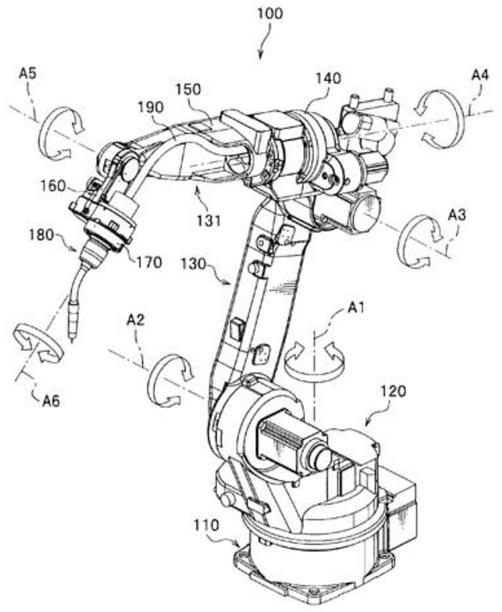
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 6
INTRODUCTION TO ROBOTICS LAB

WALEED ELBADRY

MARCH 2022



WHAT WE LEARNED SO FAR

2D – HOMOGENOUS TRANSFORM

ROTATION + TRANSLATION

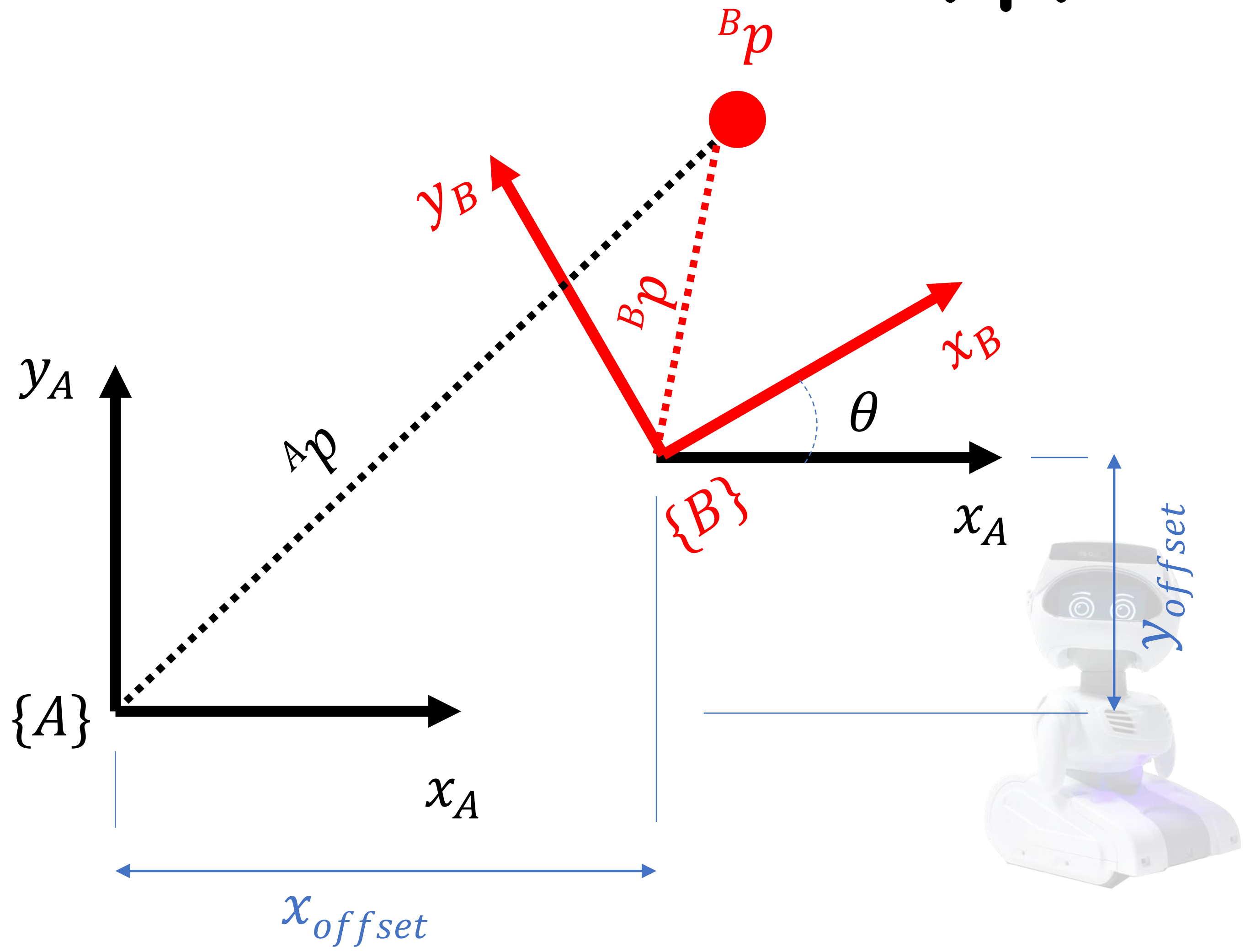
$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} {}^{B}\boldsymbol{x} \\ {}^{B}\boldsymbol{y} \end{bmatrix} + \begin{bmatrix} x_{offset} \\ y_{offset} \end{bmatrix}$$

STANDARD FORM

t

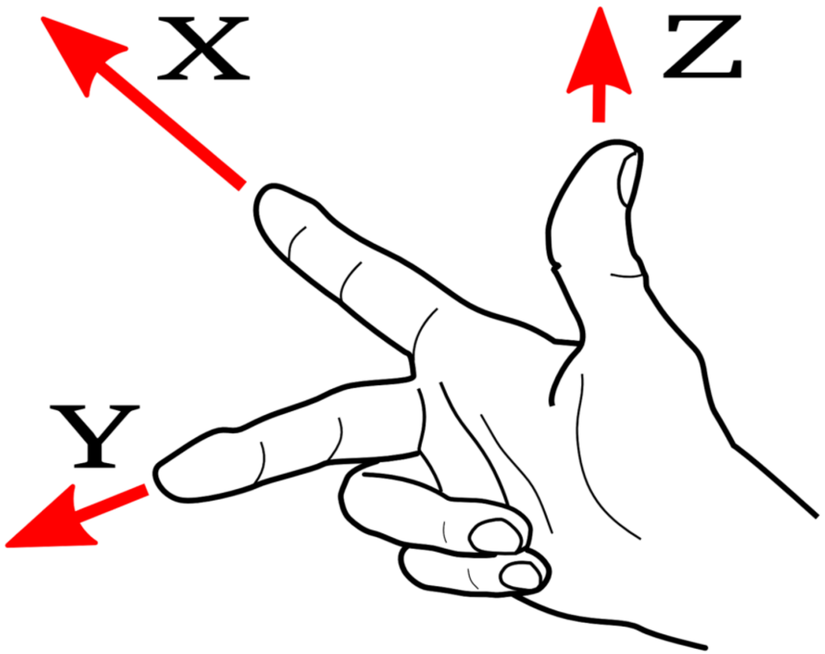
$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boxed{\mathbf{1}} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & x_{offset} \\ sin\theta & cos\theta & y_{offset} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}\boldsymbol{x} \\ {}^{B}\boldsymbol{y} \\ \boxed{\mathbf{1}} \end{bmatrix}$$

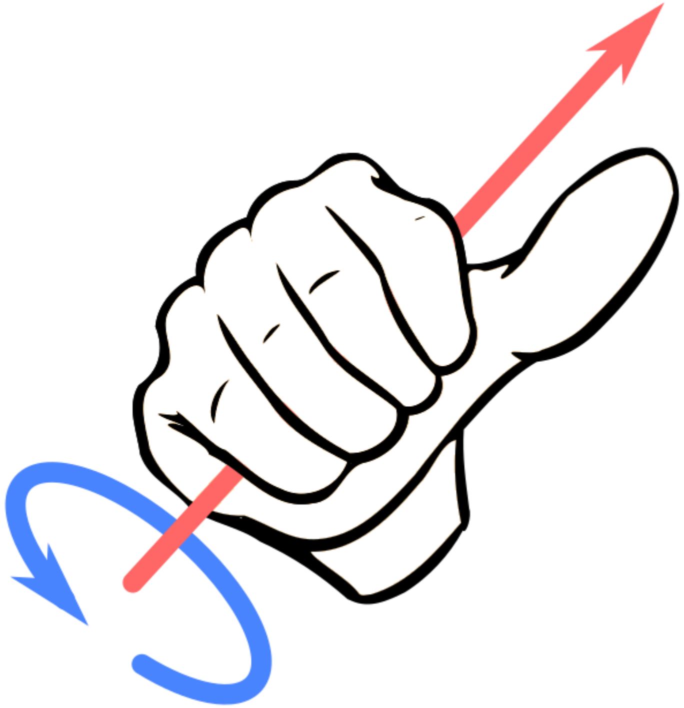
R *Scaling*



WHAT WE LEARNED SO FAR

3D – HOMOGENOUS TRANSFORM





Right–hand rule

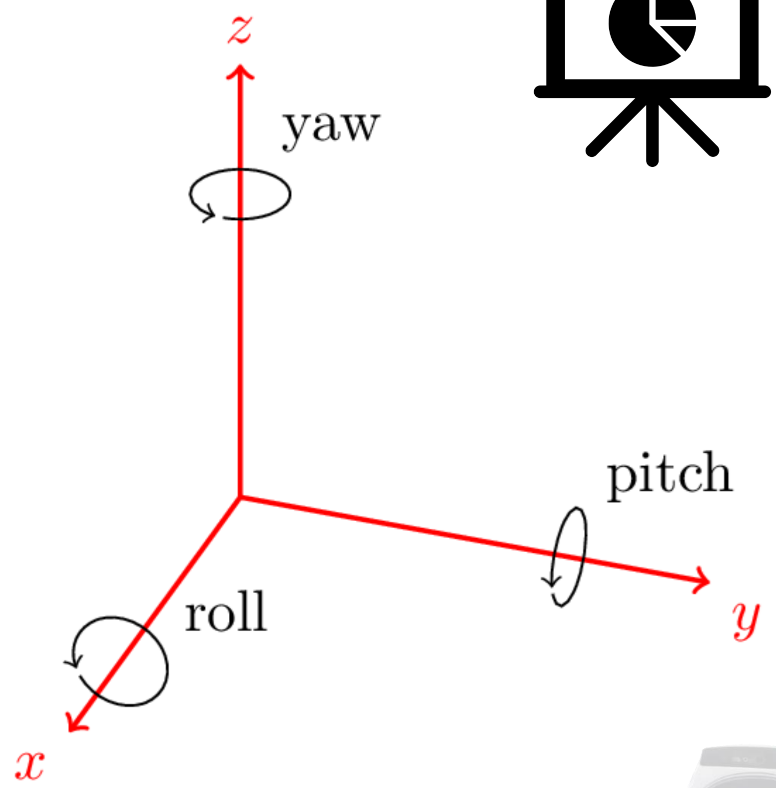
WHAT WE LEARNED SO FAR

3D – HOMOGENOUS TRANSFORM

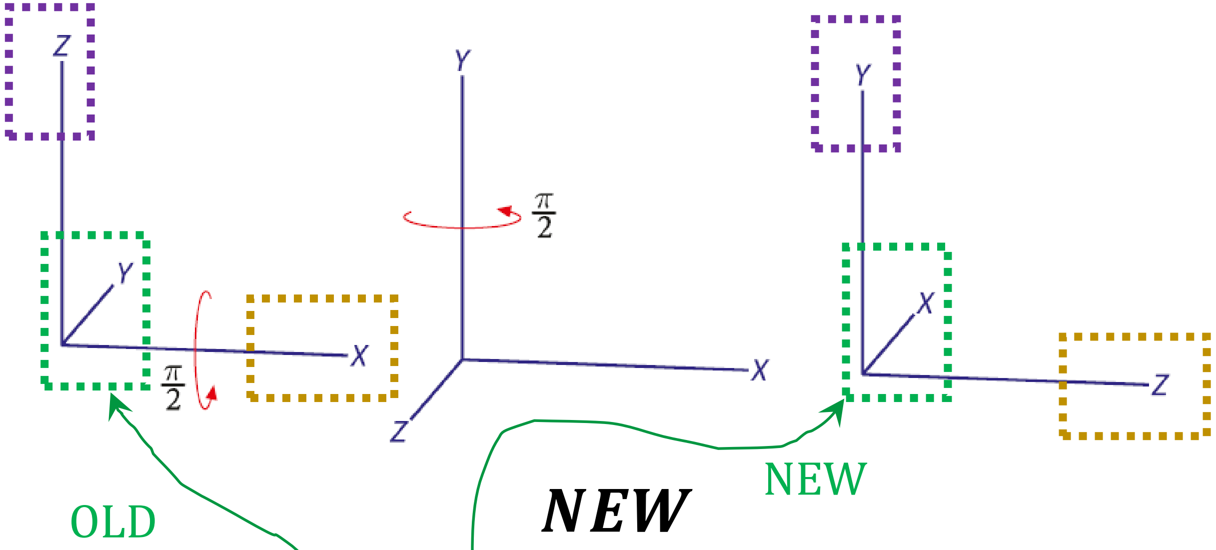
$$\boldsymbol{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

$$\boldsymbol{R}_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

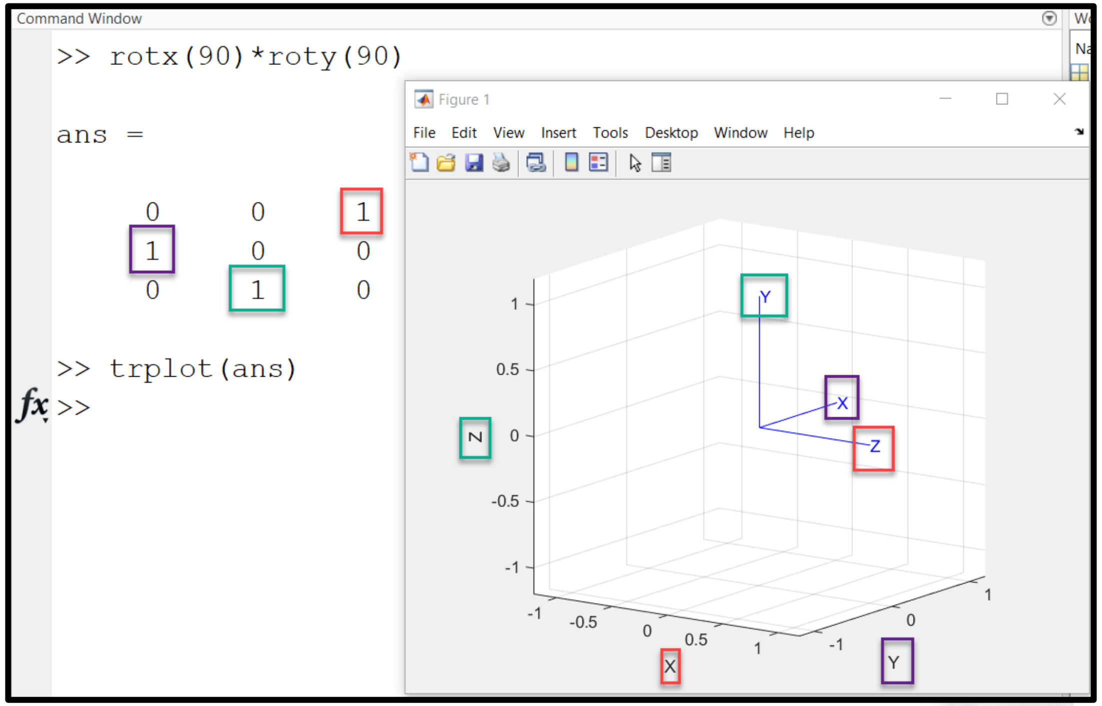


WHAT WE LEARNED SO FAR



NEW

$$R_x\left(\frac{\pi}{2}\right) R_y\left(\frac{\pi}{2}\right) = \begin{array}{cc} \begin{matrix} x & y & z \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array} \ \textbf{OLD}$$



WHAT WE LEARNED SO FAR

$\boldsymbol{R}^{3X3}$

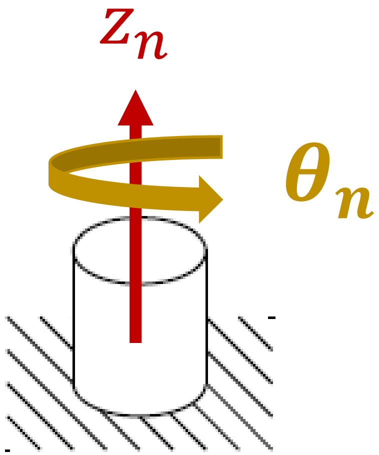
$\boldsymbol{t}^{3X1}$

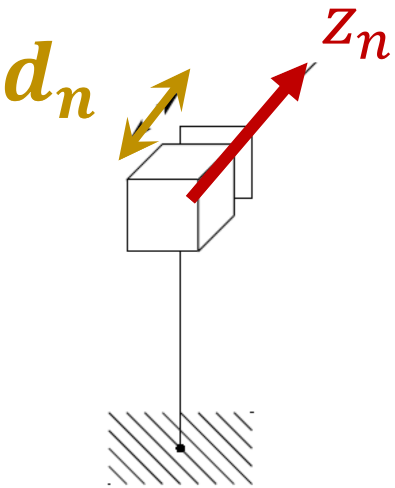
$$ {}_{B}^{A}T = \begin{bmatrix} 1 & 0 & 0 & a_x \\ 0 & \cos(\theta) & -\sin(\theta) & a_y \\ 0 & \sin(\theta) & \cos(\theta) & a_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ P = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} $$

DENAVIT-HARTENBERG RULES

1 $The\ \boldsymbol{z-axis}\ is\ the\ direction\ of\ translation\ or\ rotation$



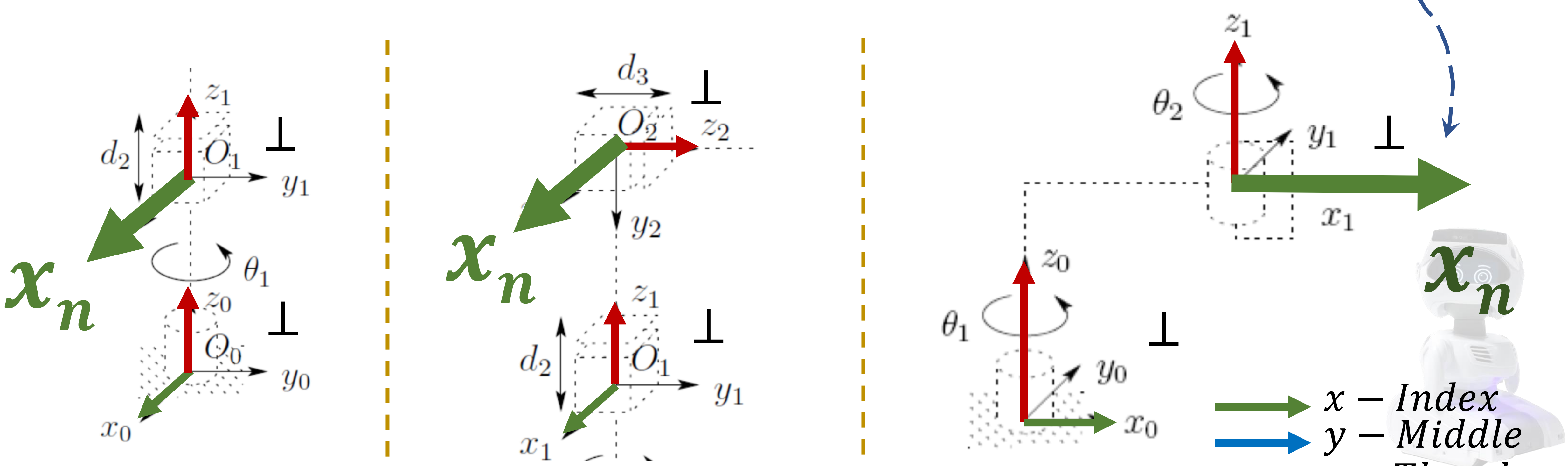


$\boldsymbol{Don't\ ever\ break\ the\ Right\ Hand\ Rule}$

DENAVIT-HARTENBERG RULES

2 *The* $\boldsymbol{x_n} - \boldsymbol{axis}$ *is prependicular* $\perp$ *to both* $\boldsymbol{z_n}$ *and* $\boldsymbol{z_{n-1}}$ *axes*

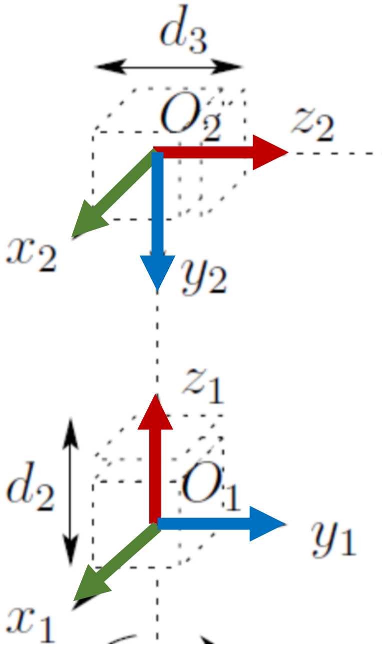
For ***Parallel*** z_{n-1} *and* z_n, *pick* $x - axis$ *direction from* $z_{n-1} \rightarrow z_n$

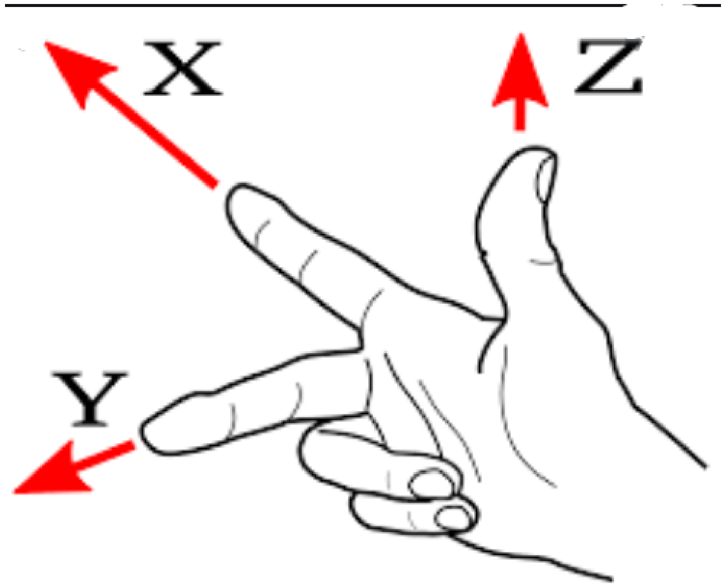


$\alpha = 0^o$ $\alpha = -90^o$ $\alpha = 0^o$

DENAVIT-HARTENBERG RULES

③ $The\ \boldsymbol{y_n} - \boldsymbol{axis}\ must\ follow\ the\ RHR\ (better\ to\ always\ use\ \boldsymbol{ZXY})$





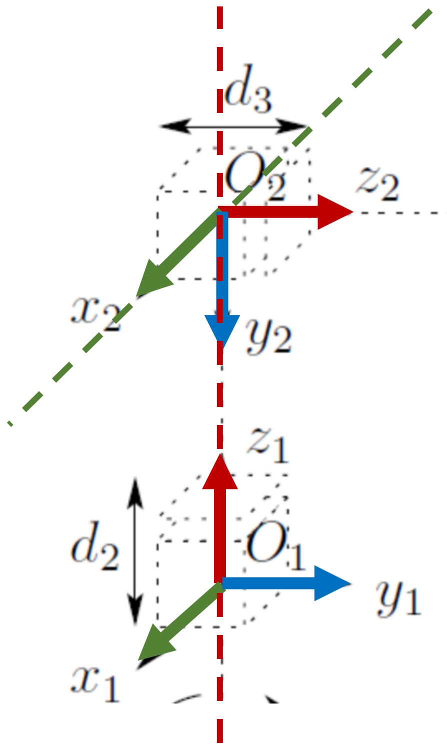
x − $Index$

y − $Middle$

z − $Thumb$

DENAVIT-HARTENBERG RULES

4 *The* $\boldsymbol{x_n - axis}$ *must* $\boldsymbol{intersect}$ *with* z_{n-1}



Pay Attention

if there is $\boldsymbol{an\ offset}$ *in the* $x - direction$

$x - Index$
$y - Middle$
$z - Thumb$

WHAT WE LEARNED SO FAR

$DH - PARAMETERS$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

BACK TO NOTATION

FORWARD KINEMATICS

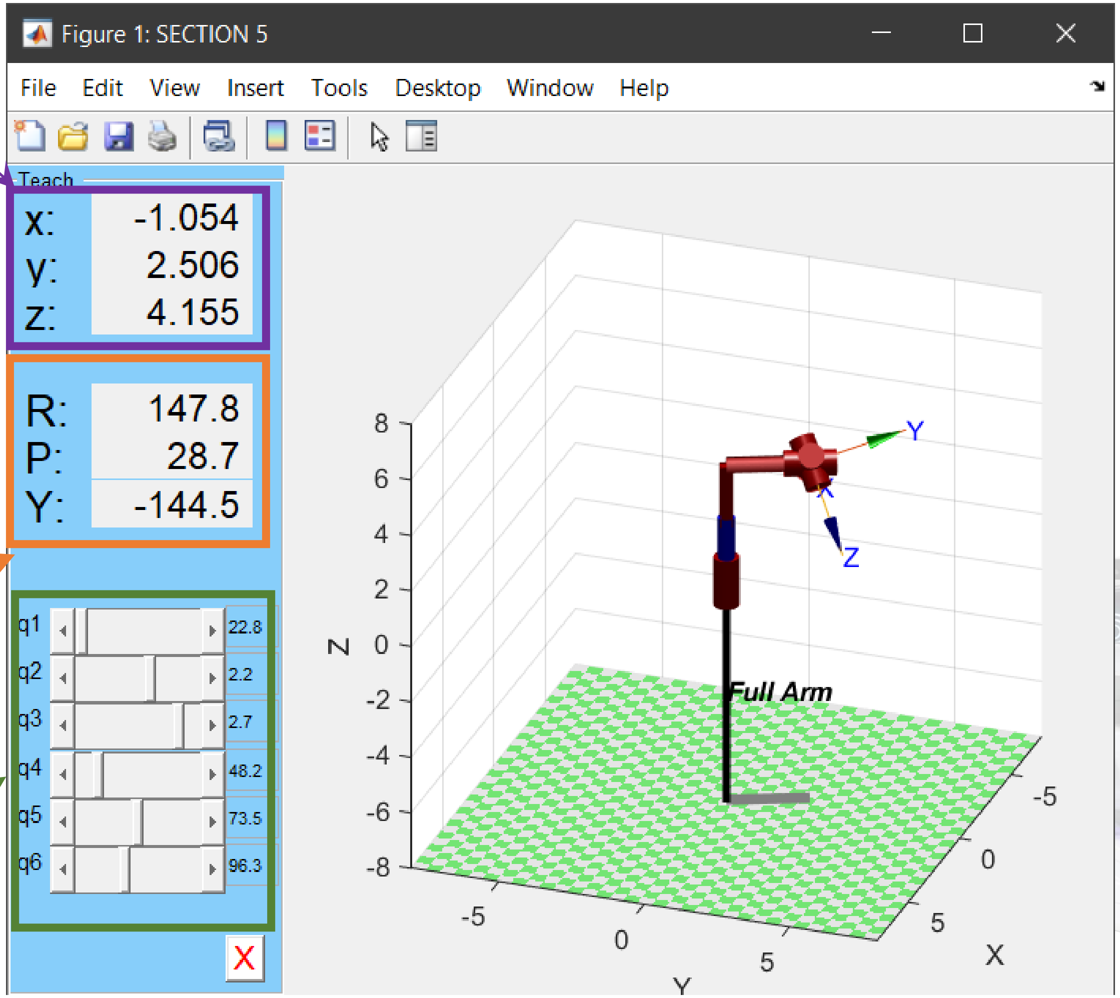
TRANSLATION

$$\prod_{i=1}^{6} A_i = \prod_{i=1}^{6} \begin{bmatrix} \boxed{\begin{matrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i \\ 0 & s\alpha_i & c\alpha_i \end{matrix}} & \boxed{\begin{matrix} a_i c\theta_i \\ a_i s\theta_i \\ d_i \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

ORIENTATION

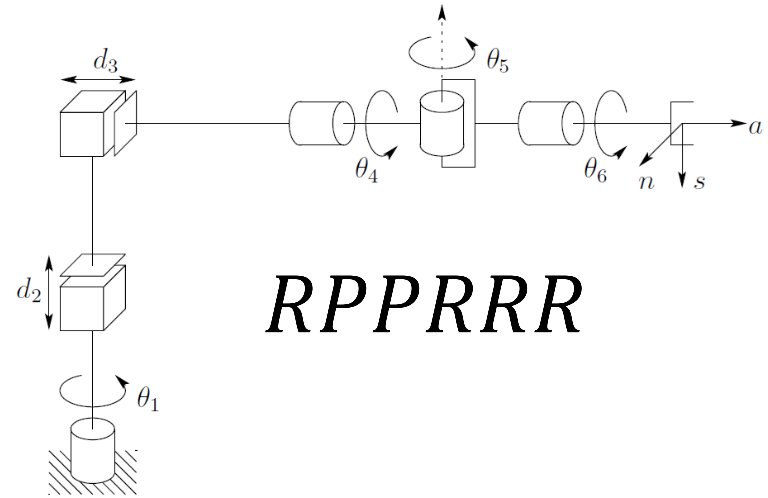
$$\boxed{[\theta_1 d_2\ d_3\ \theta_4\ \theta_5\ \theta_6]}$$

INPUT

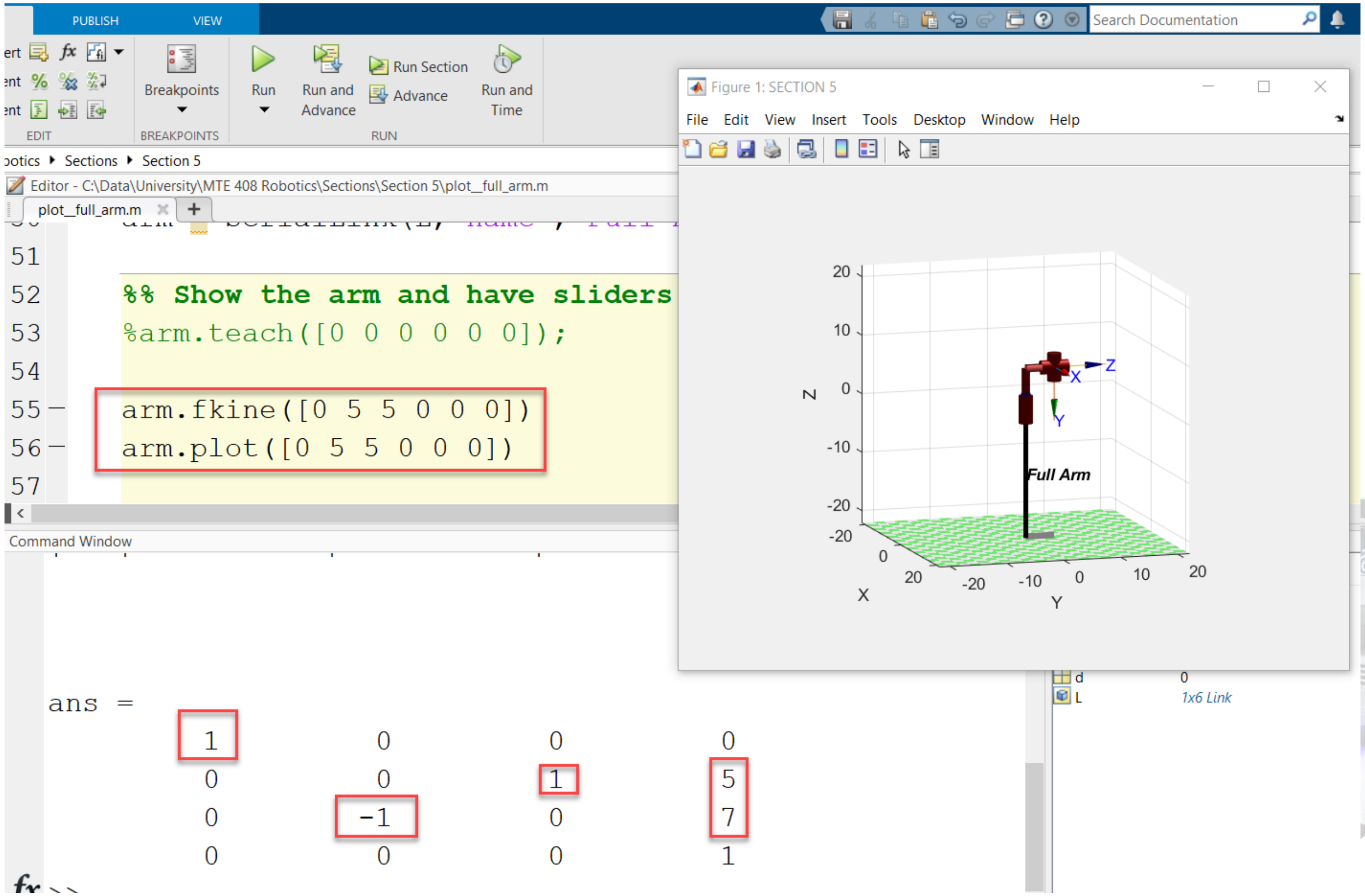


BACK TO NOTATION

FORWARD KINEMATICS



RPPRRR



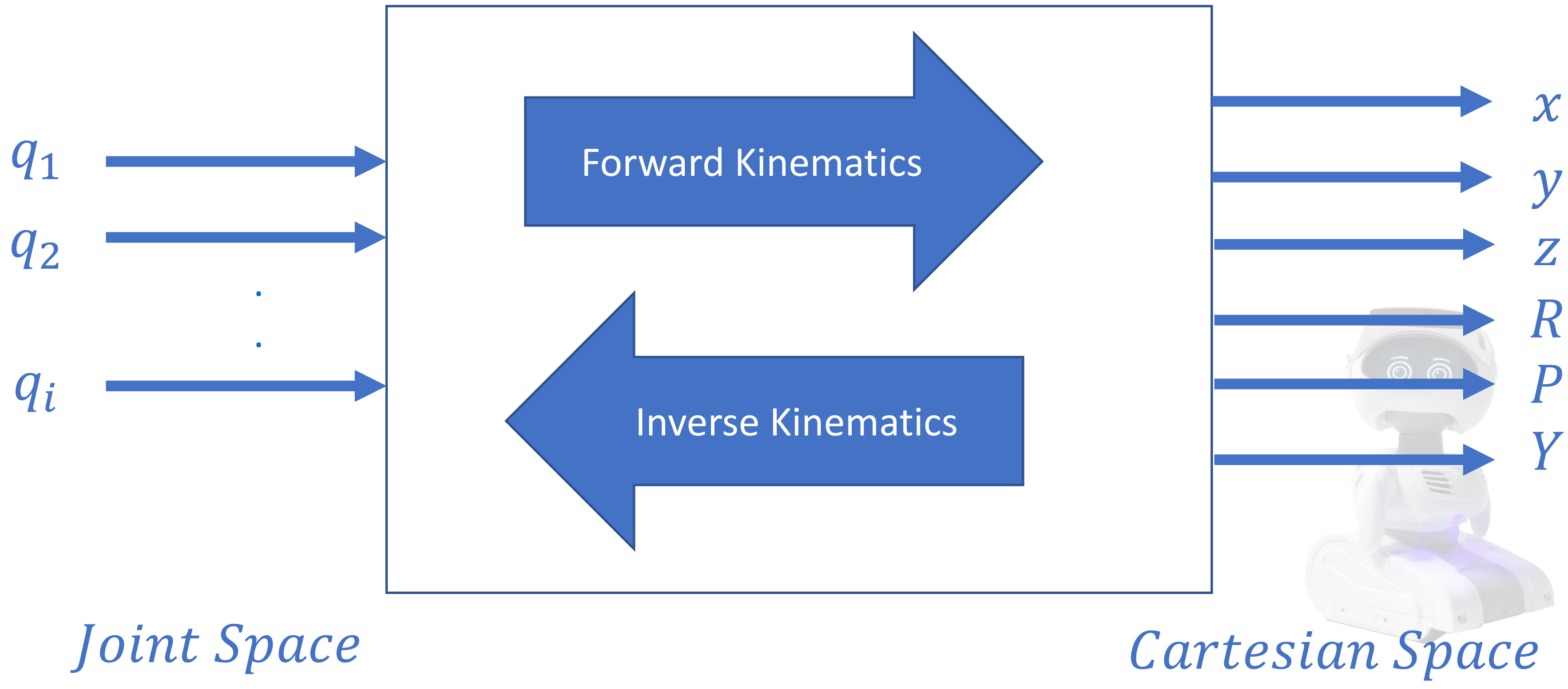
INVERSE KINEMATICS

$\boldsymbol{Given\ End\ Effector}$

$X, Y, Z, Roll, Pitch, Yaw$

$\boldsymbol{Find}$

$$q_i \in \begin{cases} \theta_i \\ d_i \end{cases}$$



INVERSE KINEMATICS

Simple Inverse Kinematics

We want to find:

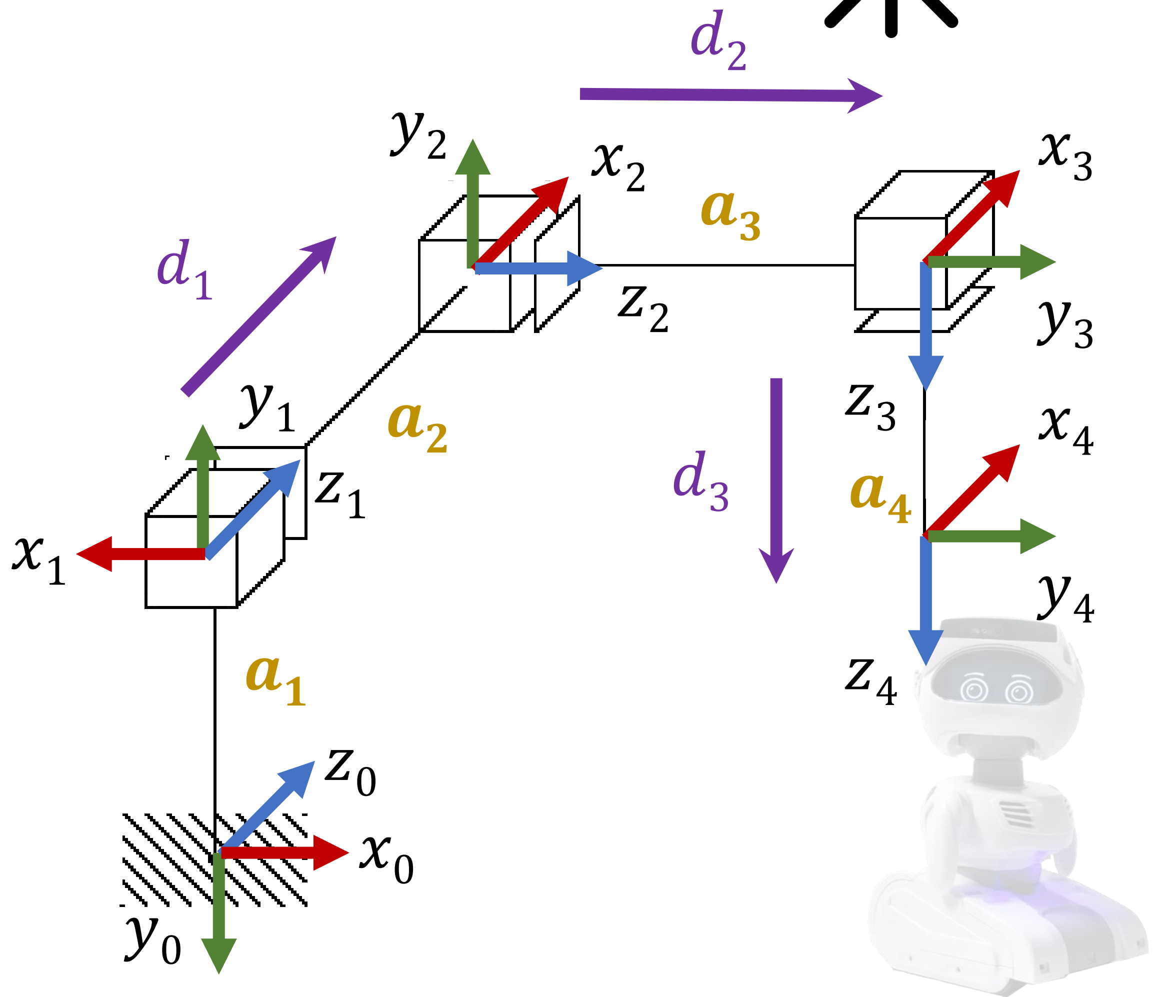
$\boldsymbol{d_1}([{}^0_4x\ {}^0_4y\ {}^0_4z\ a_1 a_2 a_3 a_4])$

$\boldsymbol{d_2}([{}^0_4x\ {}^0_4y\ {}^0_4z\ a_1 a_2 a_3 a_4])$

$\boldsymbol{d_3}([{}^0_4x\ {}^0_4y\ {}^0_4z\ a_1 a_2 a_3 a_4])$

Gripper Orientatation is ignored

$\boxed{\textbf{\textit{WHY?}}}$

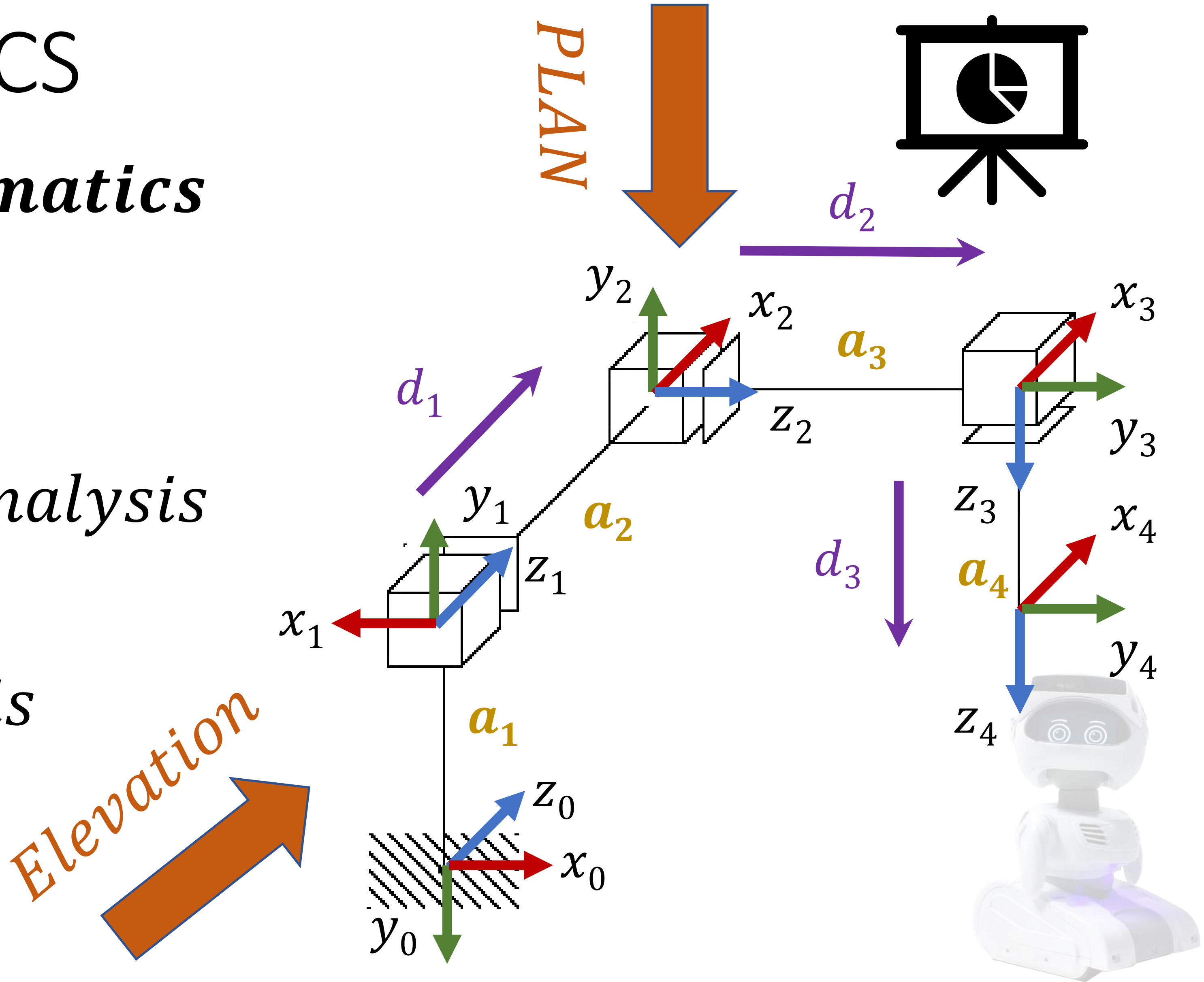


INVERSE KINEMATICS

Simple Inverse Kinematics

Geometric Analysis

1. ***Elevation*** *View analysis*
2. ***Plan*** *View analysis*



INVERSE KINEMATICS

Geometric Analysis

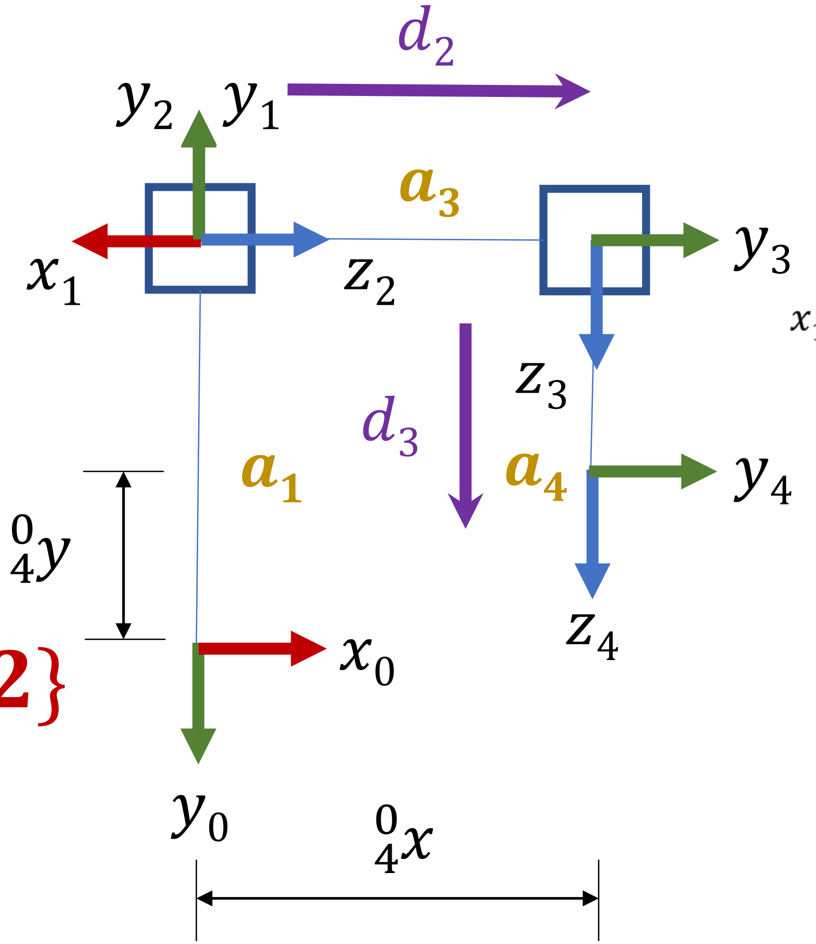
① ***Elevation*** *View analysis*

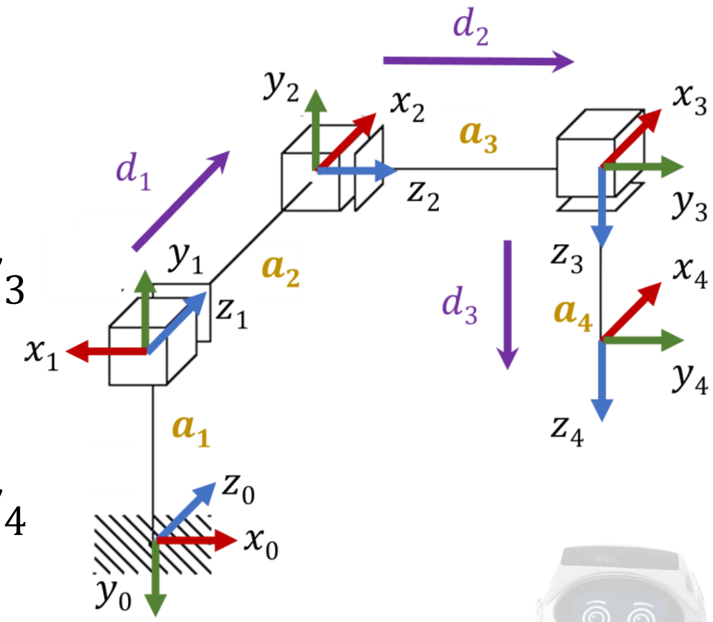
$$ {}^{0}_{4}x = a_3 + d_2 $$

$$ \boldsymbol{d_2 = {}^{0}_{4}x - a_3 \rightarrow \{1\}} $$

$$ -{}^{0}_{4}y = a_1 - (a_4 + d_3) $$

$$ \boldsymbol{d_3 = {}^{0}_{4}y + a_1 - a_4 \rightarrow \{2\}} $$





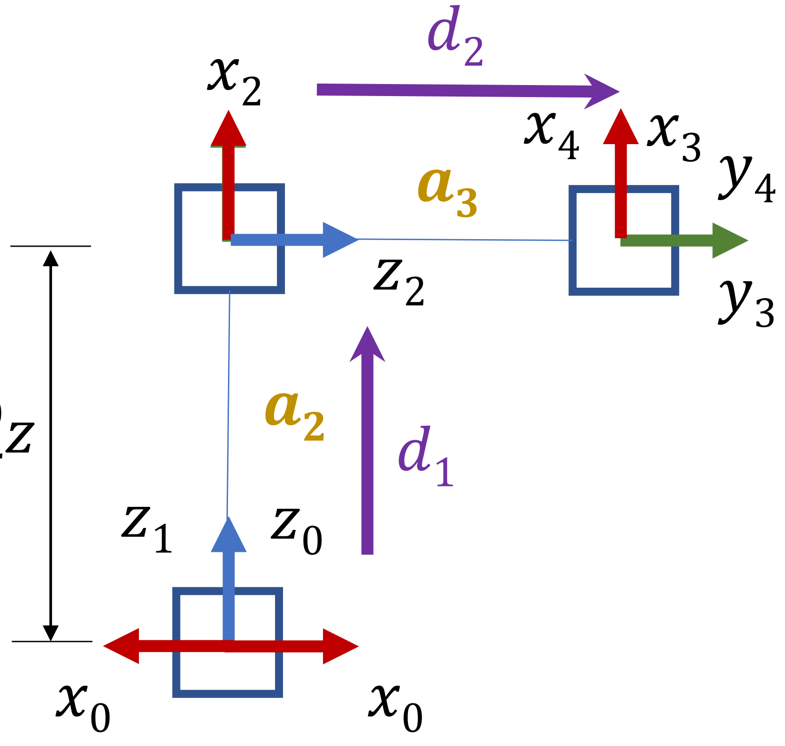
INVERSE KINEMATICS

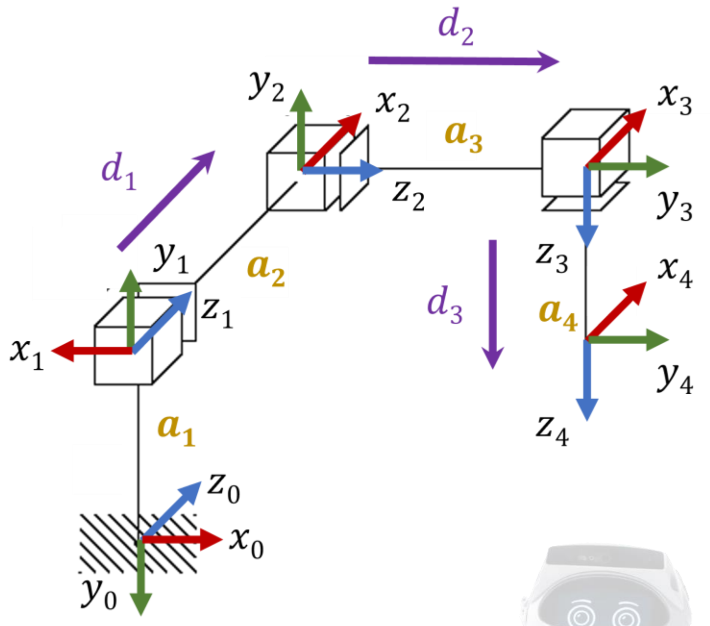
Geometric Analysis

2 ***Plan** View analysis*

$${}^{0}_{4}z = a_2 + d_1$$

$$\boldsymbol{d_1 = {}^{0}_{4}z - a_2 \rightarrow \{3\}}$$





INVERSE KINEMATICS

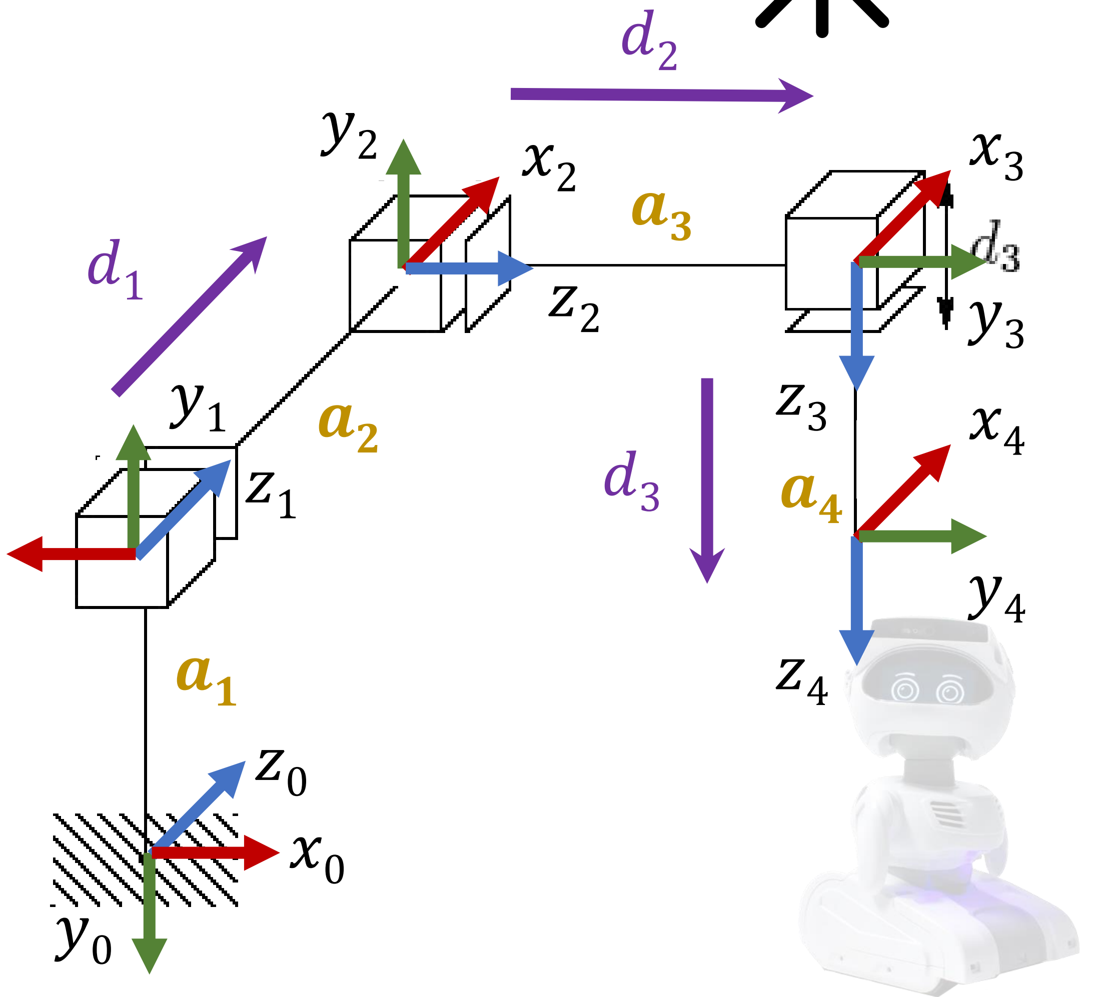
Final Solution

$$d_1 = {}^0_4z - a_2 \rightarrow \{3\}$$

$$d_2 = {}^0_4x - a_3 \rightarrow \{1\}$$

$$d_3 = {}^0_4y + a_1 - a_4 \rightarrow \{2\}$$

$$\begin{bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} {}^0_4\boldsymbol{x} \\ {}^0_4\boldsymbol{y} \\ {}^0_4\boldsymbol{z} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{a_3} \end{bmatrix}$$



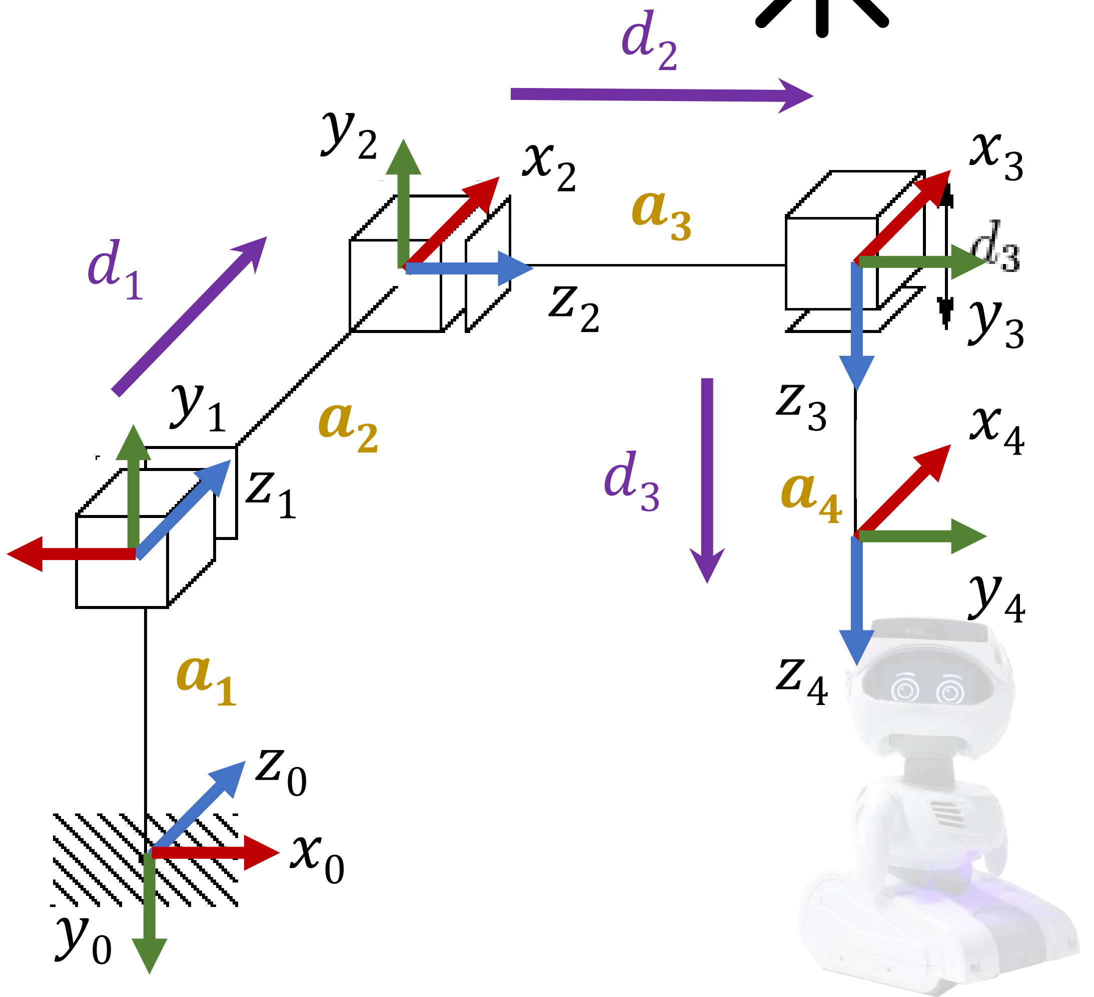
INVERSE KINEMATICS

Assignment

Using Peter Croke Toolbox:

Create the cartesian robot

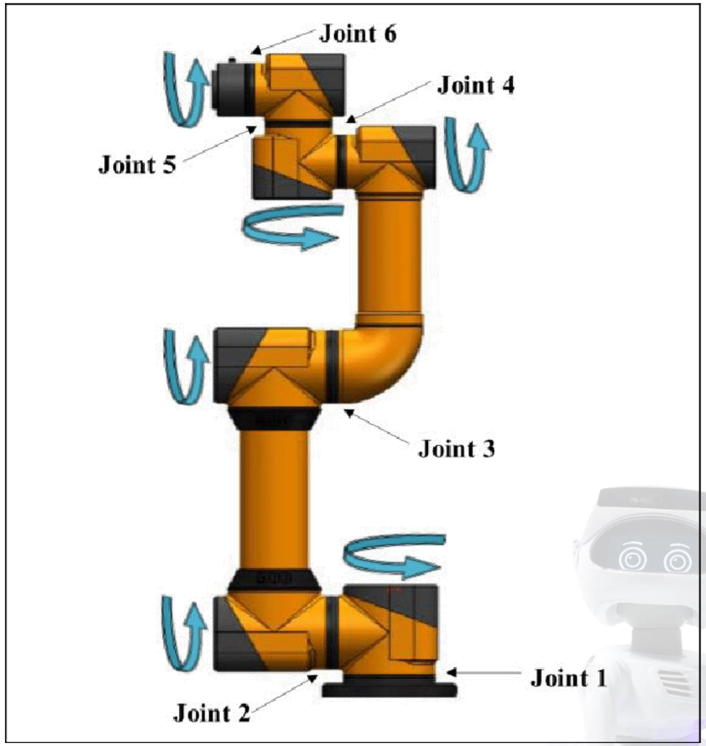
Find the forward and inverse kinematics



INVERSE KINEMATICS

Assumptions for 6 DOF

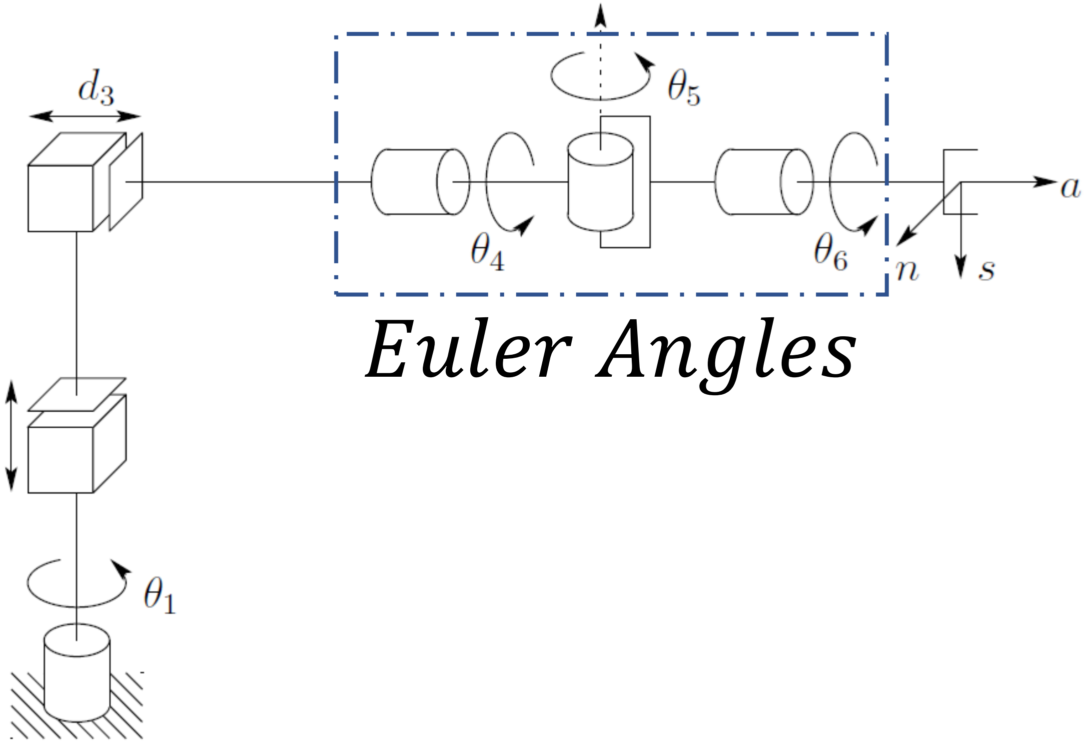
1. *The first three joints determine the* ***end effector position***
2. *The last three joints determine the* ***end effector orientation***
3. *The manipulator is* ***6 DOF***
4. *The last three joints are* ***spherical wrist***



INVERSE KINEMATICS

Assumptions for 6 DOF

4 *The last three joints are* ***spherical wrist***



NEXT SECTION : *Inverse Kinematics for Articulated Arm*