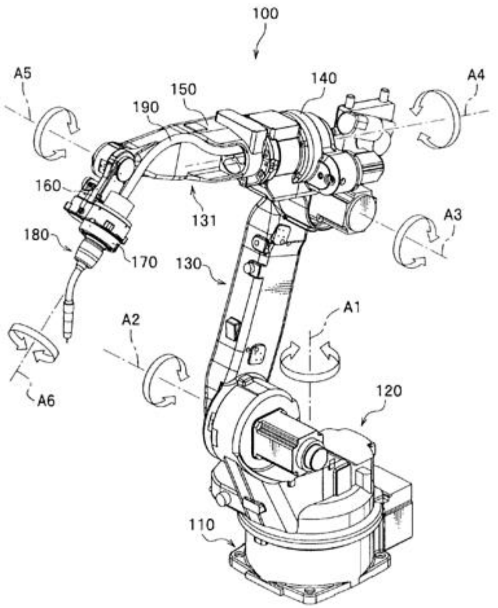
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 3
INTRODUCTION TO ROBOTICS LAB

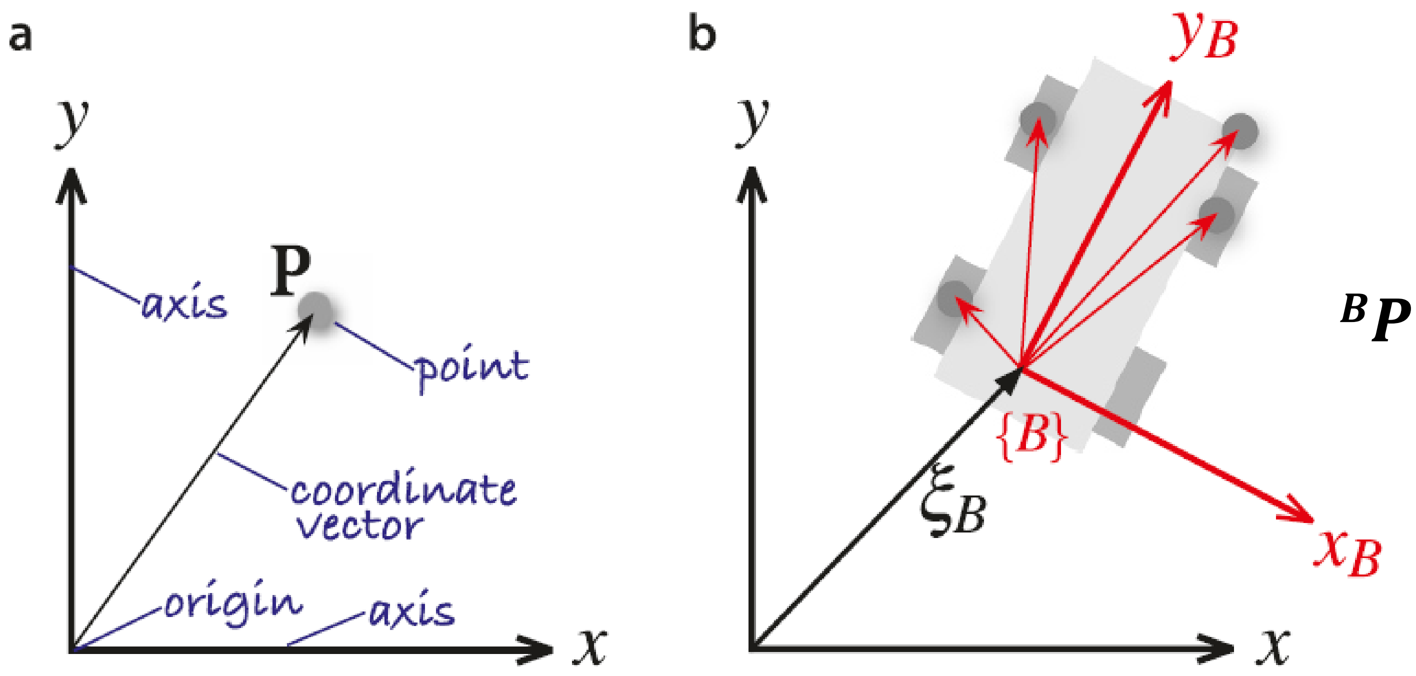
WALEED ELBADRY

MARCH 2022



SUMMARY

A ***POINT*** *is represented as a vector*



$$ {}^{A}P = \begin{bmatrix} P_x \\ P_y \end{bmatrix} $$

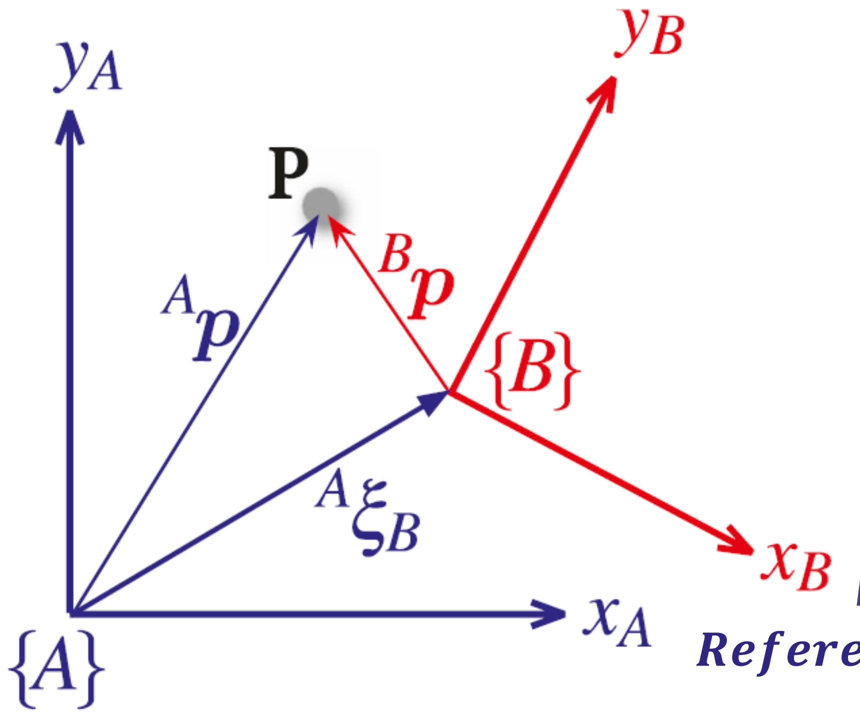
${}^{\boldsymbol{B}}\boldsymbol{P}$ … *Point P with respect to frame A*

$\boldsymbol{x_B}$ … $X - Axis\ of\ frame\ B$

$\boldsymbol{y_B}$ … $Y - Axis\ of\ frame\ B$

SUMMARY

*What is the **POSE** ?*



$$\xi \rightarrow Pronounced\ (Ksi)(Xi)$$

$$\xi \ldots Pose$$

$$^{\boldsymbol{A}}\boldsymbol{\xi}_{\boldsymbol{B}} \ldots Pose\ of\ Frame\ \boldsymbol{B}\ relative\ to\ \boldsymbol{A}$$

Reference frame

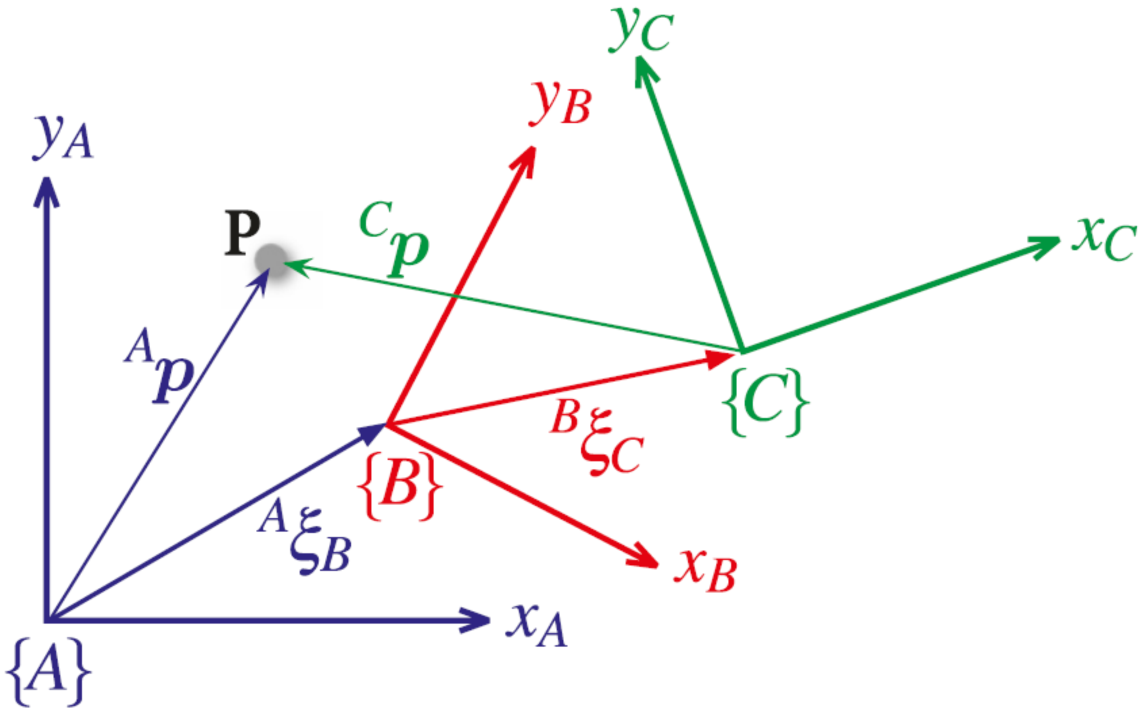
Transformed frame

$$^AP = \begin{bmatrix} P_{x_A} \\ P_{y_A} \end{bmatrix} \quad ^BP = \begin{bmatrix} P_{x_B} \\ P_{y_B} \end{bmatrix}$$

$$^AP = {}^{\boldsymbol{A}}\boldsymbol{\xi}_{\boldsymbol{B}}{}^BP$$

SUMMARY

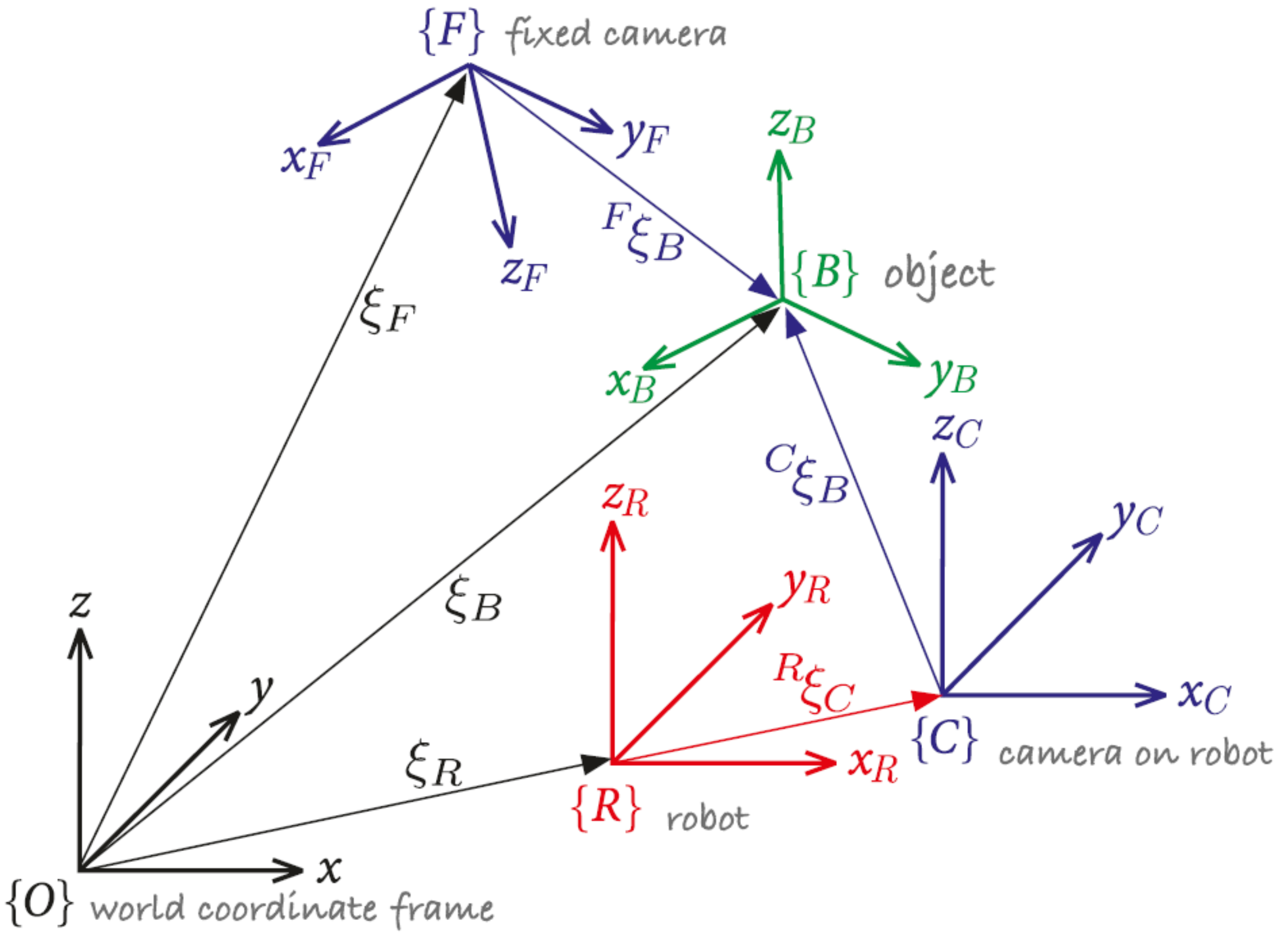
***Relative** poses*



$$ {}^{A}P = \underbrace{{}^{A}\xi_{\cancel{B}}\,{}^{\cancel{B}}\xi_{C}}_{\textbf{Compound}\ \text{Relative Poses}}\,{}^{C}P $$

SUMMARY

***Relative** poses*

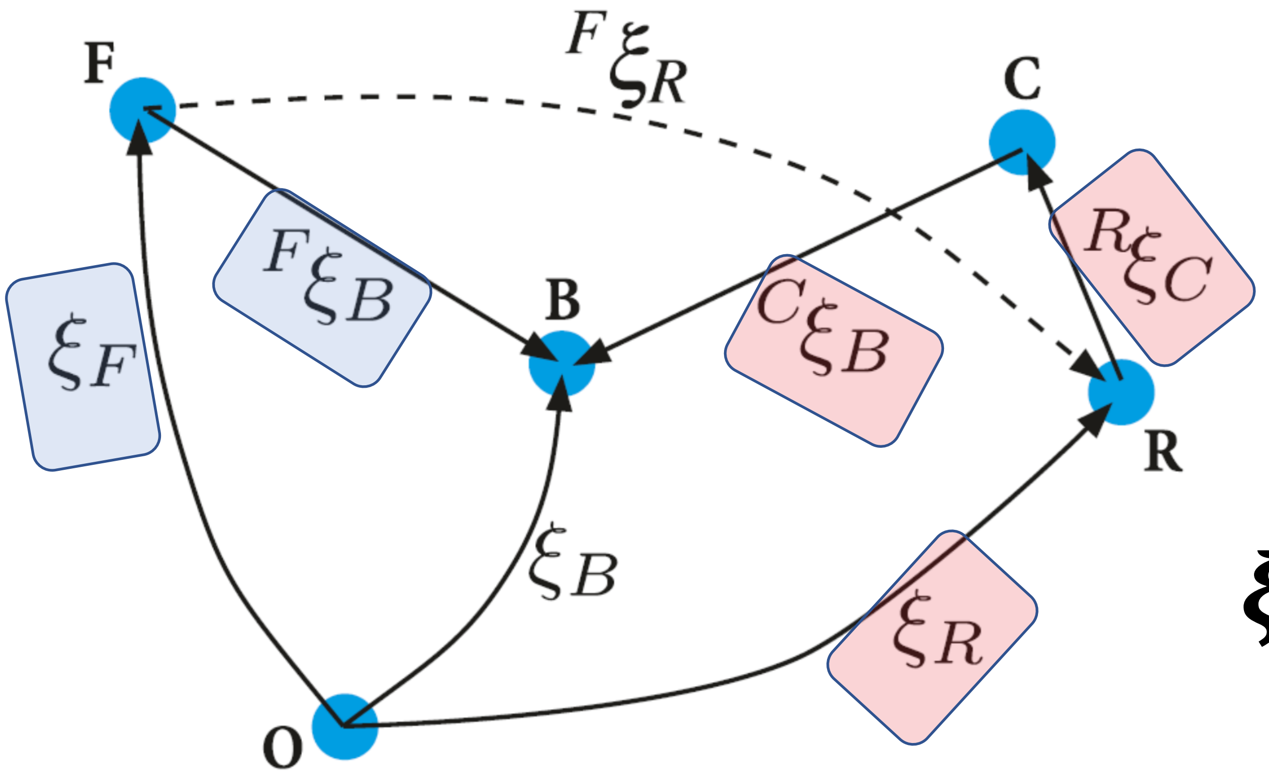


$$\cancel{O}\xi_F\,{}^F\xi_B = \cancel{O}\xi_R\,{}^R\xi_C\,{}^C\xi_B$$

$$\xi_F\,{}^F\xi_B = \xi_R\,{}^R\xi_C\,{}^C\xi_B$$

SUMMARY

Relative *poses*



Directed Graph

$$\xi_B = \xi_F\,{}^F\xi_B = \xi_R\,{}^R\xi_C\,{}^C\xi_B$$

SUMMARY

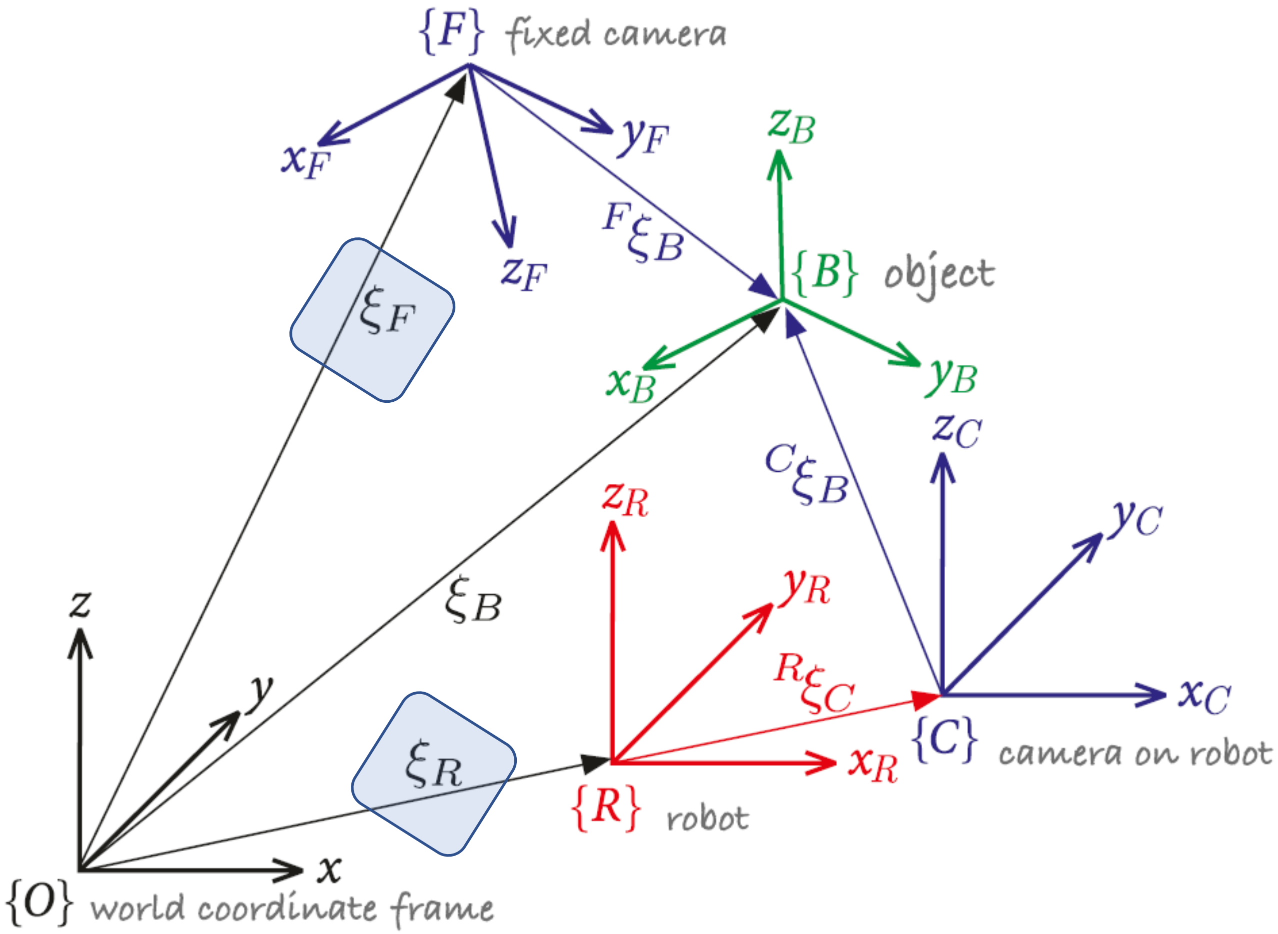
***Relative** poses*

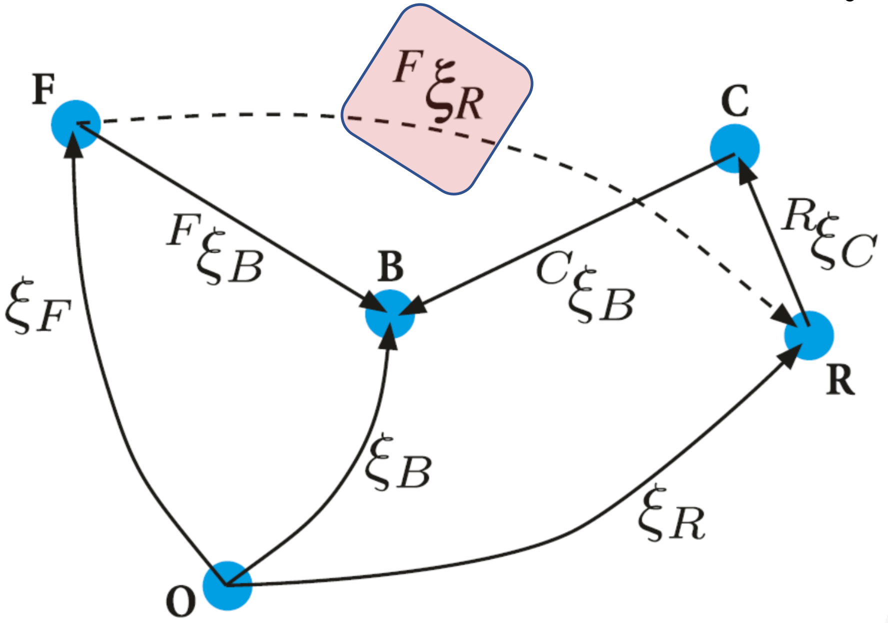
$$Inv({}^{F}\xi_{B}) = {}^{B}\xi_{F}$$

$${}^{F}\xi_{B}\,{}^{B}\xi_{C} \neq {}^{B}\xi_{C}\,{}^{F}\xi_{B}$$

SUMMARY

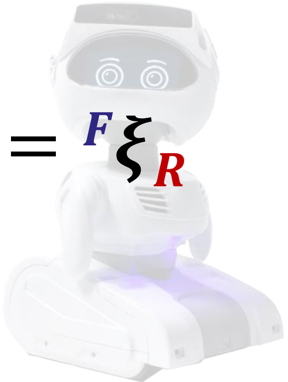
***Relative** poses*





$$Inv({}^O\xi_F)\,{}^O\xi_F\,{}^F\xi_R = I\,{}^F\xi_R = {}^F\xi_R$$

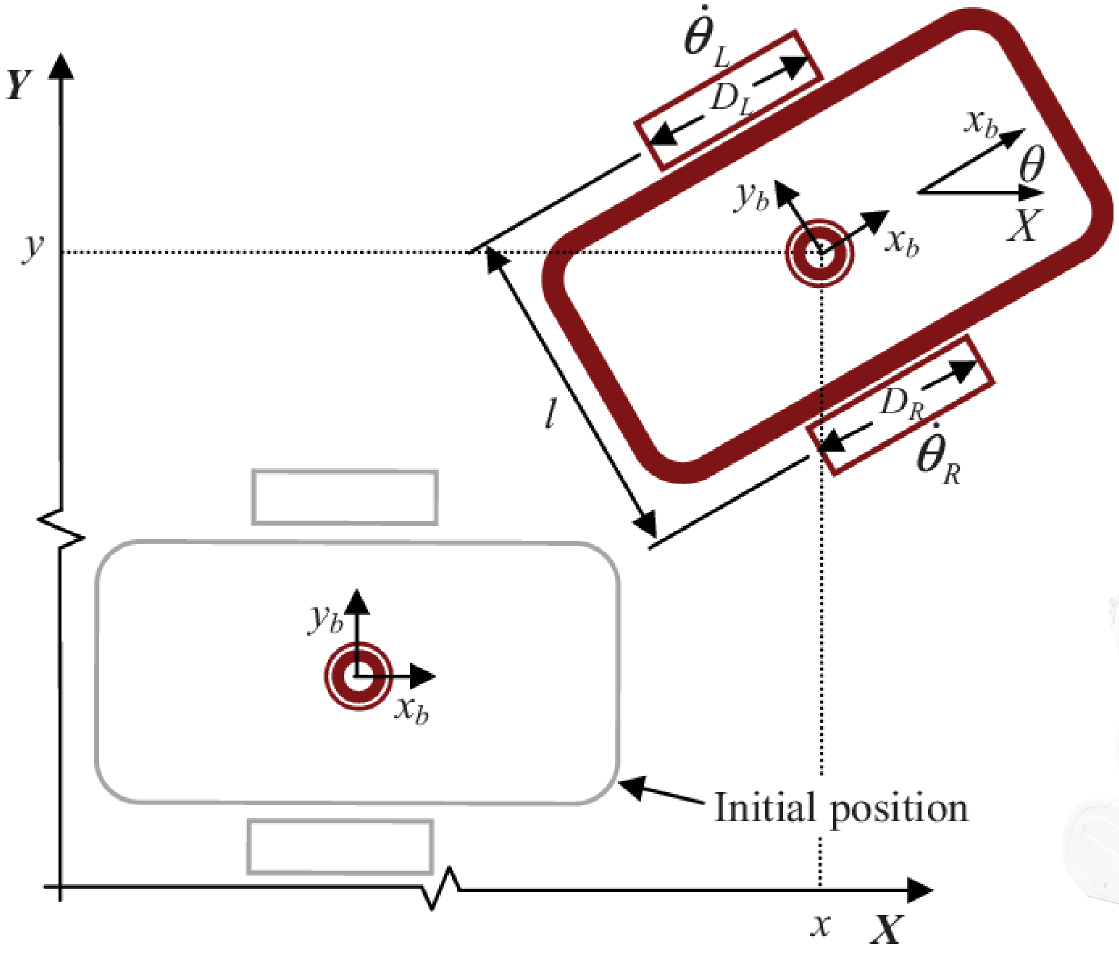
$${}^F\xi_R = Inv({}^O\xi_F)\,{}^O\xi_R$$



SUMMARY

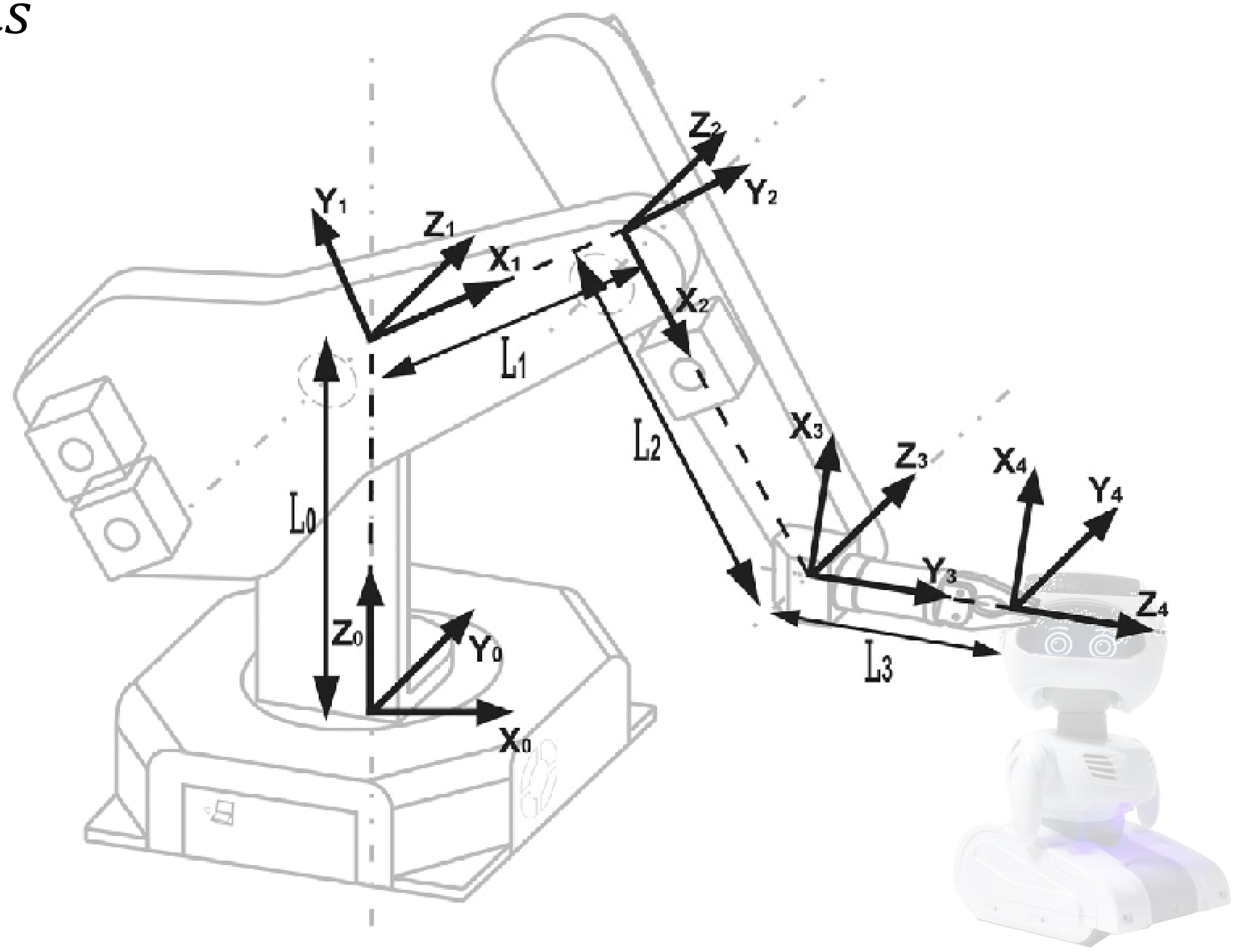
Homogenous Transformation Problems

2D
Mobile Robots



SUMMARY

Homogenous Transformation Problems



3*D*

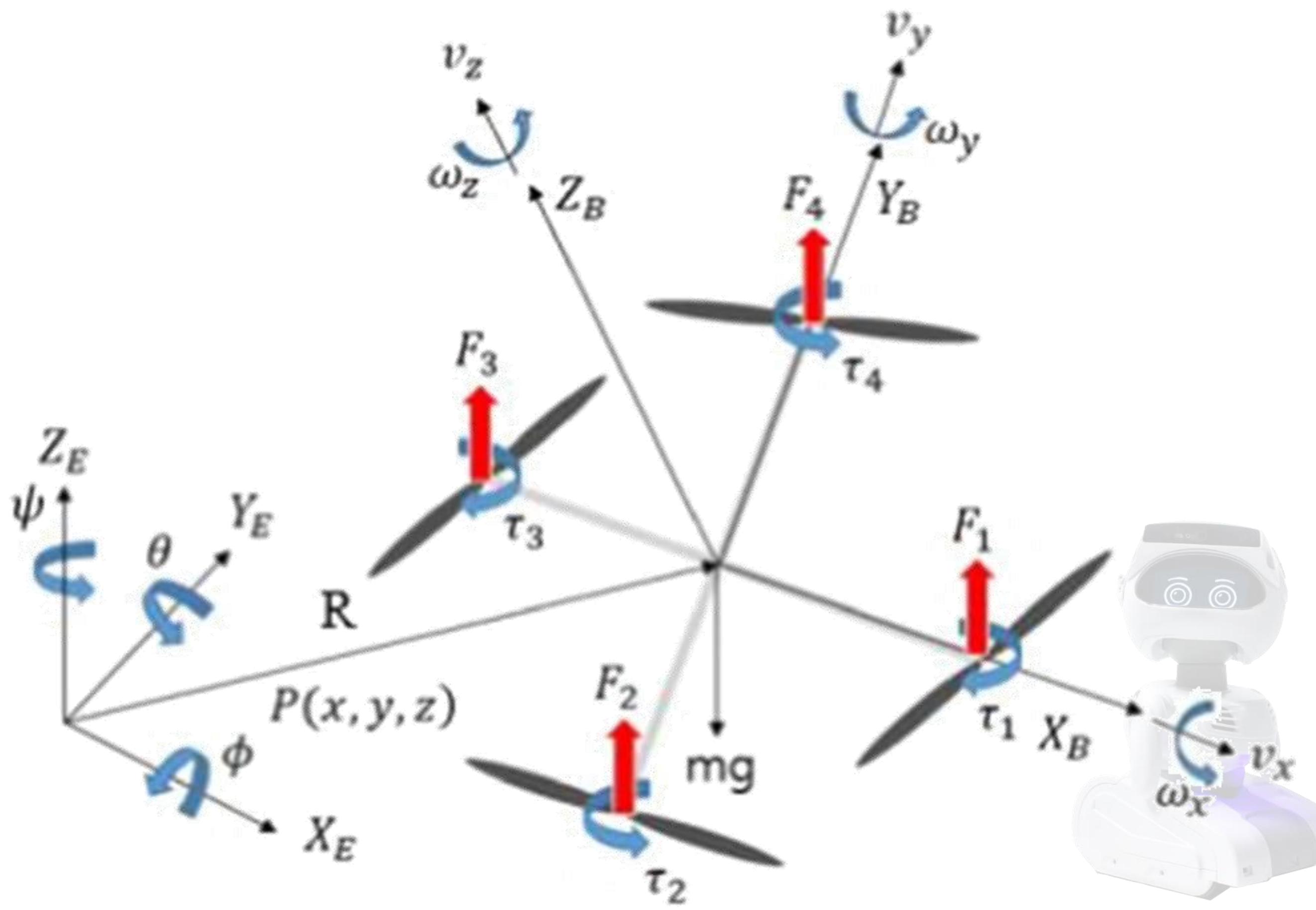
Industrial Robots

SUMMARY

Homogenous Transformation Problems

3D

Flying Robots



SUMMARY

$2D - HOMOGENOUS\ TRANSFORM$

$\boldsymbol{ROTATION + TRANSLATION}$

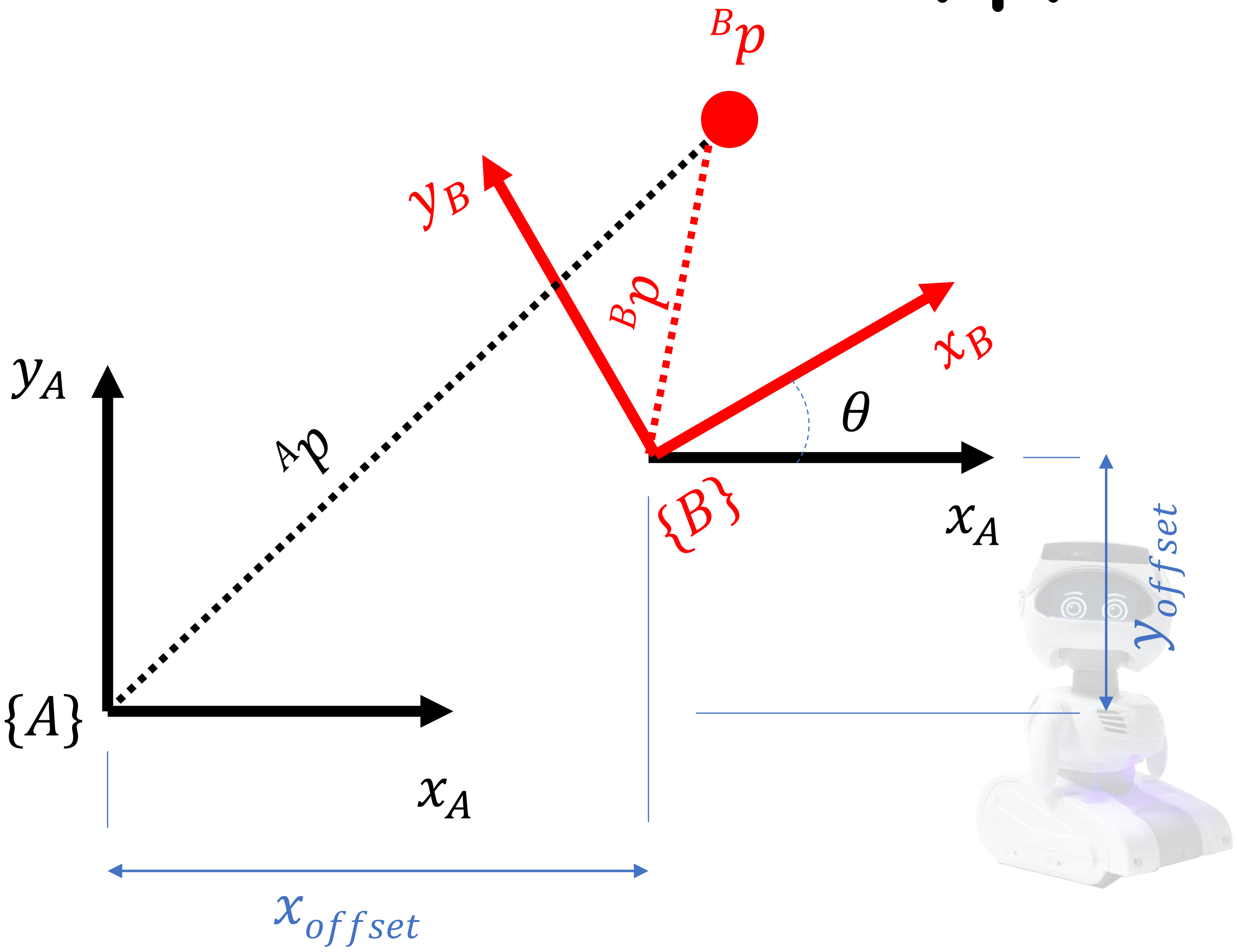
$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} {}^{B}\boldsymbol{x} \\ {}^{B}\boldsymbol{y} \end{bmatrix} + \begin{bmatrix} x_{offset} \\ y_{offset} \end{bmatrix}$$

$\boldsymbol{STANDARD\ FORM}$

t

$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boxed{\boldsymbol{1}} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & x_{offset} \\ sin\theta & cos\theta & y_{offset} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}\boldsymbol{x} \\ {}^{B}\boldsymbol{y} \\ \boxed{\boldsymbol{1}} \end{bmatrix}$$

R $Scaling$



SUMMARY

$2D\ - HOMOGENOUS\ TRANSFORM$

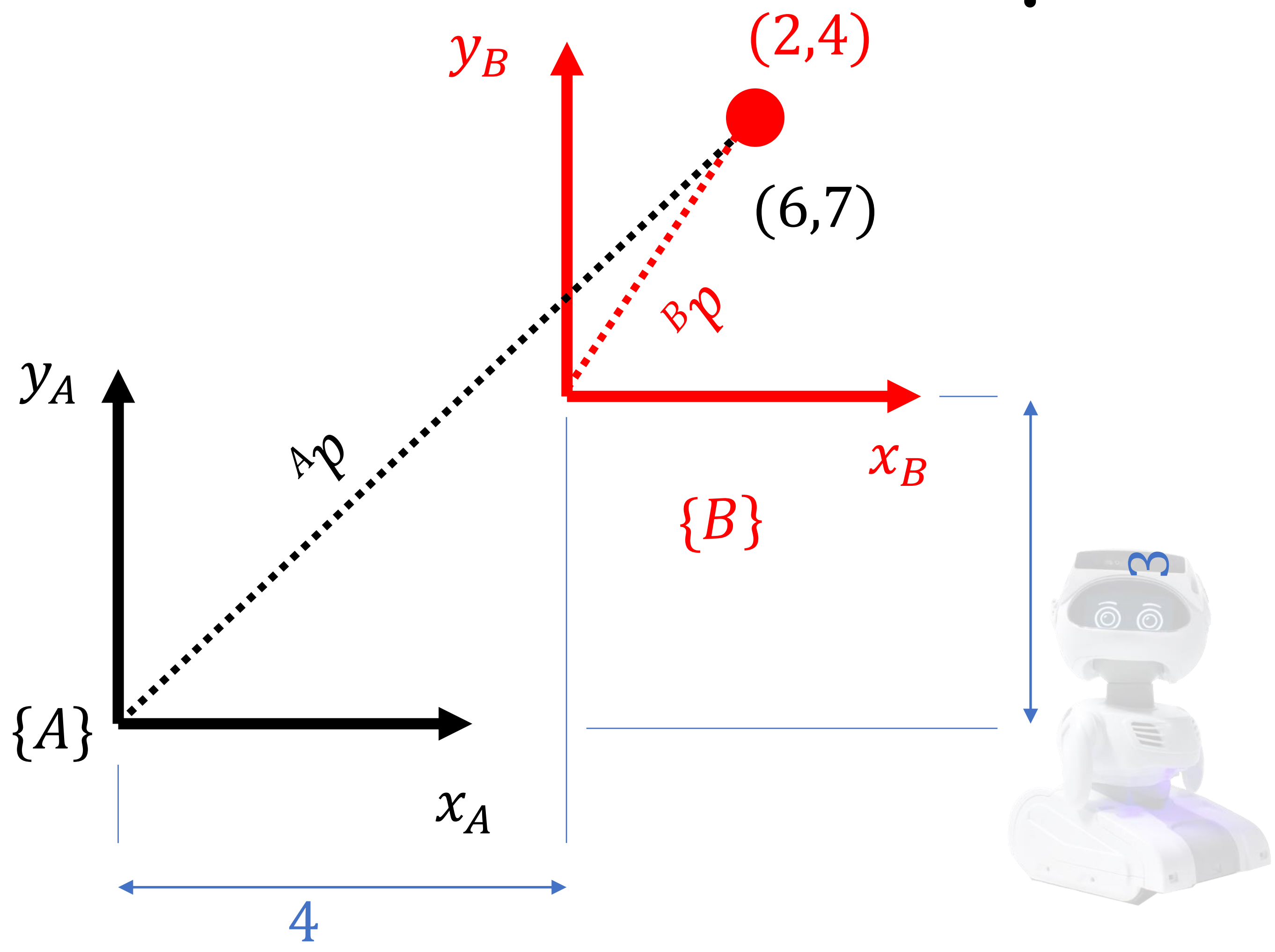
$Example\ 1$

$${}^{B}\boldsymbol{p} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad \boldsymbol{\theta_z} = 0^o$$

$$[{}^{A}\boldsymbol{p}] = \begin{bmatrix} {}^{A}R_B & {}^{A}t_B \\ 0 & 1 \end{bmatrix} [{}^{B}\boldsymbol{p}]$$

$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} \cos(0^o) & -\sin(0^o) & 4 \\ \sin(0^o) & \cos(0^o) & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{6} \\ \boldsymbol{7} \\ \boldsymbol{1} \end{bmatrix}$$



SUMMARY

$2D\ - HOMOGENOUS\ TRANSFORM$

$\boldsymbol{example\_1\_translation}$

$${}^{B}\boldsymbol{p} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad \boldsymbol{\theta_z} = 0^{o}$$

$\boldsymbol{commands}$

$SE2(x, y, thetaZ)$

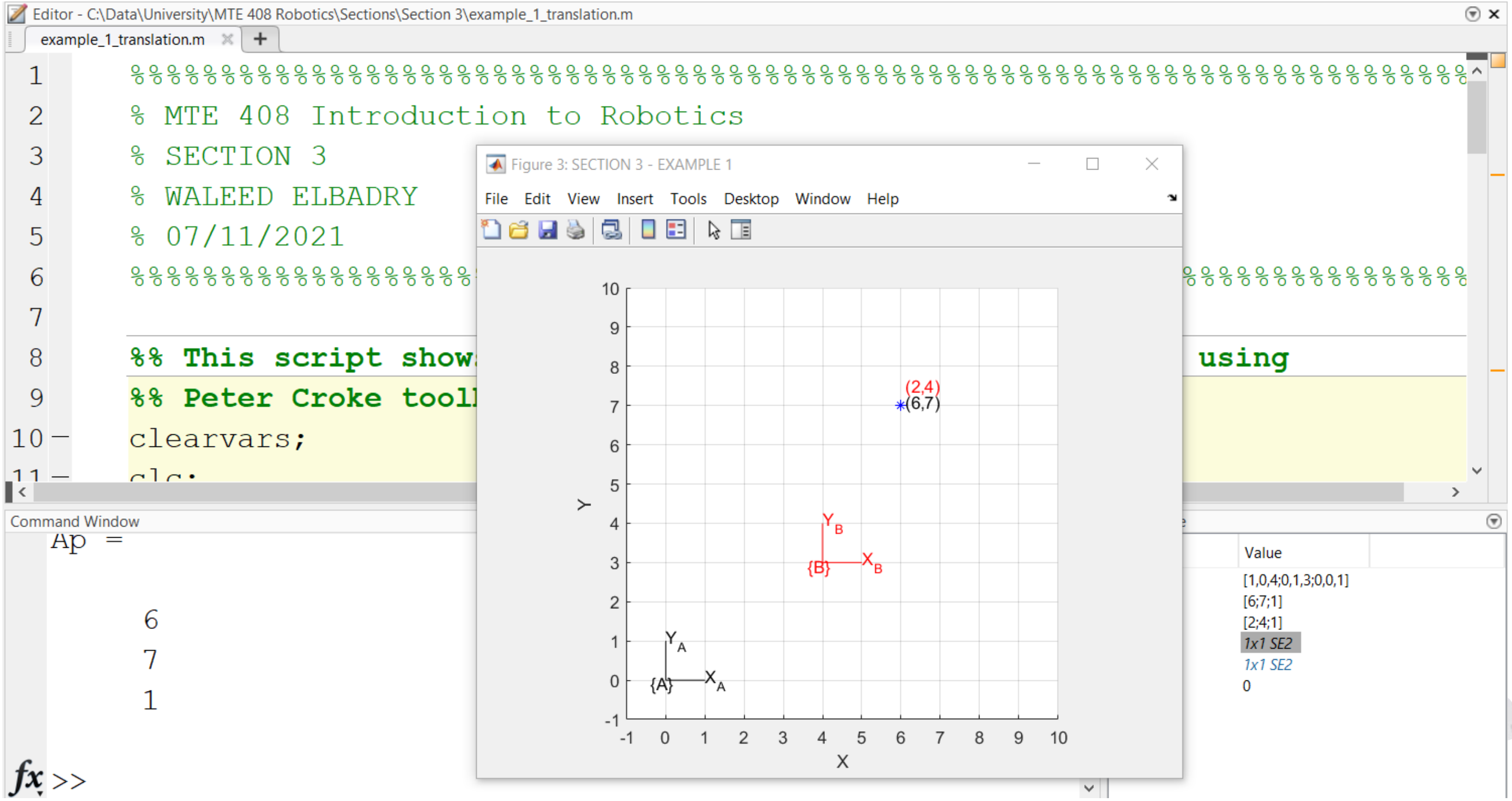
$SE2.T$

$trplot2(T, 'frame', 'color')$

$point\_plot$

$plot$

$text$



SUMMARY

$2D - HOMOGENOUS\ TRANSFORM$

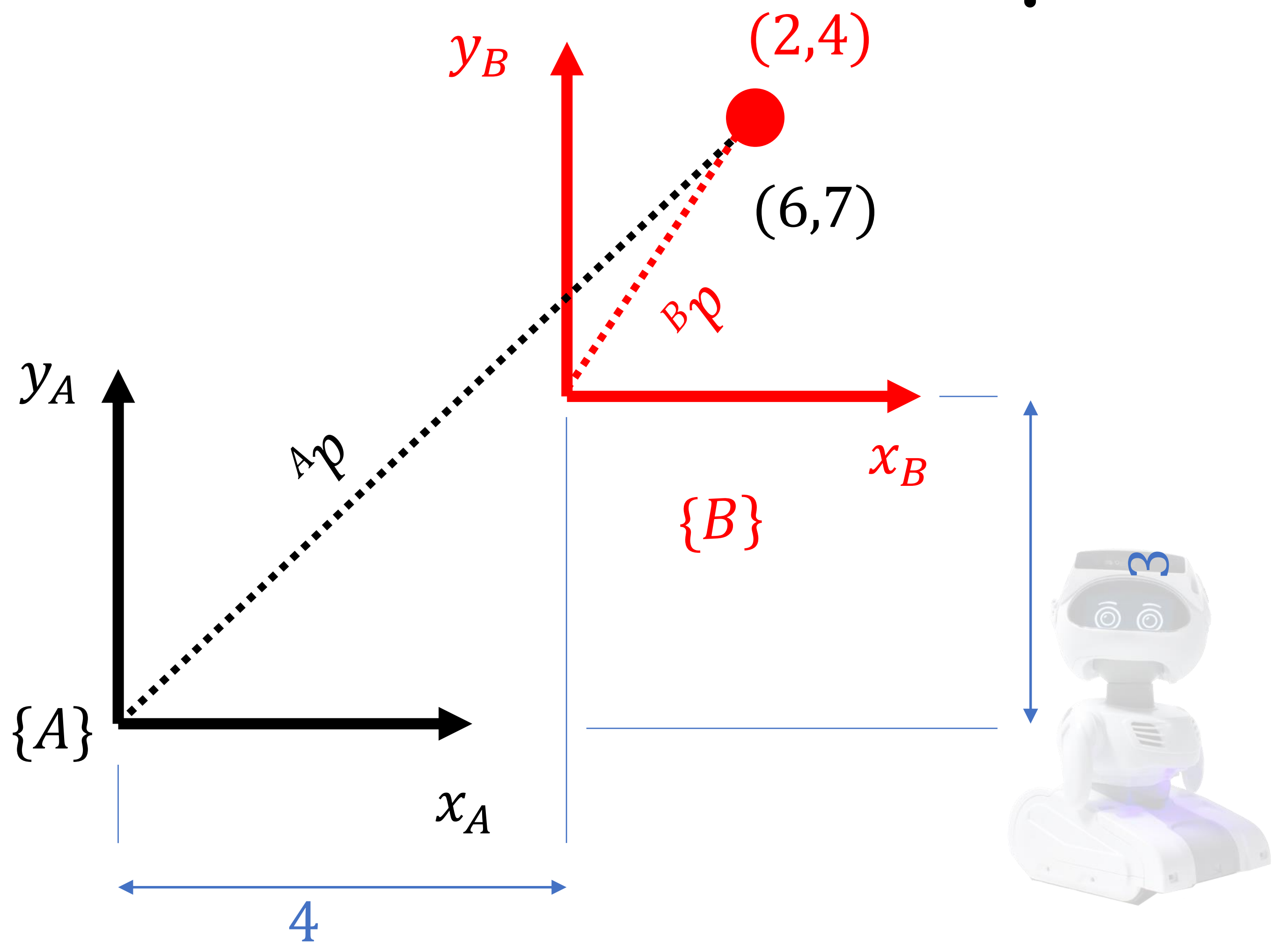
$Example\ 1$

$$ {}^{B}\boldsymbol{p} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad \boldsymbol{\theta_z} = 0^o $$

$$ [{}^{A}\boldsymbol{p}] = \begin{bmatrix} {}^{A}R_B & {}^{A}t_B \\ 0 & 1 \end{bmatrix} [{}^{B}\boldsymbol{p}] $$

$$ \begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} \cos(0^o) & -\sin(0^o) & 4 \\ \sin(0^o) & \cos(0^o) & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} $$

$$ \begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{6} \\ \boldsymbol{7} \\ \boldsymbol{1} \end{bmatrix} $$



SUMMARY

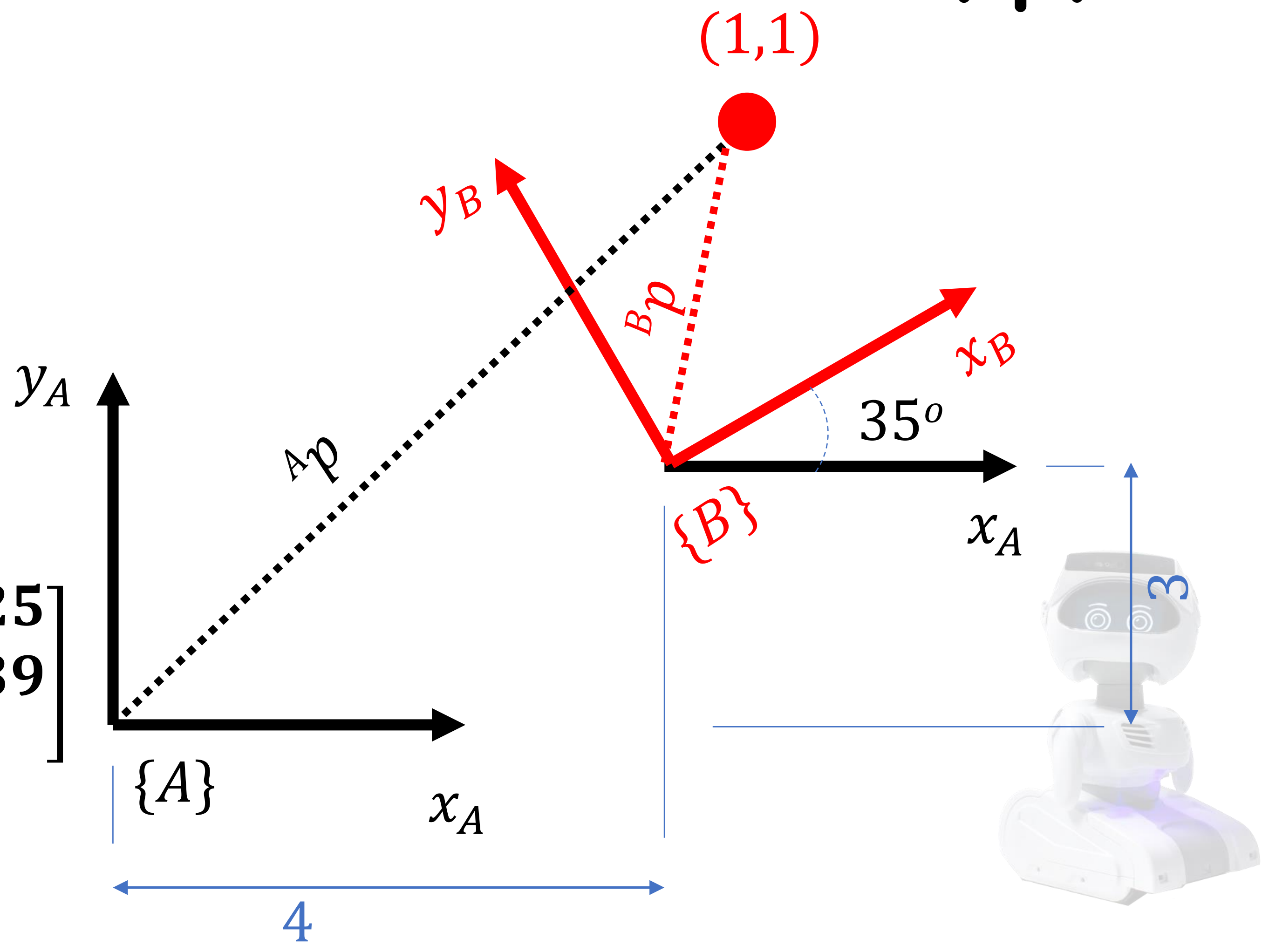
$2D\ - HOMOGENOUS\ TRANSFORM$

Example 2

$${}^{B}\boldsymbol{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \boldsymbol{\theta}_{\boldsymbol{z}} = 35^{o}$$

$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} \cos(35^{o}) & -\sin(35^{o}) & 4 \\ \sin(35^{o}) & \cos(35^{o}) & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{A}\boldsymbol{x} \\ {}^{A}\boldsymbol{y} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} 0.819 & -0.574 & 4 \\ 0.574 & 0.819 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{4.25} \\ \boldsymbol{4.39} \\ \boldsymbol{1} \end{bmatrix}$$



SUMMARY

$2D\ - HOMOGENOUS\ TRANSFORM$

$$\boldsymbol{example\_2\_transformation\_2d}$$

$${}^{B}\boldsymbol{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \boldsymbol{\theta_z} = 35^{o}$$

$\boldsymbol{commands}$

$SE2(x, y, thetaZ)$

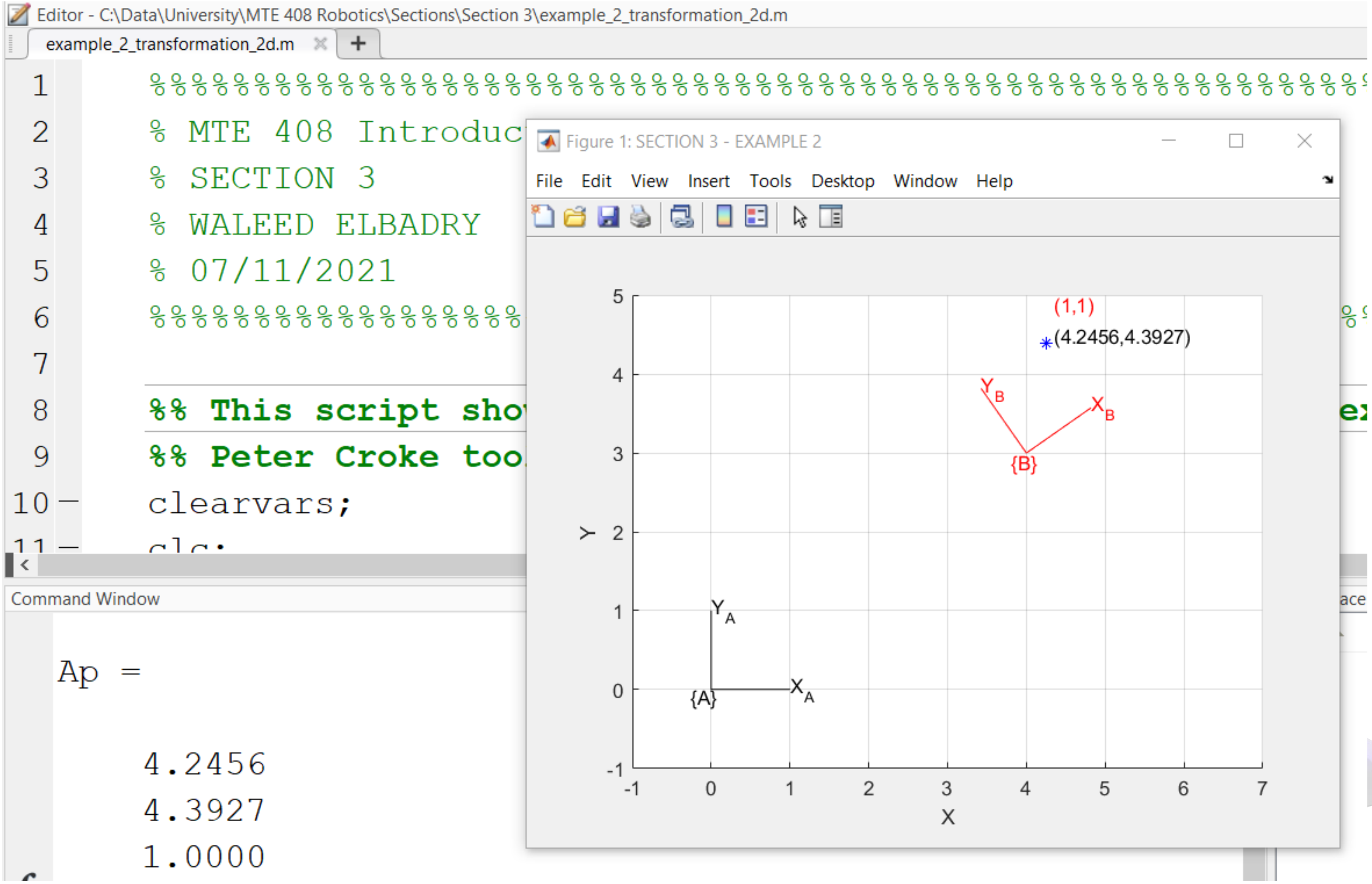
$SE2.T$

$trplot2(T, 'frame', 'color')$

$point\_plot$

$plots$

$text$



SUMMARY

Assignment

Assume any missing data, compute $^{A}\xi_{C}$, $^{B}\xi_{C}$, *and* $^{A}\xi_{B}$ ***transformation and find the*** $^{A}P_{C}$, $^{B}P_{C}$