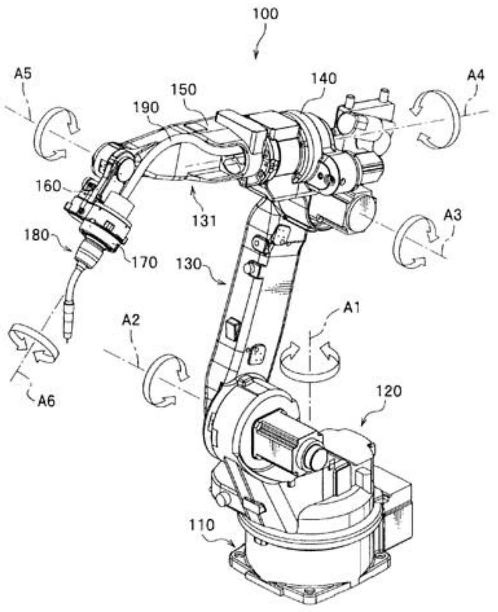
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS


# SESSION 4
# INTRODUCTION TO ROBOTICS LAB

WALEED ELBADRY

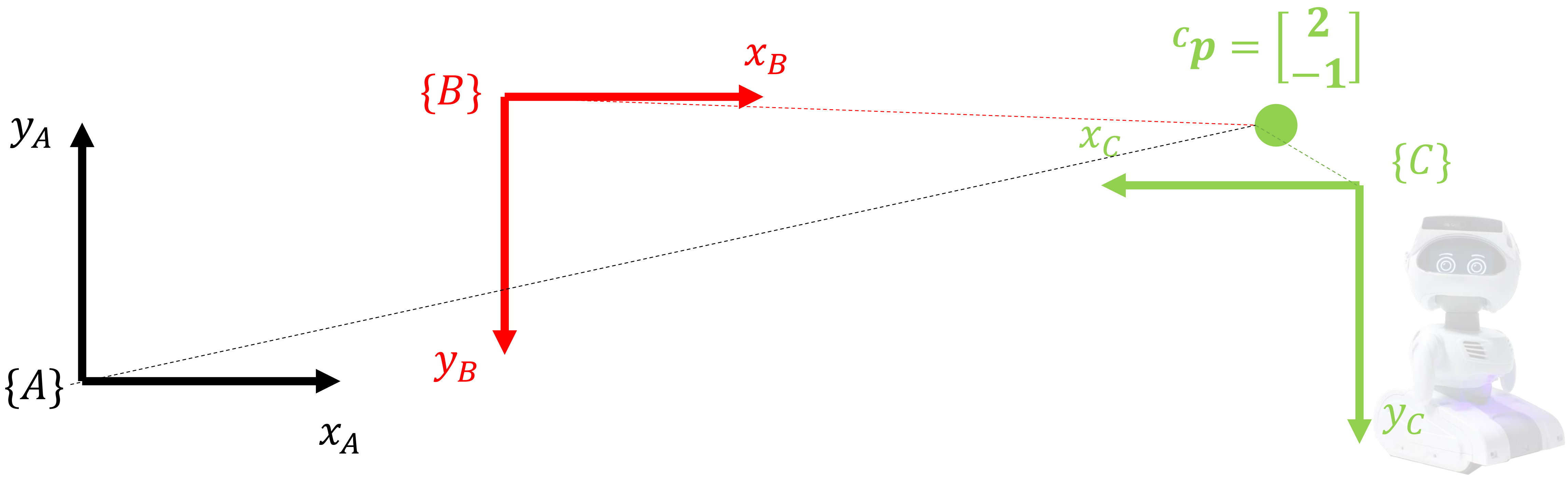
MARCH 2022



# 2D COORIDINATE SYSTEM

*Assignment*

*Assume any missing data, compute* ${}^{A}\xi_{C}, {}^{B}\xi_{C}$, *and* ${}^{A}\xi_{B}$ ***transformation and find the*** ${}^{A}P, {}^{B}P$



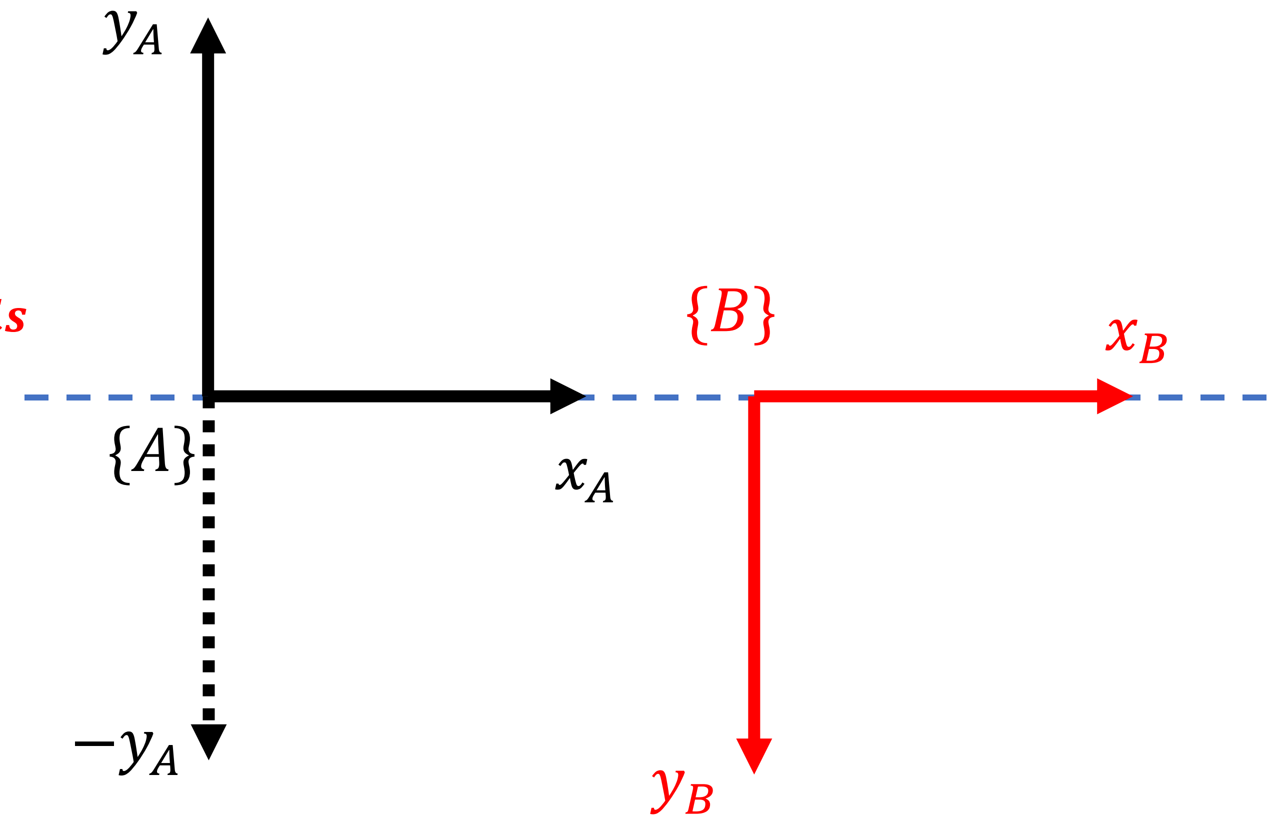
# 2D COORIDINATE SYSTEM

***Hint***

$Flip\ around\ x - axis$

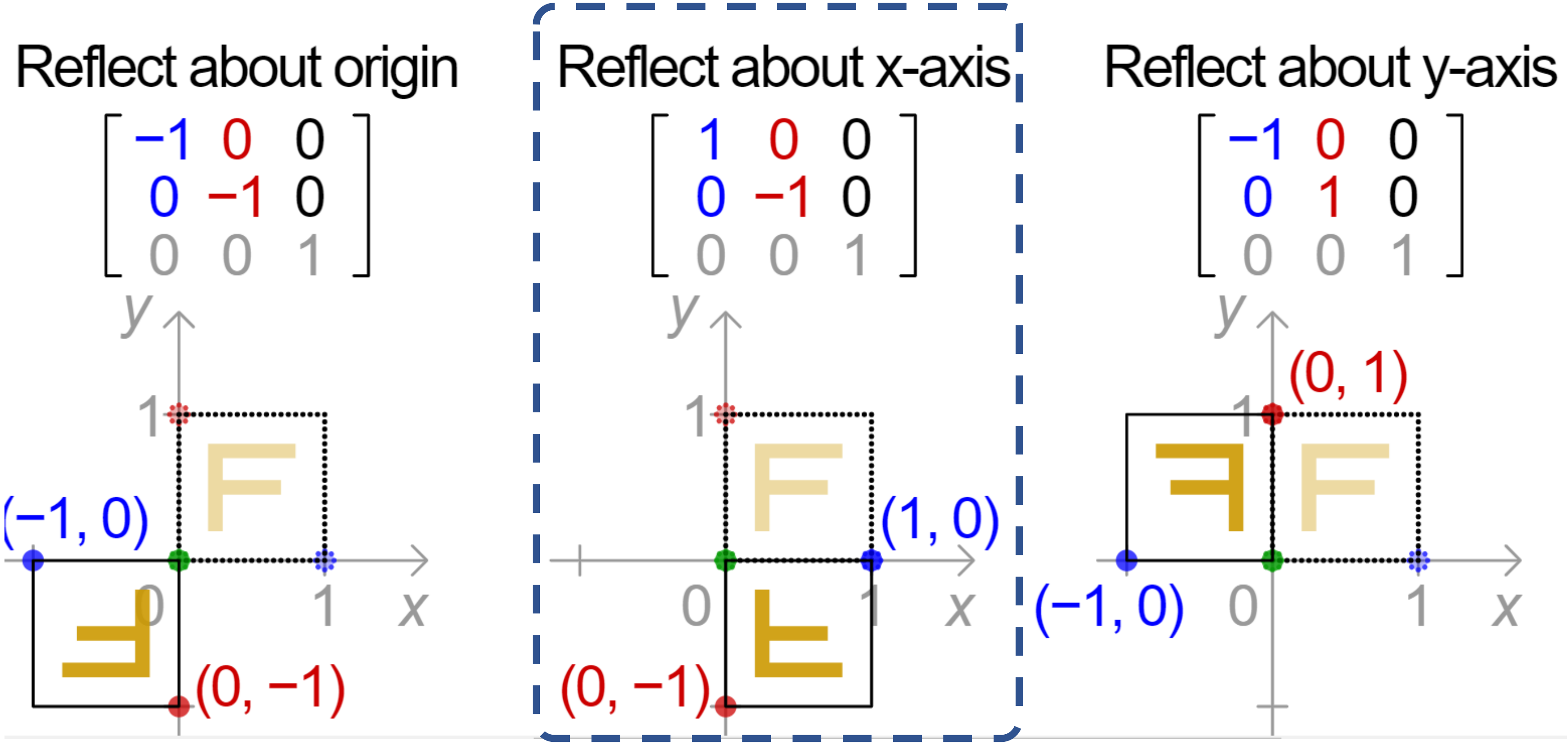
$$\boldsymbol{f}_x = \begin{bmatrix}1 & 0\\0 & \boxed{-1}\end{bmatrix}$$

$\boldsymbol{flipped\ y - axis}$



# 2D COORIDINATE SYSTEM

***Hint***




***Source*** : *Wikipedia*

# 2D COORIDINATE SYSTEM

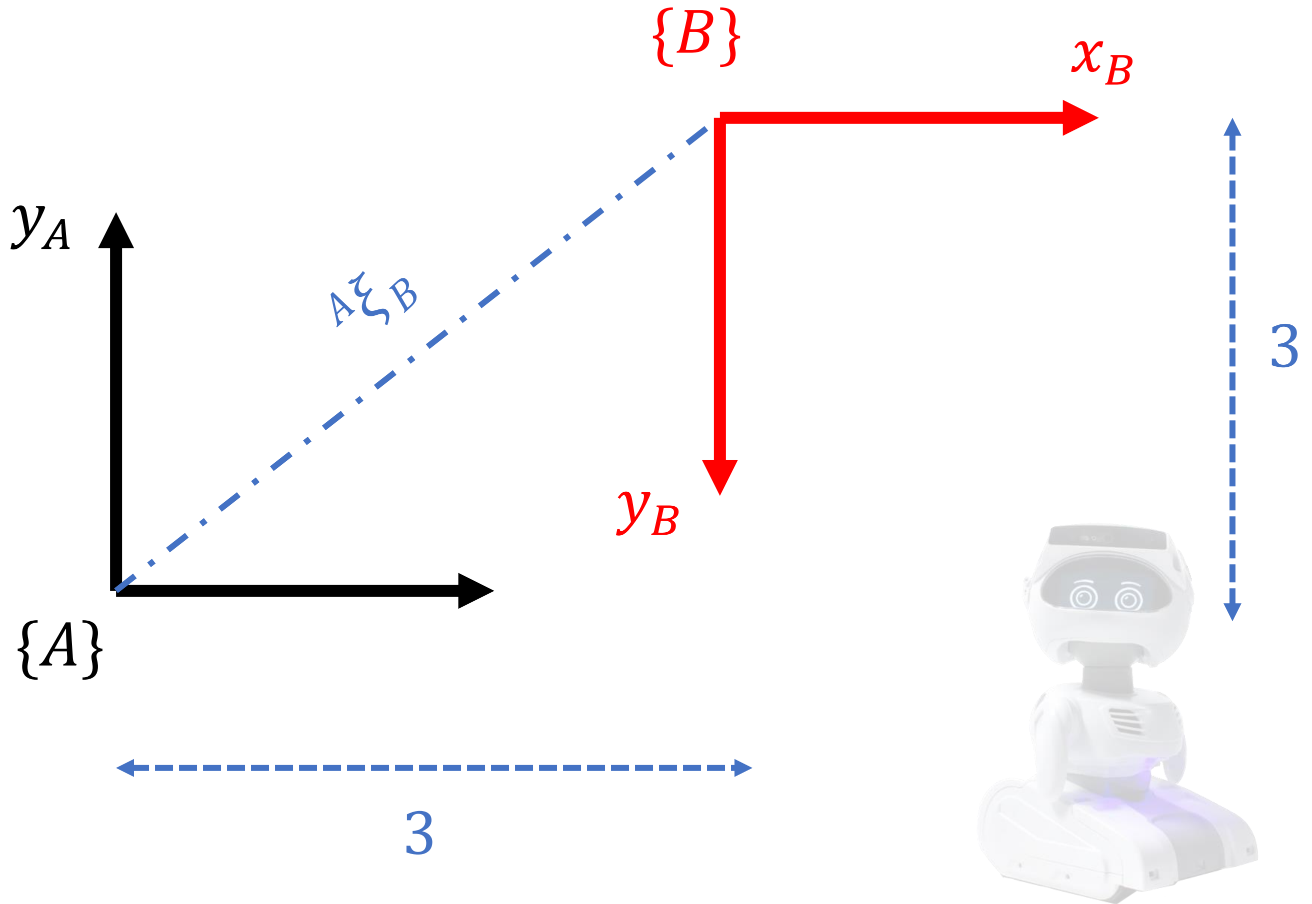
***Hint***

$$\boldsymbol{f}_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \boldsymbol{\theta}_z = 0^o$$

$$^A\boldsymbol{R}_B = \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$^A\boldsymbol{R}_B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$^A\boldsymbol{\xi}_B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$



# 2D COORIDINATE SYSTEM

```
%% Frame {B}
TB = [1   0 3;...
      0  -1 3;...
      0   0 1];
disp('Frame B Transformation Matrix:')
TB
```

Command Window

```
Error using matlab.graphics.primitive.Transform/set
Invalid value for Matrix property.

Error in trplot2 (line 265)
    set(hg, 'Matrix', se2t3(T));

Error in RTBPose/plot (line 867)
                        [varargout{1:nargout}] =
```

*Since the $flip$ is about* $\boldsymbol{x - axis}$,
*we* ***violated the 2D constraints***

***Rotation about z, Translation on x and y***

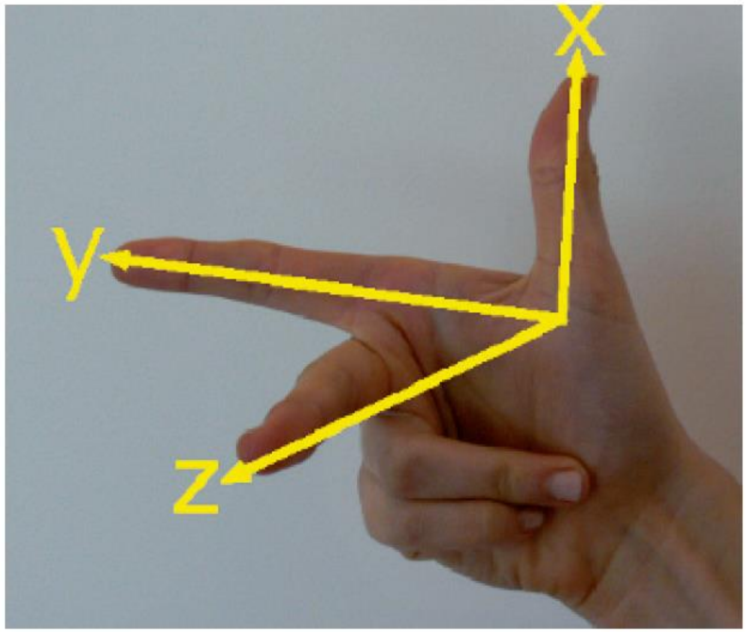
$$^{f1}\xi_{f2} = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & t_x \\ sin(\theta_z) & \cos(\theta_z) & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

# 3D COORIDINATE SYSTEM

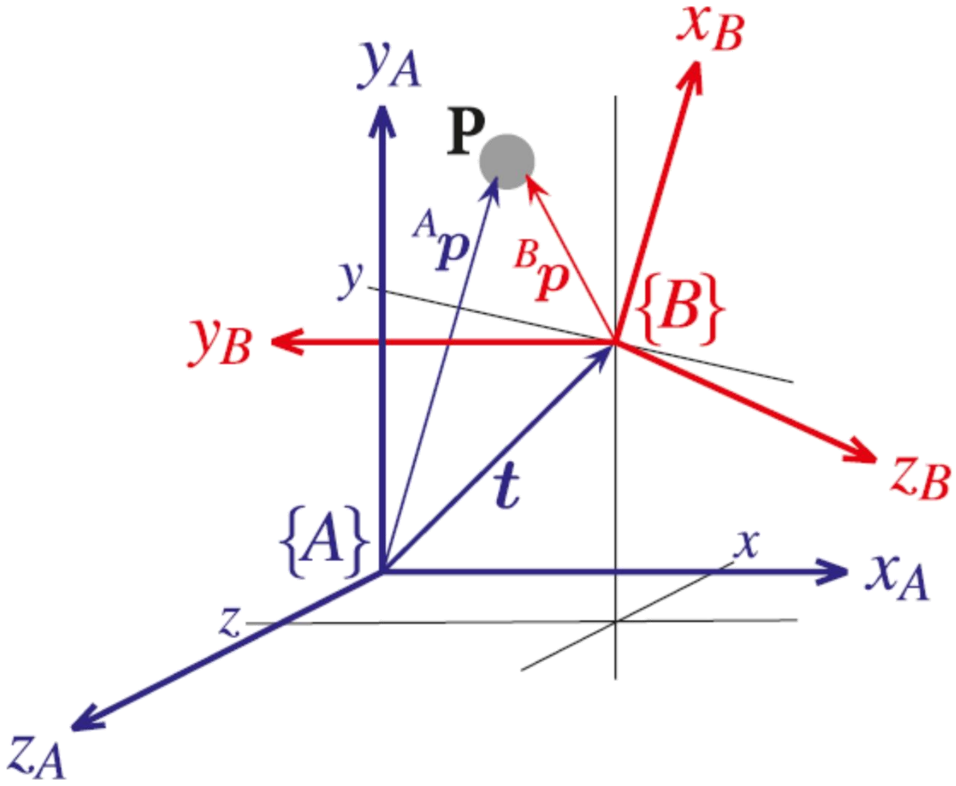
$$\boldsymbol{Point\ in\ 3D - space}$$

$$p = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\hat{x} = \hat{y} \times \hat{z} \qquad \hat{y} = \hat{x} \times \hat{z} \qquad \hat{z} = \hat{x} \times \hat{y}$$



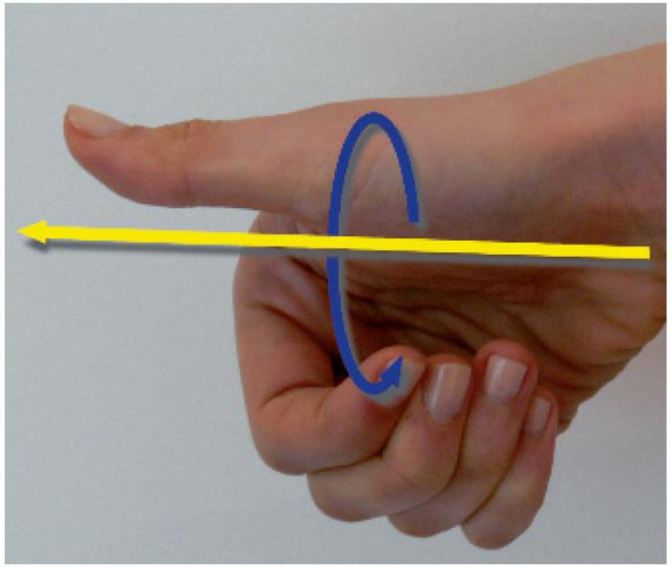

**Right–hand** rule



# 3D ORIENTATION

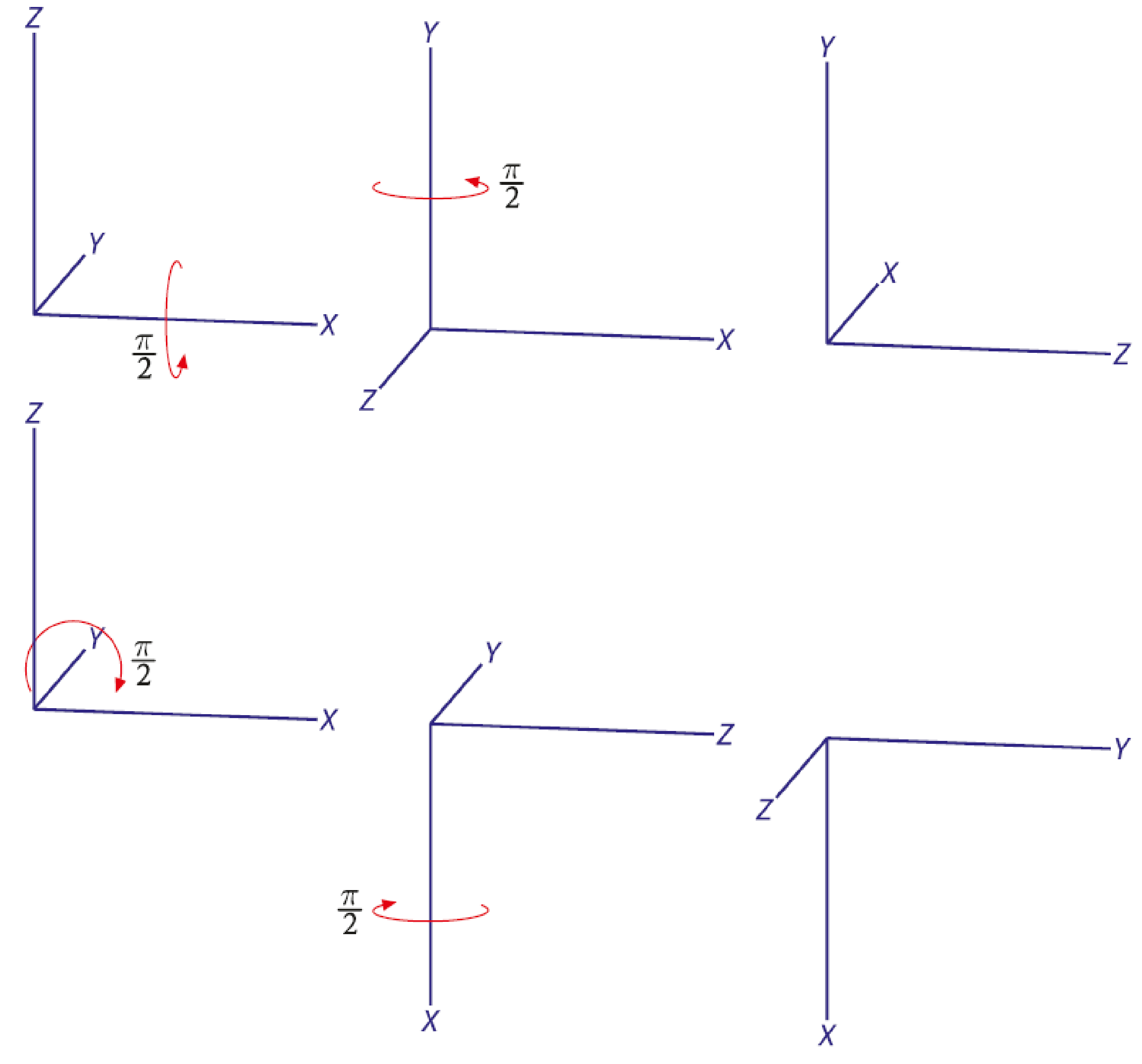
*"Any two independent orthonormal coordinate frames can be related by a sequence of rotations (not more than* ***three****) about coordinate axes, where no two successive rotations may be about the same axis"*

**Euler's rotation theorem** (Kuipers 1999)



***Rotation*** *about an axis*

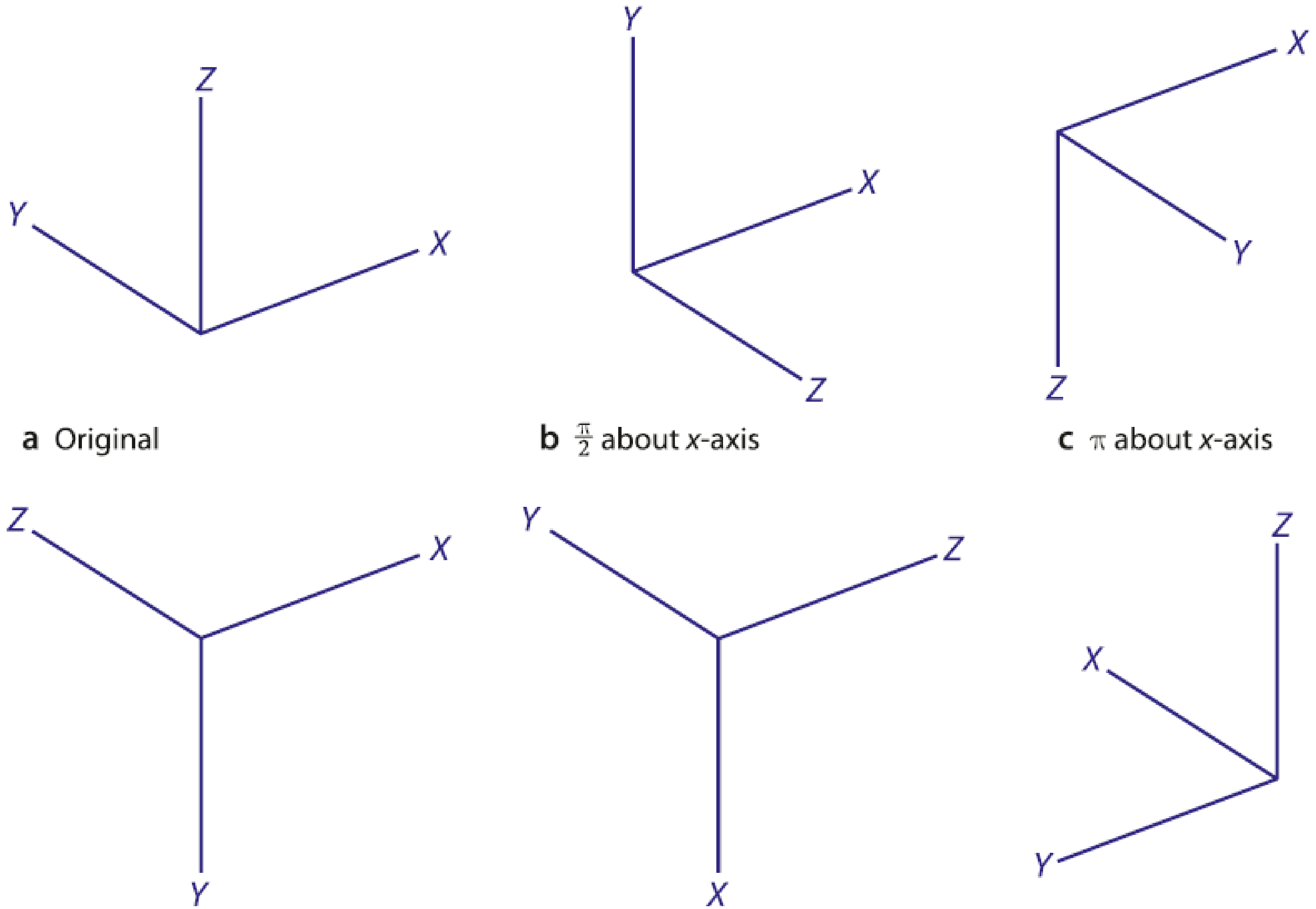
# 3D ORIENTATION




$$R_x R_y \neq RyRx$$

**Order of Roration**

# 3D ORIENTATION




a Original

b $\frac{\pi}{2}$ about $x$-axis

c $\pi$ about $x$-axis

d $-\frac{\pi}{2}$ about $x$-axis

e $\frac{\pi}{2}$ about $y$-axis

f $\frac{\pi}{2}$ about $z$-axis

# 3D ORIENTATION

$$ {}^{\boldsymbol{A}}\boldsymbol{P} = {}^{A}\boldsymbol{R}_{B}{}^{\boldsymbol{B}}\boldsymbol{P} $$

$$ \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = {}^{A}\boldsymbol{R}_{B} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} $$

$$ {}^{A}\boldsymbol{R}_{B} \subset \mathbb{R}^{3x3} $$

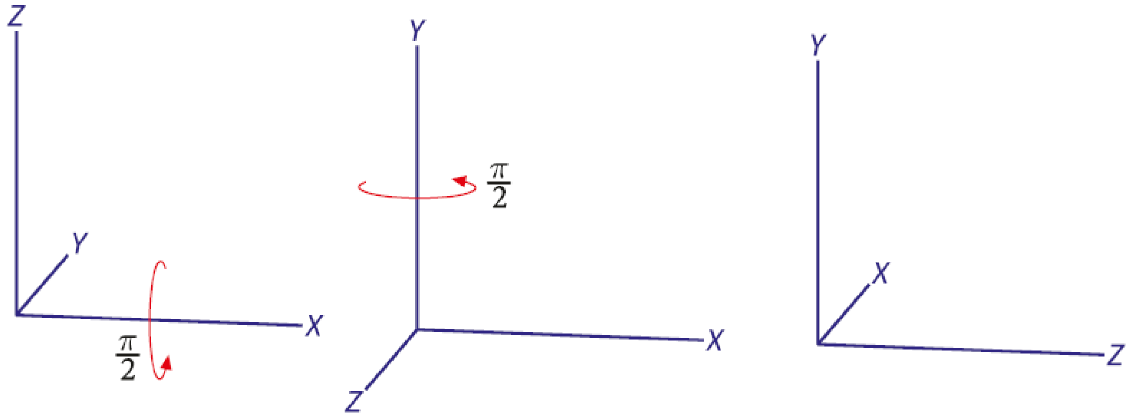
$$ \boldsymbol{R}^{-1} = \boldsymbol{R}^{T} $$

$$ \boldsymbol{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} $$

$$ \boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} $$

$$ \boldsymbol{R}_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

# 3D ORIENTATION

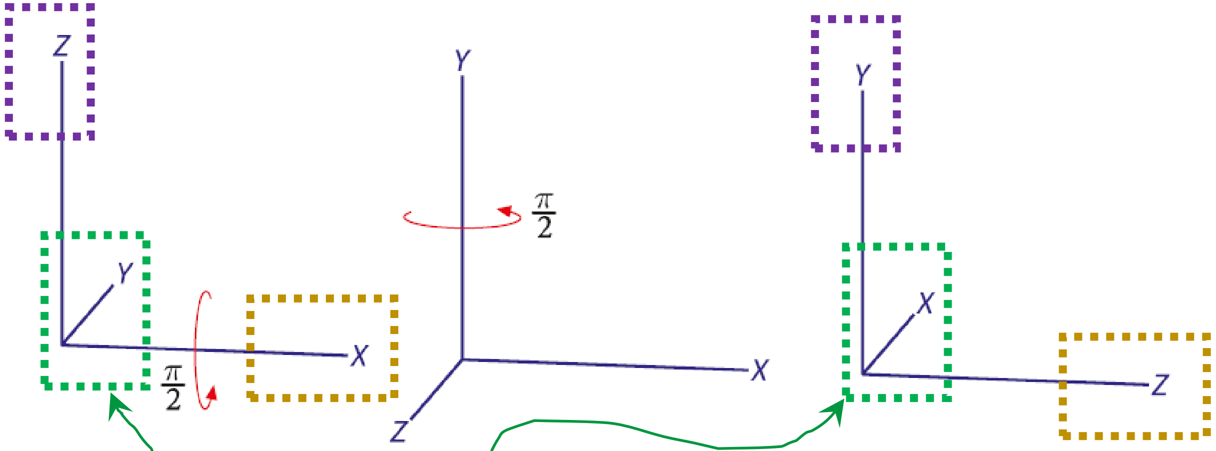



$$\boldsymbol{R}_x(90^0)\boldsymbol{R}_y(90^0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(90^0) & -\sin(90^0) \\ 0 & \sin(90^0) & \cos(90^0) \end{bmatrix} \begin{bmatrix} \cos(90^0) & 0 & \sin(90^0) \\ 0 & 1 & 0 \\ -\sin(90^0) & 0 & \cos(90^0) \end{bmatrix}$$

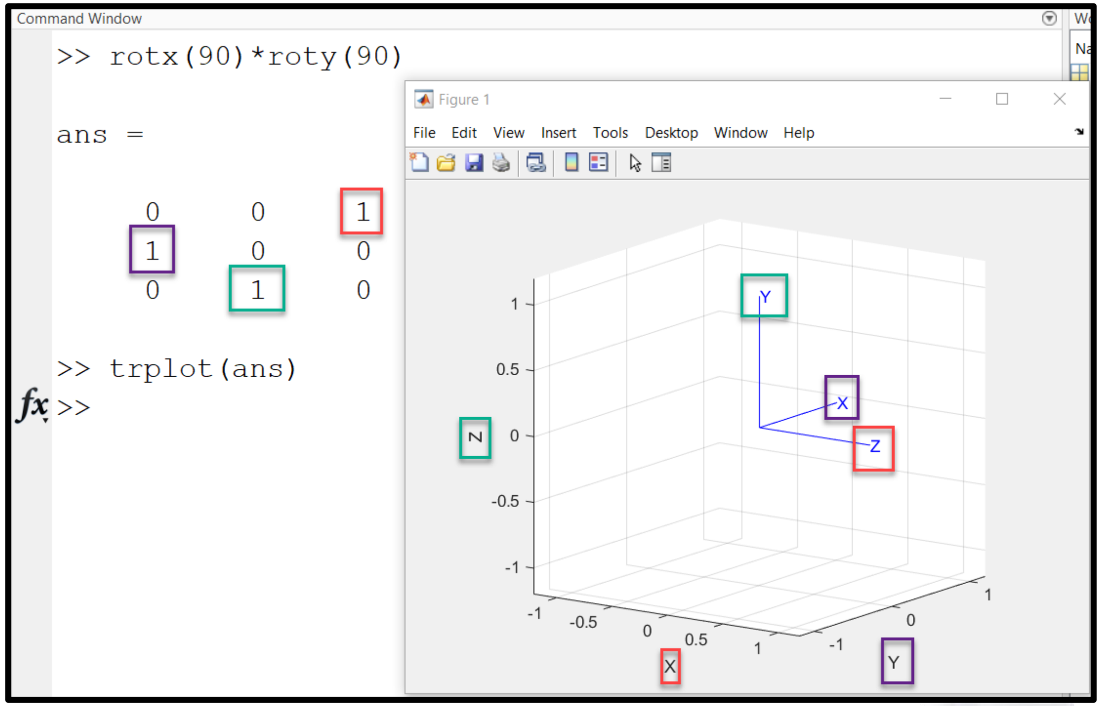
$$\boldsymbol{R}_x(90^0)\boldsymbol{R}_y(90^0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

***WHAT DOES IT MEAN ?***

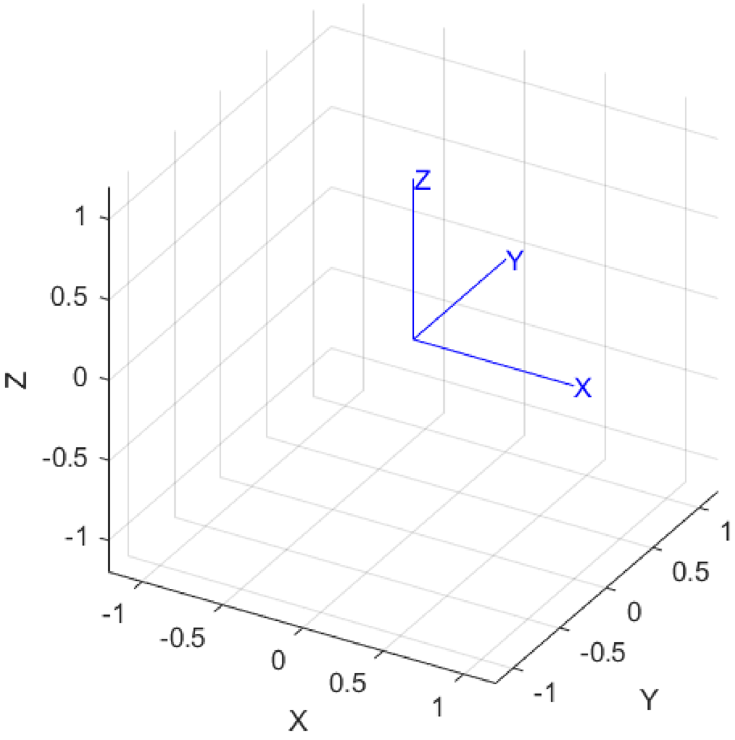
# 3D ORIENTATION



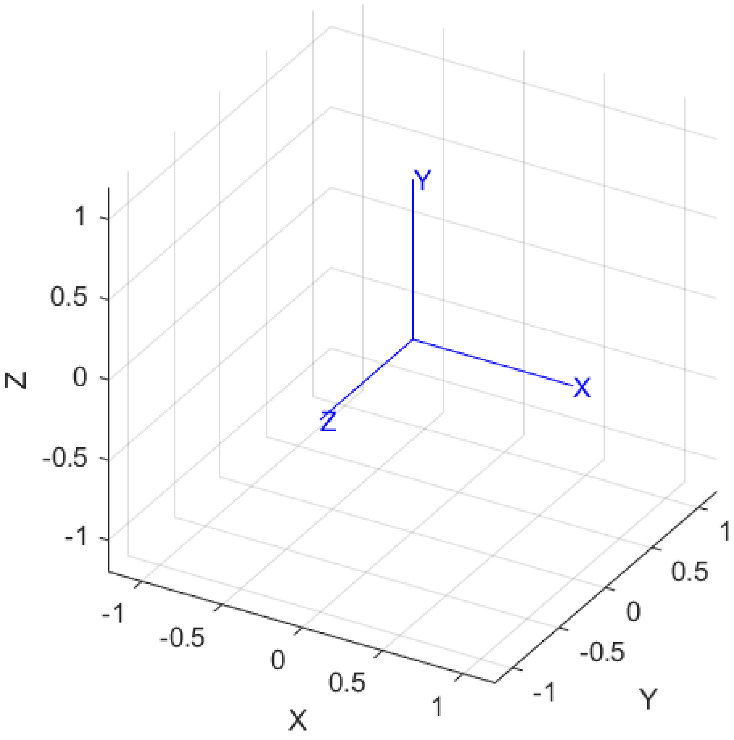

$$
R_x\left(\frac{\pi}{2}\right) R_y\left(\frac{\pi}{2}\right) = \begin{array}{c} \textbf{NEW} \\ \begin{array}{ccc} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{array} \\ \begin{bmatrix} 0 & 0 & \boxed{1} \\ \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \end{bmatrix} \end{array} \begin{array}{c} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{array} \ \textbf{OLD}
$$



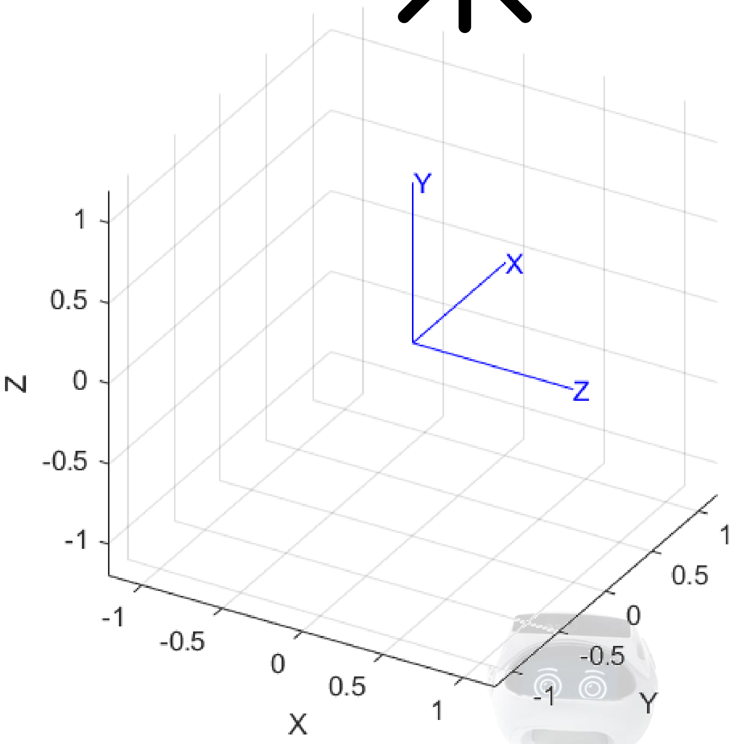
# 3D ORIENTATION




```
>> base = [1 0 0;0 1 0;0 0 1];
>> trplot(base)
>> view(30,30)
```

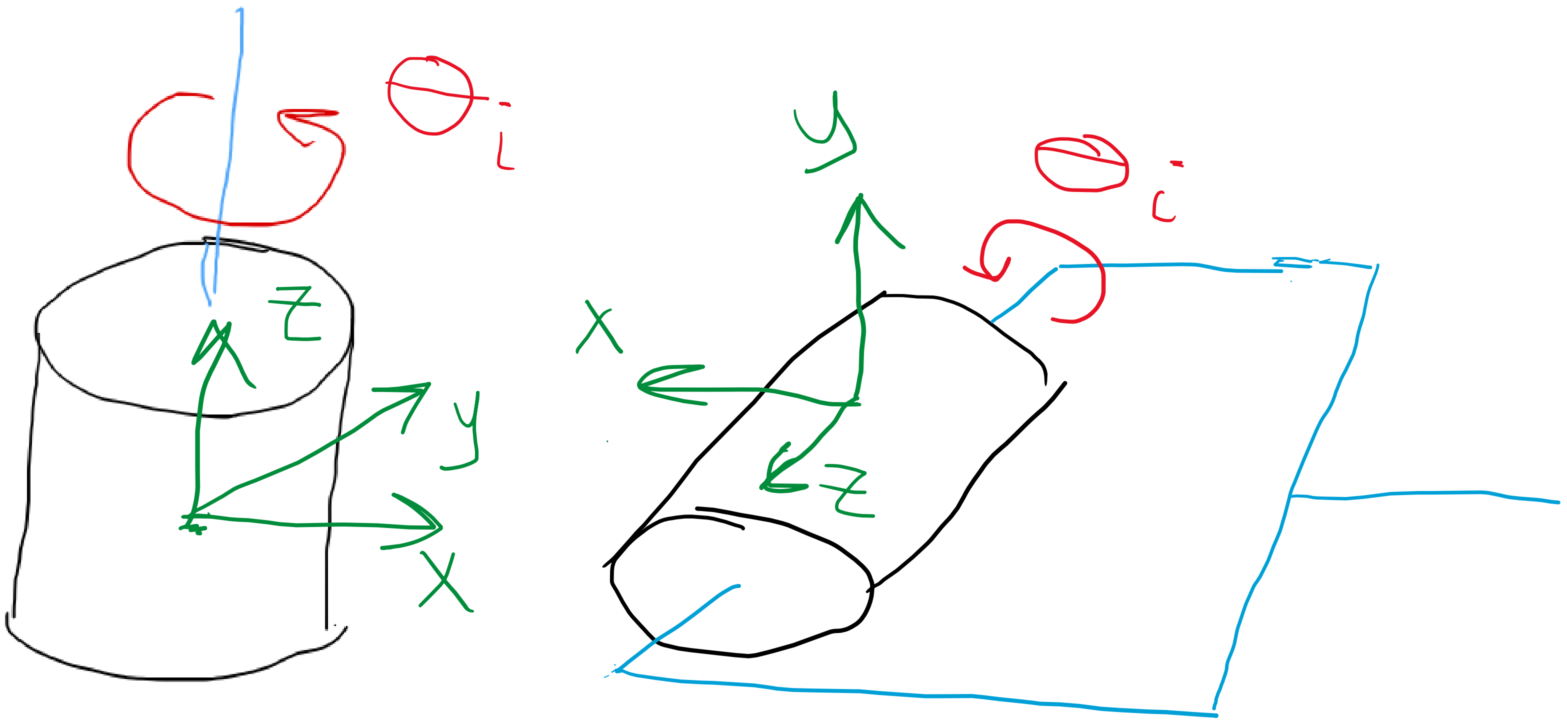



```
>> Rx = rotx(90)
>> trplot(Rx)
>> view(30,30)
```



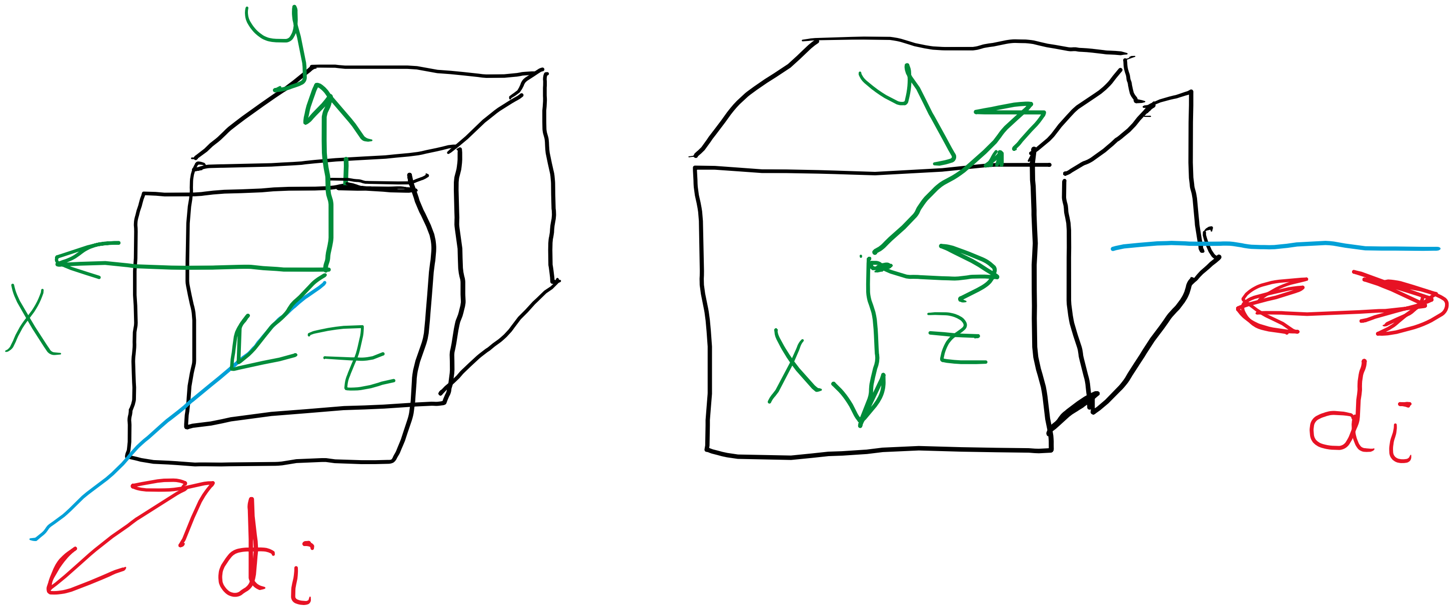

```
>> RxRy = rotx(90)*roty(90)
>> trplot(RxRy)
>> view(30,30)
```

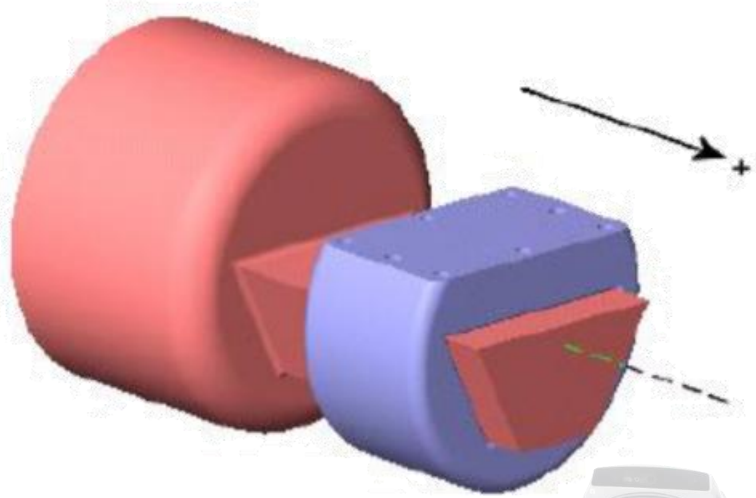
# ROBOT JOINTS




*Revolute Joint (R)*

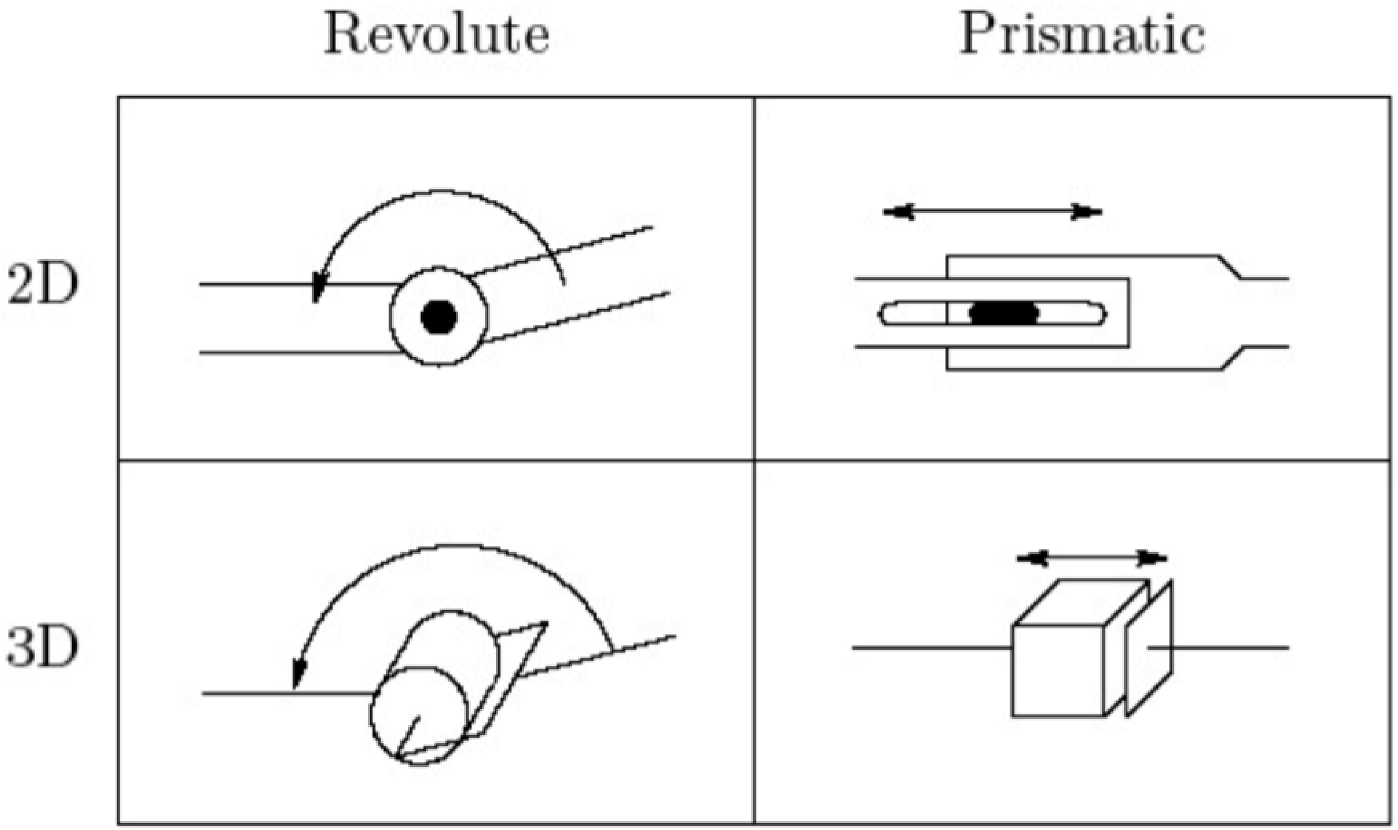
# ROBOT JOINTS



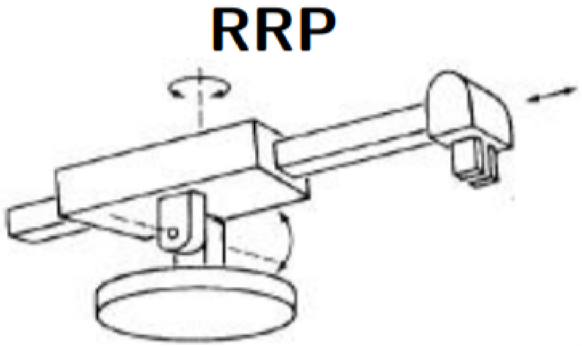



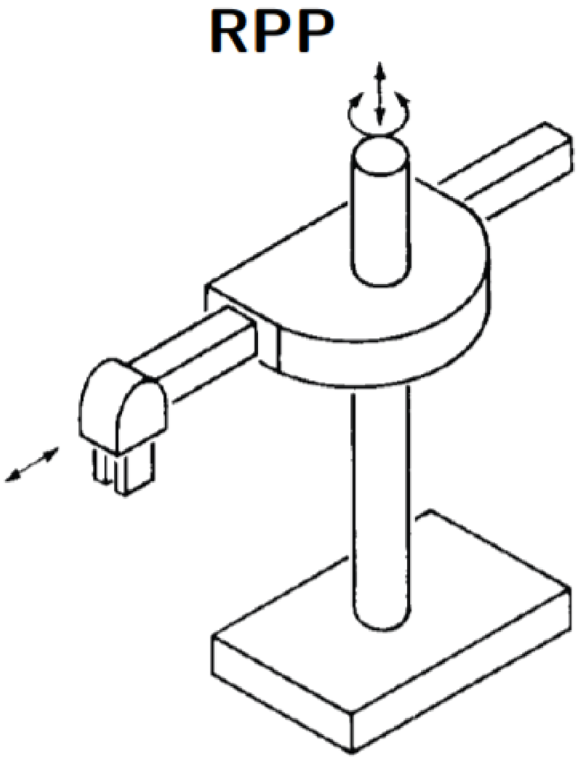
*Prismatic Joint (P)*

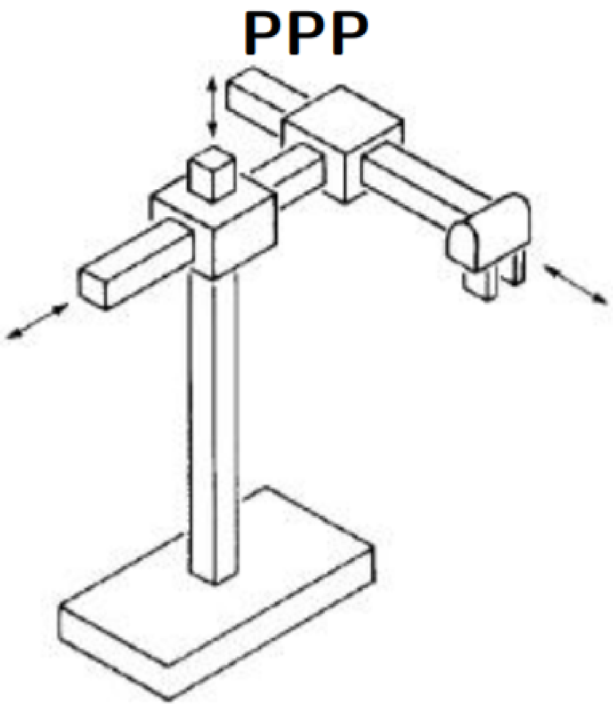
# ROBOT JOINTS

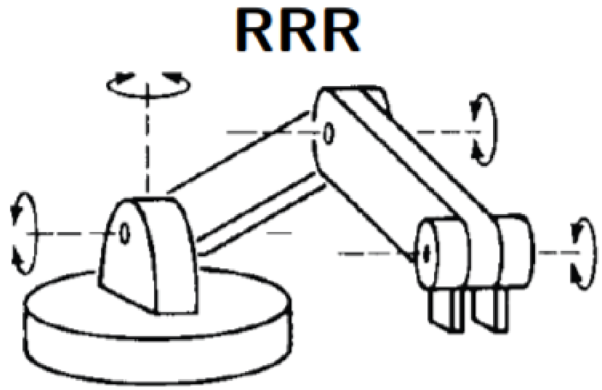


# ROBOT JOINTS



# KINEMATIC CONFIGURATION

$Articulated\ Arm\ (RRR)$

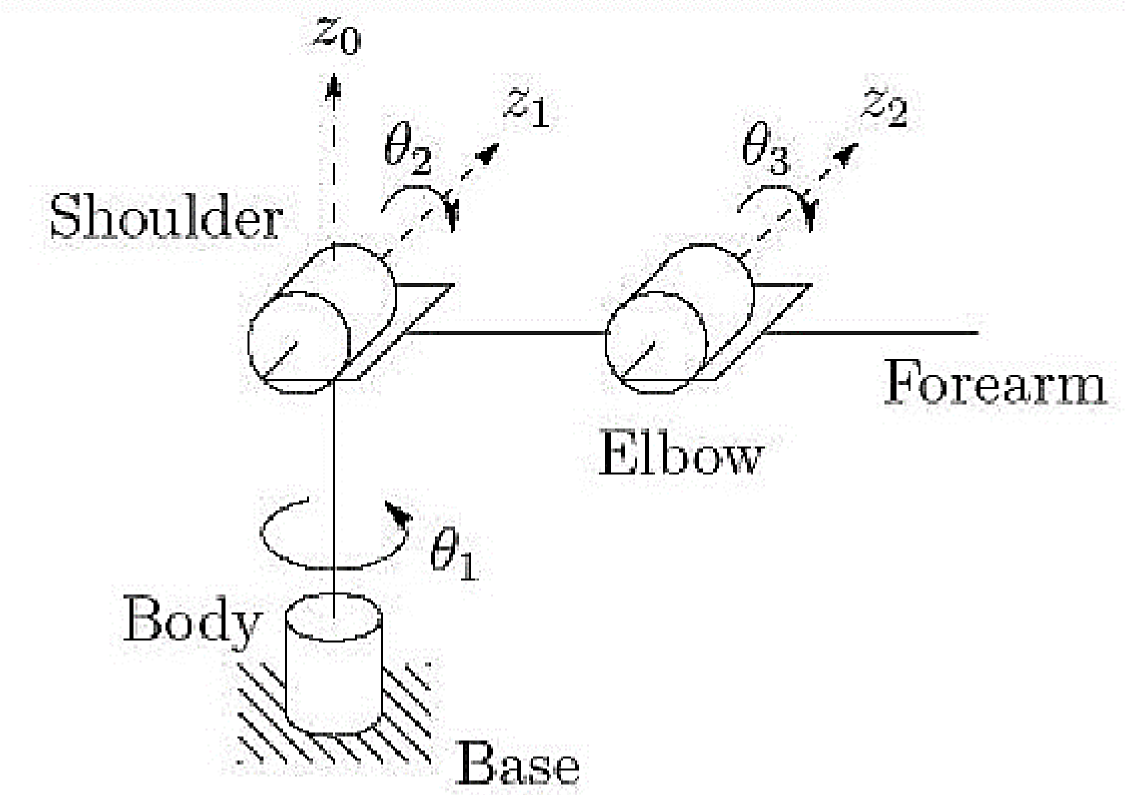
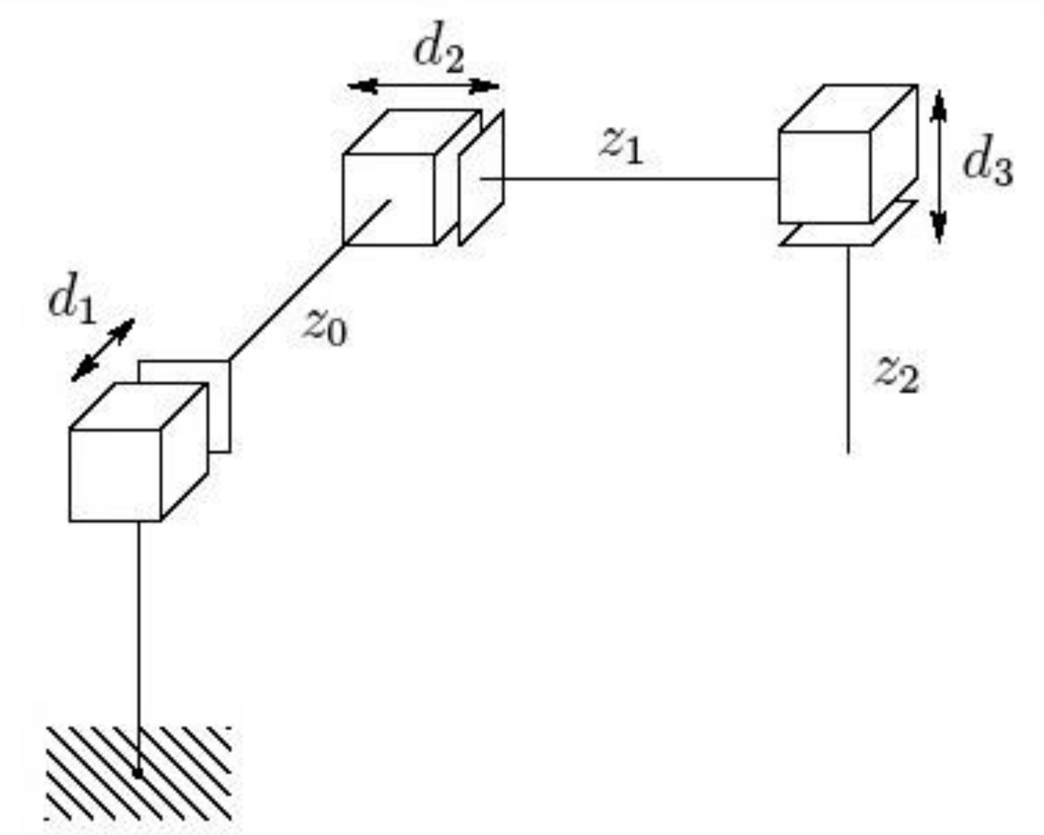



ABB IRB1400 Anthropomorphic Robot

# KINEMATIC CONFIGURATION

## $Cartesian\ Robot\ (PPP)$

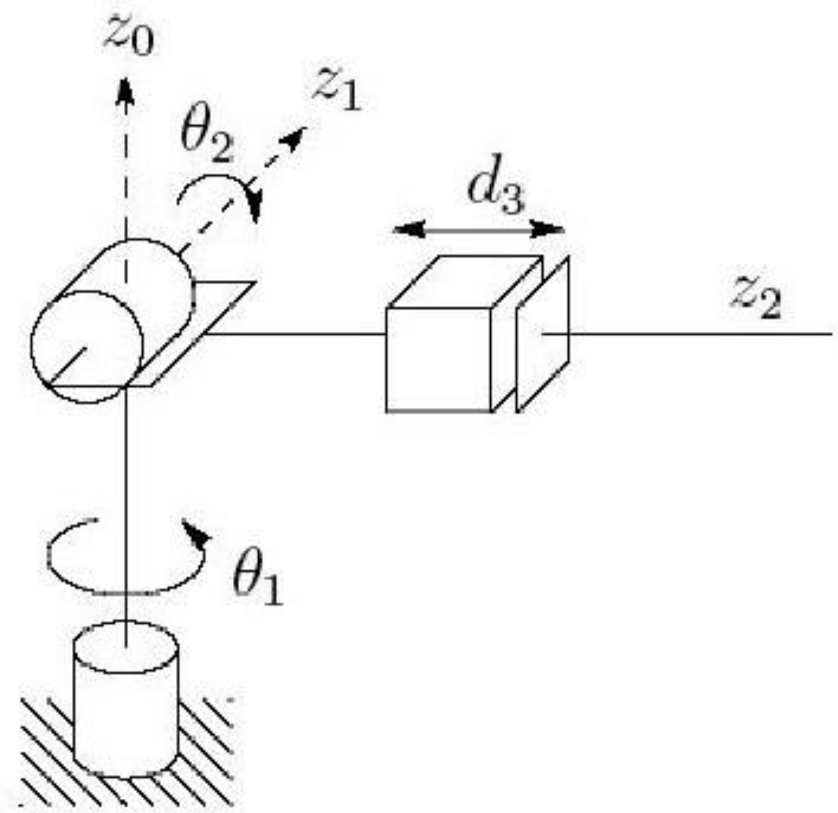


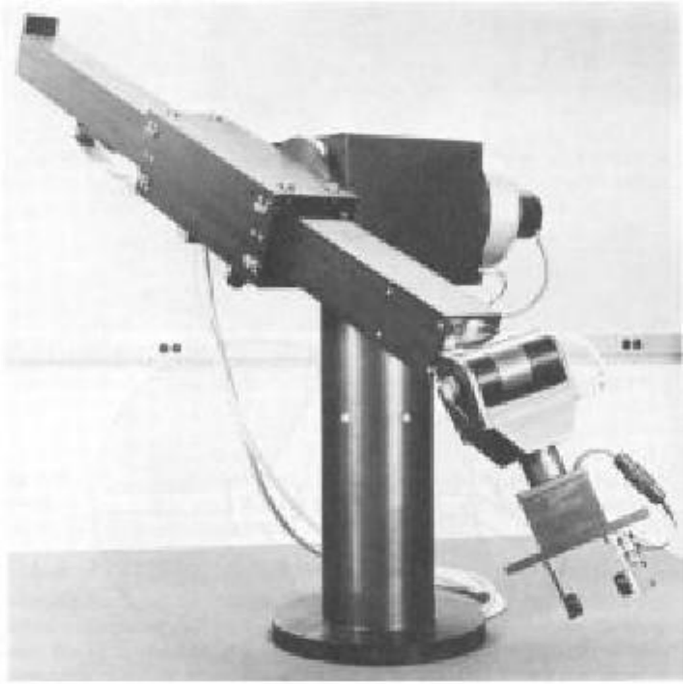


Epson Cartesian Robot

# KINEMATIC CONFIGURATION

$Spherical\ Robot\ (RRP)$

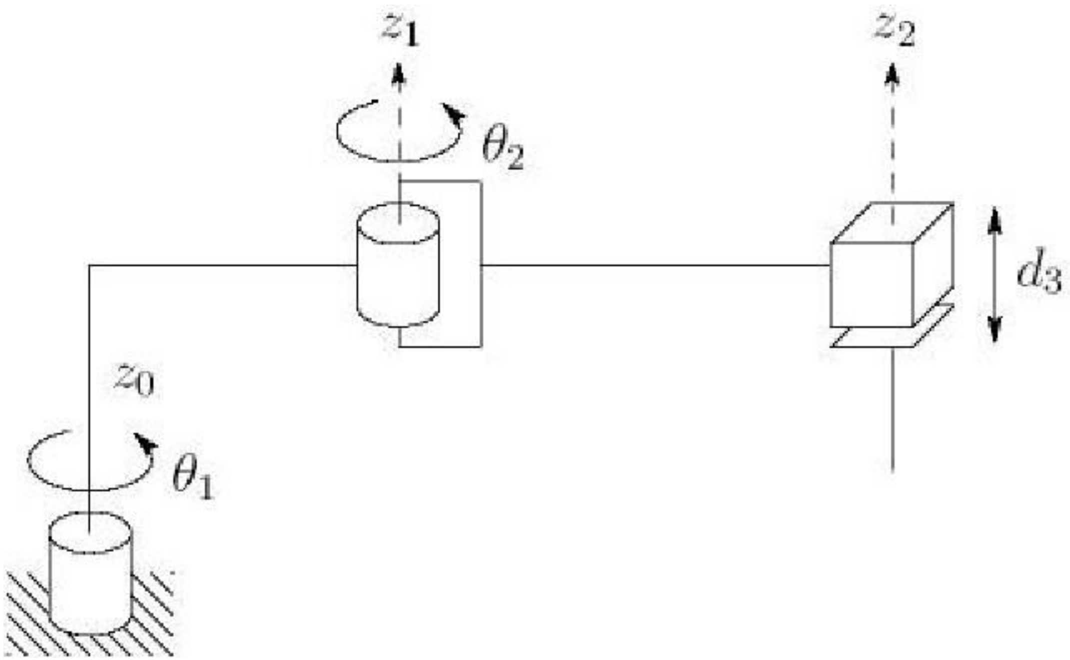


Stanford Arm

# KINEMATIC CONFIGURATION

## $SCARA\ Robot\ (RRP)$

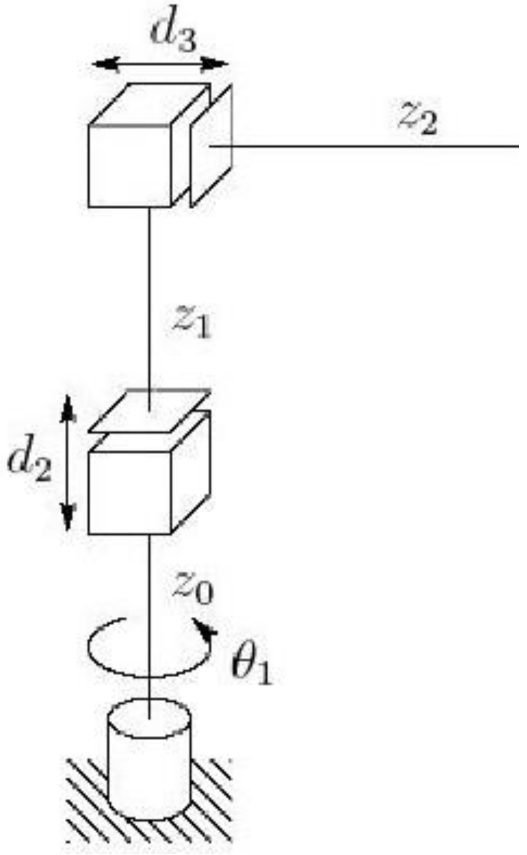


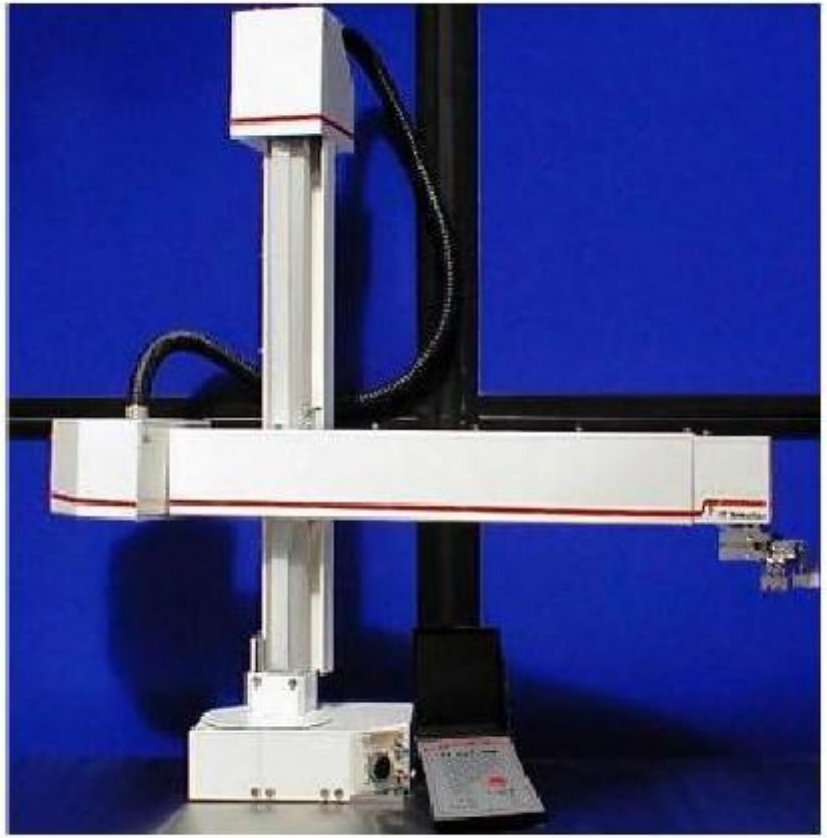


Adept Cobra i600

# KINEMATIC CONFIGURATION

## $SCARA\ Robot\ (RPP)$



Seiko RT3300 Robot

# 3D-HOMOGENEOUS TRANSFORM

$\boldsymbol{R}^{3X3}$

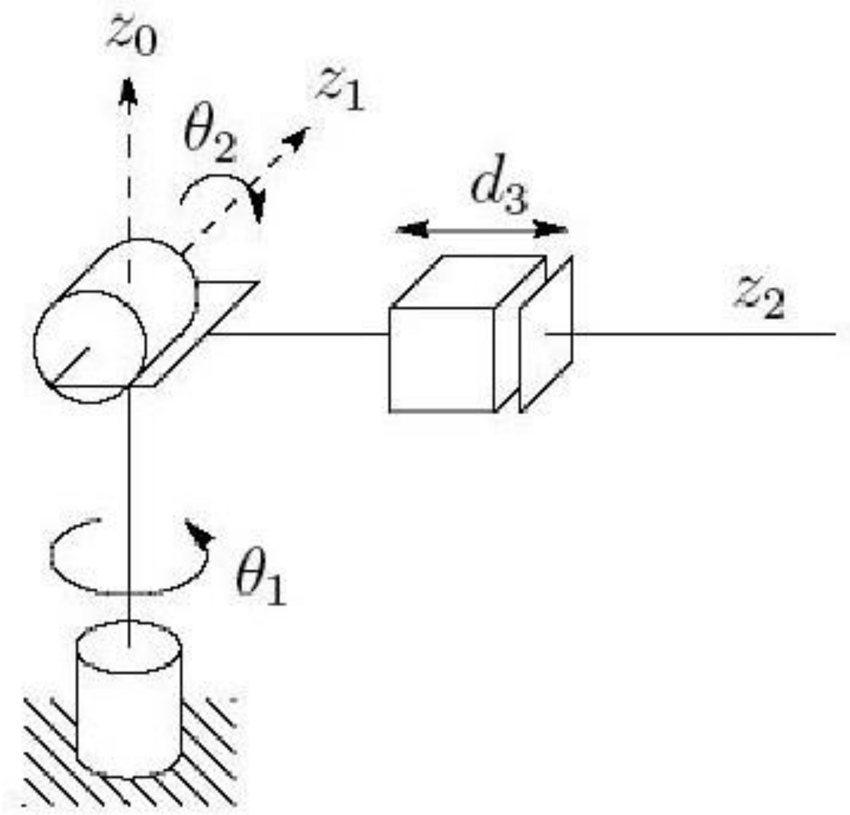
$$ {}^{A}_{B}T = \begin{bmatrix} 1 & 0 & 0 & a_x \\ 0 & \cos(\theta) & -\sin(\theta) & a_y \\ 0 & \sin(\theta) & \cos(\theta) & a_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

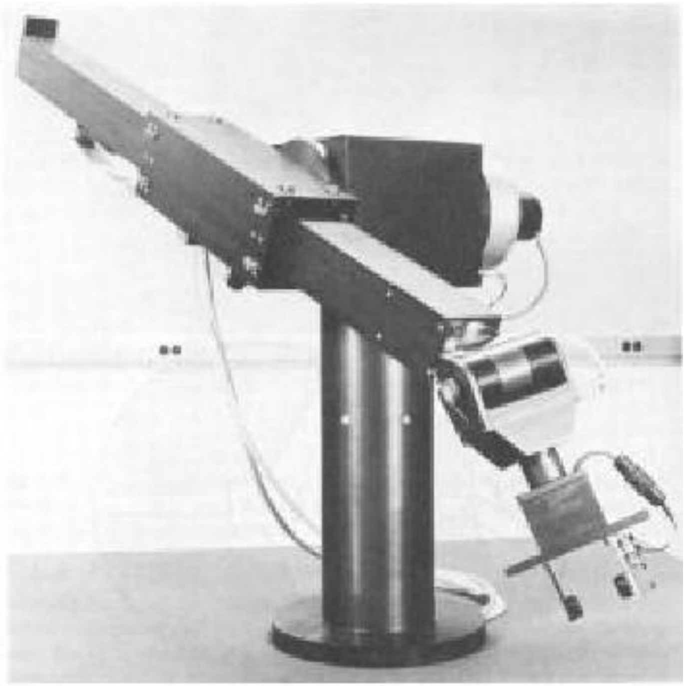
$\boldsymbol{t}^{3X1}$

$$ P = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} $$

# KINEMATIC DIAGRAM (HT)

*Spherical Robot (RRP)*



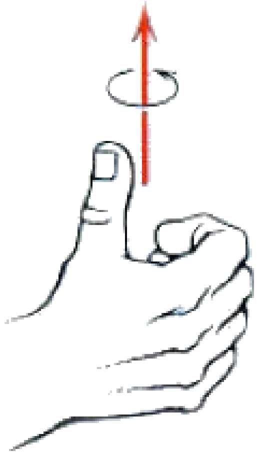
Stanford Arm

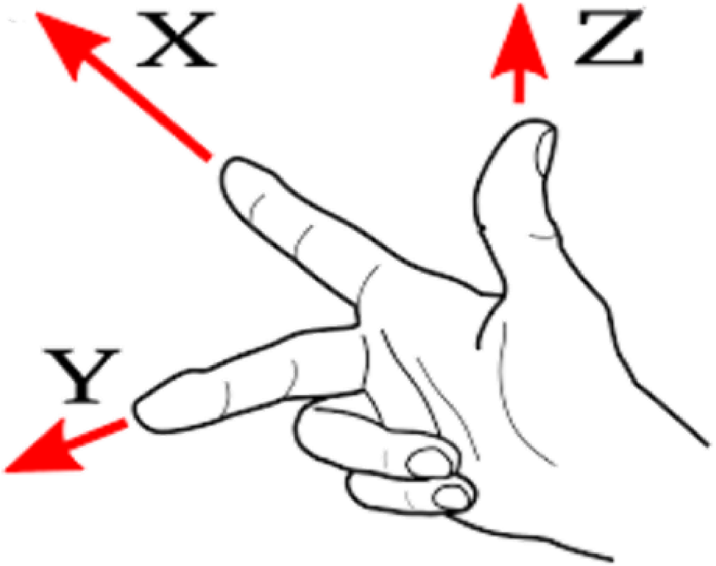
# KINEMATIC DIAGRAM (HT)

## *Spherical Robot (RRP)*

Note : From our discussion in the session, don't be confused with the axis assignment. You can pick any finger arrangement to axes but **you must keep this arrangement** when assigning frames to the whole mechanism

In this problem, I picked **ZXY** , but you may use any arrangement of preference.





# KINEMATIC DIAGRAM (HT)

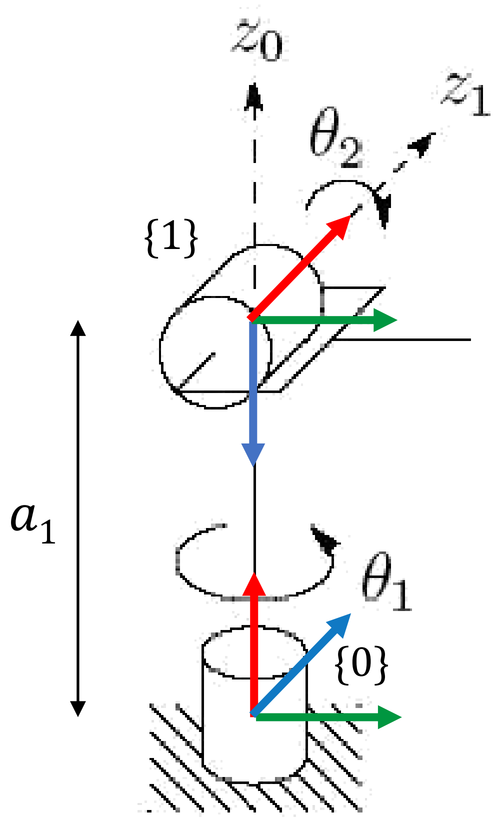
## *Spherical Robot (RRP)*

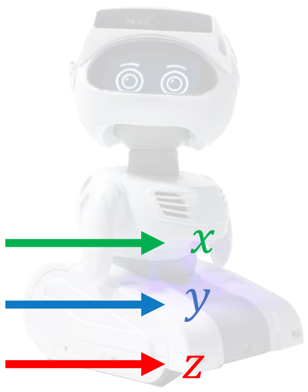
$${}^0_1T = Rz\,{}^0_1R + {}^0_1t$$

$$R_z = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(-90) & -s(-90) \\ 0 & s(-90) & c(-90) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$${}^0_1t = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \qquad {}^0_1R_c = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 \\ s\theta_1 & 0 & c\theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$



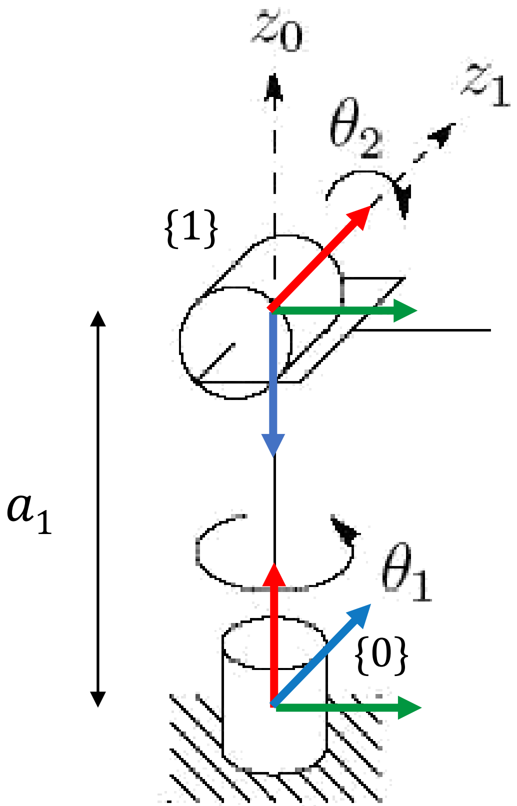
# KINEMATIC DIAGRAM (HT)

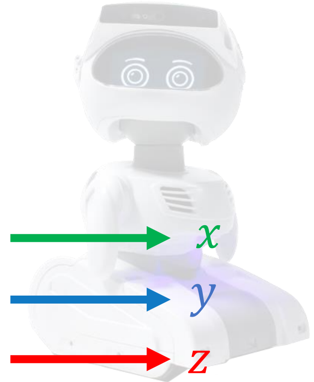
## *Spherical Robot (RRP)*

$${}^0_1T = Rz\,{}^0_1R + {}^0_1t$$

$${}^0_1t = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \qquad {}^0_1R_c = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 \\ s\theta_1 & 0 & c\theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# KINEMATIC DIAGRAM (HT)

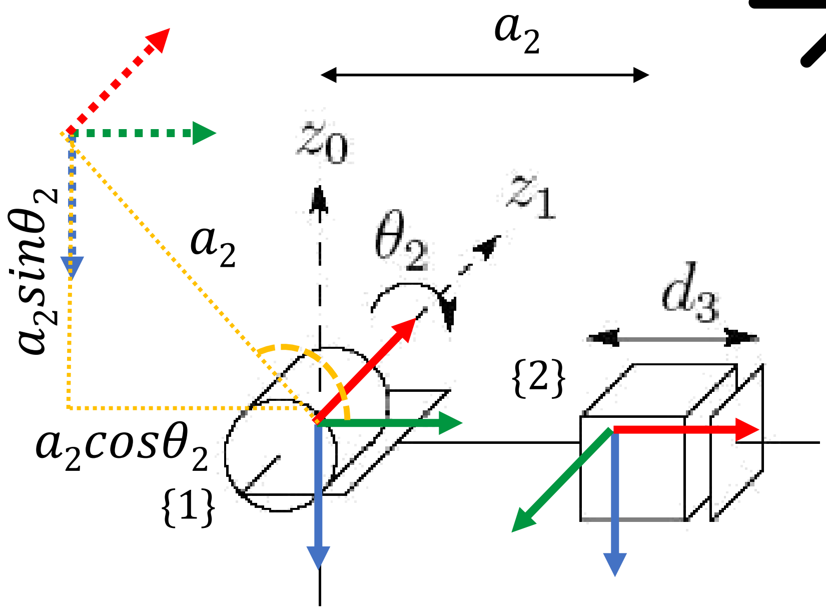
## *Spherical Robot (RRP)*

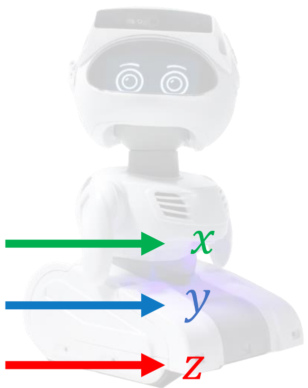
$$ {}^1_2T = Rz\,{}^1_2R + {}^1_2t $$

$$ R_z = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2R = \begin{bmatrix} c(90) & 0 & s(90) \\ 0 & 1 & 0 \\ -s(90) & 0 & c(90) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} $$

$$ {}^1_2t = \begin{bmatrix} a_2c\theta_2 \\ -a_2s\theta_2 \\ 0 \end{bmatrix} \qquad {}^1_2R_c = \begin{bmatrix} 0 & -s\theta_2 & c\theta_2 \\ 0 & c\theta_2 & s\theta_2 \\ -1 & 0 & 0 \end{bmatrix} $$



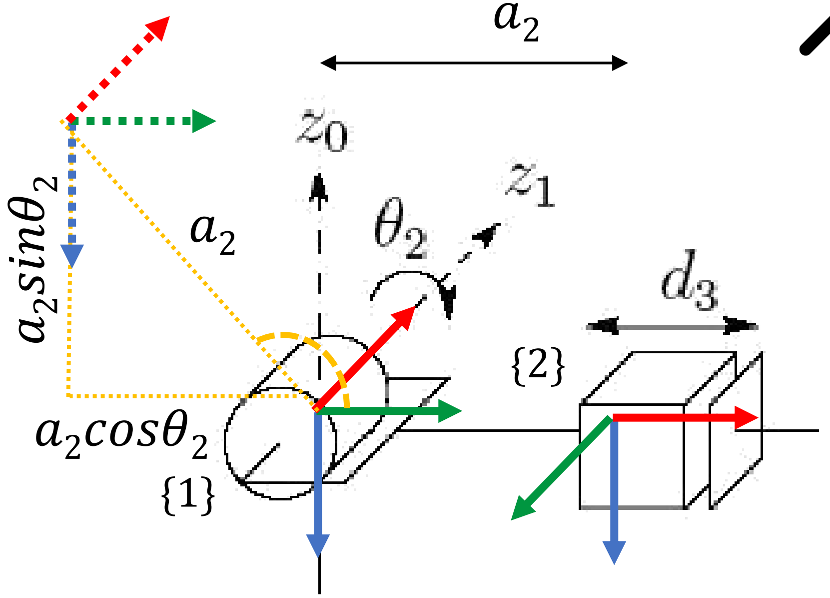
# KINEMATIC DIAGRAM (HT)

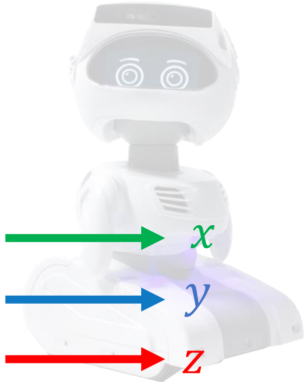
## *Spherical Robot (RRP)*

$${}^1_2T = Rz\,{}^1_2R + {}^1_2t$$

$${}^1_2t = \begin{bmatrix} a_2c\theta_2 \\ -a_2s\theta_2 \\ 0 \end{bmatrix} \quad {}^1_2R_c = \begin{bmatrix} 0 & -s\theta_2 & c\theta_2 \\ 0 & c\theta_2 & s\theta_2 \\ -1 & 0 & 0 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} 0 & -s\theta_2 & c\theta_2 & a_2c\theta_2 \\ 0 & c\theta_2 & s\theta_2 & -a_2s\theta_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# KINEMATIC DIAGRAM (HT)

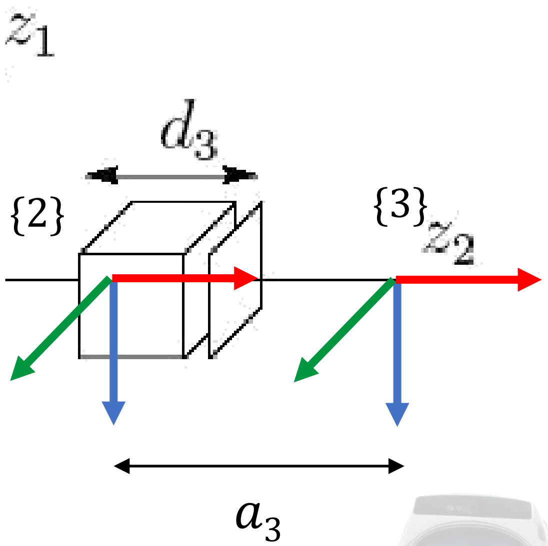
## *Spherical Robot (RRP)*

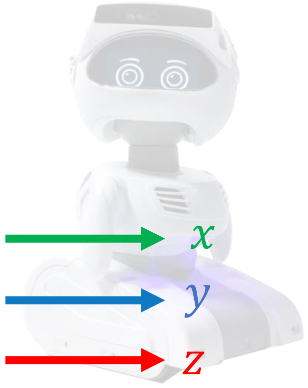
$${}^{2}_{3}T = Rz\,{}^{2}_{3}R + {}^{2}_{3}t$$

$$R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \# \, No \; rotation$$

$${}^{2}_{3}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \# \, Same \; frame \; orientation$$

$${}^{2}_{3}t = \begin{bmatrix} 0 \\ 0 \\ a_3 + d_3 \end{bmatrix} \qquad {}^{2}_{3}R_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



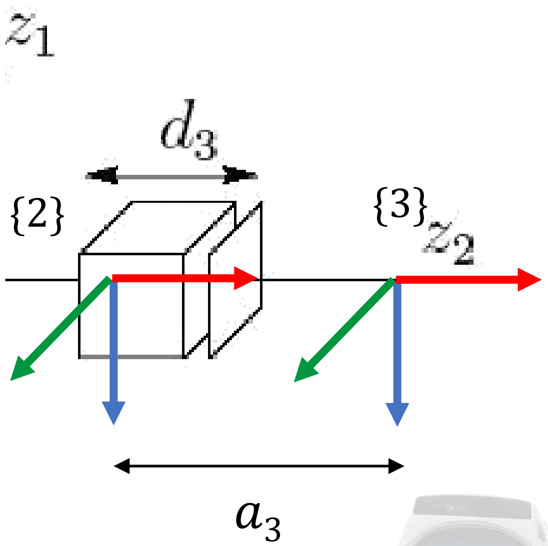
# KINEMATIC DIAGRAM (HT)

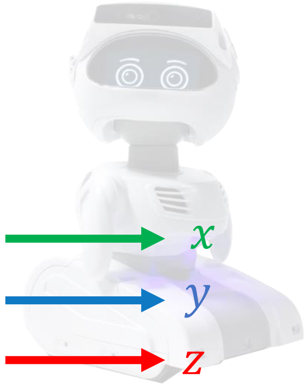
## *Spherical Robot (RRP)*

$${}^{2}_{3}T = Rz\,{}^{2}_{3}R + {}^{2}_{3}t$$

$${}^{2}_{3}t = \begin{bmatrix} 0 \\ 0 \\ a_3 + d_3 \end{bmatrix} \qquad {}^{2}_{3}R_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



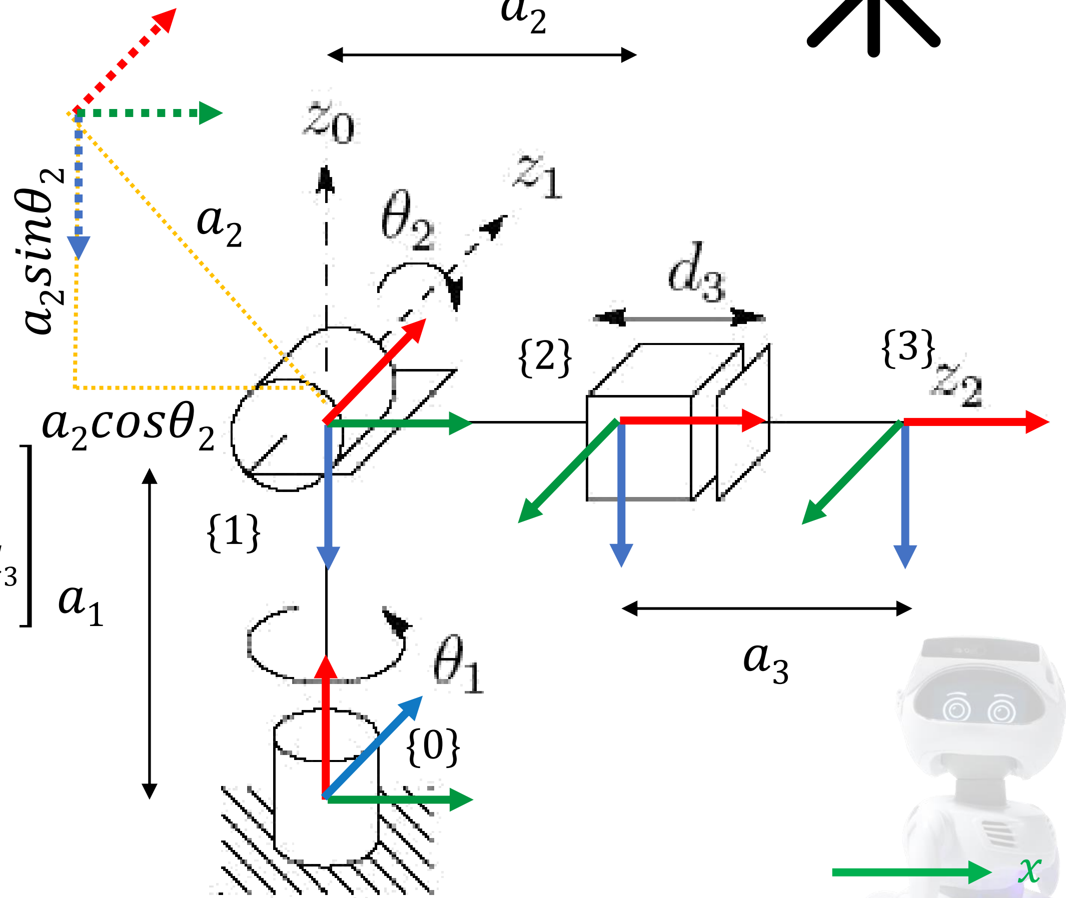



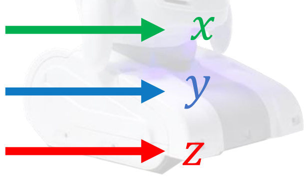
# KINEMATIC DIAGRAM (HT)

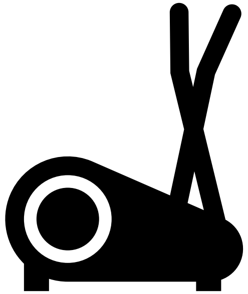
## *Spherical Robot (RRP)*

$$ {}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T $$

$$ {}^{0}_{3}T = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -s\theta_2 & c\theta_2 & a_2 c\theta_2 \\ 0 & c\theta_2 & s\theta_2 & -a_2 s\theta_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$





*Find the homogeneous transform for all stated confihurations in section. Share your answer with you colleagues*

# ***NEXT SECTION*** *: D − H PARAMETERS*