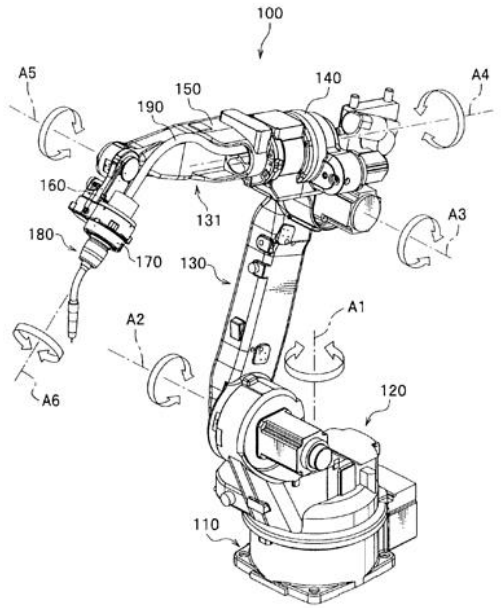
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT

MTE 408 ROBOTICS

SESSION 5
INTRODUCTION TO ROBOTICS LAB

WALEED ELBADRY

MARCH 2022



SUMMARY

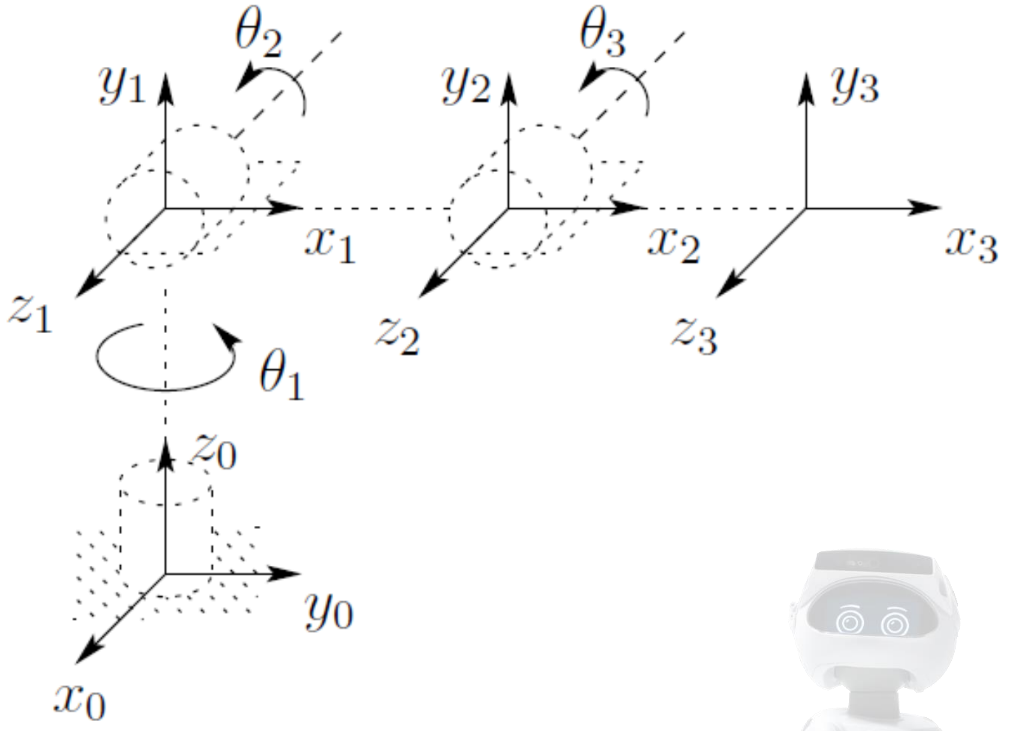
Fot ***joint i***

The ***joint variable*** $q_i = \begin{cases} \theta_i \text{ joint } i \text{ revolute} \\ d_i \text{ joint } i \text{ prismatic} \end{cases}$

$${}^{i}_{j}T = {}^{i}_{i+1}T\ {}^{i+1}_{i+2}T\ {}^{i+2}_{i+3}T \dots {}^{j-1}_{j}T\ ,\boldsymbol{i < j}$$

$${}^{i}_{j}T = I\ , i = j\ (\boldsymbol{Identical\ Frames})$$

$${}^{i}_{j}T = \left({}^{j}_{i}T\right)^{-1}, i < j\ (\boldsymbol{Matrix\ Inverse\ exists})$$



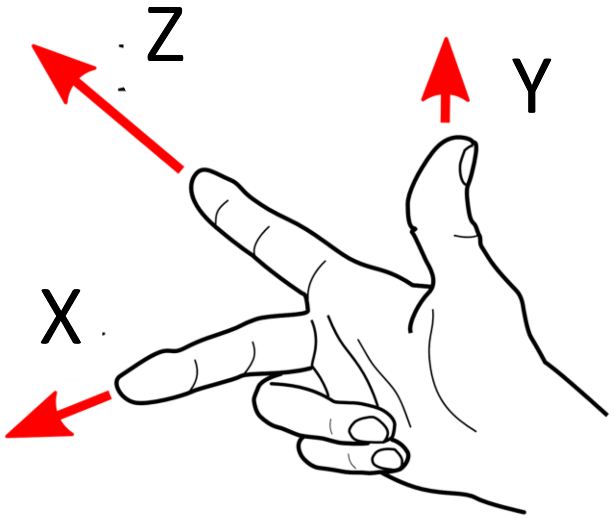
$$A_n = {}^{n-1}_{n}T\ (textbooks)$$

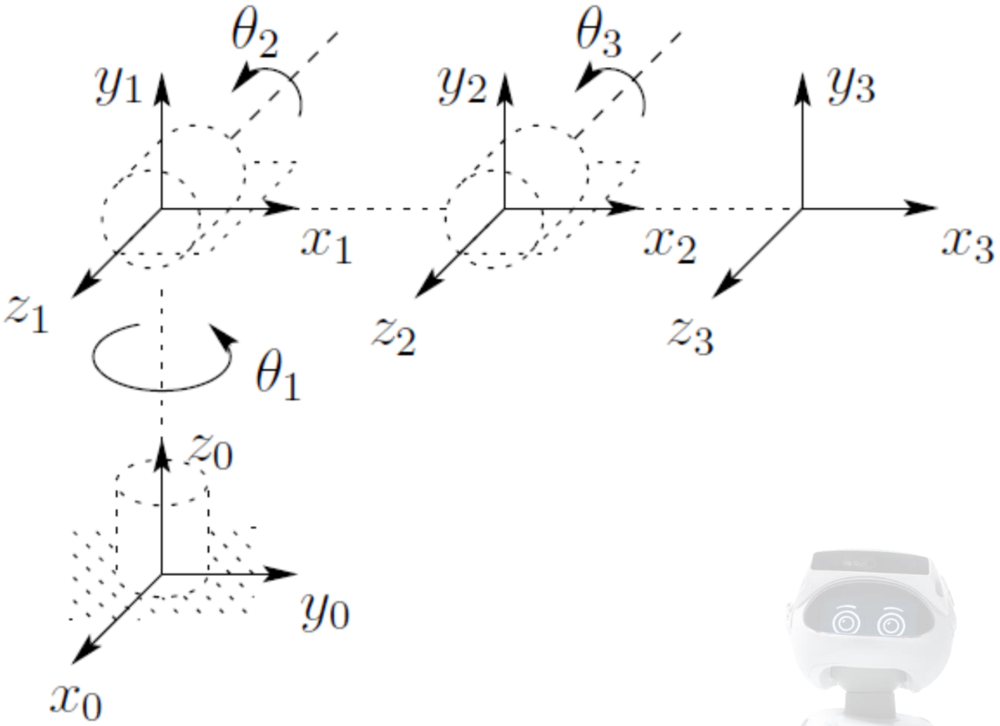
SUMMARY

$$ {}^{i}_{j}T = \begin{bmatrix} R^{3\ X\ 3} & d^{3\ X 1} \\ 0 & 1 \end{bmatrix} $$

*In the **homogeneous transformation** method*
*We can pick **arbitrary frame attached to each link***

*Here we **picked*** $\rightarrow$





DENAVIT-HARTENBERG REPRESENTATION

We should follow **a standard** frame assignment

${}_{j}^{i}T \ is \ represnted \ by \ a \ product \ of \ four \ transformations$

$$A_n = R_{z,\theta_{n-1}} t_{z,d_{n-1}} t_{x,a_n} R_{x,\alpha_n}$$

$$A_n = \underbrace{\begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{R_{z,\theta_{n-1}} \\ Joint\ Angle}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{t_{z,d_{n-1}} \\ Link\ Offset}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{t_{x,a_n} \\ Link\ Length}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta_i & -s\theta_i & 0 \\ 0 & s\theta_i & c\theta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{R_{x,\alpha_n} \\ Link\ Twist}}$$

DENAVIT-HARTENBERG REPRESENTATION

We should follow **a standard** frame assignment

${}^{i}_{j}T\ is\ represnted\ by\ a\ product\ of\ four\ transformations$

$${}^{i}_{j}T = R_{z,\theta_i} t_{z,d_i} t_{x,aj} R_{x,\alpha_j}$$

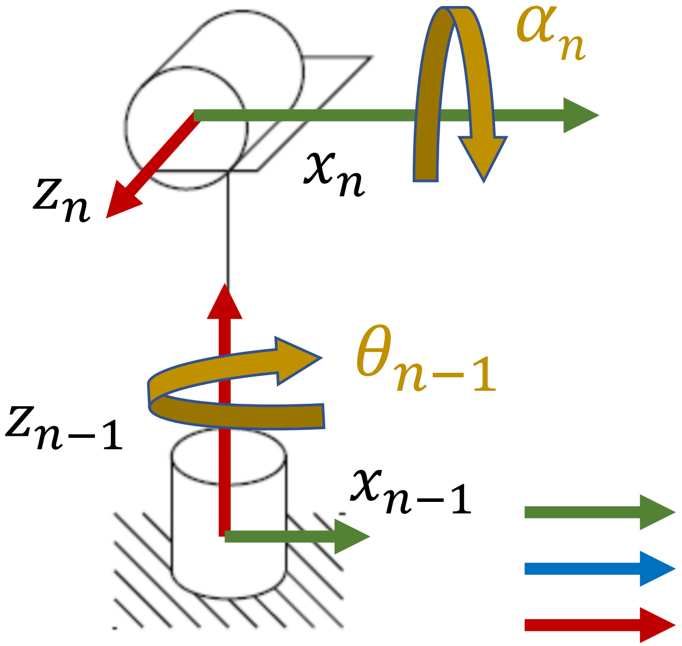
$${}^{i}_{j}T = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_j & s\theta_i s\alpha_j & a_j c\theta_i \\ s\theta_i & c\theta_i c\alpha_j & -c\theta_i s\alpha_j & a_j s\theta_i \\ 0 & s\alpha_j & c\alpha_j & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DENAVIT-HARTENBERG REPRESENTATION

ANATOMY OF **D − H ROTATIONS**

θ … *Joint variable around* z_{n-1}

α … *Rotate* z_{n-1} *around* $\boldsymbol{x_n}$ *to become* z_n



$\boldsymbol{n}$	$\boldsymbol{\theta}$	$\boldsymbol{\alpha}$
1	θ_{n-1}	90^o

${}^0_1H$

Ex : θ_1 *is a variable* (***motor angle***)

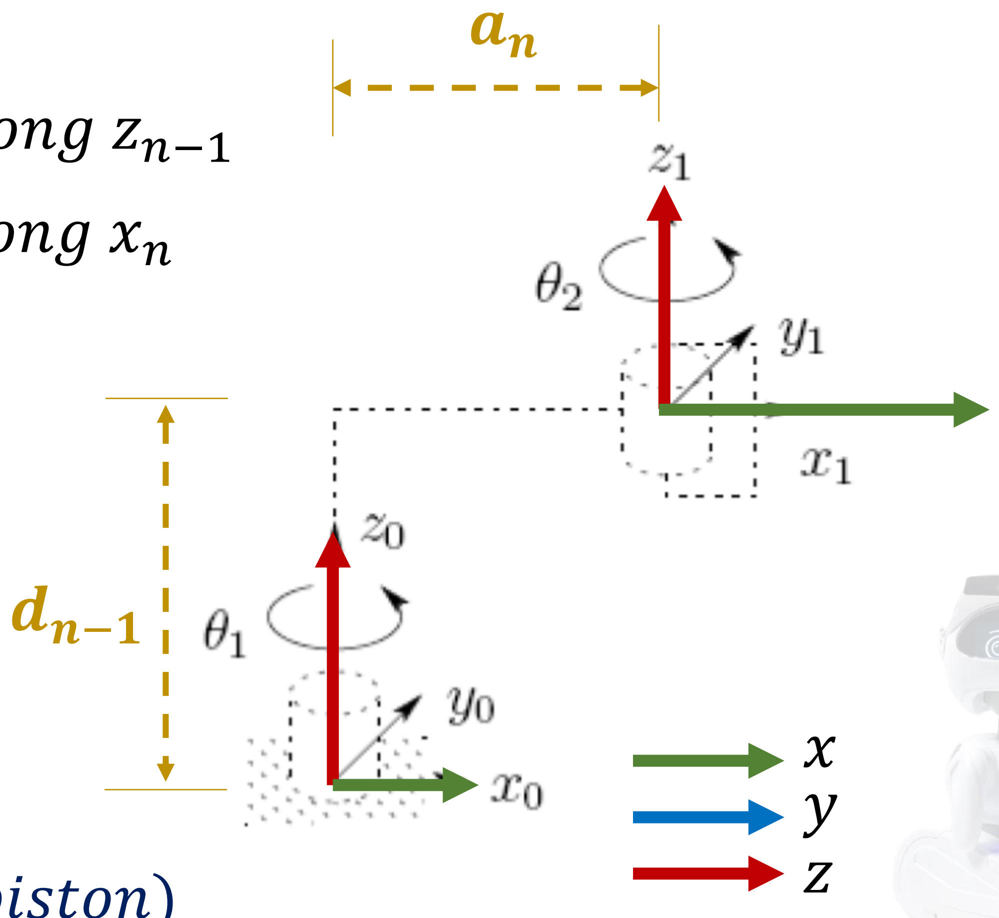
z_{n-1} *rotates to become* z_n *around* x_n

DENAVIT-HARTENBERG REPRESENTATION

ANATOMY OF **D − H DISPLACEMENTS**

$d \ldots Displacement\ between\ two\ frames\ along\ z_{n-1}$

$a \ldots Displacement\ between\ two\ frames\ along\ x_n$



n	d	a
1	d_{n-1}	a_n

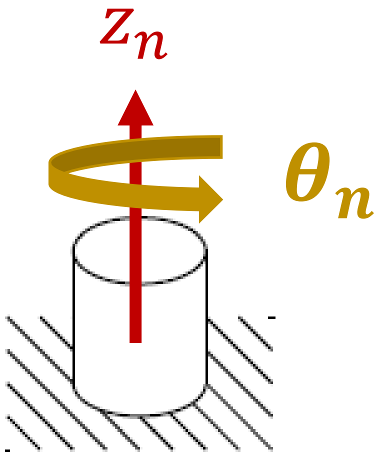
${}^0_1H$

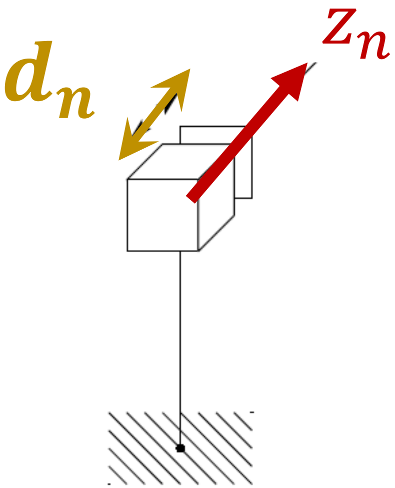
$Offset$

$$d \rightarrow \begin{cases} \boldsymbol{d_n}\ variable\ if\ joint\ is\ prismatic\ (like\ piston) \\ Link\ length\ if\ it\ is\ just\ a\ link\ between\ joints \end{cases}$$

DENAVIT-HARTENBERG RULES

① *The* $\boldsymbol{z-axis}$ *is the direction of translation or rotation*



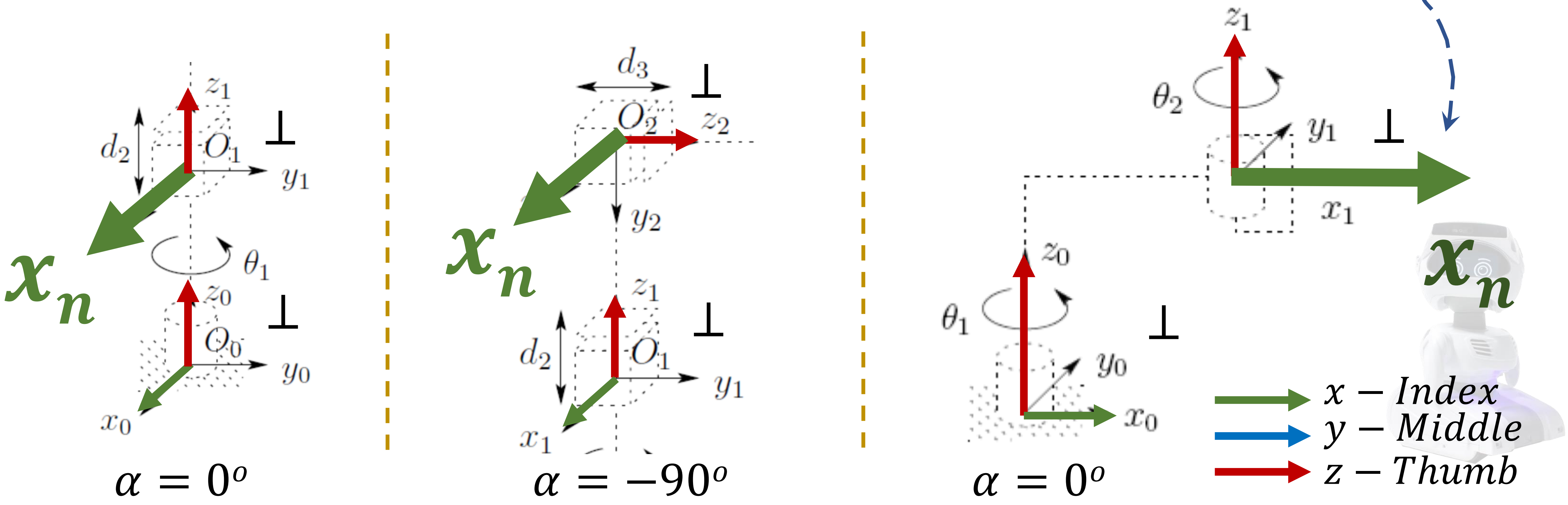


Don't ever break the Right Hand Rule

DENAVIT-HARTENBERG RULES

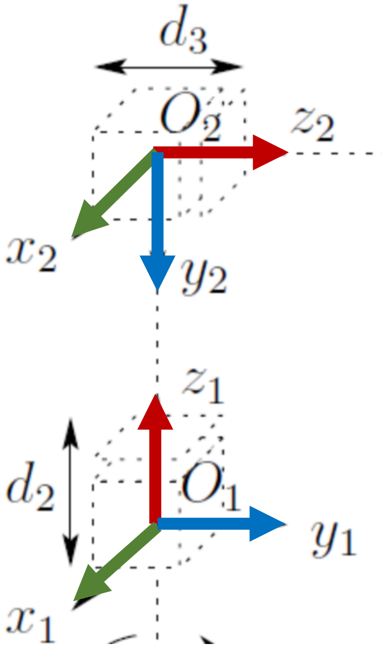
2 *The* $\boldsymbol{x_n} - \boldsymbol{axis}$ *is prependicular* $\perp$ *to both* $\boldsymbol{z_n}$ *and* $\boldsymbol{z_{n-1}}$ *axes*

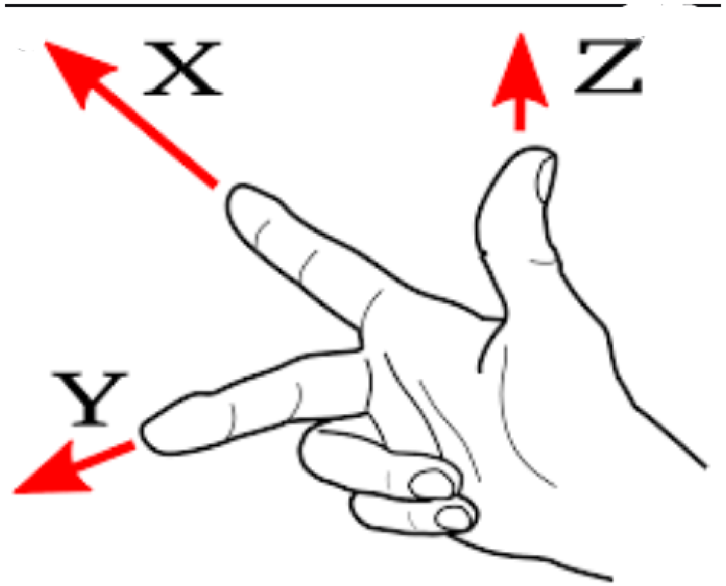
For ***Parallel*** z_{n-1} *and* z_n, *pick* $x - axis$ *direction from* $z_{n-1} \rightarrow z_n$



DENAVIT-HARTENBERG RULES

③ $The\ \boldsymbol{y_n} - \boldsymbol{axis}\ must\ follow\ the\ RHR\ (better\ to\ always\ use\ \boldsymbol{ZXY})$

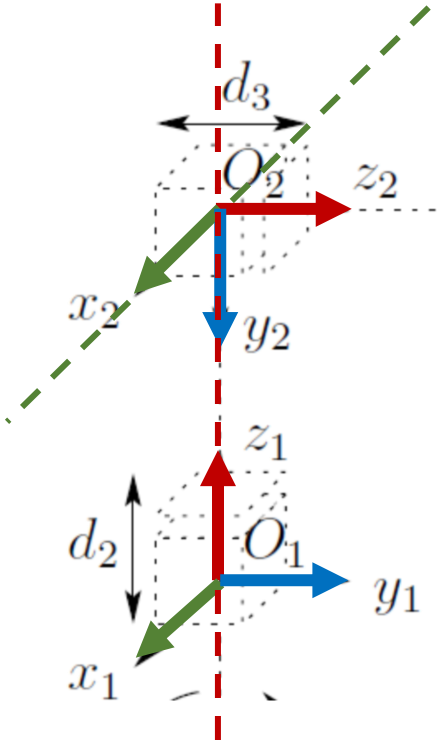




$x - Index$
$y - Middle$
$z - Thumb$

DENAVIT-HARTENBERG RULES

4 *The* $\boldsymbol{x_n - axis}$ *must* $\boldsymbol{intersect}$ *with* z_{n-1}



Pay Attention

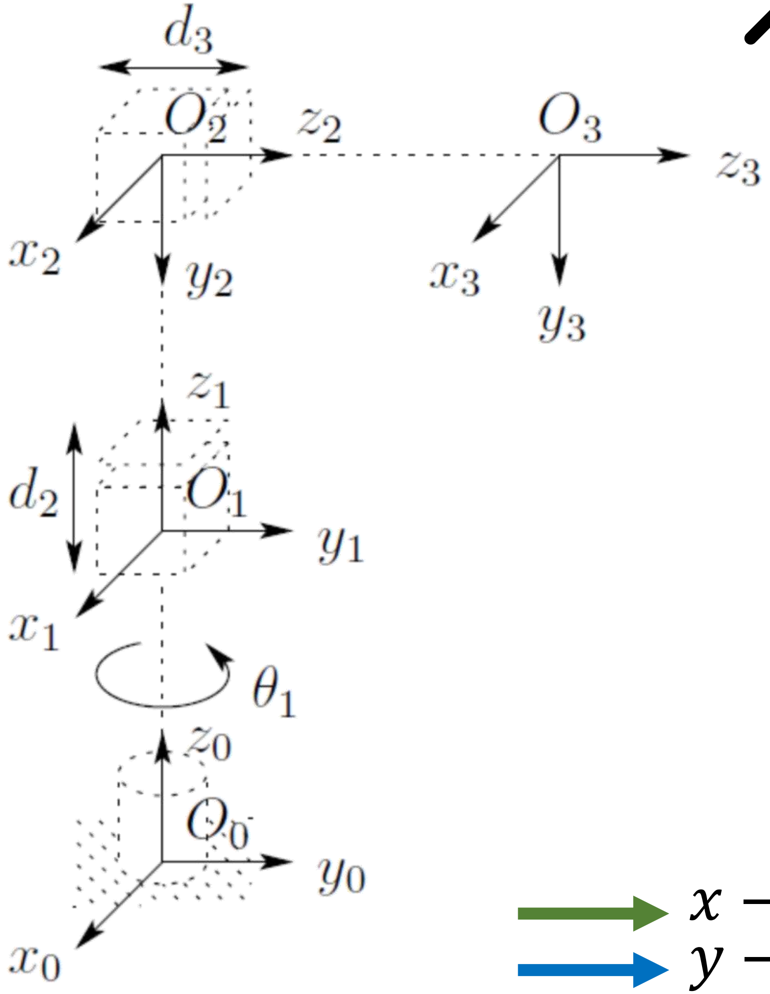
if there is $\boldsymbol{an\ offset}$ *in the* $x - direction$

$x - Index$
$y - Middle$
$z - Thumb$

DENAVIT-HARTENBERG RULES

Find the homogeneous transformation

${}^{0}_{3}T$ using **DH representation**



$x - Index$

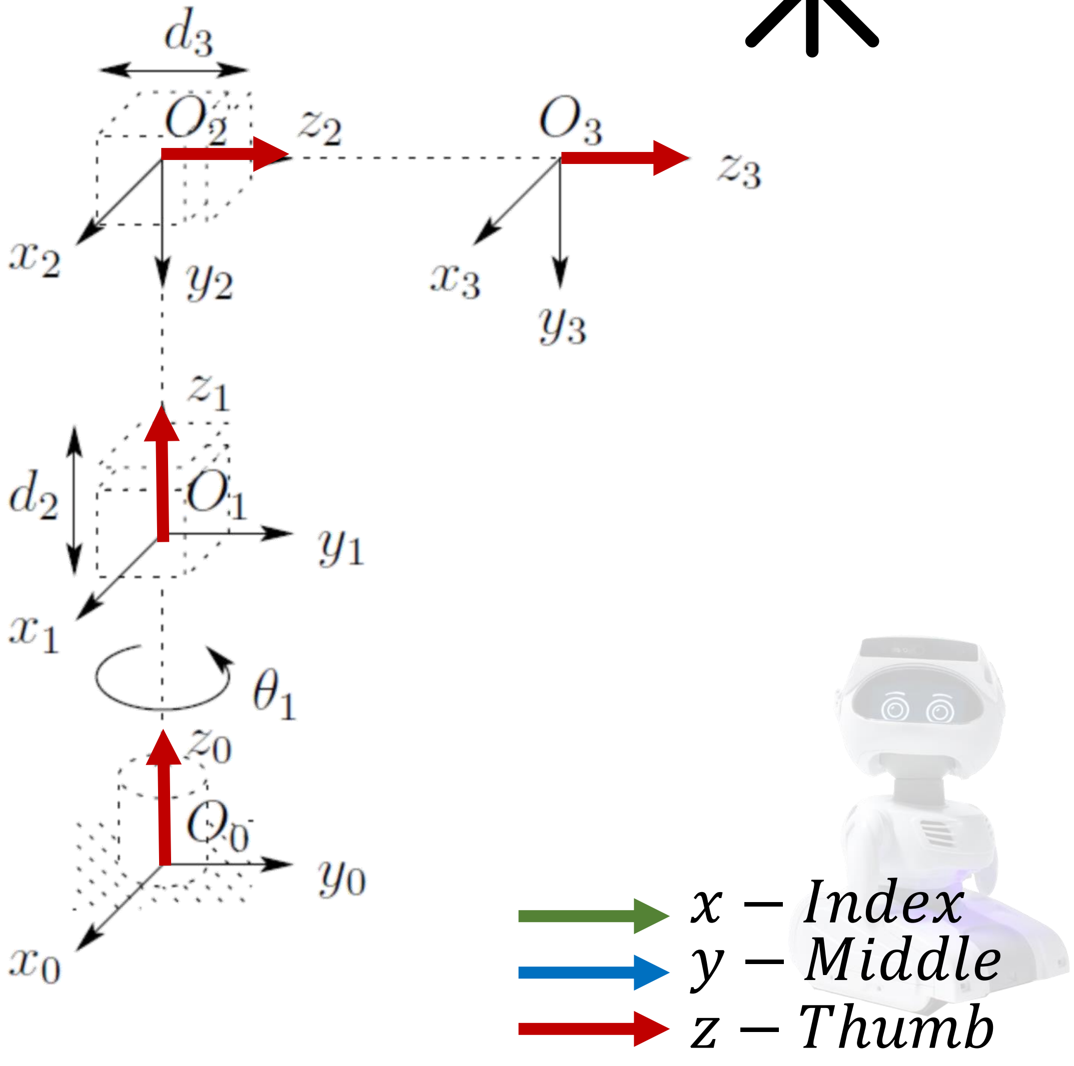
$y - Middle$

$z - Thumb$

DENAVIT-HARTENBERG RULES

Solution

1. *Setting* $\boldsymbol{z-axis}$ *as the axis of rotation or translation in all frames*



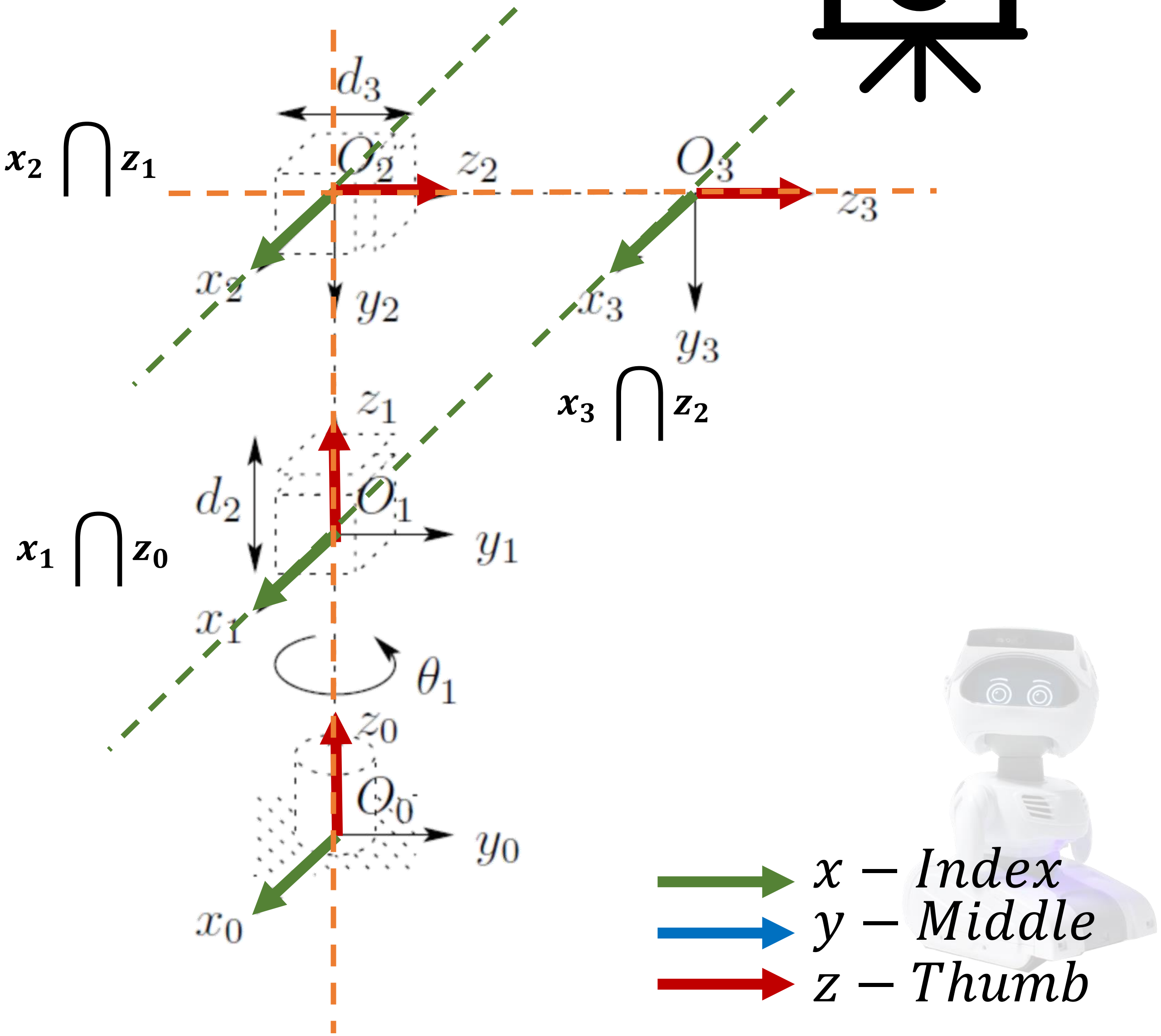
DENAVIT-HARTENBERG RULES

Solution

2. *Setting* $\boldsymbol{x_n}$ *to be* $\perp$ *on* $\boldsymbol{z_{n-1}}$ *and* $\boldsymbol{z_n}$ *axes*

3. *Setting* x_n *to* $\cap$ *with with* $\boldsymbol{z_{n-1}}$

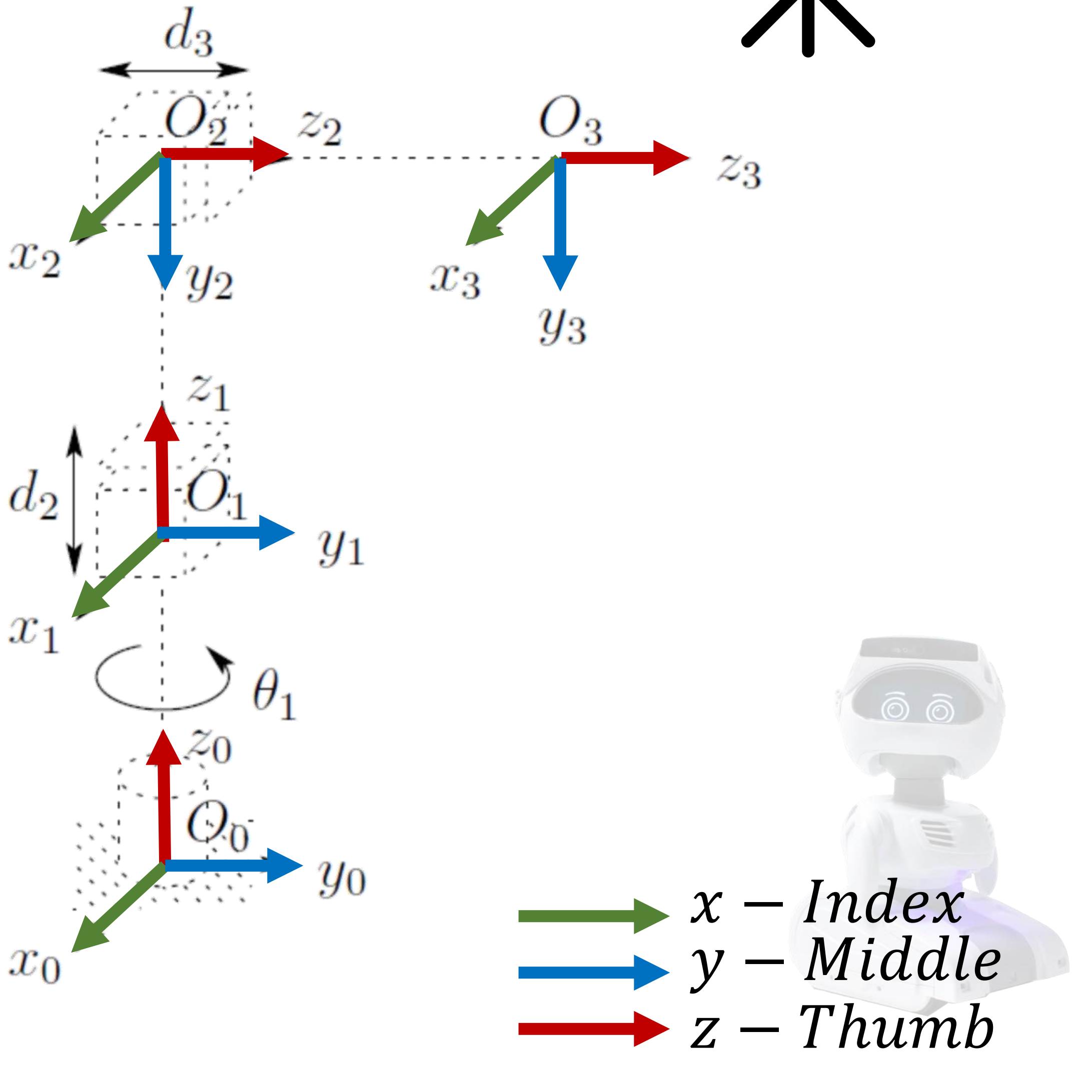
$\perp$ *prependicular* $\bigcap$ *intersect*



DENAVIT-HARTENBERG RULES

Solution

4. *Assign* $\boldsymbol{Y-axis}$ *with respect to the RHR*



DENAVIT-HARTENBERG RULES

Solution

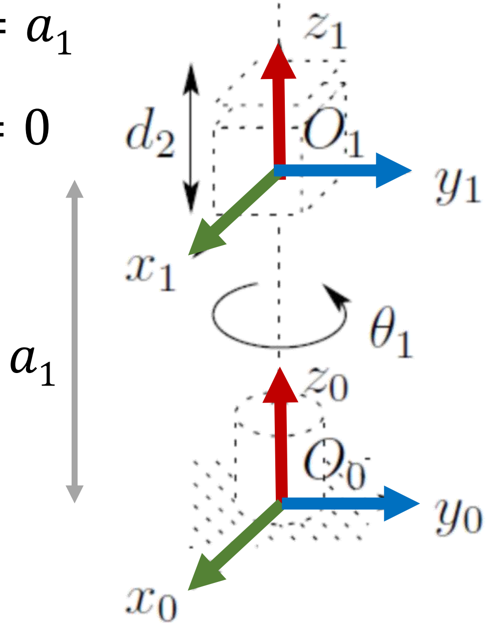
θ … *Joint variable around* $z_0 = \theta_1$(*revolute joint*)

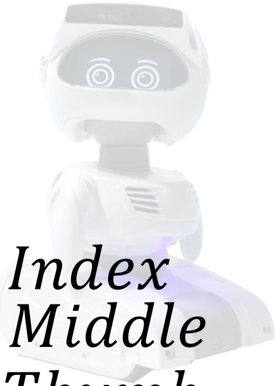
α … *Rotate* z_0 *around* x_1 *to become* $z_1 = 0$ (*same direction*)

d … *Displacement between two frames along* $z_0 = a_1$

a … *Displacement between two frames along* $x_1 = 0$

n	θ	d	α	a
1	$\boldsymbol{\theta_1}$	$\boldsymbol{a_1}$	$\mathbf{0^o}$	**0**





x − *Index*
y − *Middle*
z − *Thumb*

DENAVIT-HARTENBERG RULES

Solution

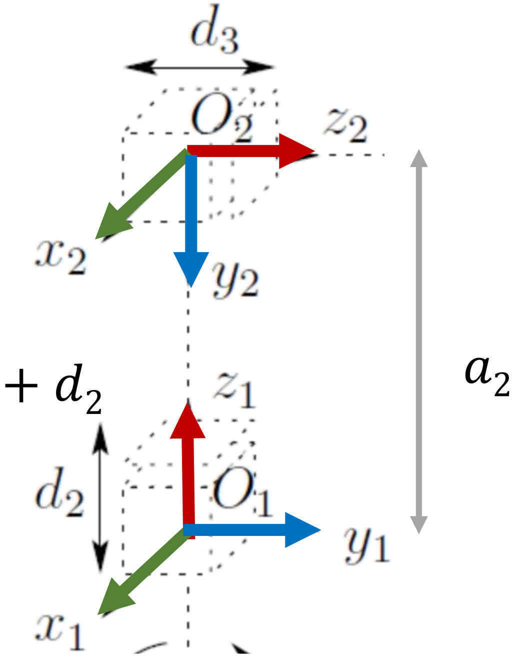
θ … *Joint variable around* $z_1 = 0$ *(prismatic joint)*

α … *Rotate* z_1 *around* x_2 *to become* $z_2 = -90^o$

d … *Displacement between two frames along* $z_1 = a_2 + d_2$

a … *Displacement between two frames along* $x_2 = 0$

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	$\mathbf{0^o}$	$\boldsymbol{a_2 + d_2}$	$\mathbf{-90^o}$	**0**



x – *Index*
y – *Middle*
z – *Thumb*

DENAVIT-HARTENBERG RULES

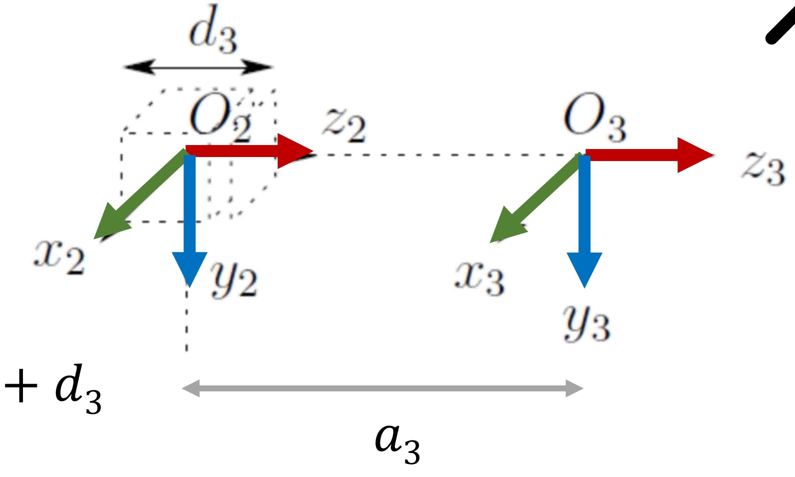
Solution

θ … *Joint variable around* $z_2 = 0$ *(prismatic joint)*

α … *Rotate* z_2 *around* x_3 *to become* $z_3 = 0^o$

d … *Displacement between two frames along* $z_2 = a_3 + d_3$

a … *Displacement between two frames along* $x_2 = 0$



n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	0^o	$a_2 + d_2$	-90^o	0
3	$\mathbf{0^o}$	$\boldsymbol{a_3 + d_3}$	$\mathbf{0^o}$	**0**

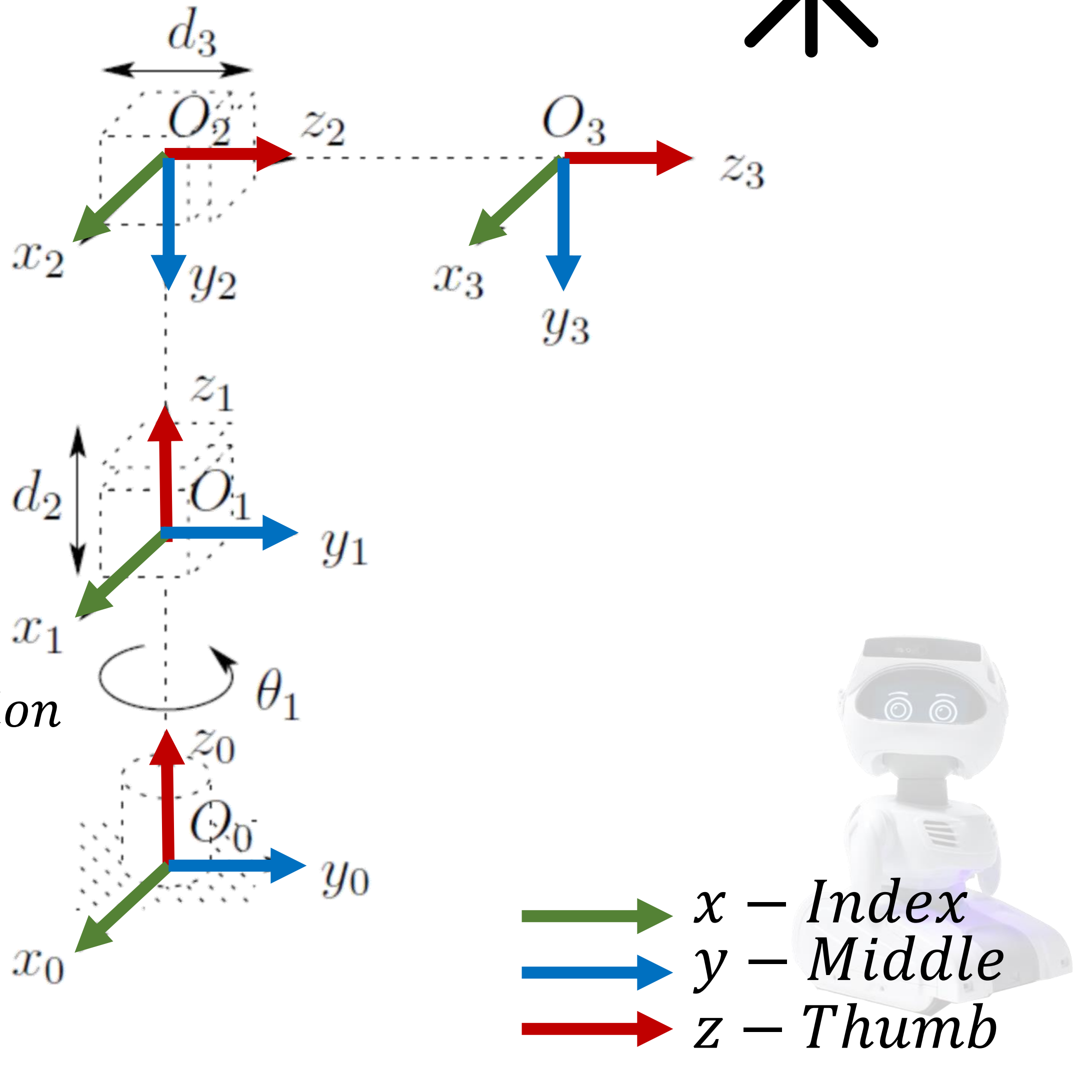
$x - Index$
$y - Middle$
$z - Thumb$

DENAVIT-HARTENBERG RULES

Solution

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	0^o	$a_2 + d_2$	-90^o	0
3	0^o	$a_3 + d_3$	0^o	0

Next step *is filling the three homogenous transformation matrices*



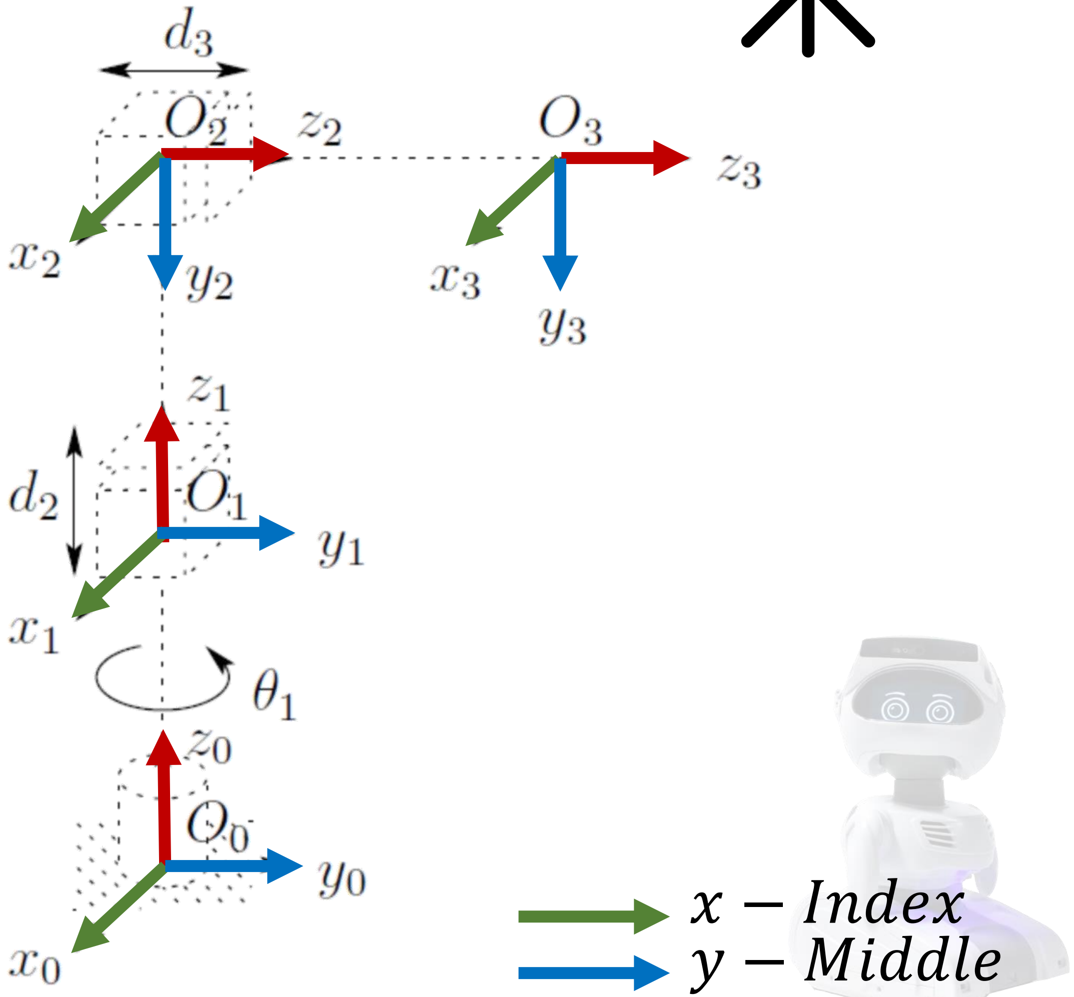
DENAVIT-HARTENBERG RULES

Solution

n	θ	d	α	a
1	$\boldsymbol{\theta_1}$	$\boldsymbol{a_1}$	$\boldsymbol{0^o}$	**0**
2	0^o	$a_2 + d_2$	-90^o	0
3	0^o	$a_3 + d_3$	0^o	0

$$ {}_1^0T = \begin{bmatrix} c\theta_1 & -s\theta_1 c0^o & s\theta_1 s0^o & 0c\theta_1 \\ s\theta_1 & c\theta_1 c0^o & -c\theta_1 s0^o & 0s\theta_1 \\ 0 & s0^o & c0^o & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_1^0T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



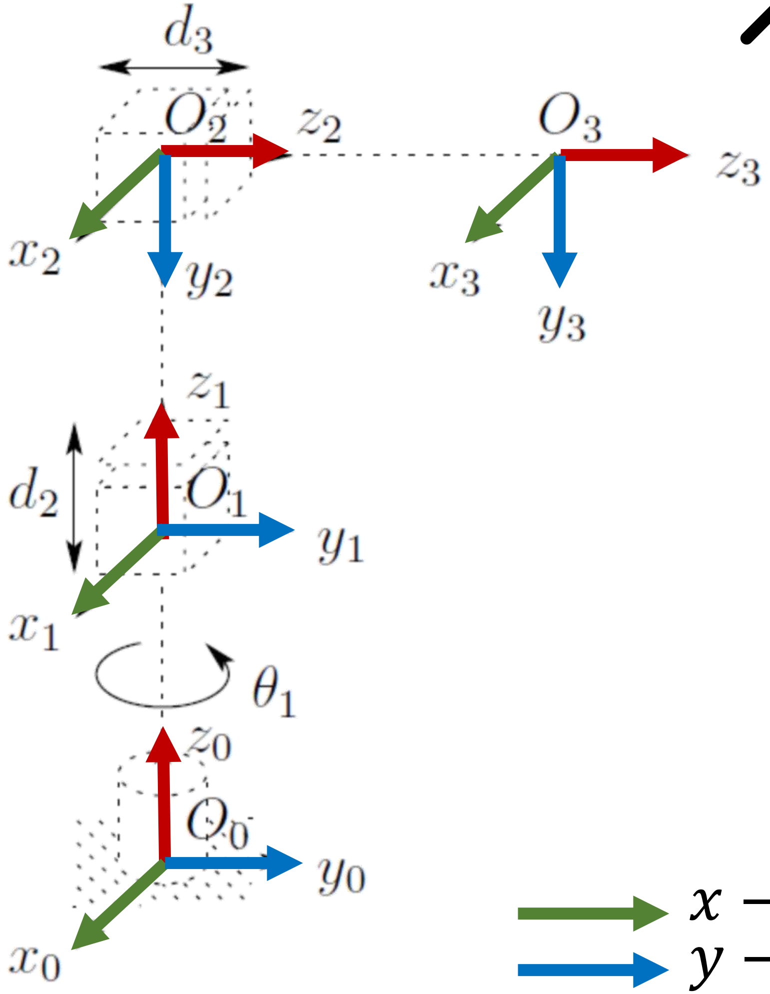
DENAVIT-HARTENBERG RULES

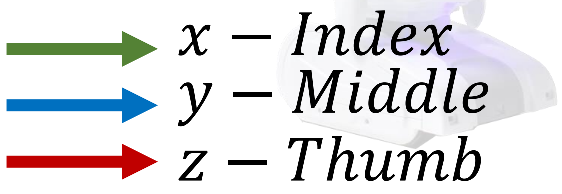
Solution

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	$\mathbf{0^o}$	$\boldsymbol{a_2 + d_2}$	$\mathbf{-90^o}$	**0**
3	0^o	$a_3 + d_3$	0^o	0

$$
{}^1_2T = \begin{bmatrix} c0 & -s0c(-90) & s0s(-90) & 0c0 \\ s0 & c0c(-90) & -c0s(-90) & 0s0 \\ 0 & s(-90) & c(-90) & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$





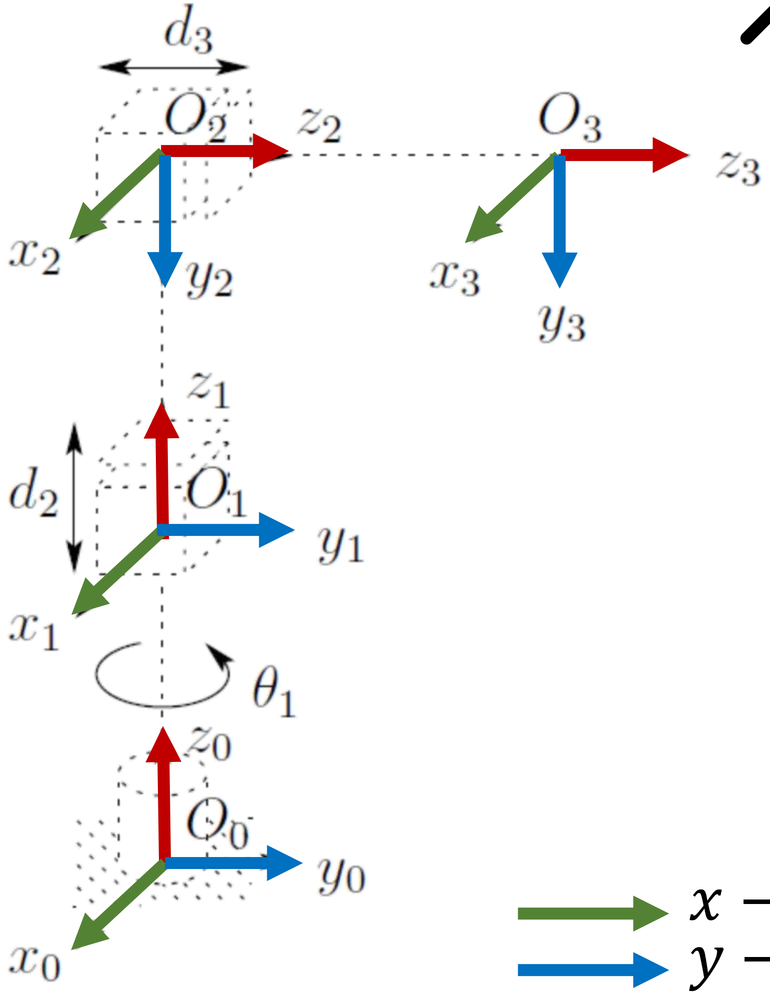
DENAVIT-HARTENBERG RULES

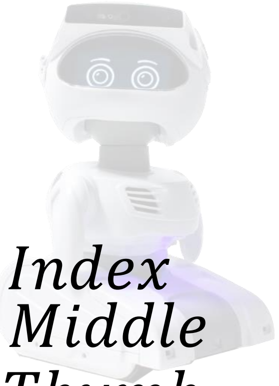
Solution

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	0^o	$a_2 + d_2$	-90^o	0
3	$\boldsymbol{0^o}$	$\boldsymbol{a_3 + d_3}$	$\boldsymbol{0^o}$	**0**

$$ {}^2_3T = \begin{bmatrix} c0 & -s0c0 & s0s0 & 0c0 \\ s0 & c0c0 & -c0s0 & 0s0 \\ 0 & s0 & c0 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$





$x - Index$
$y - Middle$
$z - Thumb$

DENAVIT-HARTENBERG RULES

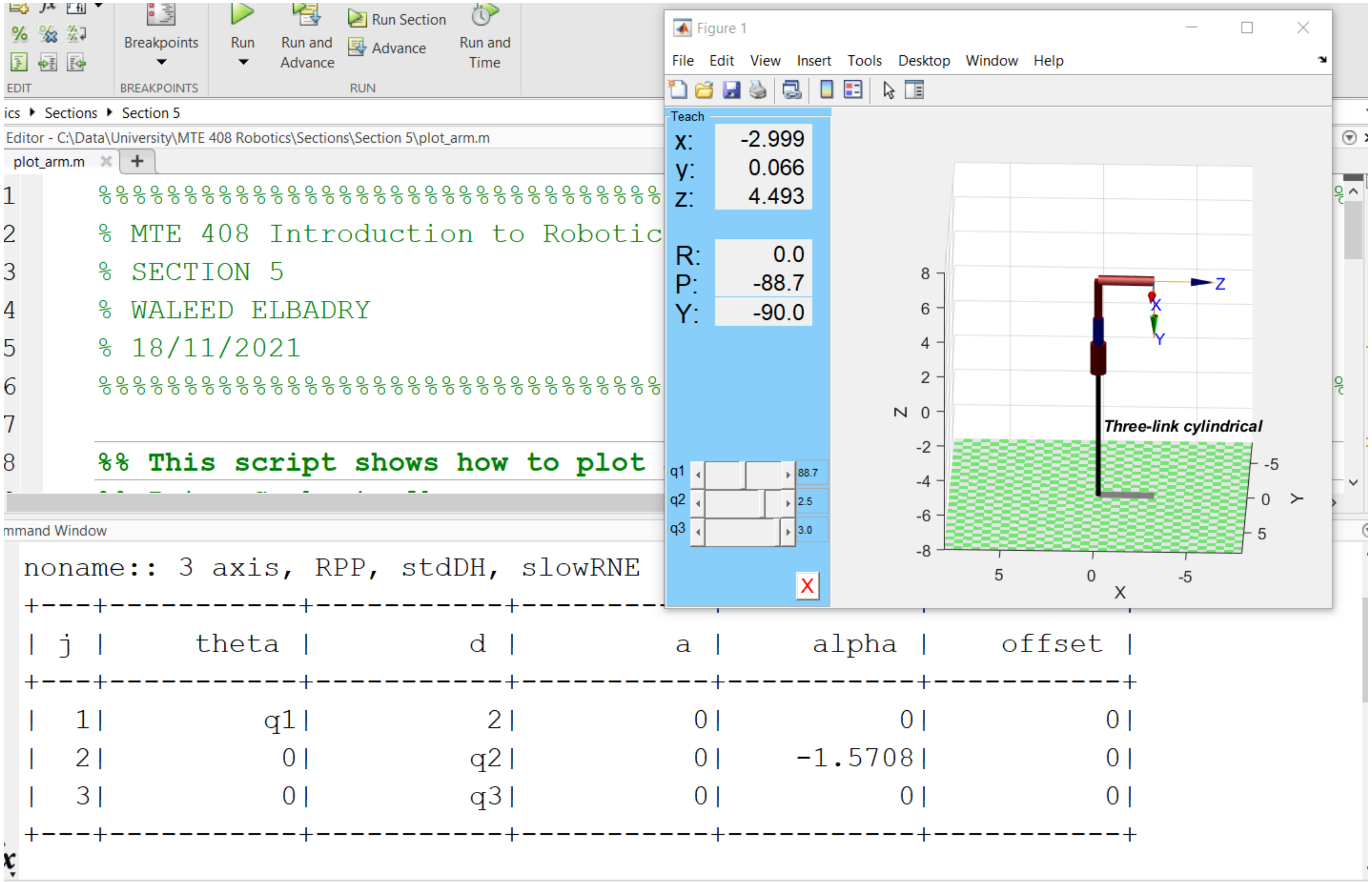
Solution

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	0^o	$a_2 + d_2$	-90^o	0
3	0^o	$a_3 + d_3$	0^o	0

$$ {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T $$

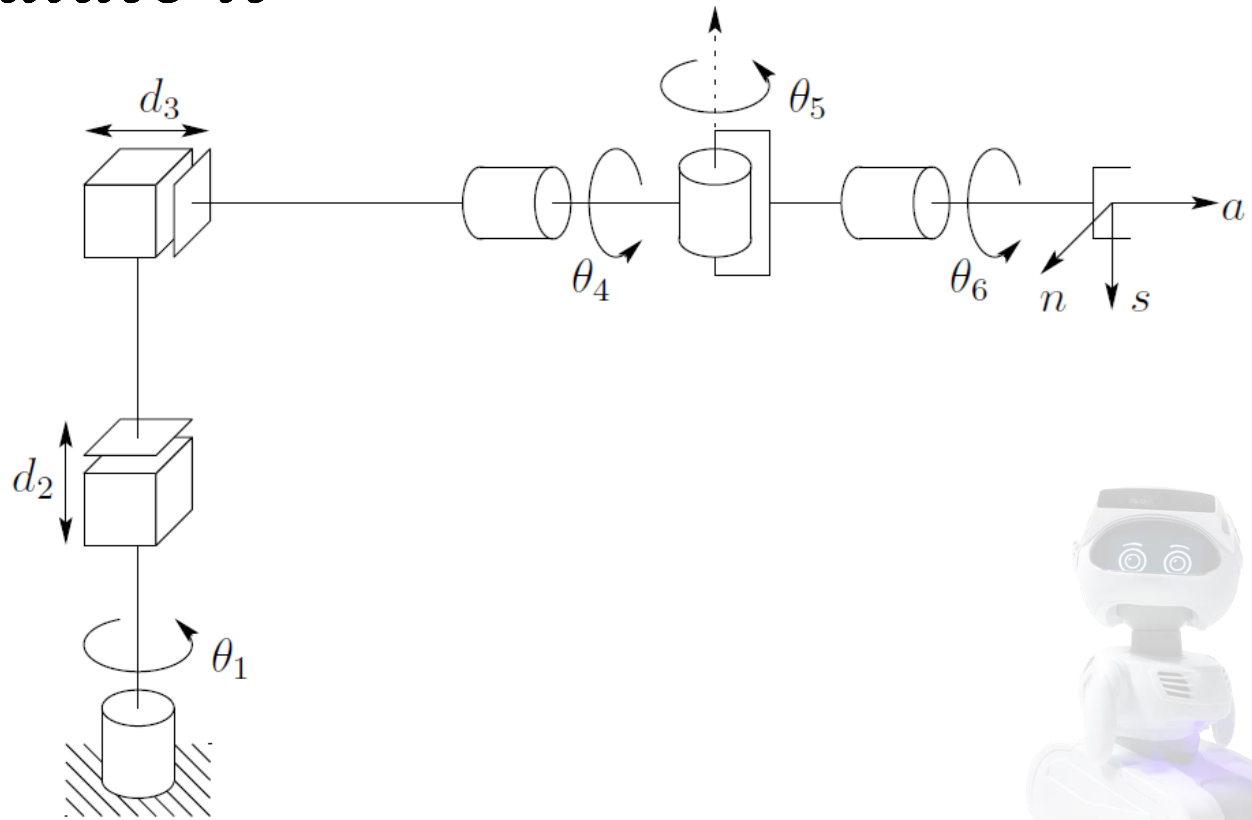
$$ {}^0_3T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{c\theta_1} & \mathbf{0} & \boldsymbol{-s\theta_1} & \boldsymbol{-s\theta_1(a_3 + d_3)} \\ \boldsymbol{s\theta_1} & \mathbf{0} & \boldsymbol{c\theta_1} & \boldsymbol{c\theta_1(a3 + d3)} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} & \boldsymbol{a_1 + a_2 + d_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} $$

DH SIMULATION USING PETER CORKE



$Verify\ the\ solution\ and\ simulate\ it$

n	θ	d	α	a
1	θ_1	a_1	0^o	0
2	0^o	$a_2 + d_2$	-90^o	0
3	0^o	$a_3 + d_3$	0^o	0
4	θ_4	0	-90^o	0
5	θ_5	0	90^o	0
6	θ_6	$a_6 + d_6$	0^o	0



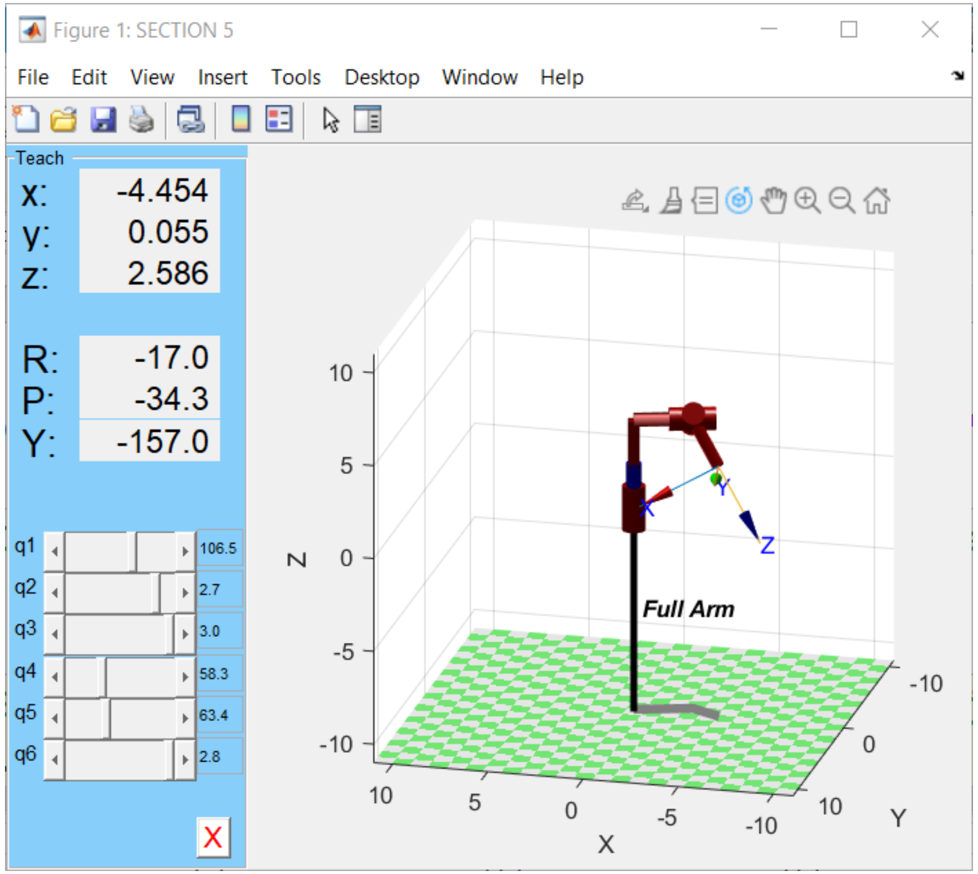
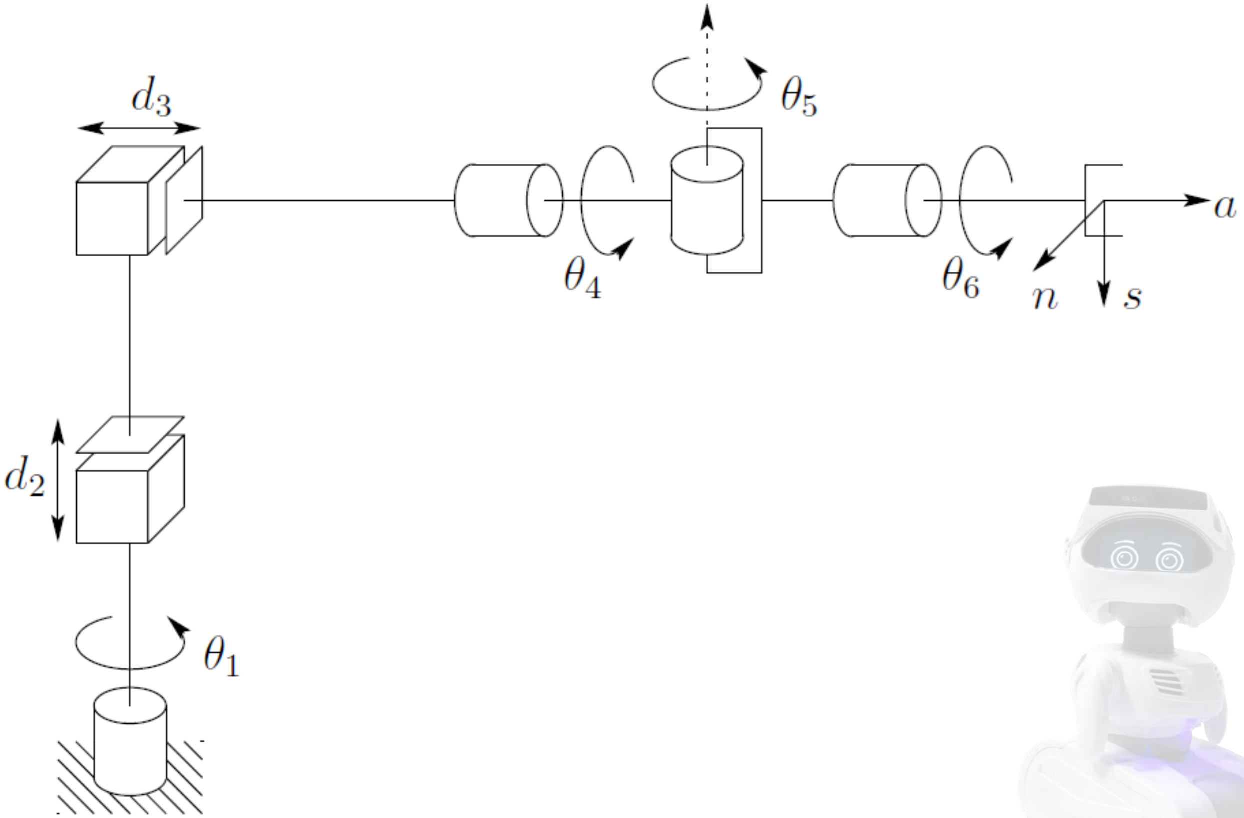


Figure 1: SECTION 5
File Edit View Insert Tools Desktop Window Help
Teach
x: -4.454
y: 0.055
z: 2.586
R: -17.0
P: -34.3
Y: -157.0
q1 106.5
q2 2.7
q3 3.0
q4 58.3
q5 63.4
q6 2.8
Full Arm
X
Y
Z
10
5
0
-5
-10


d_3
θ_5
θ_4
θ_6
a
n
s
d_2
θ_1

NEXT SECTION : *Inverse Kinematics*