MTE 408 | ROBOTICS

MOTOR SIZING
PART II: MOTOR SELECTION 2

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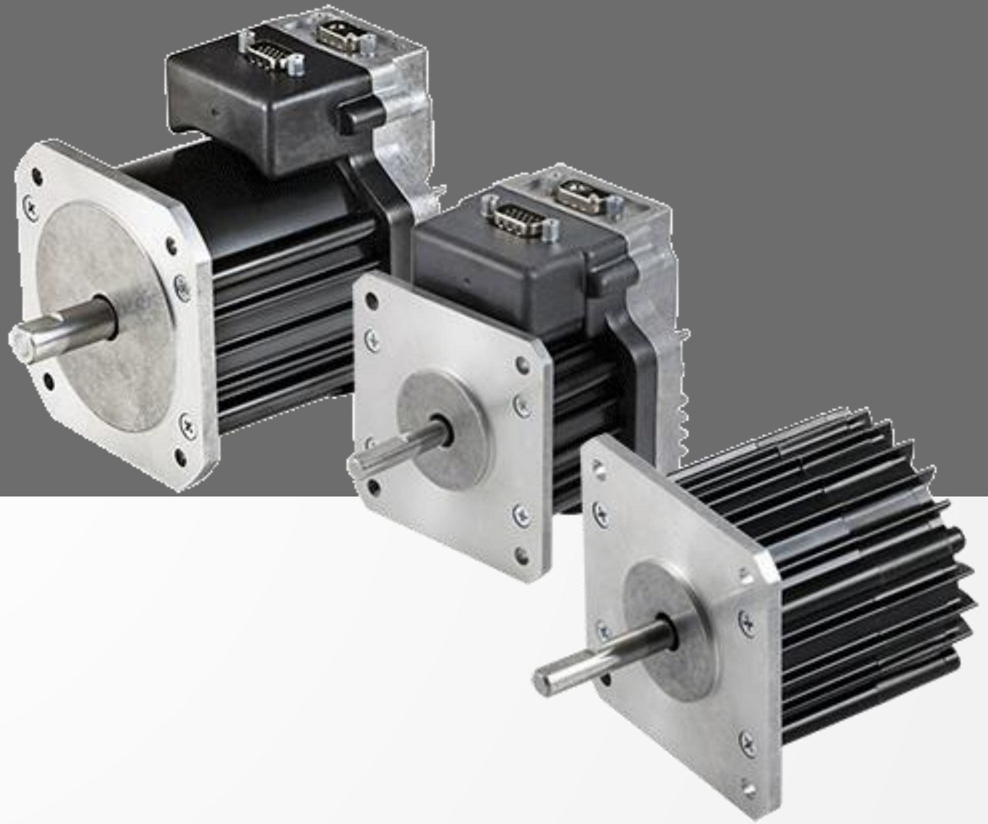
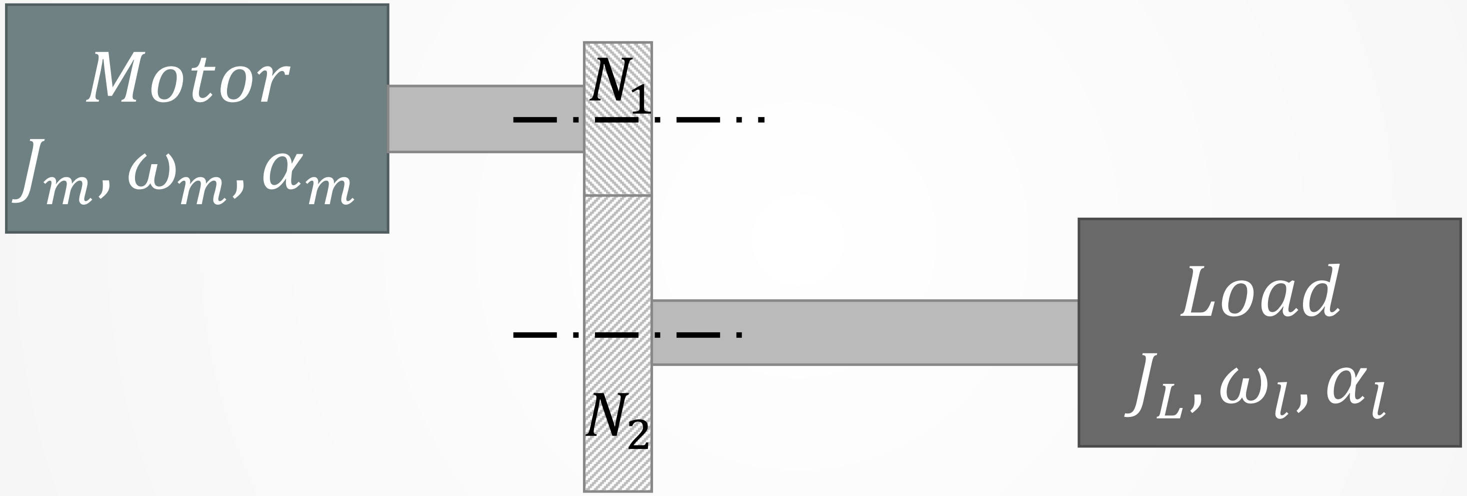
Motor Sizing

And its relationship to trajectory generation



Reflected Inertia (Real Case)

$$\alpha_m^{max} = i.\alpha_L^{max}$$



$$T_m = \left(J_m + J_{gearbox} + \frac{J_L}{\eta i^2}\right).\alpha_m^{max} = J_{system}.i.\alpha_L^{max}$$

$$T_h \ldots Resistive\ and\ Holding\ torque \rightarrow T_{h(reflected)} = \frac{T_h}{\eta.i}$$

Total Torque Required

$$T_m = \left(J_m + J_{gearbox} + \frac{J_L}{\eta . i^2}\right) . \alpha_m^{max} = J_{system} . i . \alpha_L^{max}$$

$$T_{h(reflected)} = \frac{T_h}{\eta . i}$$

$$\therefore T_{total} = T_m + T_{h(reflected)} = \left(J_m + J_{gearbox} + \frac{J_L}{\eta . i^2}\right) . i . \alpha_L^{max} + \frac{T_h}{\eta . i}$$

$$\therefore T_{total} = \left(J_m . i + J_{gearbox} . i + \frac{J_L}{\eta . i}\right) . \alpha_L^{max} + \frac{T_h}{\eta . i}$$

$$T_{total}^{max} = \frac{dT_{total}}{di} = 0 \rightarrow T_{total}^{max} = \left(J_m + J_{gearbox} - \frac{J_L}{\eta . i^2}\right) . \alpha_L^{max} - \frac{T_h}{\eta . i^2} = 0$$

Optimal Gear Ratio

$$T_{total}^{max} = \frac{dT_{total}}{di} = 0 \rightarrow T_{total}^{max} = \left(J_m + J_{gearbox} - \frac{J_L}{\eta.i^2}\right).\alpha_L^{max} - \frac{T_h}{\eta.i^2} = 0$$

$$T_{total}^{max} = (J_m + J_{gearbox}).\alpha_L^{max} - \frac{J_L.\alpha_L^{max} + T_h}{\eta.i^2} = 0$$

$$\frac{J_L.\alpha_L^{max} + T_h}{\eta.i^2} = (J_m + J_{gearbox}).\alpha_L^{max}$$

$$\frac{J_L.\alpha_L^{max} + T_h}{\eta.(J_m + J_{gearbox}).\alpha_L^{max}} = i_{optimal}^2 \rightarrow \quad i_{optimal} = \sqrt{\frac{J_L.\alpha_L^{max} + T_h}{\eta.(J_m + J_{gearbox}).\alpha_L^{max}}}$$

Optimal Motor Power

$$i_{optimal} = \sqrt{\frac{J_L . \alpha_L^{max} + T_h}{\eta . (J_m + J_{gearbox}) . \alpha_L^{max}}}$$

$$T_{total} = \left(J_m + J_{gearbox} + \frac{J_L}{\eta . i_{optimal}^2}\right) . i_{optimal} . \alpha_L^{max} + \frac{T_h}{\eta . i}$$

$$\therefore T_{total} = \frac{2}{\eta . i} . (T_h + J_L . \alpha_L^{max})$$

$$\because P_{total} = T_{total} . \omega_m^{max} \rightarrow \left[\frac{2}{\eta . i} . (T_h + J_L . \alpha_L^{max})\right] . i . \omega_L^{max}$$

$$P_{total}^{optimal} = \left[\frac{2}{\eta} . (T_h + J_L . \alpha_L^{max})\right] . \omega_L^{max} \rightarrow \textit{Motor power is a function of load only}$$

Design Procedure (Revisited)

1. *Design the mechanical mechanism*

2. *Motion planning and trajectory generation.*

3. *Obtain* $J_L, \omega_L^{max}, \alpha_L^{max}$ *and* T_h *(From motion study on SolidWORKS)*

4. *Compute the motor power* $\rightarrow P_{total}^{optimal} = \left[\frac{2}{\eta}.(T_h + J_L.\alpha_L^{max})\right].\omega_L^{max}$

5. *Select the motor from catalog* $\rightarrow P_m^{rated} \geq P_{catalog}^{rated}$

6. *Get estimated gear box inertia* $\rightarrow J_{g.b} \cong 0.3 * J_m$

7. *Calculate the optimal gear ratio* $\rightarrow i_{optimal} = \sqrt{\frac{J_L.\alpha_L^{max} + T_h}{\eta.(J_m + J_{gearbox}).\alpha_L^{max}}}$

Design Procedure (Revisited)

$$7.\ \textit{Calculate the optimal gear ratio} \rightarrow i_{optimal} = \sqrt{\frac{J_L.\alpha_L^{max} + T_h}{\eta.\left(J_m + J_{gearbox}\right).\alpha_L^{max}}}$$

Optional -> $\textit{Select gearbox from catalog} \rightarrow J_{g.b}^{new}$

$$8.\ \textit{Confirm Intertia mismatch} \rightarrow I_{mismatch} = \frac{J_L}{i_{optimal}^2 \cdot J_m}$$

$$9.\ \textit{Rated speed factor} \rightarrow \frac{i.\omega_L^{max}}{\omega_m^{max}} \leq 0.8 - 1$$

$$10.\ \textit{Torque factor} \rightarrow \frac{\textit{RMS Loading Torque}\ (T_L^{RMS})}{\textit{Motor Rated Torque}(T_m^{Rated})} \leq 0.8 - 1$$

$$11.\ \textit{Maximum Torque factor} \rightarrow \frac{T_L^{max}}{i.T_m^{max}} \leq 0.8 - 1$$

Design Procedure (Revisited)

$$7.\ \textit{Calculate the optimal gear ratio} \rightarrow i_{optimal} = \sqrt{\frac{J_L . \alpha_L^{max} + T_h}{\eta . (J_m + J_{gearbox}) . \alpha_L^{max}}}$$

$$8.\ \textit{Confirm Intertia mismatch} \rightarrow I_{mismatch} = \frac{J_L}{i_{optimal}^2 . J_m}$$

$$9.\ \textit{Rated speed factor} \rightarrow \frac{i . \omega_L^{max}}{\omega_m^{max}} \leq 0.8 - 1$$

$$10.\ \textit{Torquefactor} \rightarrow \frac{\textit{RMS Loading Torque } (T_L^{RMS})}{\textit{Motor Rated Torque}(T_m^{Rated})} \leq 0.8 - 1 \longrightarrow \times i$$

$$11.\ \textit{Maximum Torquefactor} \rightarrow \frac{T_L^{max}}{i . T_m^{max}} \leq 0.8 - 1$$

Example

Problem: Choose a servo motor of a precise mechanism given the following data:

$$J_L = 100\ Kg.m^2, T_r = 120\ N.m\ , \omega_L^{max} = 40^o s^{-1}$$
$$\alpha_L^{max} = 25^o\ s^{-1}, \eta = \ 72\%$$

Estimate the optimum reduction ratio of the gearbox.

SOLUTION

A. Compute the required motor power (P_m)

$$P_m = \frac{2}{\eta}(M_r + J_L.\alpha_L^{max}).\omega_L^{max}$$

$$= \frac{2}{0.72}\left[120 + (100)(25)\left(\frac{\pi}{180}\right)\right](40)\left(\frac{\pi}{180}\right)$$

$$\boldsymbol{P_m = 320\ w}$$

B. Fetching from catalog the closest power match to be the candidate motor for selection

$$P_{mc} = 370\ w\ , \omega_{mc}^{max} = 314\frac{rad}{s}, J_{mc} = 4.08\ x\ 10^{-3}\ Kg.m^2$$

C. Selection of gearbox inertia

$$J_{gb1} = \underline{0.03}\ J_{mc} = 0.12\ x\ 10^{-3}\ Kg.m^2$$

Take the factor as 0.3 in exam

Example

Problem: Choose a servo motor of a precise mechanism given the following data:

$$J_L = 100\ Kg.m^2, T_r = 120\ N.m\ , \omega_L^{max} = 40^o s^{-1}$$
$$\alpha_L^{max} = 25^o\ s^{-1}, \eta = \ 72\%$$

Estimate the optimum reduction ratio of the gearbox.

SOLUTION

$$\boldsymbol{P_m = 320\ w} \qquad P_{mc} = 370\ w\ , \omega_{mc}^{max} = 314\frac{rad}{s}, J_{mc} = 4.08\ x\ 10^{-3}\ Kg.m^2 \qquad J_{gb1} = 0.12\ x\ 10^{-3}\ Kg.m^2$$

D. Finding the optimum reduction ratio

$$i_{opt} = \sqrt{\frac{T_r + J_L.\alpha_L^{max}}{\eta(J_{mc} + J_{gb1})\alpha_L^{max}}}$$

$$= \sqrt{\frac{120 + (100)(0.44)}{(0.72)(4.08 + 0.12)(10^{-3})(0.44)}}$$

$$\boldsymbol{i_{opt} = 351.0789 \cong 350}$$

E. Fetching from catalog the closest match to optimum gearbox ratio

$$J_{gb} = 10^{-4}\ Kg.m^2\ , \eta_n = 70\% = 0.7$$

Calculating the **torque required**

$$T_m^{req} = \left(J_{mc} + J_{gb} + \frac{J_L}{{i_{opt}}^2\eta_n}\right).i_{opt}.\alpha_L^{max} + \frac{T_r}{i_{opt}\eta_n}$$

$$= \left[(4.08 + 0.1)(10^{-3}) + \frac{100}{(350)^2(0.7)}\right](350)(0.44) + \frac{120}{(350)(0.7)}$$

Example

Problem: Choose a servo motor of a precise mechanism given the following data:

$$J_L = 100\ Kg.m^2, T_r = 120\ N.m\ , \omega_L^{max} = 40^o s^{-1}$$
$$\alpha_L^{max} = 25^o\ s^{-1}, \eta = \ 72\%$$

Estimate the optimum reduction ratio of the gearbox.

SOLUTION

$\boldsymbol{P_m = 320\ w}$ $\quad P_{mc} = 370\ w\ , \omega_{mc}^{max} = 314\frac{rad}{s}, J_{mc} = 4.08\ x\ 10^{-3}\ Kg.m^2 \quad J_{gb1} = 0.12\ x\ 10^{-3}\ Kg.m^2$

$\boldsymbol{i_{opt} = 350}$ $\quad \boldsymbol{T_m^{req=}}1.3131\ \boldsymbol{N.m}$

F. Calculating the nominal (rated) torque

$$T_{mc}^{nom} = \frac{P_{mc}^{nom}}{\omega_{mc}^{nom}} = \frac{370}{314}$$

$$\boldsymbol{T_{mc}^{nom} = 1.18\ N.m}$$

G. Calculating motor max speed

$$\omega_m^{max} = i_{opt}.\omega_L^{max} = (350)(0.7)$$

$$\omega_m^{max} = 245\frac{rad}{s}$$

Example

Problem: Choose a servo motor of a precise mechanism given the following data:

$$J_L = 100\ Kg.m^2, T_r = 120\ N.m\ , \omega_L^{max} = 40^o s^{-1}$$
$$\alpha_L^{max} = 25^o\ s^{-1}, \eta = 72\%$$

Estimate the optimum reduction ratio of the gearbox.

SOLUTION

$\boldsymbol{P_m = 320\ w}$ $\quad P_{mc} = 370\ w\ , \omega_{mc}^{max} = 314\frac{rad}{s}, J_{mc} = 4.08\ x\ 10^{-3}\ Kg.m^2 \quad J_{gb1} = 0.12\ x\ 10^{-3}\ Kg.m^2$

$J_{gb} = 10^{-4}\ Kg.m^2\ , \eta_n = 70\% = 0.7 \quad \boldsymbol{i_{opt} = 351.0789 \cong 350} \quad \boldsymbol{T_m^{req=}}1.3131\ \boldsymbol{N.m} \quad \boldsymbol{T_m^{nom} = 1.17\ N.m}$

$\boldsymbol{\omega_m^{max} = 245\frac{rad}{s}}$

First check

$$\frac{T_r + T_L^{max}}{i.T_m^{max}} = \frac{120 + (100)(25)\left(\frac{\pi}{180}\right)}{(350)(1.17)} = \frac{164}{409} = 0.4\ (\boldsymbol{Passed})$$

Second check

$$\frac{i.\omega_L^{max}}{\omega_m^{max}} = \frac{(350) * (40 * \frac{\pi}{180})}{314} = \frac{244}{314} = 0.7\ (\boldsymbol{Passed})$$

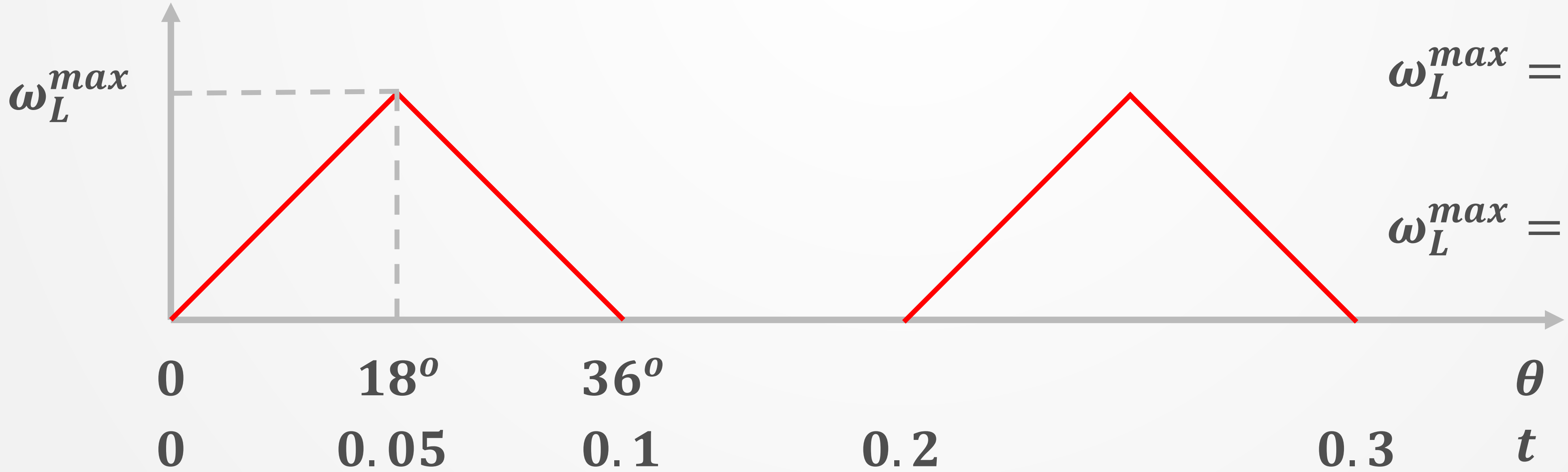
Problem

$Given\ a\ triangular\ profile\ for\ an\ indexing\ rotary\ table\ as\ shown,$

$If\ the\ polar\ moment\ of\ inertia\ of\ the\ table\ is\ 0.07\,\frac{kg}{m^2},positioning\ time$

$is\ 0.1\ s\ ,recycle\ time\ is\ 0.2\ sec\ and\ the\ angle\ for\ one\ indexing\ is\ 36^o$

$Design\ a\ servo\ system\ for\ the\ selected\ application$



$$\omega_L^{max} = \frac{18-0}{0.05} * \frac{\pi}{180}$$

$$\omega_L^{max} = 6.28\,\frac{rad}{s}$$

Problem Solution

$$J_L = 0.07\ kg.m^2 \qquad T_h = 0\ N.m^2$$

$$\omega_L^{max} = \frac{\theta_{max} - \theta_{min}}{t_{max} - t_{min}} * \frac{\pi}{180} = \frac{18 - 0}{0.05 - 0} * \frac{\pi}{180} = 6.28 \frac{rad}{s}$$

$$\alpha_L^{max} = \frac{\omega_{max} - \omega_{min}}{t_{max} - t_{min}} = \frac{6.28 - 0}{0.05 - 0} = 125.66 \frac{rad}{s^2}$$

Complete using Visual Servoing and paper

Use any servo motor catalog

Due : 08 – 12 – 2019

End of Lecture

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