ load\ inertia\ must\ be\ reflected\ to\ motor\ side\ to\ pick\ the\ suitable\ motor\ to\ maintain\ the\ desired\ velocity\ and\ accelration$

$We\ need\ to\ find\ the\ eqivalent\ inertia\ J_e$

Notation

ω_L … *Load angular velocity* $(\frac{rad}{s})$

ω_m … *Motor angular velocity* $(\frac{rad}{s})$

α_L … *Load angular accelration* $(\frac{rad}{s^2})$

α_m … *Motor angular velocity* $(\frac{rad}{s^2})$

T_L … *Load resistive torque* $(N.m)$

T_m … *Motor driving torque* $(N.m)$

J_L … *Load rotational moment of inertia* $(kg.m^2)$

J_m … *Motor shaft moment of inertia* $(kg.m^2)$

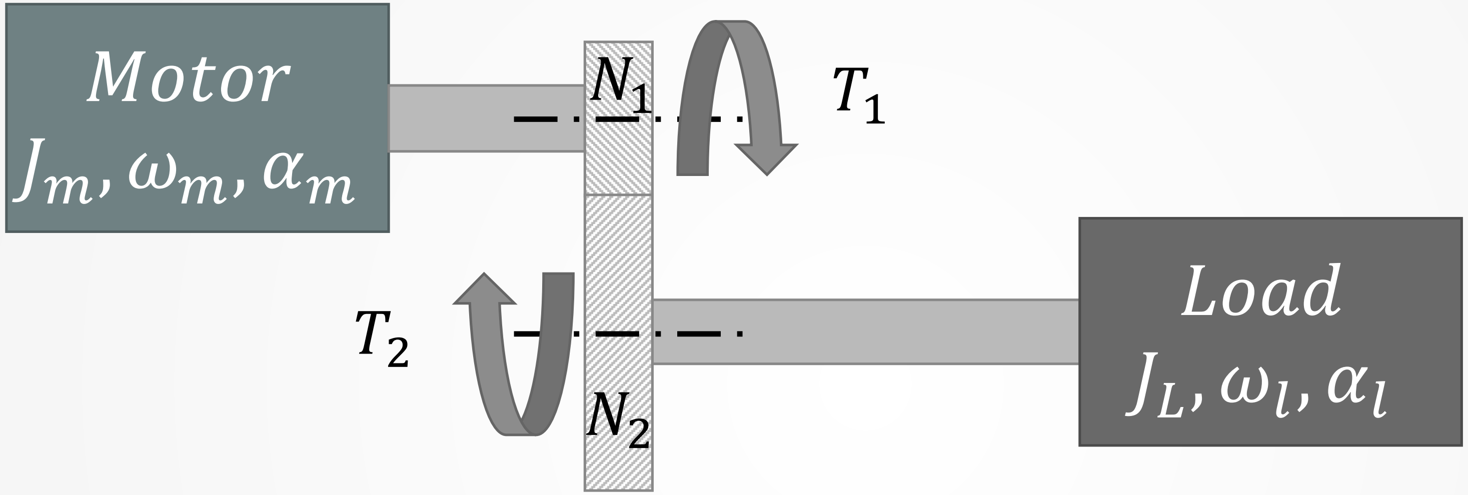
J_{gb} … *Gearbox moment of inertia* $(kg.m^2)$

i … *Gearbox ratio* > 1 (***boosting torque***)

$\omega_m > \omega_L$ *and* $T_m < T_L$

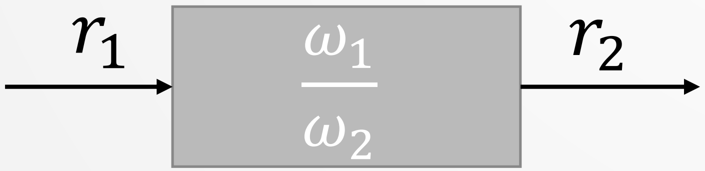
N … *Number of teeth of gear*

Reflected Inertia (Review)

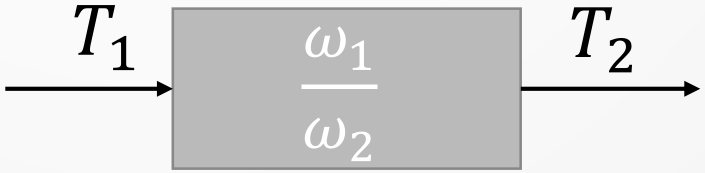


$For\ power\ transmission\ (Gears)$ $\quad P = T.\omega$

$V_1 = V_2$

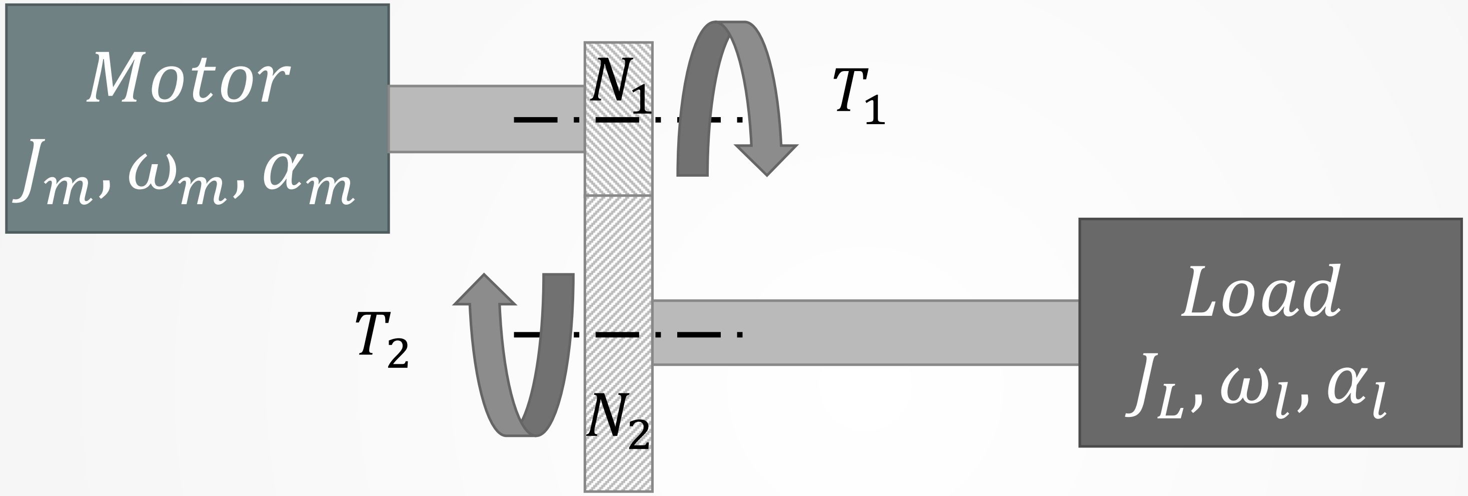


$P_1 = P_2$



$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1}$$

Reflected Inertia (Review)

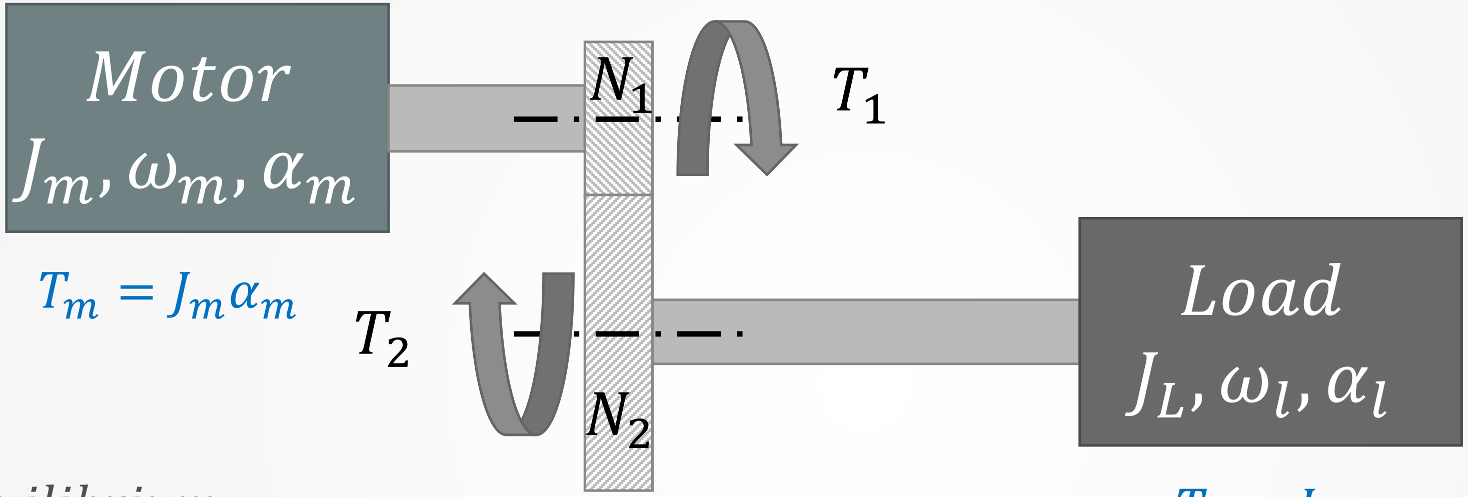


$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1} \rightarrow T_2 = T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

Assuming Gearbox inertia is ignored

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1} \rightarrow \omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

Reflected Inertia (Review)



$T_m = J_m \alpha_m$

$T_L = J_L \alpha_L$

$$T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

$$\omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

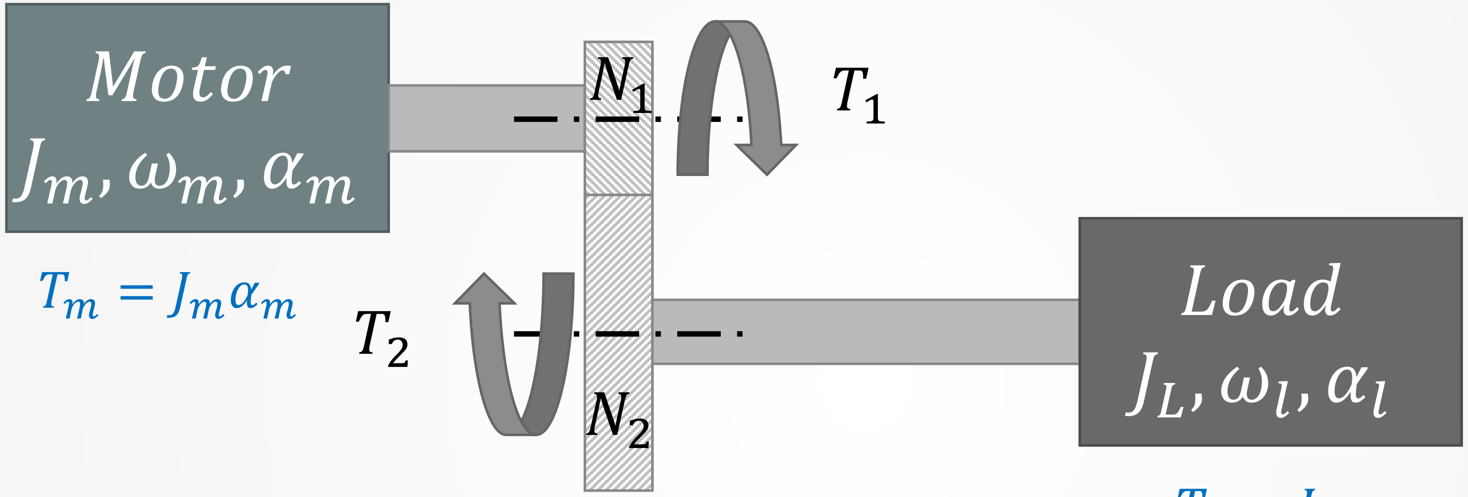
Rotaional equilibrium

$$\sum T = 0 \rightarrow T_m + T_L = J_m \alpha_m + J_L \alpha_L = 0$$

Reflection using equations 1 and 2

$$J_L \alpha_L = T_L \rightarrow J_L \boldsymbol{\alpha_m} \frac{\boldsymbol{N_1}}{\boldsymbol{N_2}} = T_m \frac{N_2}{N_1} \rightarrow \boldsymbol{J_L} \left(\frac{\boldsymbol{N_1}}{\boldsymbol{N_2}}\right)^{\boldsymbol{2}} \boldsymbol{\alpha_m} = \boldsymbol{T_m}\text{, } \textbf{\textit{but we know that}}\ T_m = J_m \alpha_m$$

Reflected Inertia (Review)



$T_m = J_m \alpha_m$

$T_L = J_L \alpha_L$

$$T_L = \frac{N_2}{N_1} T_m \rightarrow \{1\}$$

$$\omega_2 = \omega_L = \frac{N_1}{N_2} \omega_m \rightarrow \{2\}$$

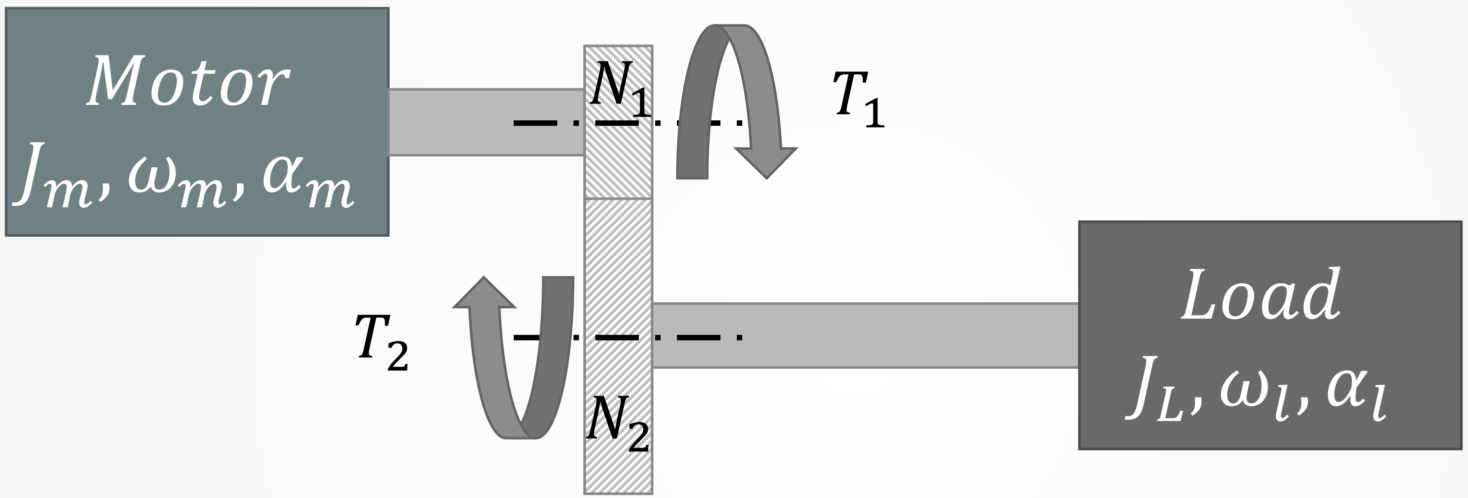
Reflection using equations 1 and 2

$$\because J_L \alpha_L = T_L \rightarrow J_L \alpha_m \frac{N_1}{N_2} = T_m \frac{N_2}{N_1} \rightarrow J_L \left(\frac{N_1}{N_2}\right)^2 \alpha_m = T_m$$

Rotaional equilibrium

$$\therefore T_m + T_{m(reflected)} = J_m \alpha_m + J_L \alpha_L = J_m \alpha_m + J_L \left(\frac{N_1}{N_2}\right)^2 \alpha_m = \left(\boldsymbol{J_m + J_L \left(\frac{N_1}{N_2}\right)^2}\right) \boldsymbol{\alpha_m} = \boldsymbol{T_{m(overall)}}$$

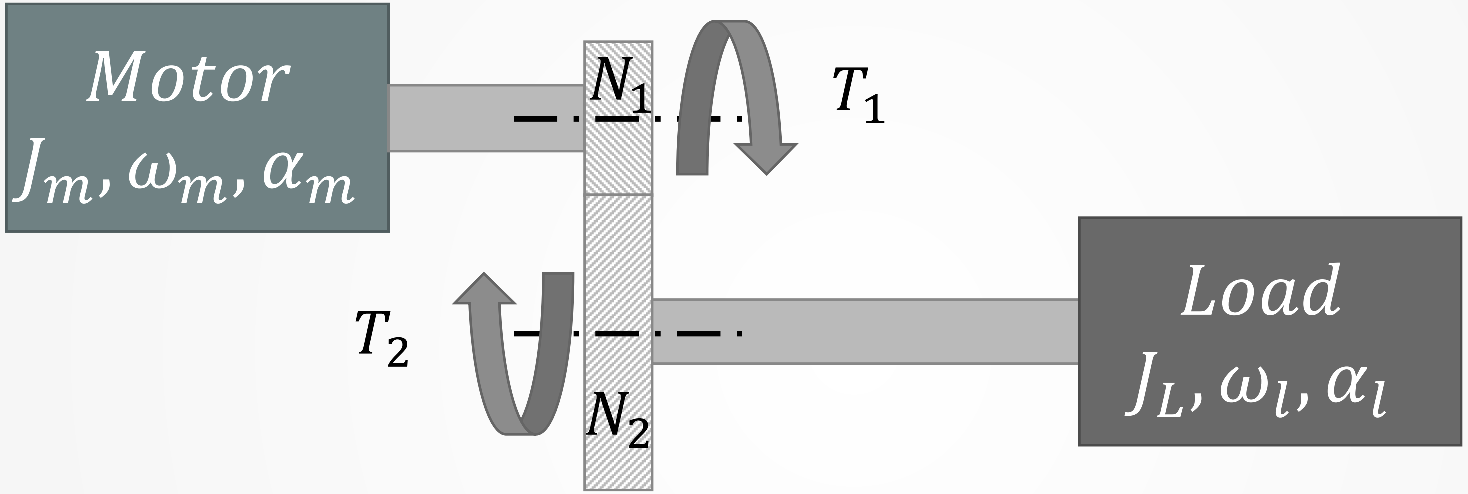
Reflected Inertia (Review)



$$\therefore J_e = J_m + J_L\left(\frac{N_1}{N_2}\right)^2 \; kg.m^2 (Equivalent\ Reflected\ Inertia)$$

$$\therefore J_e = J_m + \frac{J_L}{i^2}, \qquad i = \frac{N_2}{N_1}, i \ldots Gear\ Ratio = \frac{Driven\ No.of\ teeth}{Driver\ No.of\ teeth}$$

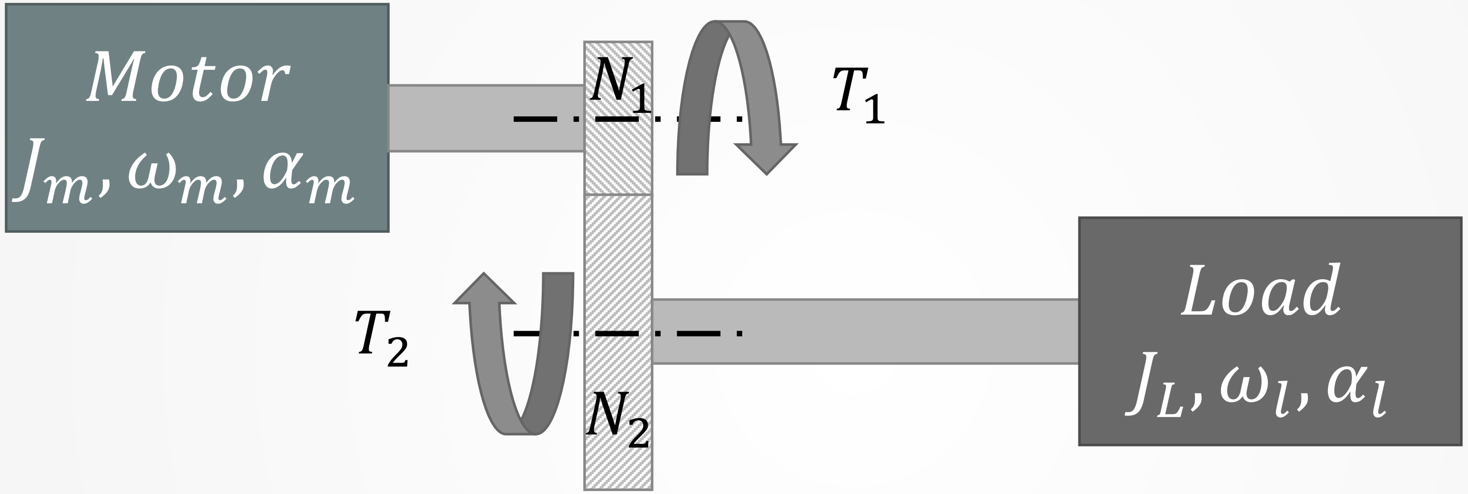
Motor Torque



$$\therefore T_m = \left(J_m + \frac{J_L}{i^2}\right).\,\boldsymbol{\alpha_m^{max}} = \left(J_m + \frac{J_L}{i^2}\right).\,\boldsymbol{(i.\,\alpha_L^{max})},$$

$$\boldsymbol{since\ \alpha_m^{max} = \frac{N_2}{N_1}\,\alpha_L^{max}} = i.\,\alpha_L^{max}\ ,\ \alpha_m^{max} > \alpha_L^{max}$$

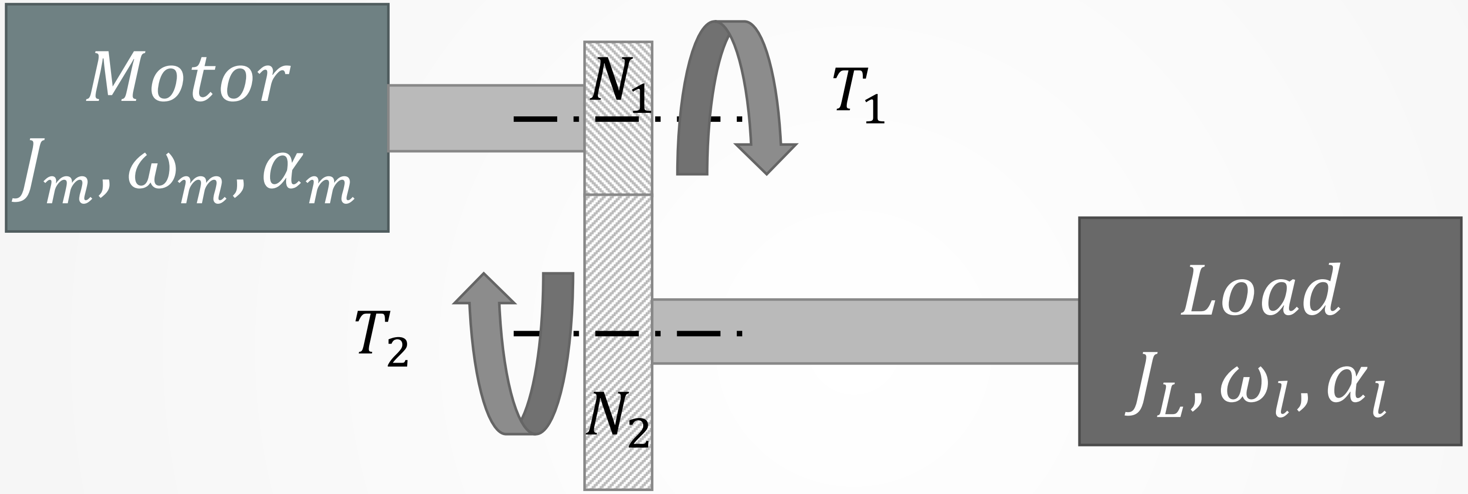
Maximum Motor Torque



$$\therefore T_m = \left(J_m + \frac{J_L}{i^{\cancel{2}}}\right).\left(\cancel{i}.\,\alpha_L^{max}\right) = J_m.\,i.\,\alpha_L^{max} + \frac{J_L}{i}.\,\alpha_L^{max}$$

$$To\ find\ \boldsymbol{T_m^{max}} = \frac{\partial T_m}{\partial i} = 0 \rightarrow J_m.\,\alpha_L^{max} - \frac{J_L}{i^2}.\,\alpha_L^{max} = 0$$

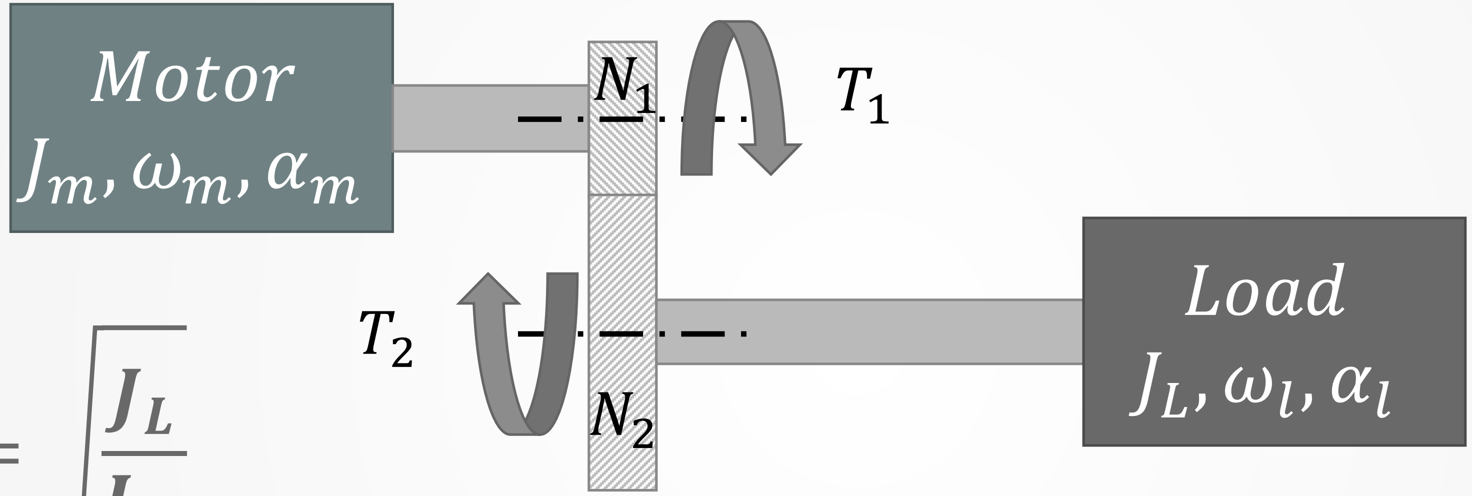
Optimal Gear Ratio



$$T_m^{max} = (J_m - \frac{J_L}{i^2}).\,\alpha_L^{max} = 0$$

$$\text{Either } \alpha_L^{max} = 0 \ (\textbf{rejected}) \text{ or } \left(J_m - \frac{J_L}{i^2}\right) = 0 \rightarrow i_{optimal} = \sqrt{\frac{J_L}{J_m}}$$

Inertia Mismatch



$$i_{optimal} = \sqrt{\frac{J_L}{J_m}}$$

$$Let \quad \frac{J_L}{i^2} = J_L^{eq} \rightarrow \ i_{opt}^2 = \frac{J_L}{J_m} \rightarrow \frac{\frac{J_L}{i_{opt}^2}}{J_m} = 1 \rightarrow \frac{J_L^{eq}}{J_m} = 1 \ (Inertia\ Mismatch)$$

Inertia Mismatch

$$\frac{J_L^{eq}}{J_m} = \mathbf{1-10}\ (\boldsymbol{Inertia\ Mismatch})$$

$$\mathbf{1-3}\ \rightarrow \boldsymbol{Precise\ servo\ systems}$$

$$\mathbf{3-6}\ \rightarrow \boldsymbol{moderate\ accuracy}\ \ (\boldsymbol{industrial\ systems})$$

$$\mathbf{6-10}\ \rightarrow \boldsymbol{rough\ handling\ systems}$$

Required Motor Power

$$\because T_m = \left(J_m + \frac{J_L}{i^2}\right).i.\alpha_L^{max} \; and \; P_m = T_m.\omega_m^{max}$$

$\omega_m^{max} = i.\omega_L^{max}$
$\alpha_m^{max} = i.\alpha_L^{max}$

$$\therefore P_m = \left(J_m + \frac{J_L}{i^2}\right).i.\alpha_L^{max}.i.\omega_L^{max}$$

$$i_{opt}^2 = \frac{J_L}{J_m}$$

$$\therefore P_m = \left(J_m.i^2 + \cancel{i^2}.\frac{J_L}{\cancel{i^2}}\right).\alpha_L^{max}.\omega_L^{max}$$

$$\therefore P_m = \left(\cancel{J_m}.\frac{J_L}{\cancel{J_m}} + J_L\right).\alpha_L^{max}.\omega_L^{max} = 2.J_L.\alpha_L^{max}.\omega_L^{max} = \frac{2}{\eta}.J_L.\alpha_L^{max}.\omega_L^{max}$$

$$\eta(efficiency) = 80\% - 90\%\text{ , } if not\ given, then\ take\ it\ as\ 72\% = 0.72$$

Design Procedure

1. ***Design the mechanical mechanism***

2. ***Motion planning and trajectory generation.***

3. ***Obtain*** J_L, ω_L^{max} ***and*** α_L^{max} *(From motion study on SolidWORKS)*

4. ***Select the desired mismatch*** *(with respect to application)*

5. ***calculate the desired power*** $\rightarrow P_m = \frac{2}{\eta} . J_L . \alpha_L^{max} . \omega_L^{max}$

6. ***Select the motor from the catalog*** *(vendor catalog)*

Next is parametric check

Design Procedure

$Parametric\ Check:$

$$I.\quad Rated\ speed\ factor \rightarrow \frac{i.\omega_L^{max}}{\omega_m^{max}} \leq 0.8-1$$

$$II.\quad Torquefactor \rightarrow \frac{RMS\ Loading\ Torque\ (T_L^{RMS})}{Motor\ Rated\ Torque(T_m^{Rated})} \leq 0.8-1$$

$$III.\quad Maximum\ Torquefactor \rightarrow \frac{\frac{T_L^{max}}{i}}{T_m^{max})} \leq 0.8-1$$

Example

Choose the servo motor operating on a robot joint with the following parameters:

$$J_L = 200\ Kg.m^2, T_f = 120\ N.m\ , \omega_L^{max} = 0.15 \frac{rad}{s}, \alpha_L^{max} = 0.44 \frac{rad}{s^2}$$

$$\eta = 0.72$$

Solution

$$\because\ P_m = \frac{2}{\eta}\left(T_f + J_L.\alpha_L^{max}\right).\omega_L^{max} = \frac{2}{0.72}(120 + 200 * 0.44) * 0.7 = 404.44\ W\ (rated\ power)$$

Selection of fitting motor from catalog

$$\because\ P_m = \frac{2}{\eta}\left(T_f + J_L.\alpha_L^{max}\right).\omega_L^{max} = \frac{2}{0.72}(120 + 200 * 0.44) * 0.7 = 404.44\ W\ (rated\ power)$$

Example

$Choose\ the\ servo\ motor\ operating\ on\ a\ robot\ joint\ with\ the\ following\ parameters$:

$$J_L = 200\ Kg.m^2, T_f = 120\ N.m\ ,\ \omega_L^{max} = 0.15\frac{rad}{s}, \alpha_L^{max} = 0.44\frac{rad}{s^2}$$

$$\eta = 0.72$$

$Solution$

$$\because\ P_m = \frac{2}{\eta}(T_f + J_L.\alpha_L^{max}).\omega_L^{max} = \frac{2}{0.72}(120 + 200 * 0.44) * 0.7 = 404.44\ W\ (rated\ power)$$

$Selection\ of\ fitting\ motor\ from\ catalog$

Model	Speed n^{rated} (min^{-1})	Torque T^{rated} $(N.m)$	Power P^{rated} (KW)	Power V^{rated} (V)	Current I^{rated} (A)	Maximum Torque T^{max} $(N.m)$	Frequency f^{rated} (Hz)	Inertia J_m $(kg.cm^2)$
MDSKS 036-23,200	4000	1.3	0.54	120	0.9	7.2	200	0.36

Example

$Choose\ the\ servo\ motor\ operating\ on\ a\ robot\ joint\ with\ the\ following\ parameters:$

$J_L = 200\ Kg.m^2, T_f = 120\ N.m\ ,\ \omega_L^{max} = 0.15\frac{rad}{s}, \alpha_L^{max} = 0.44\frac{rad}{s^2}$

$\eta = 0.72$

Solution

$$P_{m(required)}^{rated} = 404.44\ W \rightarrow P_{m(catalog)}^{rated} = 540\ W$$

$$n_{m(catalog)}^{rated} = 4000\ RPM\ \rightarrow \omega_{m(catalog)}^{rated} = \frac{2 * \pi * 4000}{60} = 418\frac{rad}{s}$$

$$i_{opt} = \sqrt{\frac{J_L}{J_m}} = \sqrt{\frac{200 * 10^4}{0.36}} = 2357\ (gear\ train)$$

Example

$Choose\ the\ servo\ motor\ operating\ on\ a\ robot\ joint\ with\ the\ following\ parameters$:

$J_L = 200\ Kg.m^2, T_f = 120\ N.m\ ,\ \omega_L^{max} = 0.15\frac{rad}{s}, \alpha_L^{rated} = 0.44\frac{rad}{s^2}$

$\eta = 0.72$

$Solution$

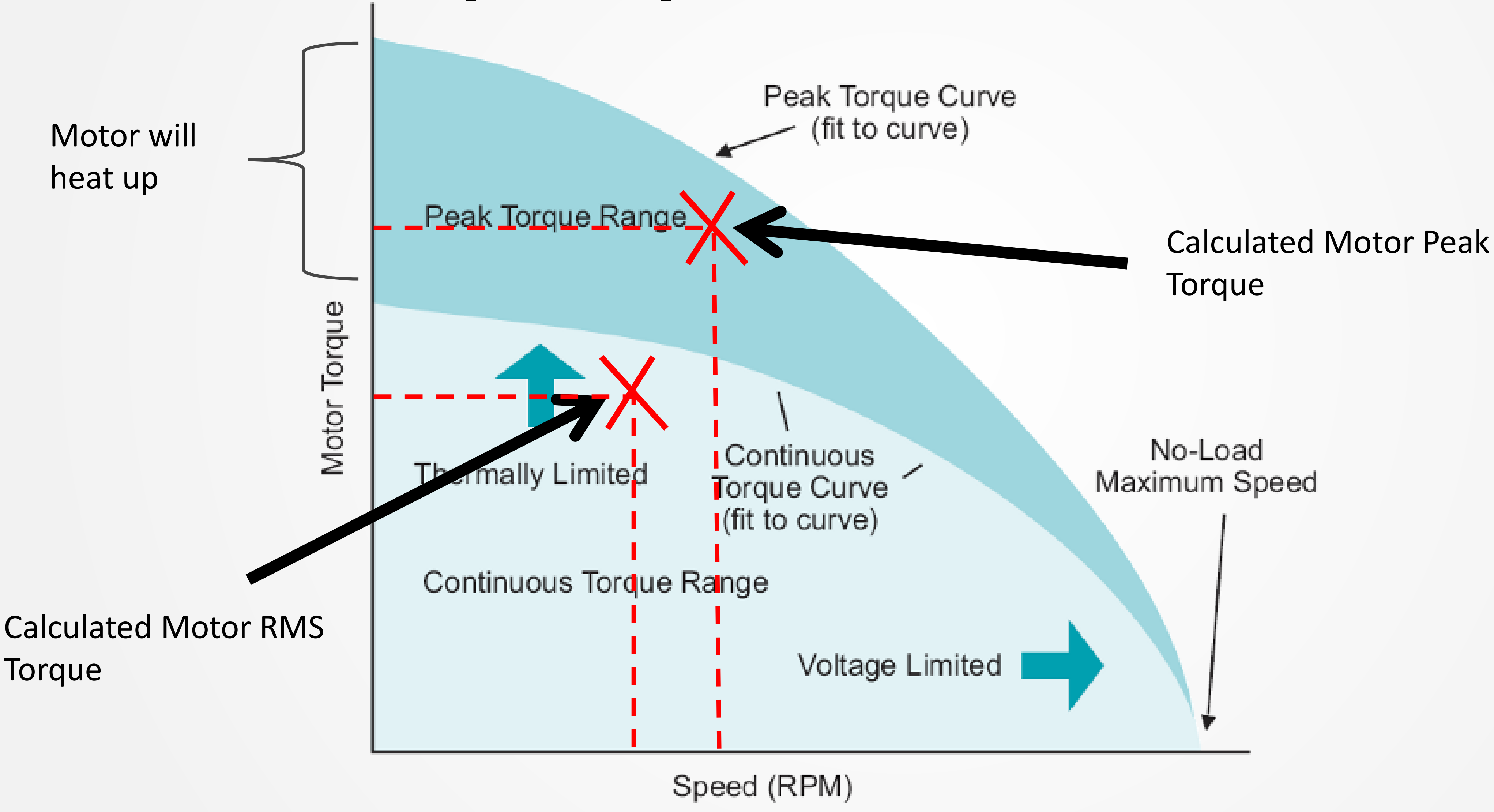
$$P_{m(catalog)}^{rated} = 540\ W \quad \omega_{m(catalog)}^{rated} = 418\frac{rad}{s} \quad i_{opt} = 373$$

$Checking$:

$$Rated\ speed\ factor \rightarrow \frac{i.\omega_L^{max}}{\omega_m^{max}} = \frac{2357 * 0.15}{418} = 3.94 \leq 0.8 - 1\ (OK)$$

$$Maximum\ Torquefactor \rightarrow \frac{\frac{T_L^{max}}{i}}{T_m^{rated}} = \frac{\frac{200 * 0.44}{2357}}{1.3} = 0.029 \leq 0.8 - 1(OK)$$

Motor Torque-Speed Chart

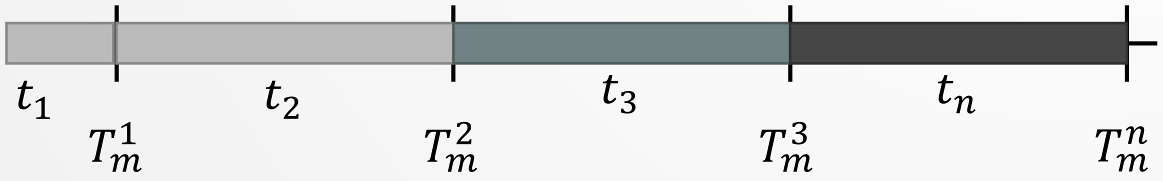


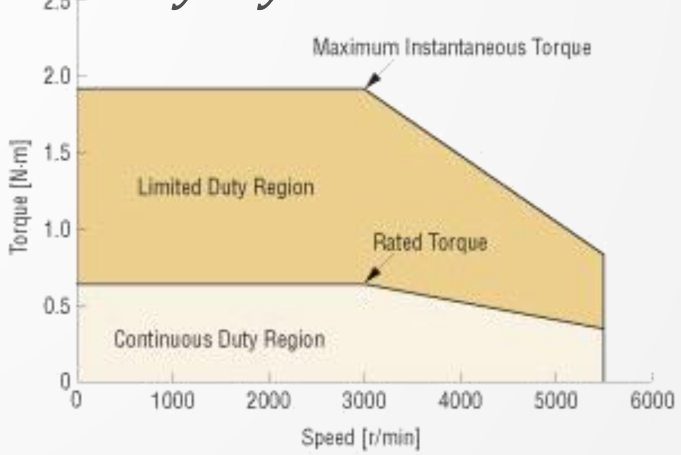
Notes

RMS Torque : *Represents the average torque of resistive load during trajectory. It is known as "continuous or rated torque"*

$$\boldsymbol{T_m^{RMS}} = \sqrt{\frac{\sum_{k=0}^{n} T_m^k t_k}{\sum_{k=1}^{n} t_k}} = \sqrt{\frac{\left(T_m^1\right)^2 t_1 + \left(T_m^2\right)^2 t_2 + \left(T_m^3\right)^2 t_3 + \cdots \left(T_m^n\right)^2 t_n}{t_1 + t_2 + t_3 + \cdots t_n}}\ (N.m)$$

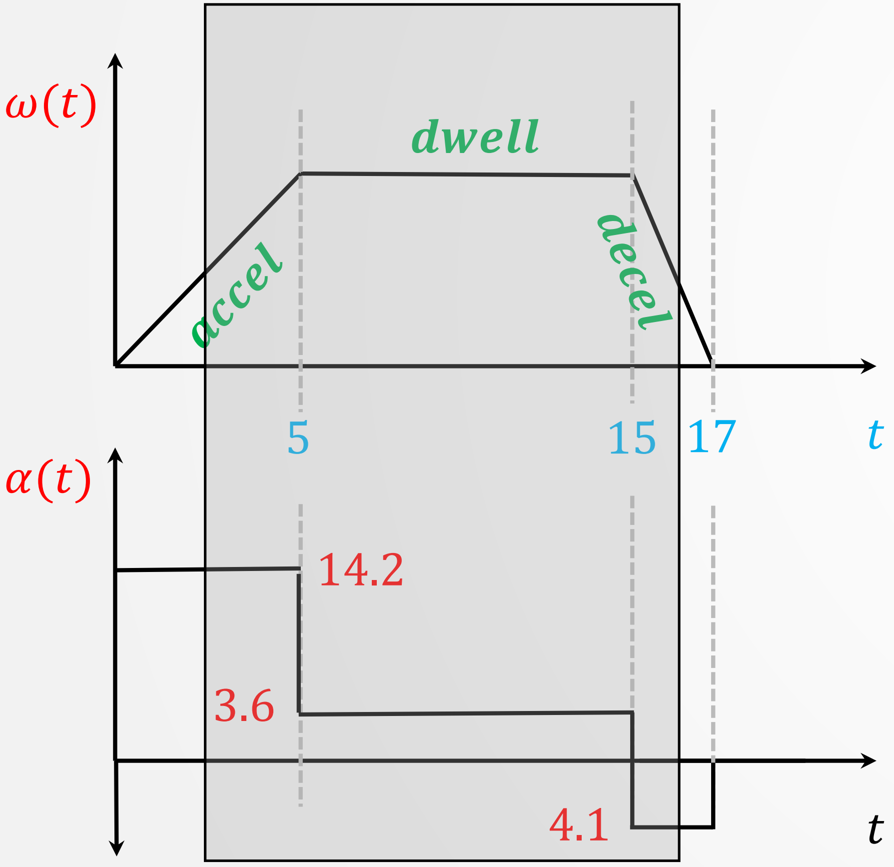
Peak Torque : *Is the worst torque reached (momentarily by the load motion). Also referred to as "intermittent torque"*





Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2\ Kg.cm^2$



SOLUTION

$$J_m = 0.2 * 10^{-4}\ Kg.m\text{^}2$$

Computing Torque for each motion segment

$$T_{accel} = (J_m)(\alpha_{accel}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} N.m$$

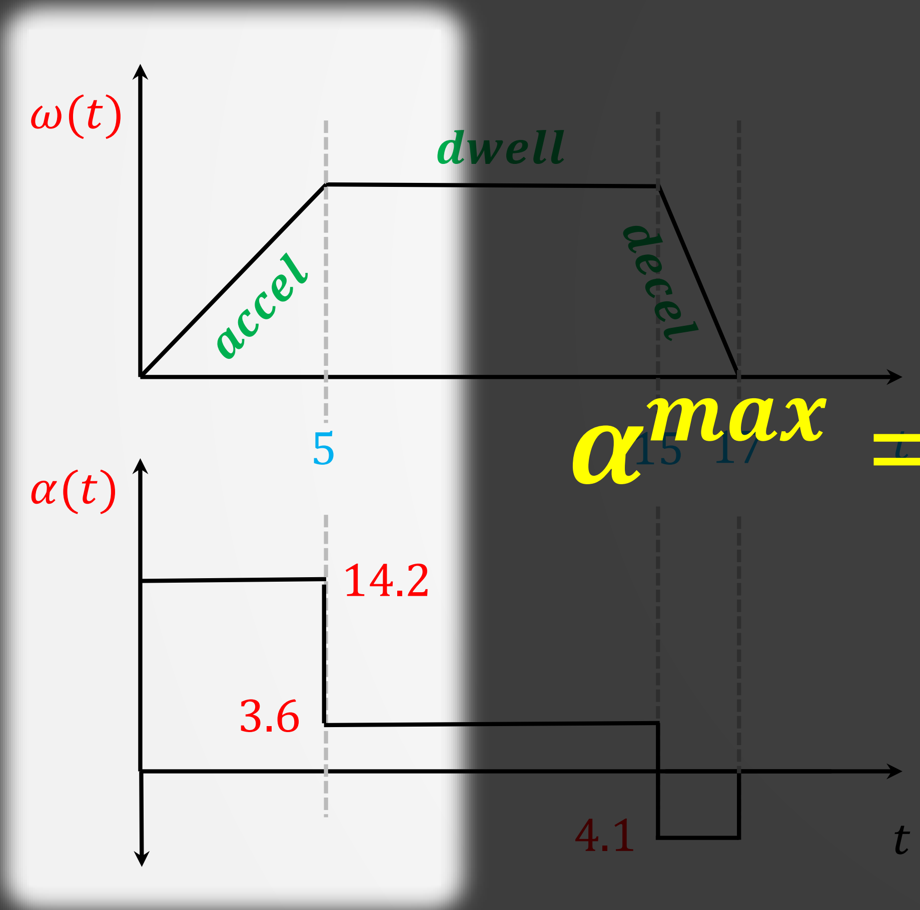
$$T_{dwell} = (J_m)(\alpha_{dwell}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} N.m$$

$$T_{decel} = (J_m)(\alpha_{decel}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} N.m$$

Show that $T_{RMS} = 1.66 * 10^{-4}\ N.m$

Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2\ Kg.cm^2$



SOLUTION

$J_m = 0.2 * 10^{-4}\ Kg.m\char`^2$

Computing Torque for each motion segment

$T_{accel} = (J_m)(\alpha_{accel}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} N.m$

$T_{dwell} = (J_m)(\alpha_{dwell}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} N.m$

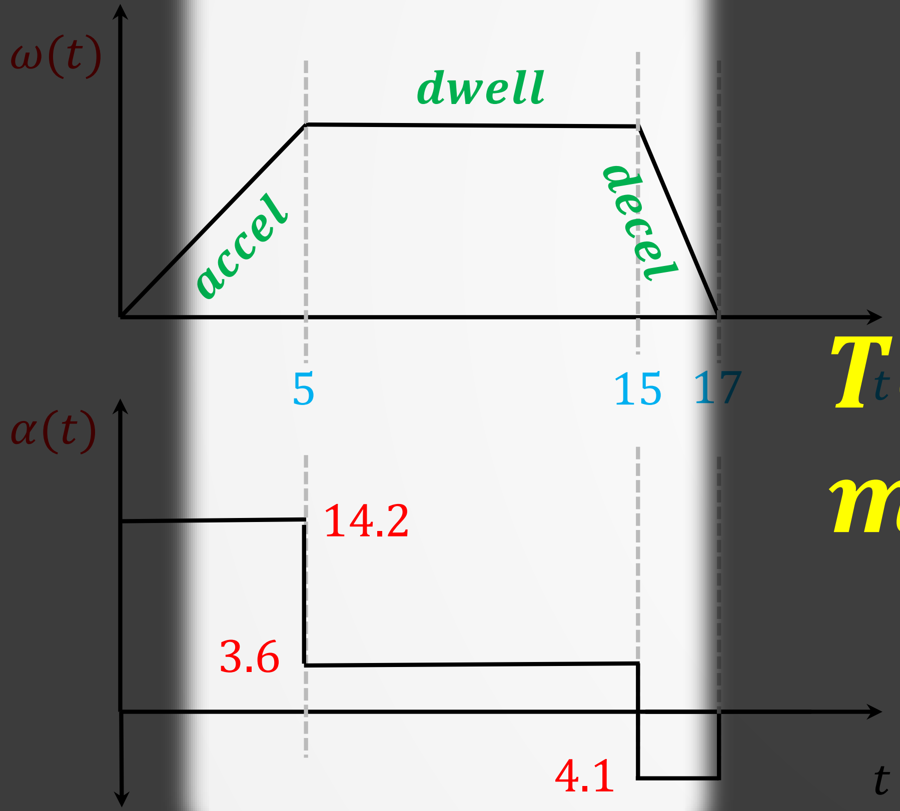
$T_{decel} = (J_m)(\alpha_{decel}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} N.m$

Show that $T_{RMS} = 1.66 * 10^{-4}\ N.m$

$$\alpha^{max} = \frac{\omega^{max}}{t} \rightarrow \omega^{max} = 71 \frac{rad}{s}$$

Example

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2\ Kg.cm^2$



SOLUTION

$J_m = 0.2 * 10^{-4}\ Kg.m^2$

Computing Torque for each motion segment

Torque needed to overecome motion friction

$T_{accel} = (J_m)(\alpha_{accel}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} N.m$

$T_{dwell} = (J_m)(\alpha_{dwell}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} N.m$

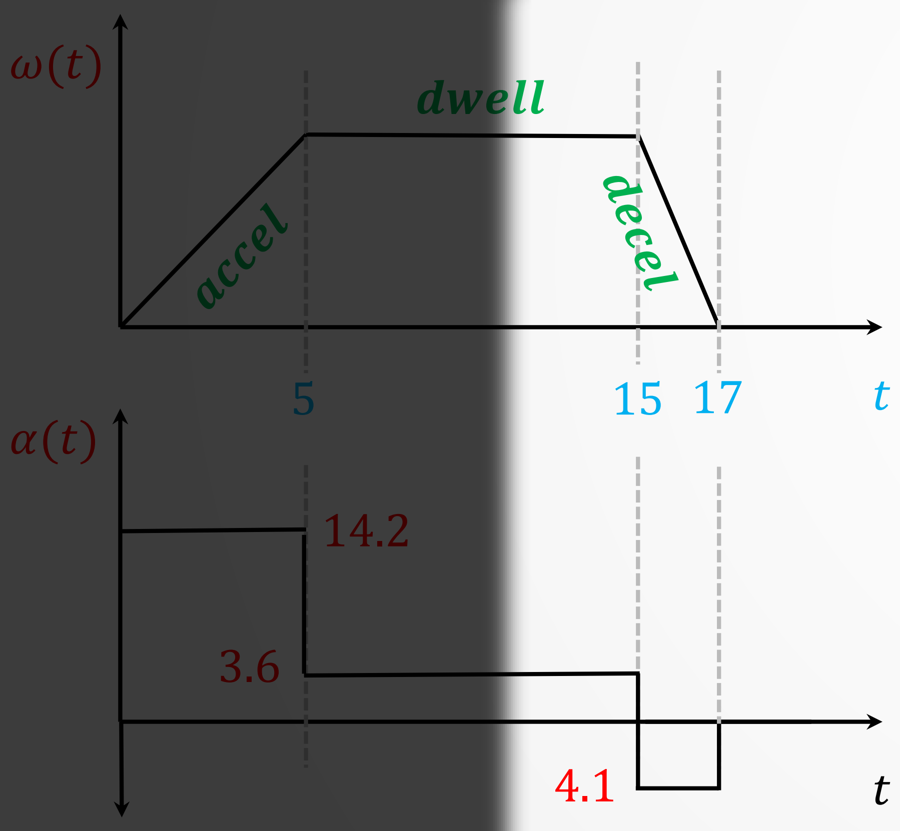
$T_{decel} = (J_m)(\alpha_{decel}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} N.m$

Show that $T_{RMS} = 1.66 * 10^{-4}\ N.m$

Example

$$\alpha^{decel} = 4.1 \rightarrow \omega^{decel} =? \frac{rad}{s}$$

Find the RMS and Peak Torque for the following motion profile if $J_m = 0.2\ Kg\,cm^2$



SOLUTION

$J_m = 0.2 * 10^{-4}\ Kg.m\text{^}2$

Computing Torque for each motion segment

$T_{accel} = (J_m)(\alpha_{accel}) = (0.2 * 10^{-4})(14.2) = 2.84 * 10^{-4} N.m$

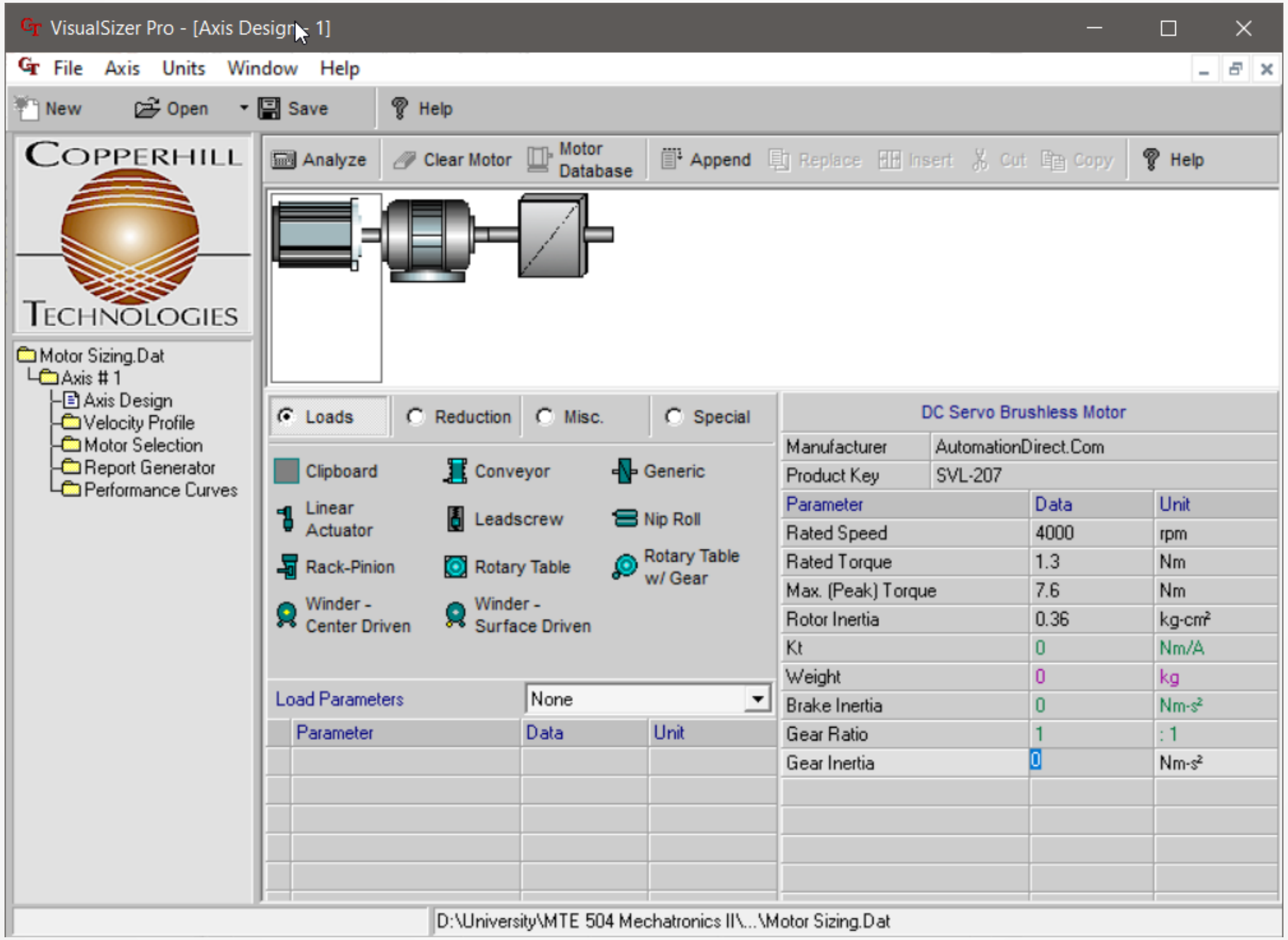
$T_{dwell} = (J_m)(\alpha_{dwell}) = (0.2 * 10^{-4})(3.6) = 0.72 * 10^{-4} N.m$

$T_{decel} = (J_m)(\alpha_{decel}) = (0.2 * 10^{-4})(4.1) = 0.82 * 10^{-4} N.m$

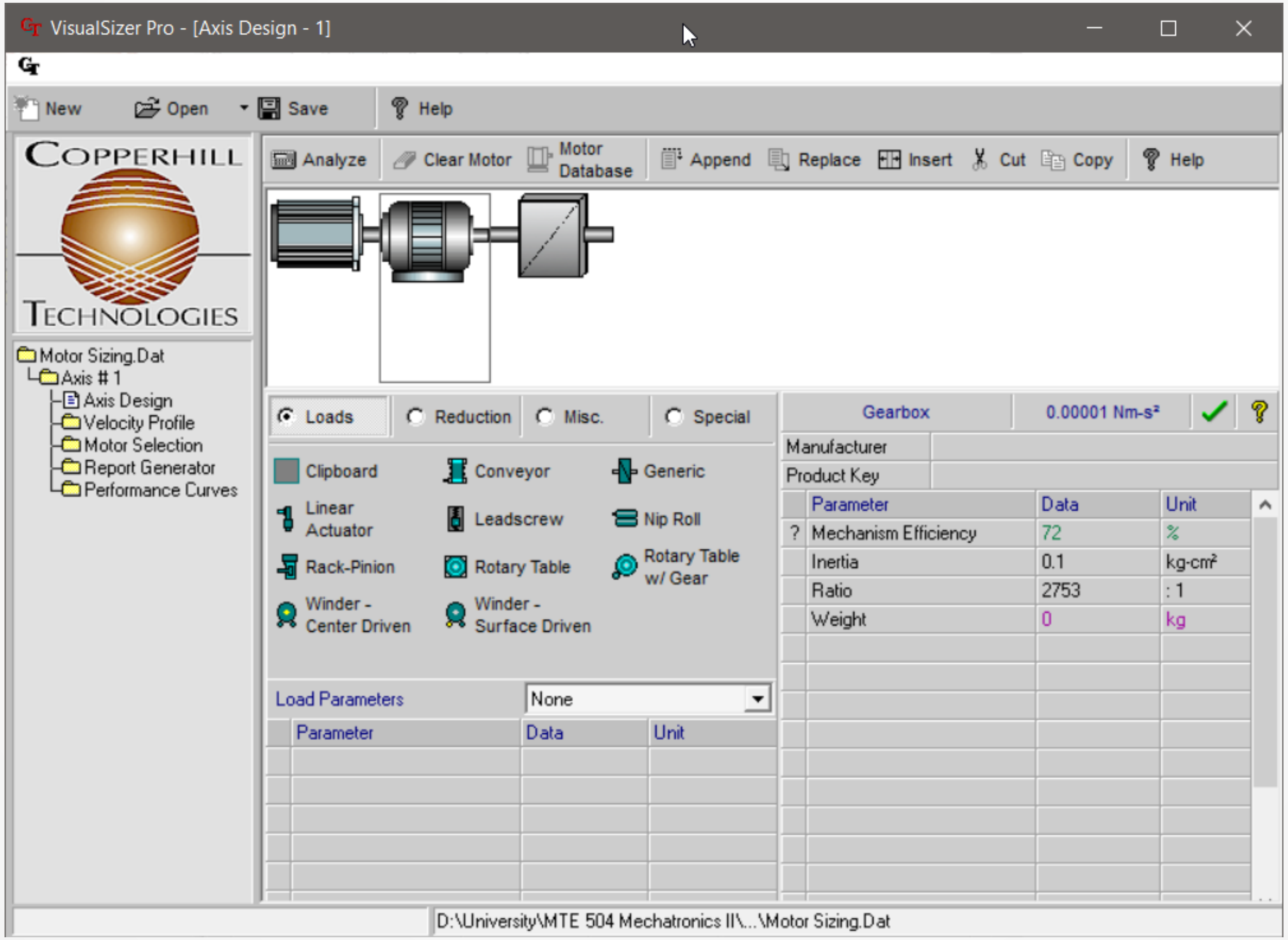
Show that $T_{RMS} = 1.66 * 10^{-4}\ N.m$

Why deceleration torque is lower than accelration torque?

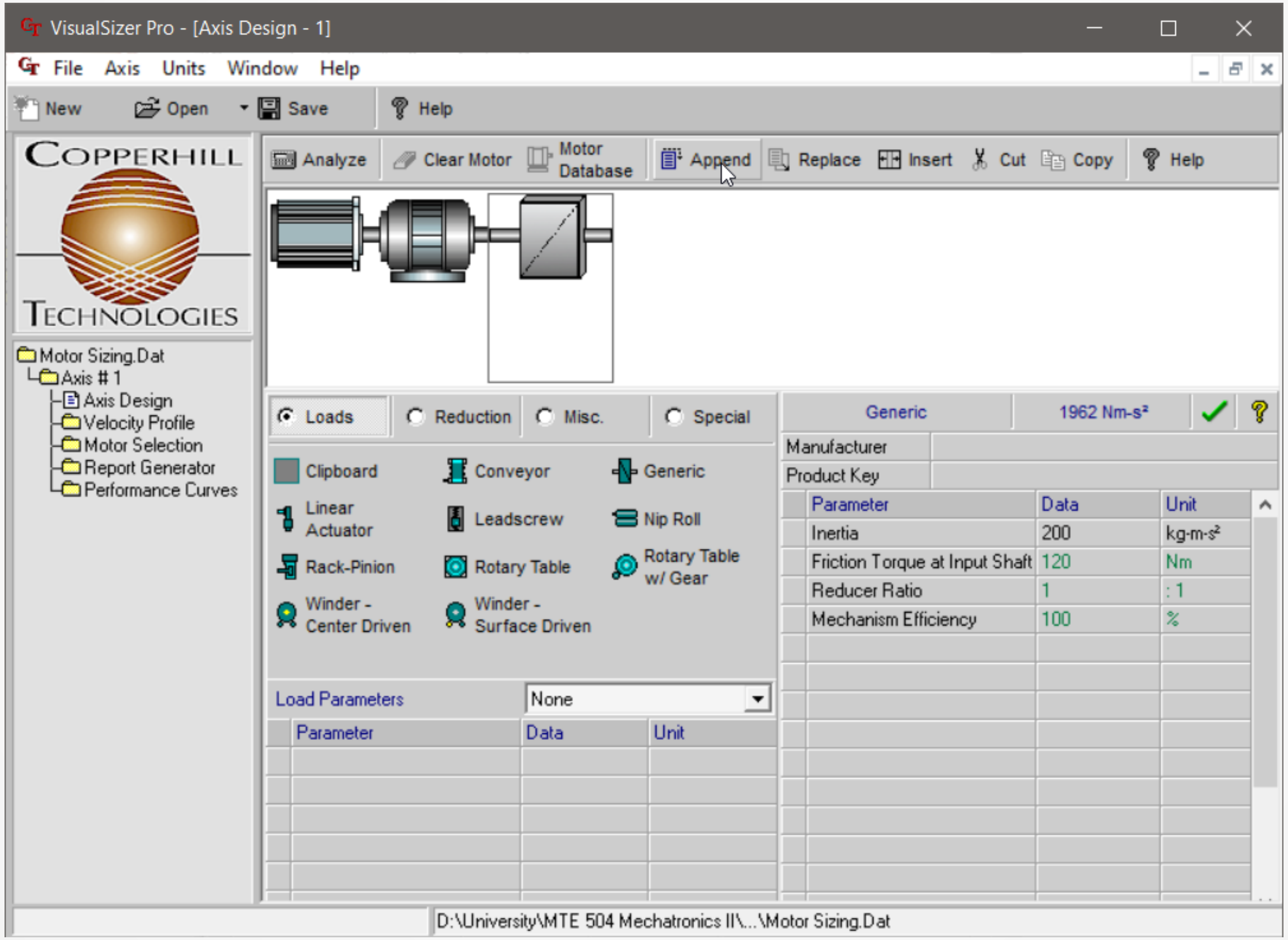
VISUAL SIZER (Axis Design)



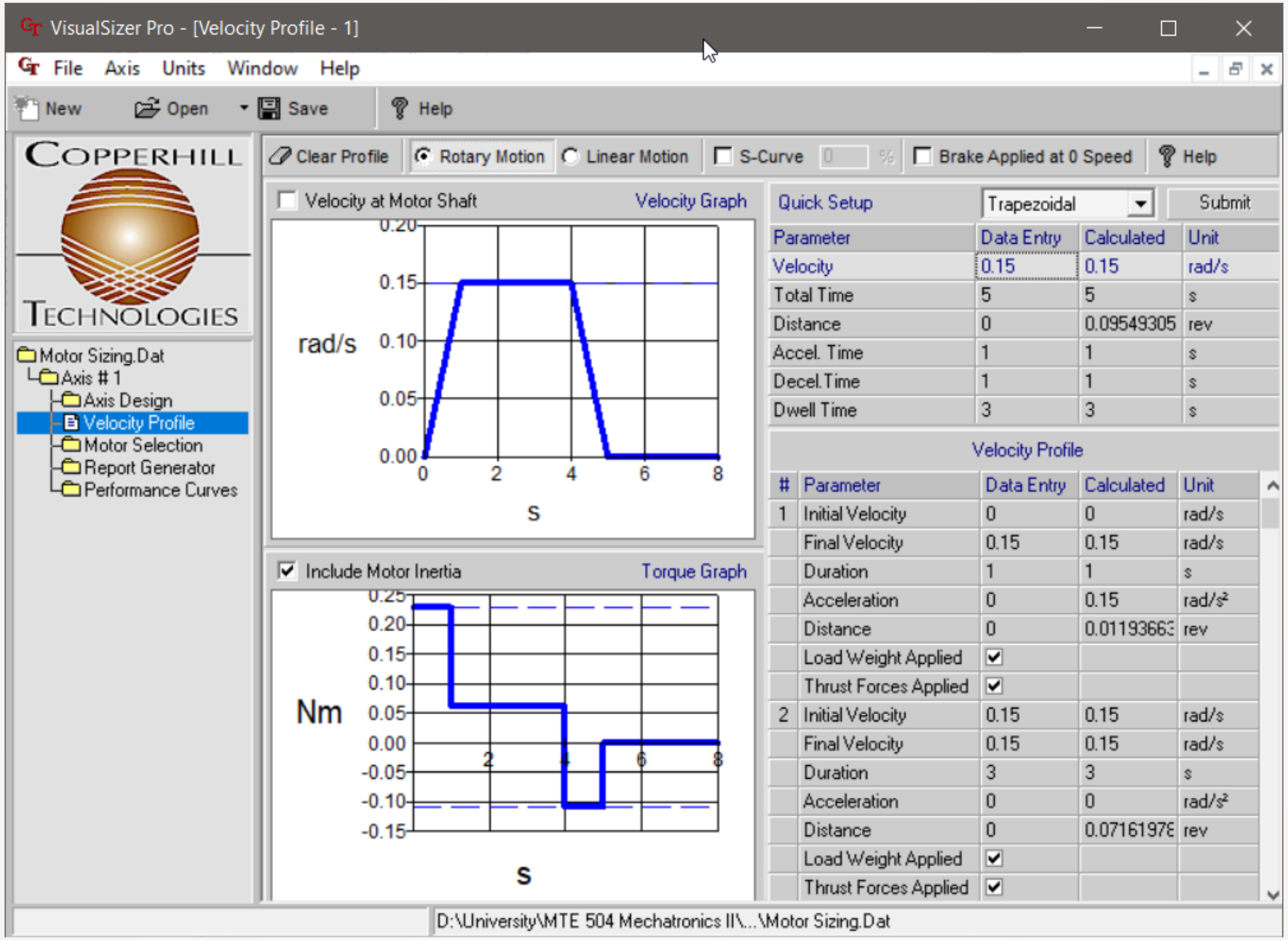
VISUAL SIZER (Axis Design)



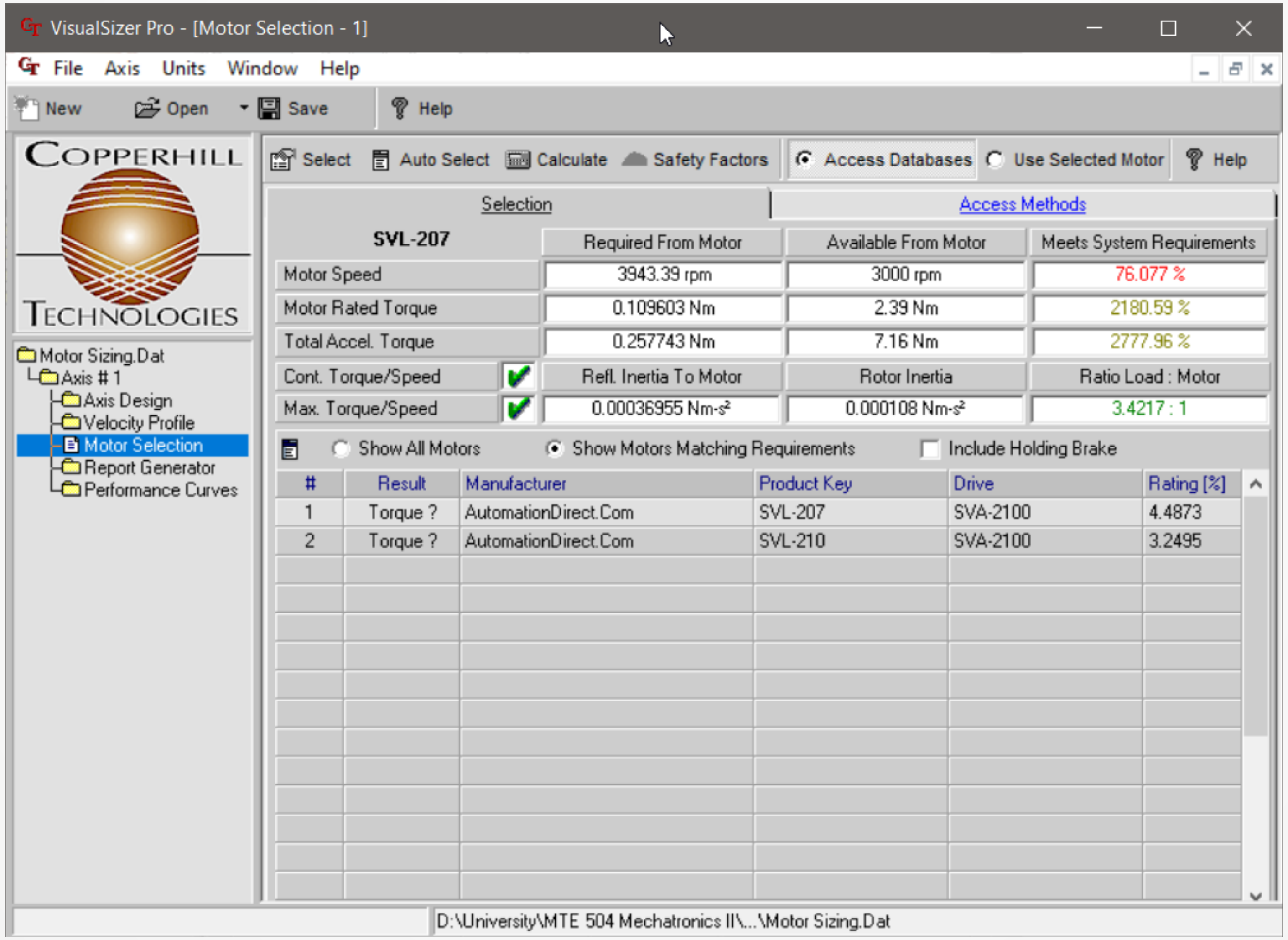
VISUAL SIZER (Axis Design)



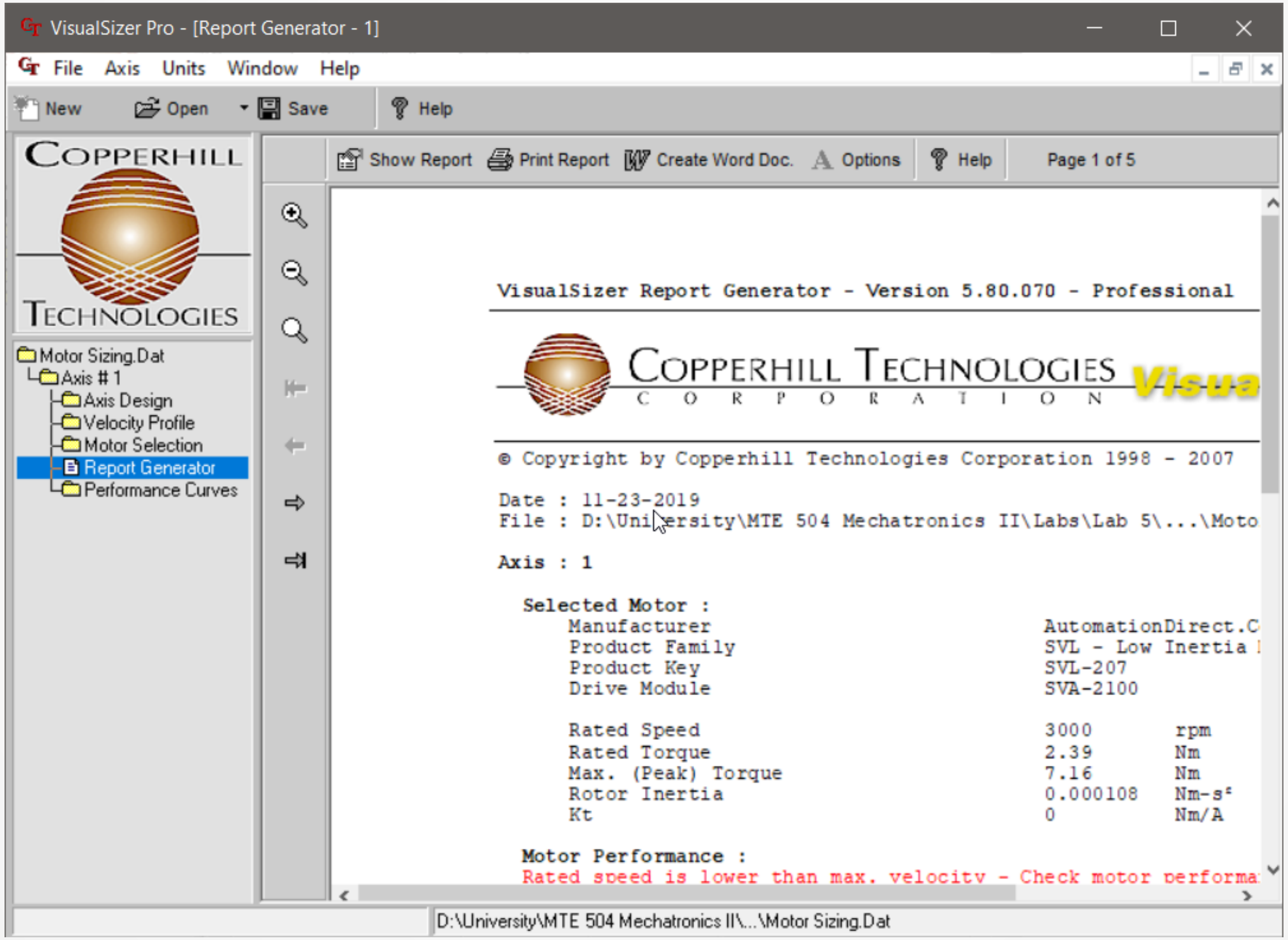
VISUAL SIZER (Trajectory)



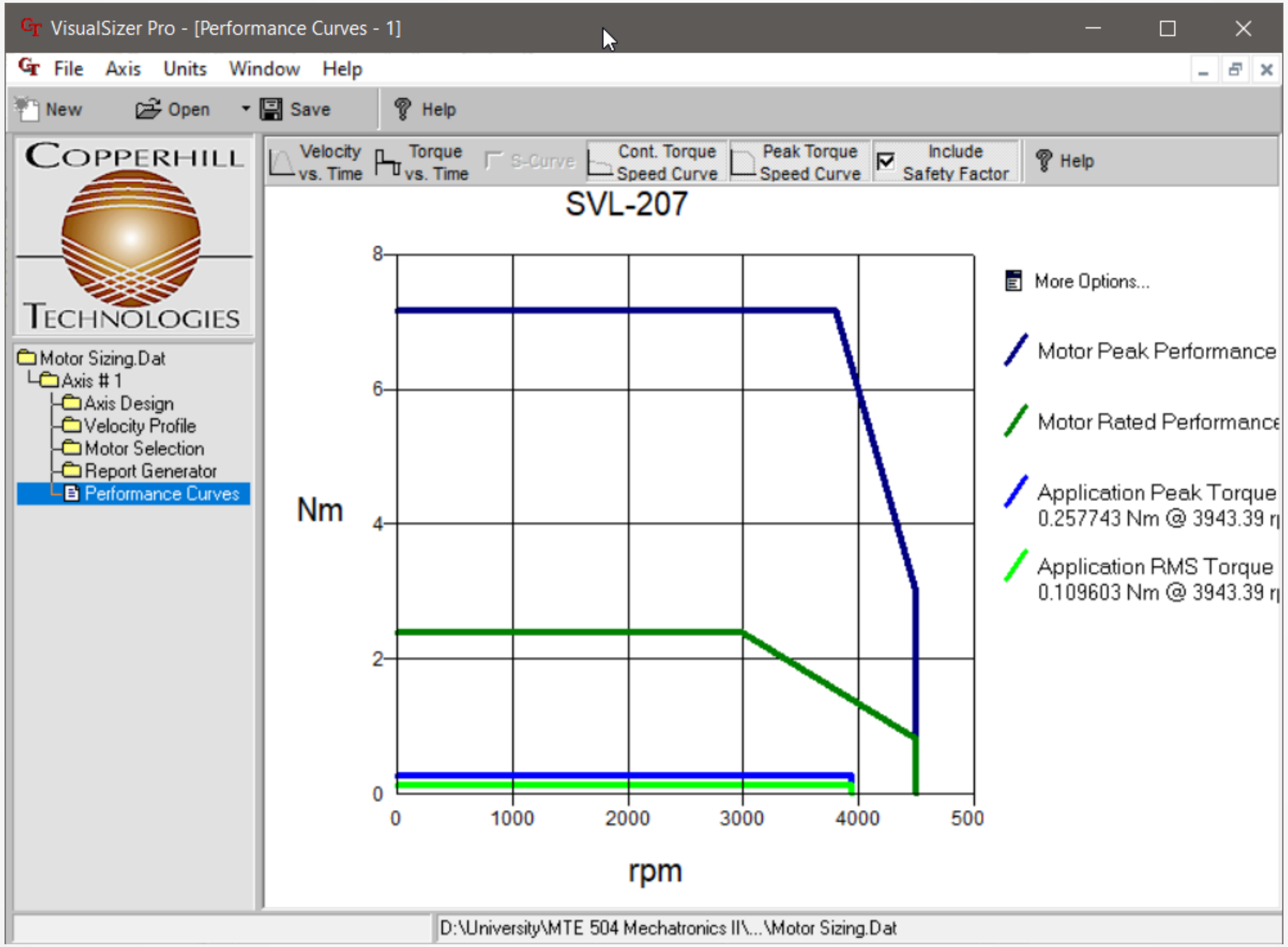
VISUAL SIZER (Motor Selection and Check)



VISUAL SIZER (Selection Report)



VISUAL SIZER (Torque-Speed Curve)



NEXT LECTURE

Motor Sizing with Gearbox Inertia

Motion Study with SolidWORKS

Custom Trajectory with LabVIEW and SolidWORKS

End of Lecture

Inquiries:

Prof. Farid Tolbah ftolbah2@yahoo.com

Eng. Waleed El-Badry waleed.elbadry@must.edu.eg