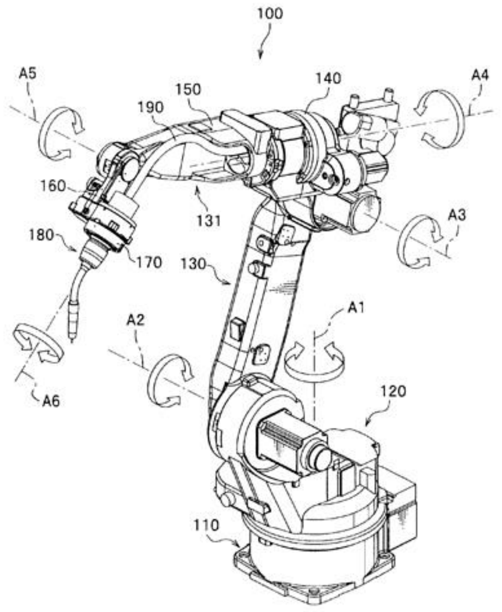
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 7
INTRODUCTION TO ROBOTICS LAB

WALEED ELBADRY

MARCH 2022



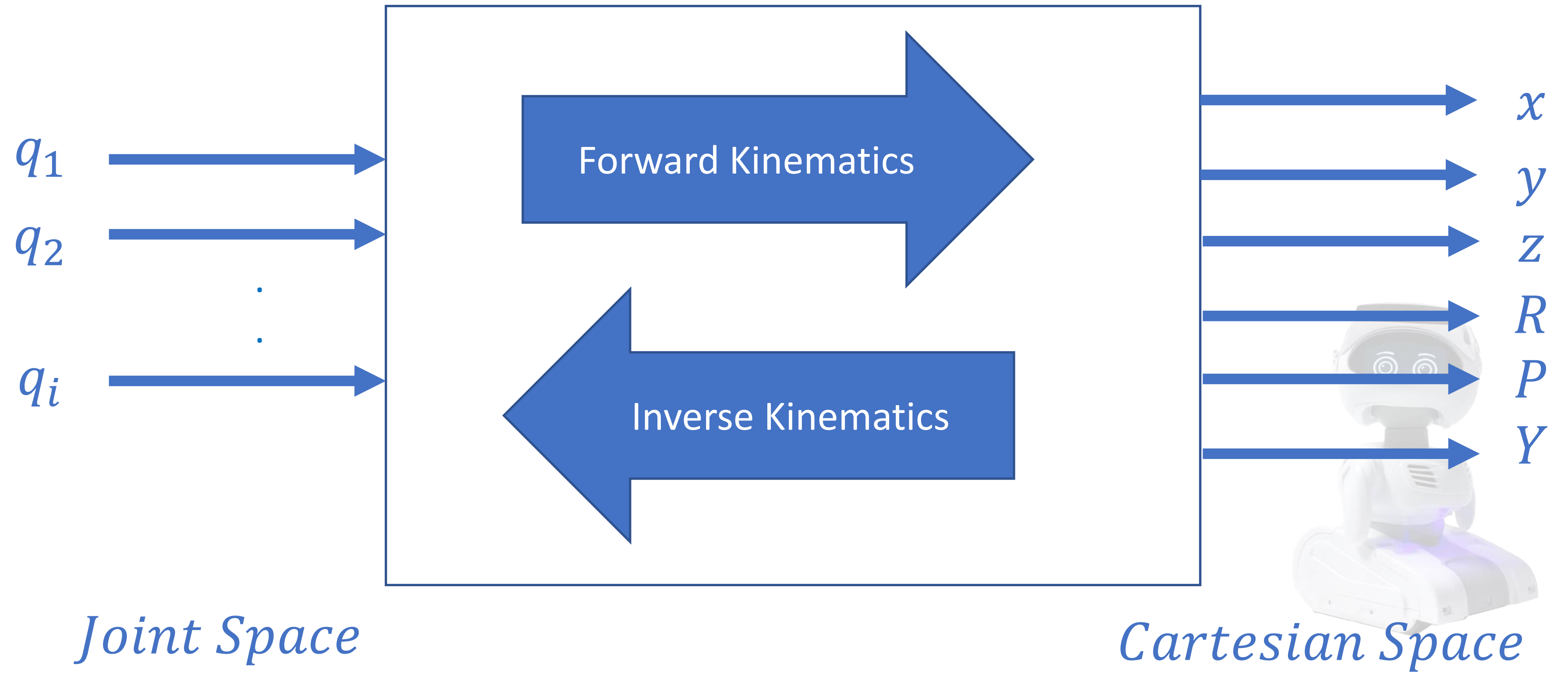
INVERSE KINEMATICS

$$\boldsymbol{Given\ End\ Effector}$$

$$X\ ,Y\ ,Z\ ,Roll\ ,Pitch,Yaw$$

$$\boldsymbol{Find}$$

$$q_i \ \in \begin{cases} \theta_i \\ d_i \end{cases}$$



INVERSE KINEMATICS

Articulated Arm (RRR)

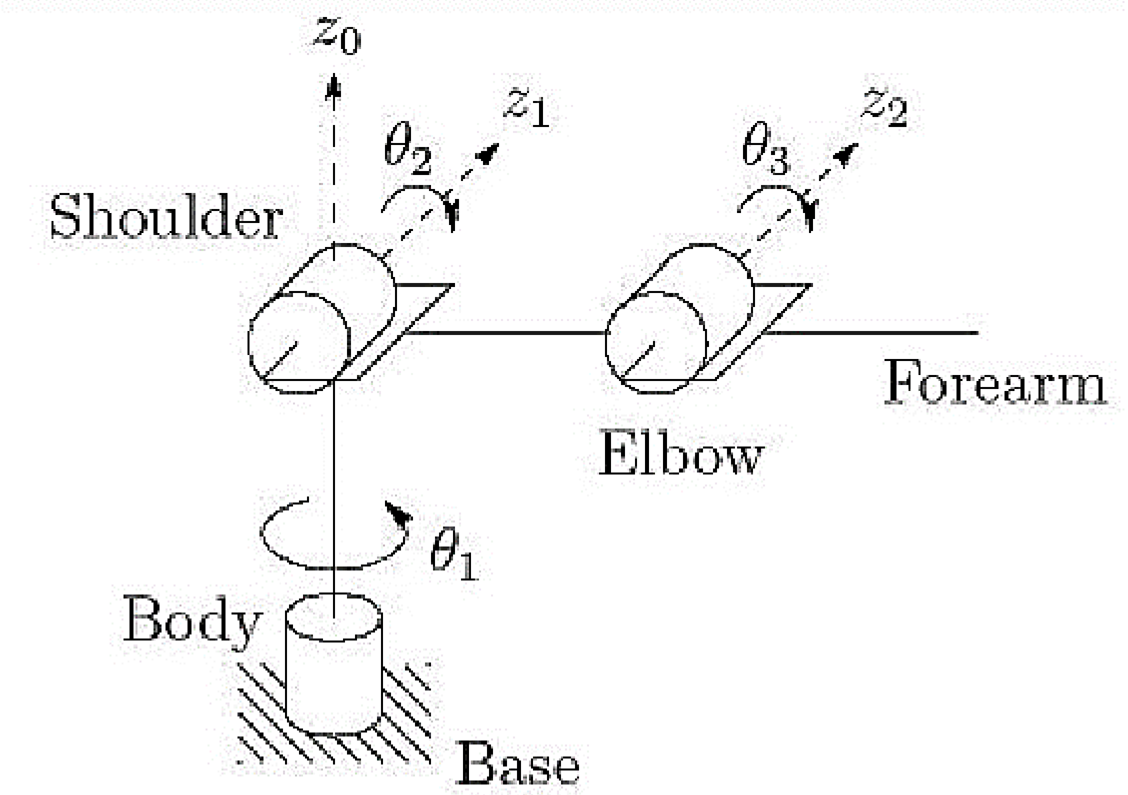


ABB IRB1400 Anthropomorphic Robot

INVERSE KINEMATICS

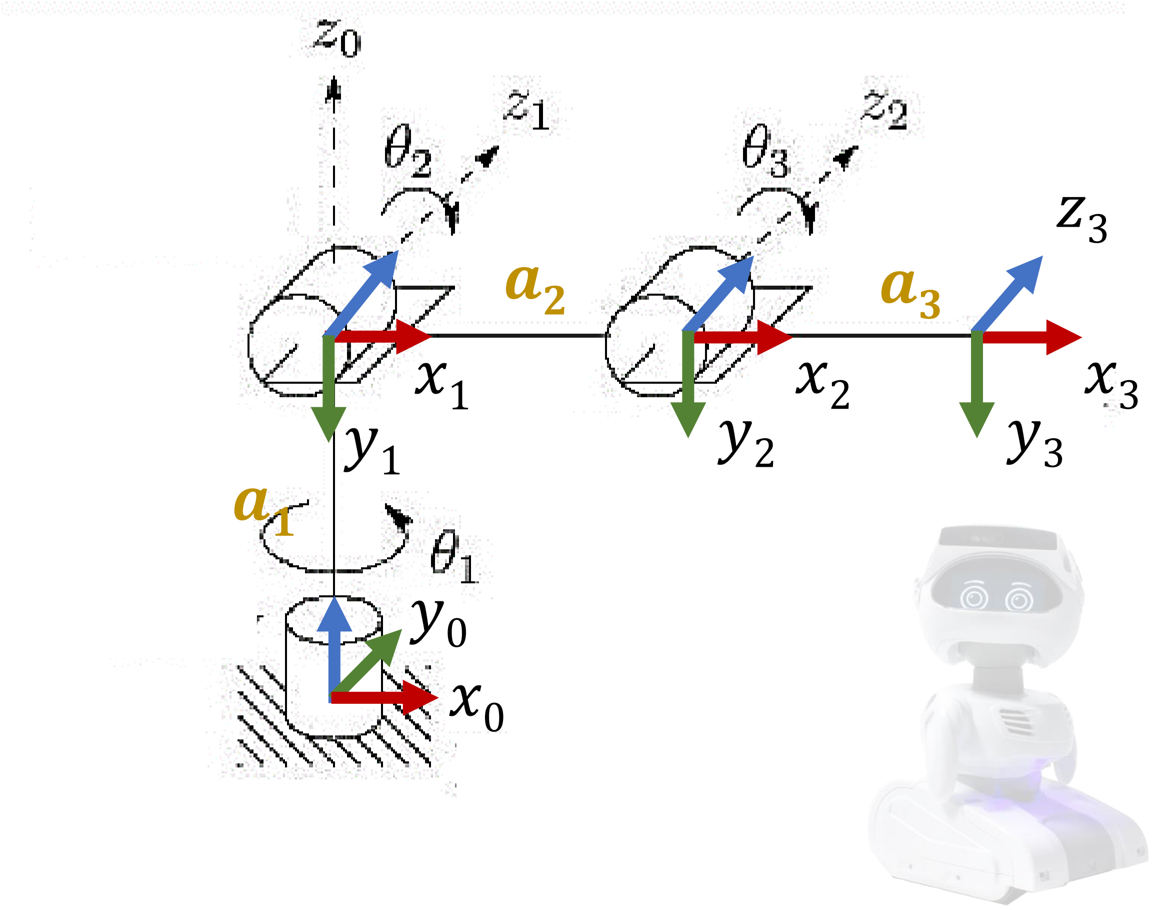
$Articulated\ Arm\ (RRR)$

$We\ want\ to\ find$:

$\boldsymbol{\theta_1}([{}^0_3x\ {}^0_3y\ {}^0_3z\ a_1 a_2 a_3])$

$\boldsymbol{\theta_2}([{}^0_3x\ {}^0_3y\ {}^0_3z\ a_1 a_2 a_3])$

$\boldsymbol{\theta_3}([{}^0_3x\ {}^0_3y\ {}^0_3z\ a_1 a_2 a_3])$

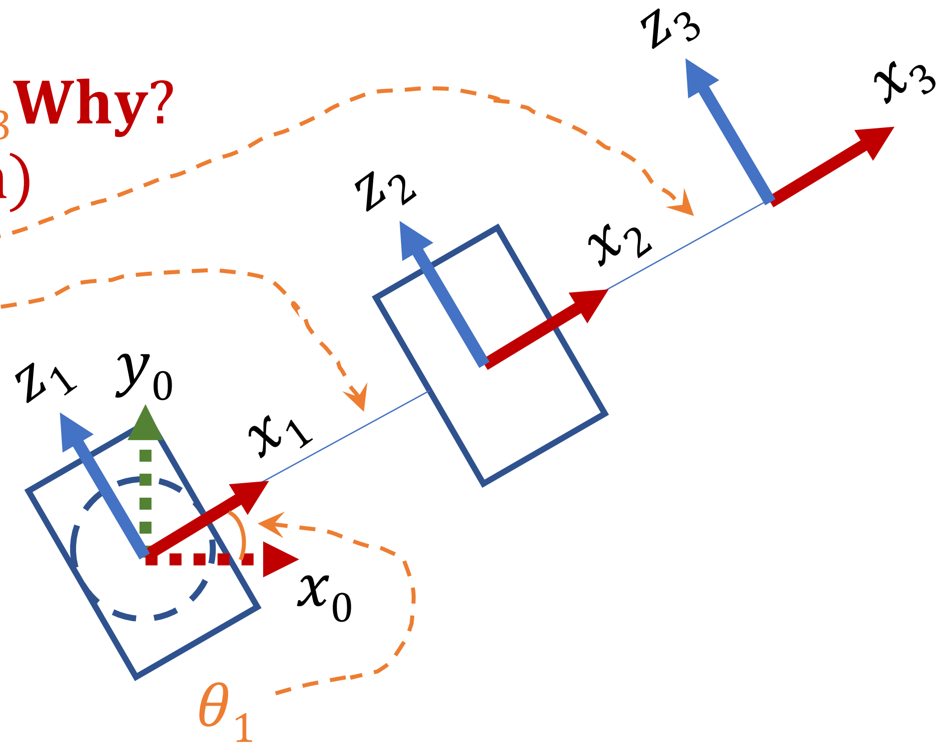


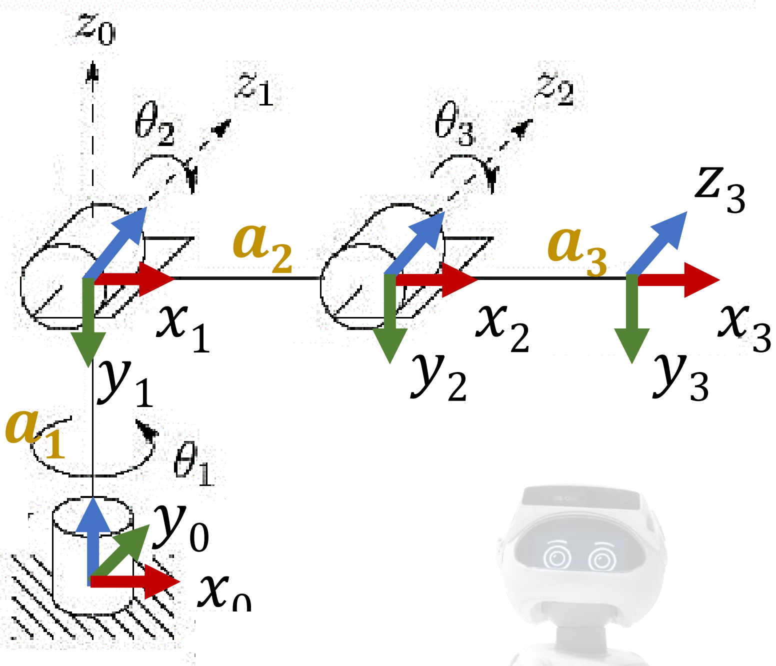
INVERSE KINEMATICS

Articulated Arm (RRR)

Plan View Analysis

NOT a_2 , a_3 **Why?**
(projection)



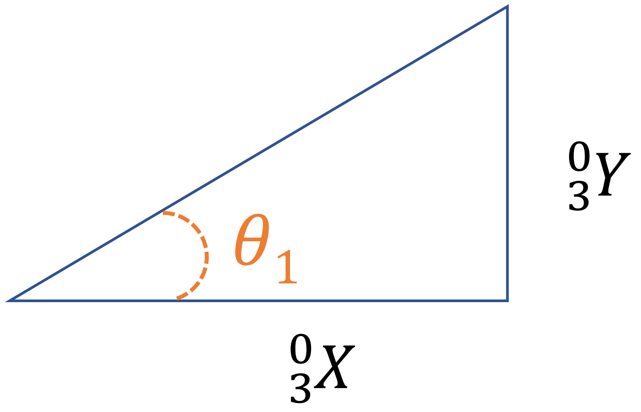


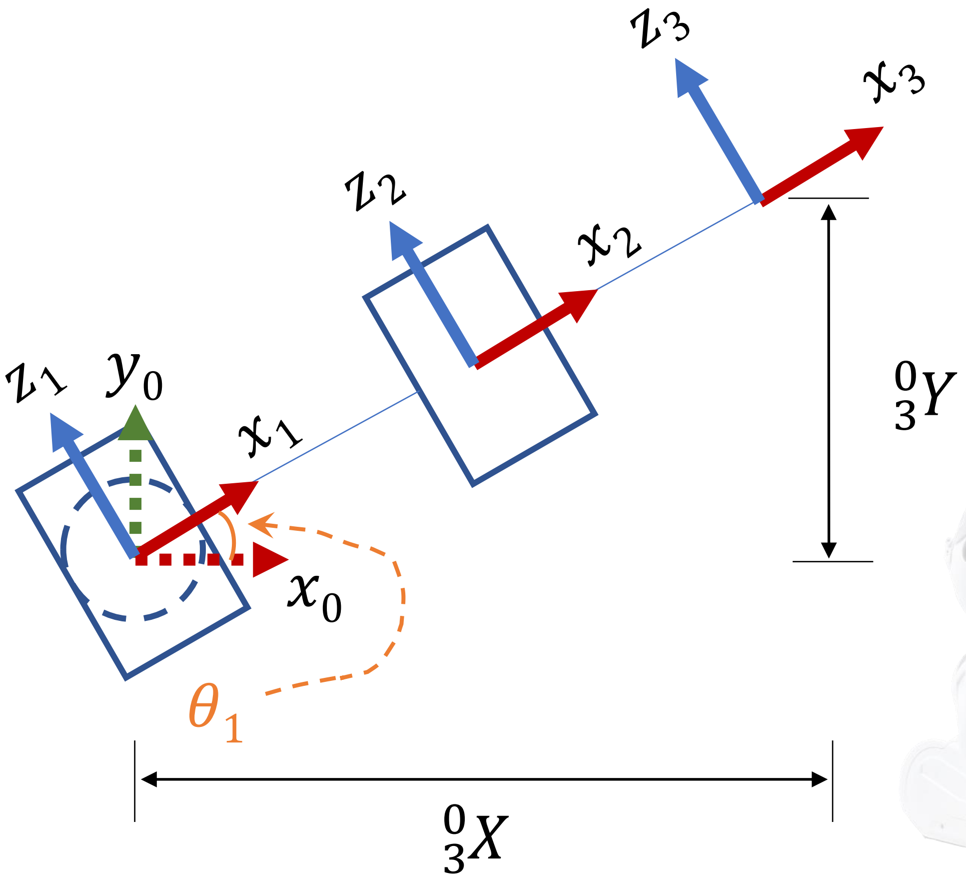
INVERSE KINEMATICS

Articulated Arm (RRR)

Plan View Analysis

$$\theta_1 = \tan^{-1}\left(\frac{{}^0_3Y}{{}^0_3X}\right)$$

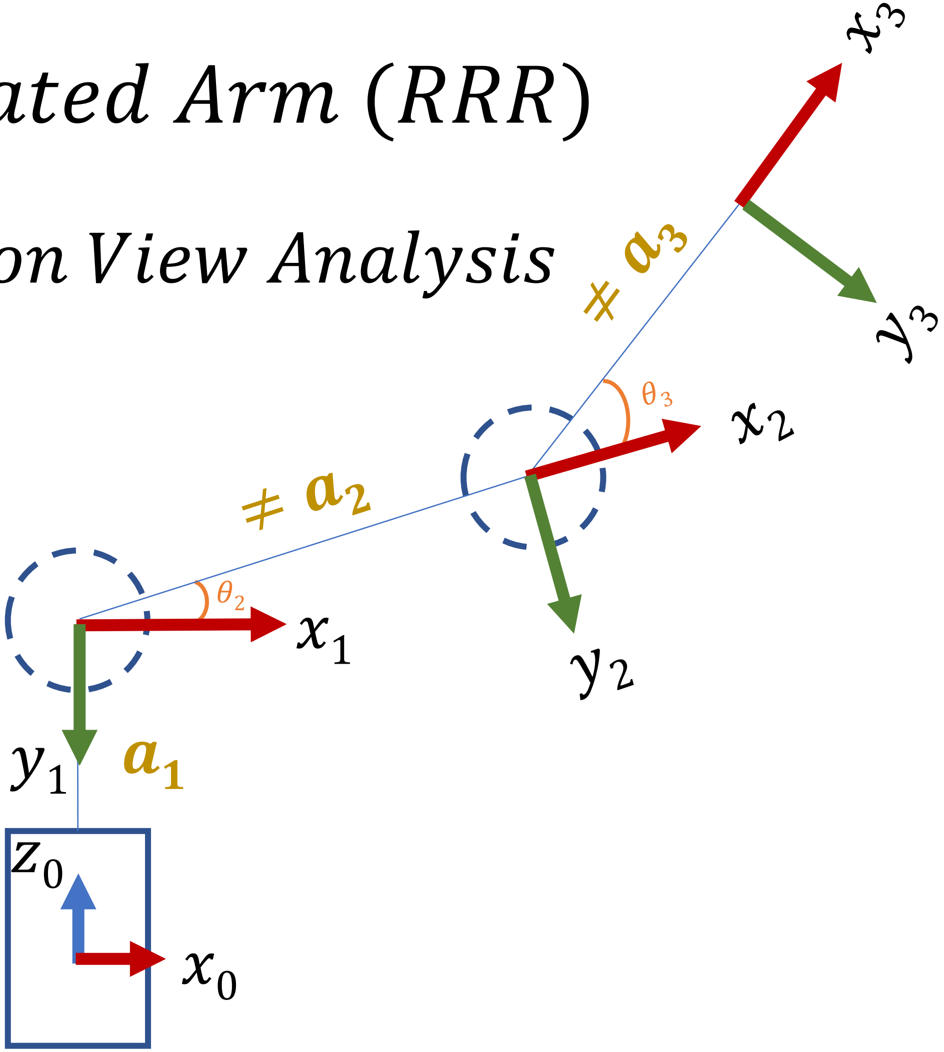


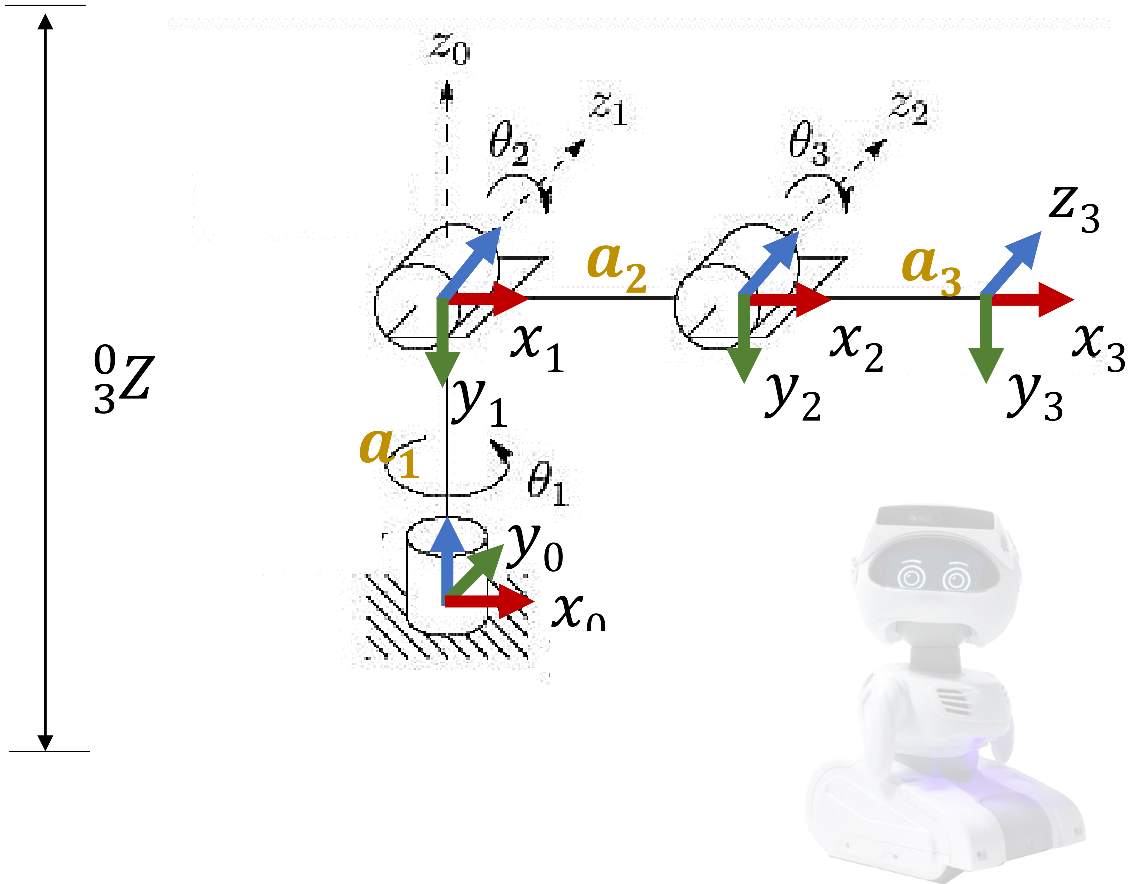


INVERSE KINEMATICS

Articulated Arm (RRR)

Elevation View Analysis





INVERSE KINEMATICS

Articulated Arm (RRR)

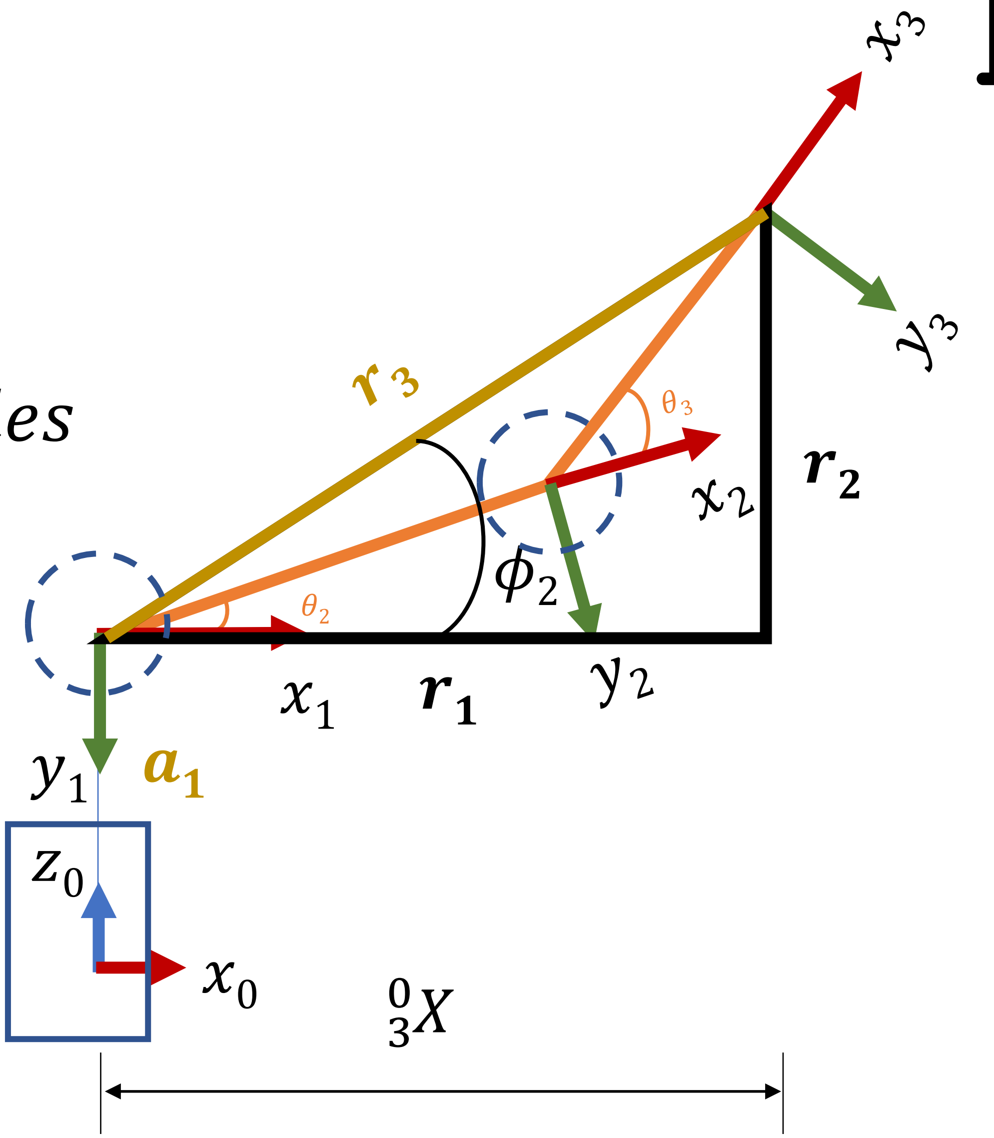
Elevation View Analysis

Let's focus on the two triangles

*The **black** triangle*

*The **orange** triangle*

*Both triangles share the same **Hypotenuse***



INVERSE KINEMATICS

Articulated Arm (RRR)

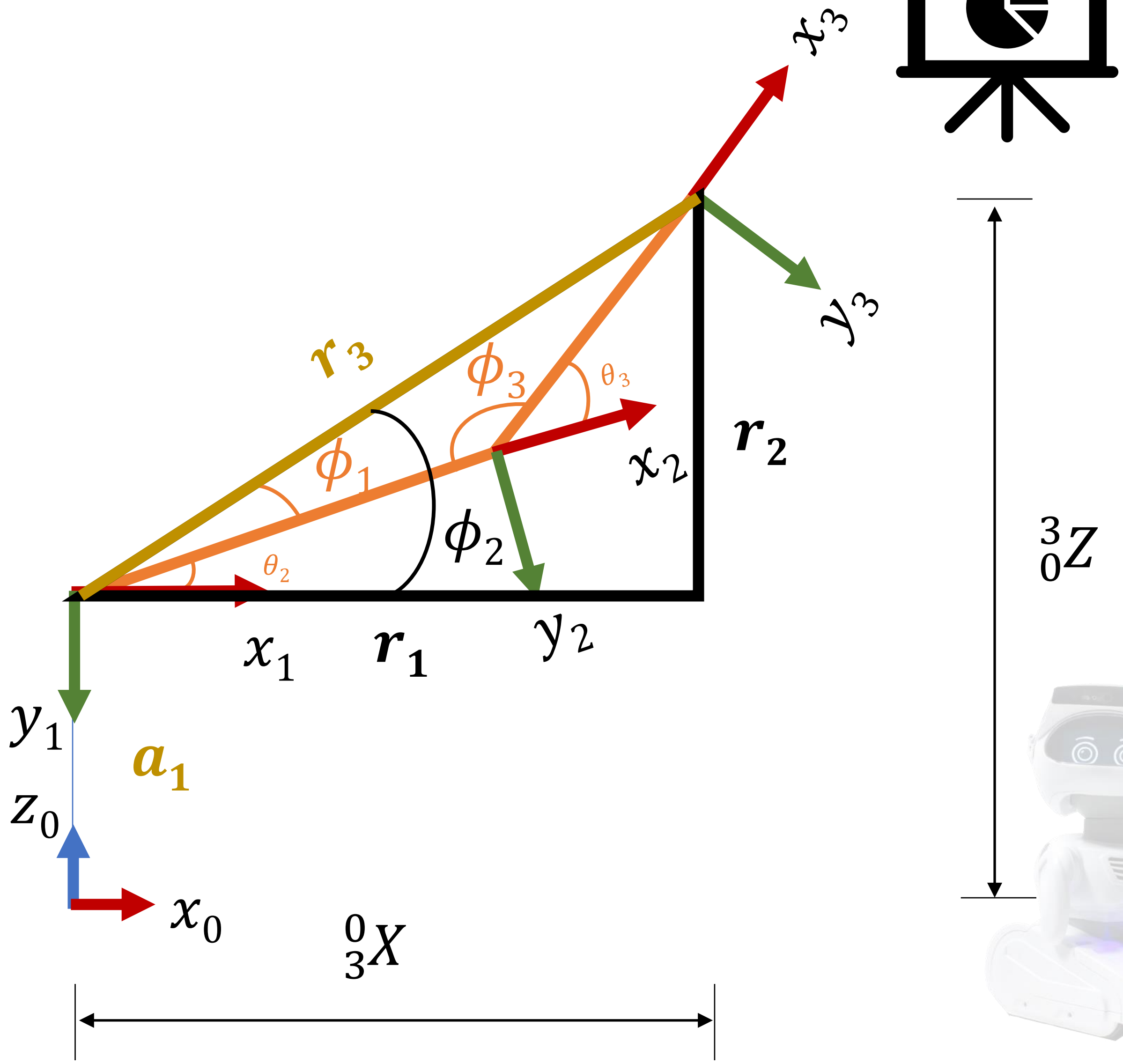
Elevation View Analysis

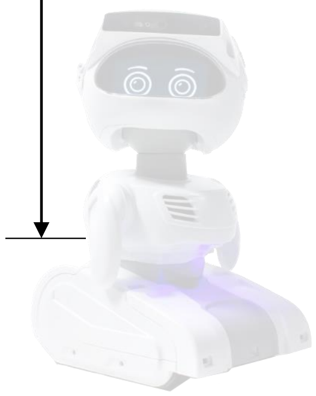
$\theta_2 = \phi_2 - \phi_1$

$\phi_2 = \tan^{-1}(\frac{r_2}{r_1})$

$r_2 = {}^0_3Z - a_1 \rightarrow \{\mathbf{1}\}$

How to get $\boldsymbol{r_1}$?



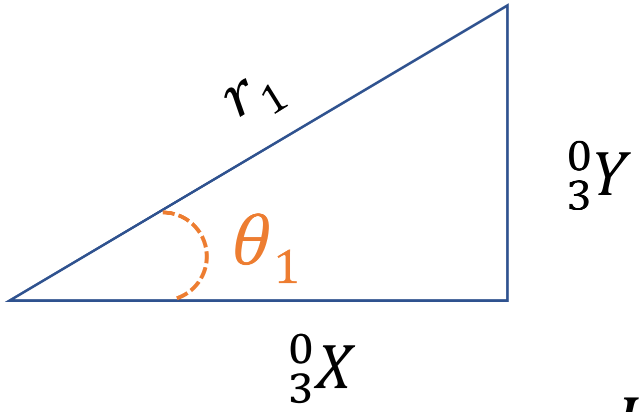


INVERSE KINEMATICS

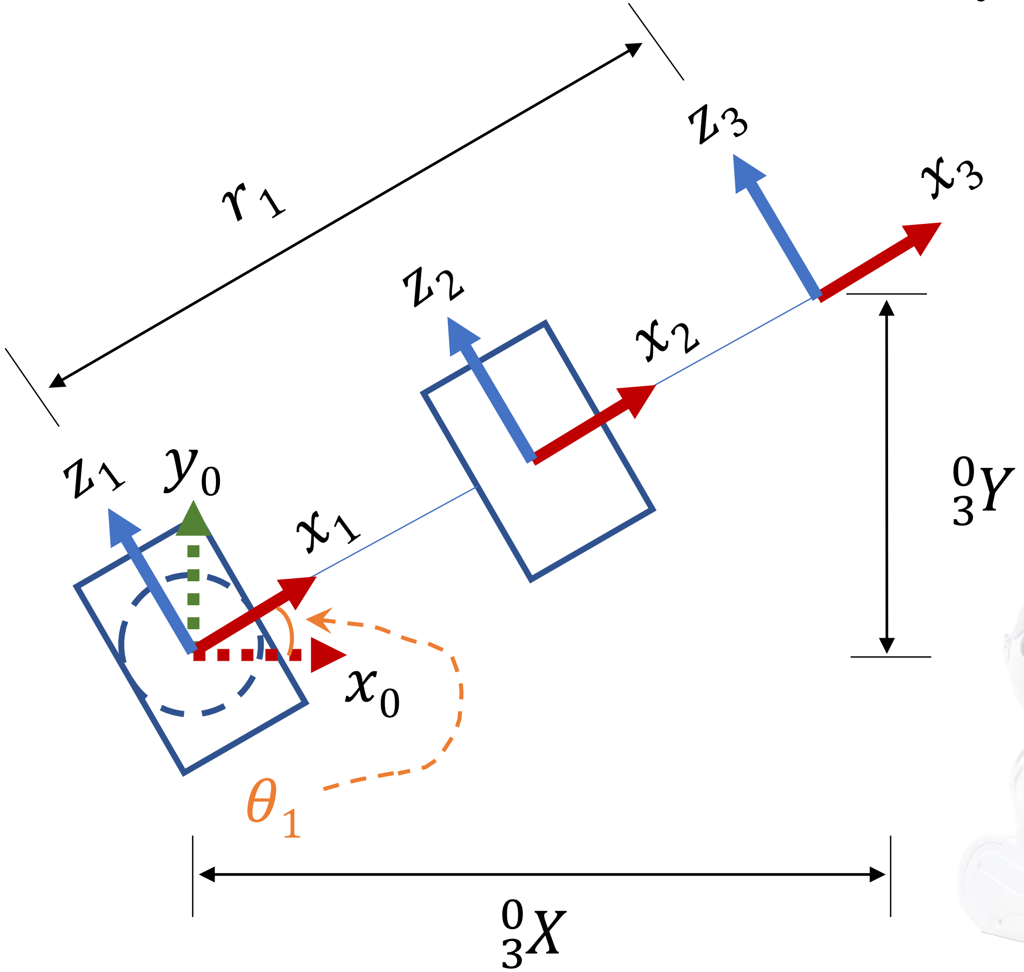
Articulated Arm (RRR)

Bak to Plan View Analysis

$$r_1 = \sqrt{({}^0_3X)^2 + ({}^0_3Y)^2} \rightarrow \{\mathbf{2}\}$$



How to get $\boldsymbol{\phi_1}$?



INVERSE KINEMATICS

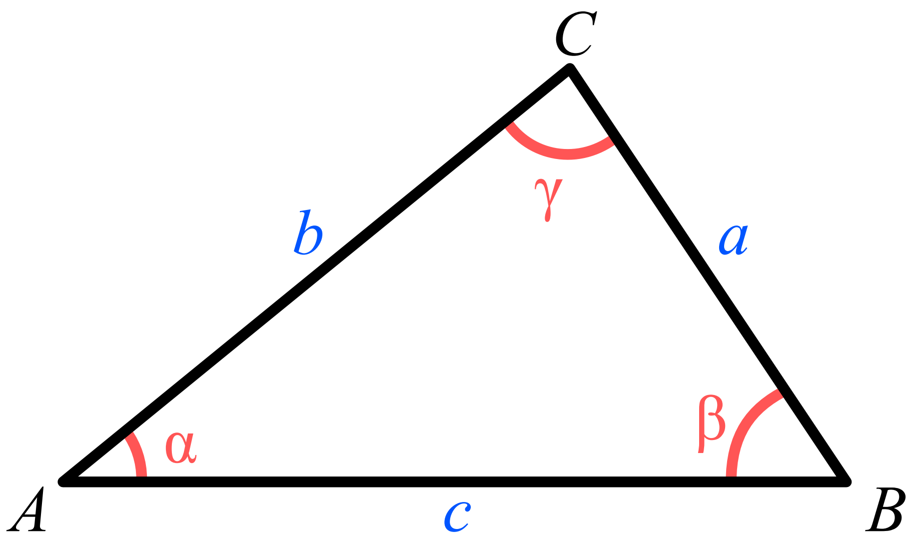
Articulated Arm (RRR)

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$
$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

$$\alpha = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \qquad \alpha = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$



INVERSE KINEMATICS

Articulated Arm (RRR)

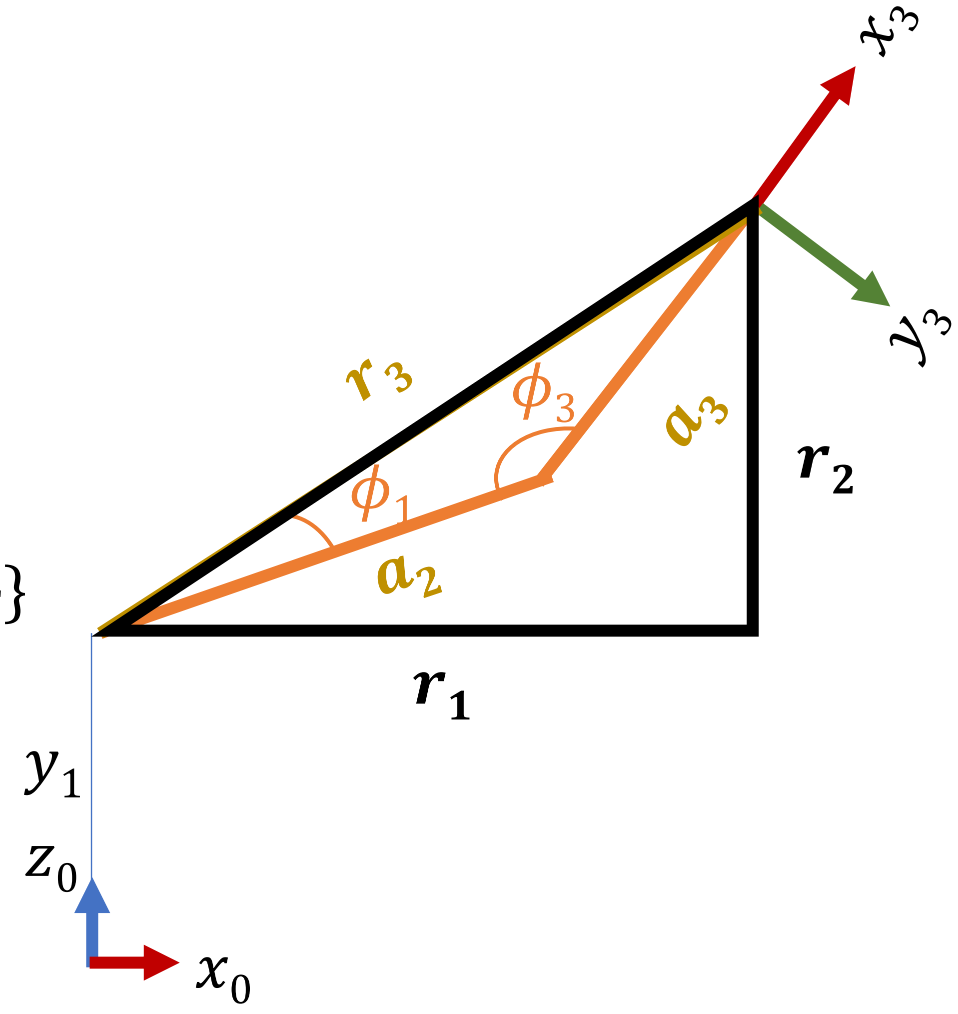
Elevation View Analysis

Law of Cosines

$$\phi_1 = \cos^{-1}\left(\frac{(a_2)^2 + (r_3)^2 - (a_3)^2}{2a_2r_3}\right) \rightarrow \{4\}$$

$$r_3 = \sqrt{(r_1)^2 + (r_2)^2} \rightarrow \{5\}$$

Let's rearrange the equations so far



INVERSE KINEMATICS

$Articulated\ Arm\ (RRR)$

$Sequence\ of\ calculations$

$$\boldsymbol{\theta_1} = \tan^{-1}\left(\frac{{}^3_0Y}{{}^3_0X}\right) \rightarrow [1]$$

$$r_1 = \sqrt{({}^3_0X)^2 + ({}^3_0Y)^2} \rightarrow \{\mathbf{1}\}$$

$$r_2 = {}^3_0Z - a_1 \rightarrow \{\mathbf{2}\}$$

$$r_3 = \sqrt{(r_2)^2 + (r_3)^2} \rightarrow \{\mathbf{3}\}$$

$$\phi_1 = \cos^{-1}\left(\frac{(a_2)^2 + (r_3)^2 - (a_3)^2}{2a_2r_3}\right) \rightarrow \{\mathbf{4}\}$$

$$\phi_2 = \tan^{-1}\left(\frac{r_2}{r_1}\right) \rightarrow \{\mathbf{5}\}$$

$$\boldsymbol{\theta_2} = \phi_2 - \phi_1 \rightarrow [2]$$

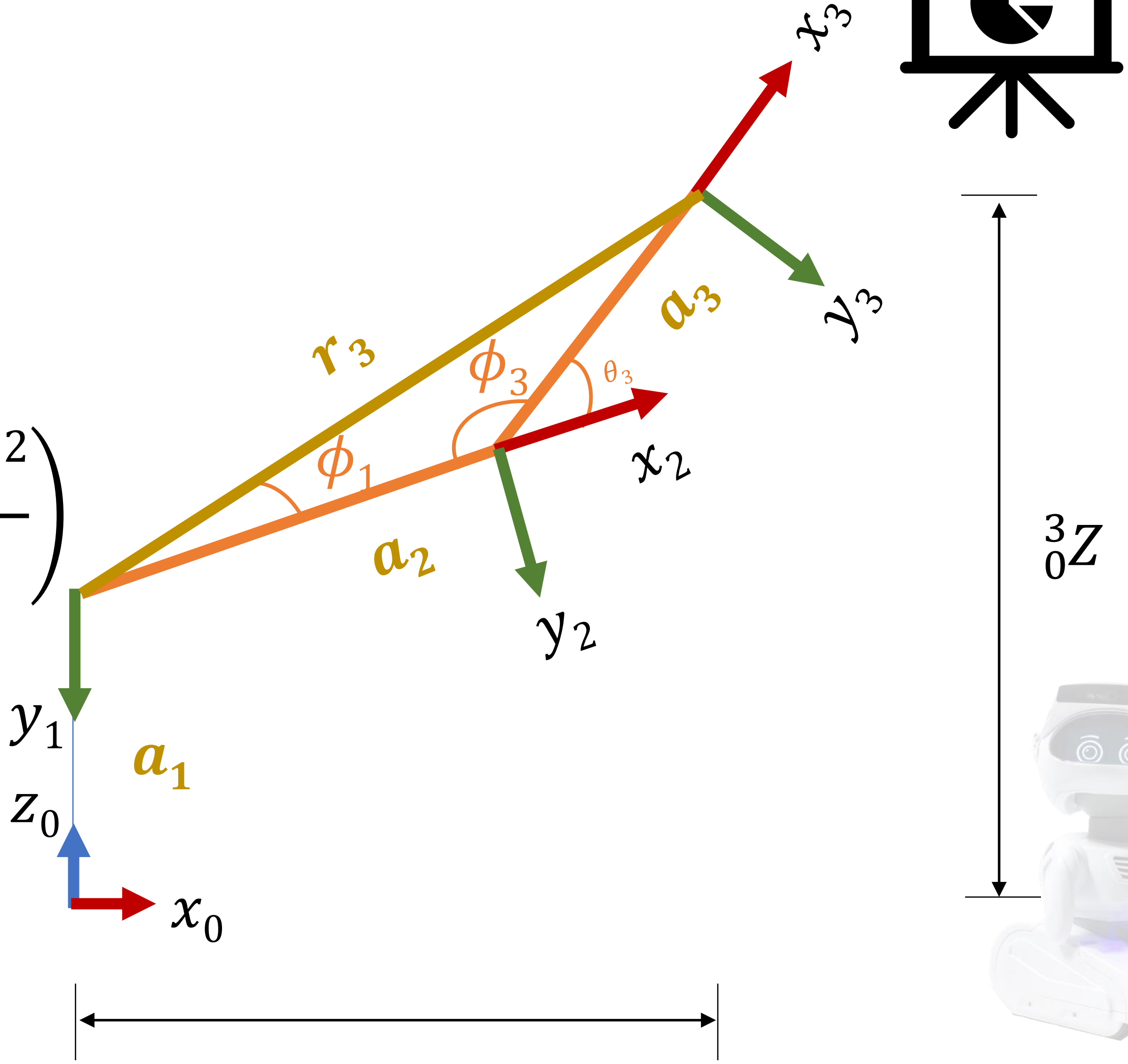
INVERSE KINEMATICS

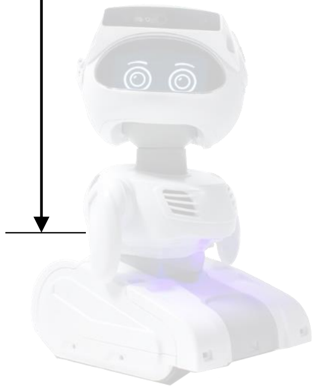
Articulated Arm (RRR)

Elevation View Analysis

$$\boldsymbol{\theta_3} = 180 - \phi_3$$

$$\phi_3 = \cos^{-1}\left(\frac{(a_2)^2 + (a_3)^2 - (r_3)^2}{2a_2a_3}\right)$$





INVERSE KINEMATICS

$Articulated\ Arm\ (RRR)$

$Final\ Sequence\ of\ calculations$

$$\boldsymbol{\theta_1} = \tan^{-1}\left(\frac{{}^3_0Y}{{}^3_0X}\right) \rightarrow [1]$$

$$r_1 = \sqrt{({}^3_0X)^2 + ({}^3_0Y)^2} \rightarrow \{\mathbf{1}\}$$

$$r_2 = {}^3_0Z - a_1 \rightarrow \{\mathbf{2}\}$$

$$r_3 = \sqrt{(a_2)^2 + (a_3)^2} \rightarrow \{\mathbf{3}\}$$

$$\phi_1 = \cos^{-1}\left(\frac{(a_2)^2 + (r_3)^2 - (a_3)^2}{2a_2r_3}\right) \rightarrow \{\mathbf{4}\}$$

$$\phi_2 = \tan^{-1}\left(\frac{r_2}{r_1}\right) \rightarrow \{\mathbf{5}\}$$

$$\boldsymbol{\theta_2} = \phi_2 - \phi_1 \rightarrow [2]$$

$$\phi_3 = \cos^{-1}\left(\frac{(a_2)^2 + (a_3)^2 - (r_3)^2}{2a_2a_3}\right) \rightarrow \{\mathbf{6}\}$$

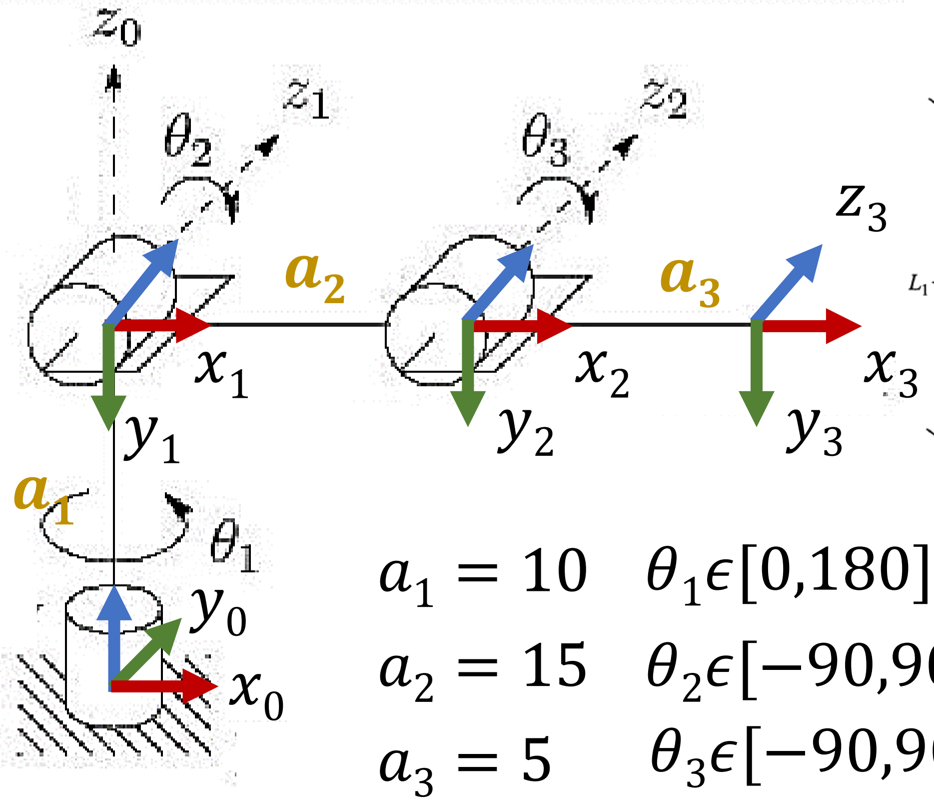
$$\boldsymbol{\theta_3} = 180 - \phi_3 \rightarrow [3]$$

INVERSE KINEMATICS

Lab Assignment

Build the articulated arm with Peter Corke Toolbox

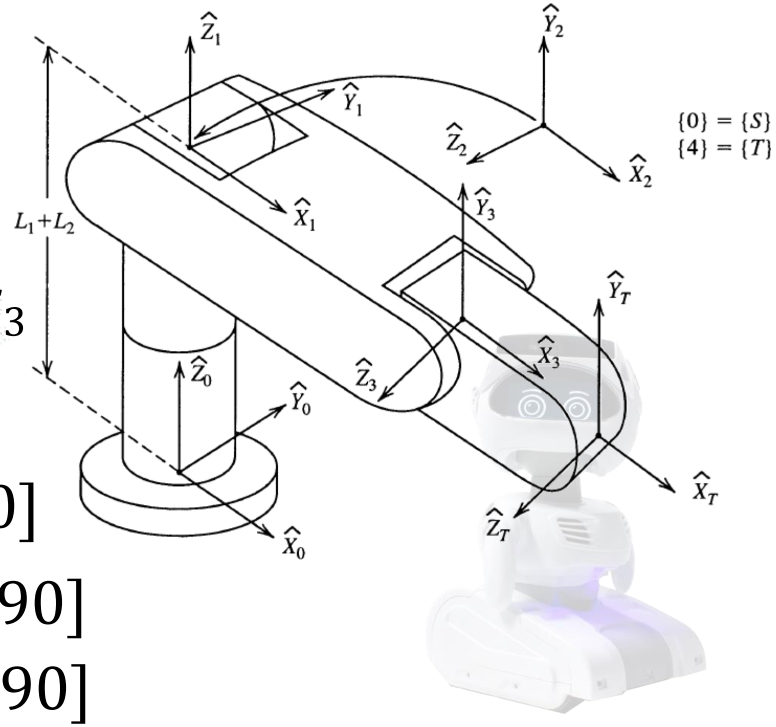
n	θ	d	α	a
1	θ_1	a_1	-90^o	0
2	θ_2	0	0^o	a_2
3	θ_3	0	0^o	a_3



$a_1 = 10 \quad \theta_1 \epsilon [0,180]$

$a_2 = 15 \quad \theta_2 \epsilon [-90,90]$

$a_3 = 5 \quad \theta_3 \epsilon [-90,90]$



***NEXT SECTION** : Numerical Inverese Kinematics*