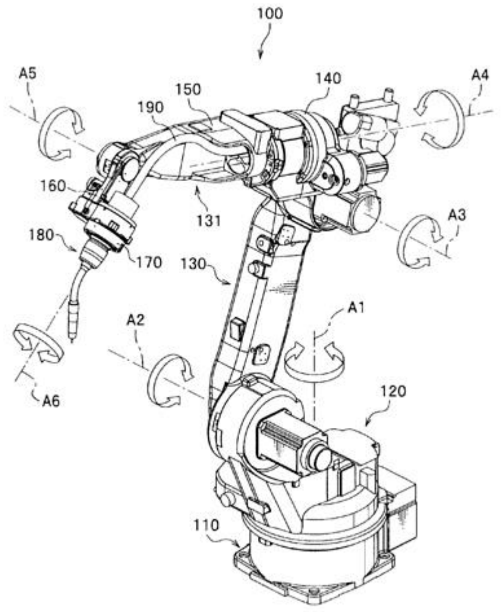
MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 9
INTRODUCTION TO ROBOTICS LAB

WALEED ELBADRY

MAY 2022



JACOBIAN

What is the Jacobian Matrix $(\boldsymbol{J})$

$\dot{X}$ … *The end effector Velocity* $(m\ x\ 1)$

$\dot{q}$ … *The joints variables angular velocity* $(n\ x\ 1)$

J … *The jacobian matrix* $(m\ x\ n)$

n … *The number of robot joints*

m … *Robot location and orientation*

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} J_{11} & \cdots & J_{1n} \\ J_{21} & \cdots & J_{2n} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ J_{61} & \cdots & J_{6n} \end{bmatrix}_{m\ x\ n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\ x\ 1}$$

Jacobian *is a matrix provides the relationship between the* ***end effector velocity*** $\dot{\boldsymbol{X}}$ *and the* ***joints angular velocities*** $\dot{\boldsymbol{q}}$

The number of rows is always 6 but the number of columns depneds on the number of joints

JACOBIAN

$$What\ is\ the\ Jacobian\ Matrix\ (\boldsymbol{J})$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \frac{\partial \theta_z}{\partial q_1} & \cdots & \frac{\partial \theta_z}{\partial q_1} \end{bmatrix}_{m\ x\ n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\ x\ 1}$$

$$\boldsymbol{Jacobian}\ is\ also\ the\ first\ derivative\ of\ the\ position\ and\ orientation$$

$$Recall\ that\ \boldsymbol{v} = \frac{dx}{dt} = \frac{x_{t+\Delta t} - x_t}{(t + \Delta t) - t}$$

JACOBIAN

Jacobian transformation table

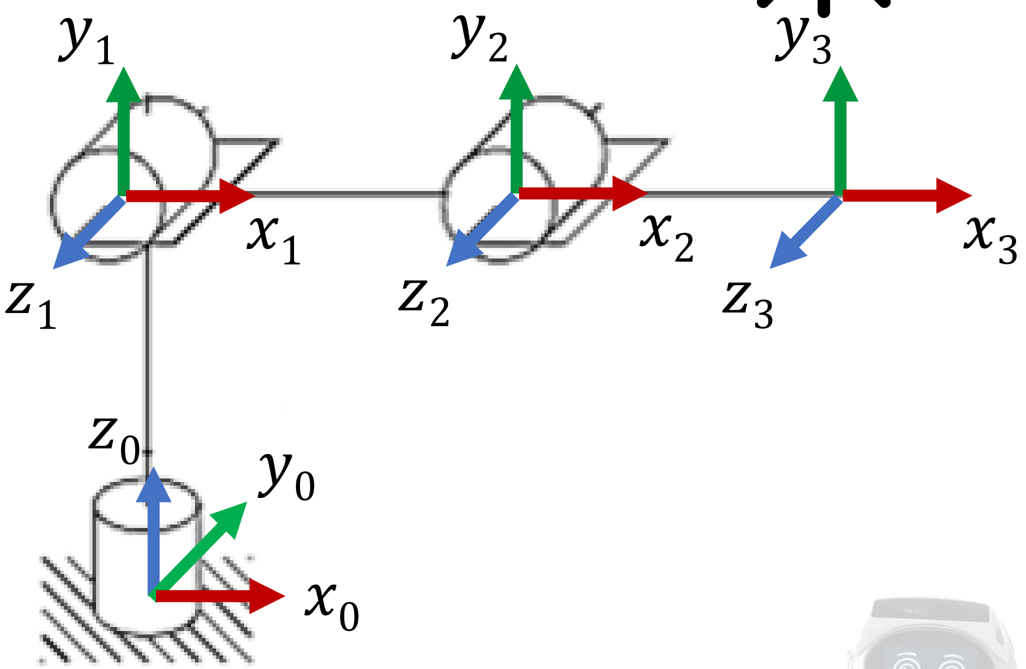
	Prismatic	***Revolute***
Linear	${}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix}$	${}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix} \times ({}^{0}_{n}d - {}^{0}_{i-1}d)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	${}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix}$

$$J_{Rev} = \begin{bmatrix}J_v\\J_\omega\end{bmatrix} = \begin{bmatrix} {}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix} \\ \begin{bmatrix}0\\0\\0\end{bmatrix} \end{bmatrix} \qquad J_{Pri} = \begin{bmatrix}J_v\\J_\omega\end{bmatrix} = \begin{bmatrix} {}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix} \times {}^{0}_{n}d - {}^{0}_{i-1}d) \\ {}^{0}_{i-1}R \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix}$$

JACOBIAN

Articulated robot

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix}_{6\,x\,1} = \underbrace{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}_{6\,x\,3}}_{3\ JOINTS\ JACOBIAN} \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}_{3\,x\,1}$$

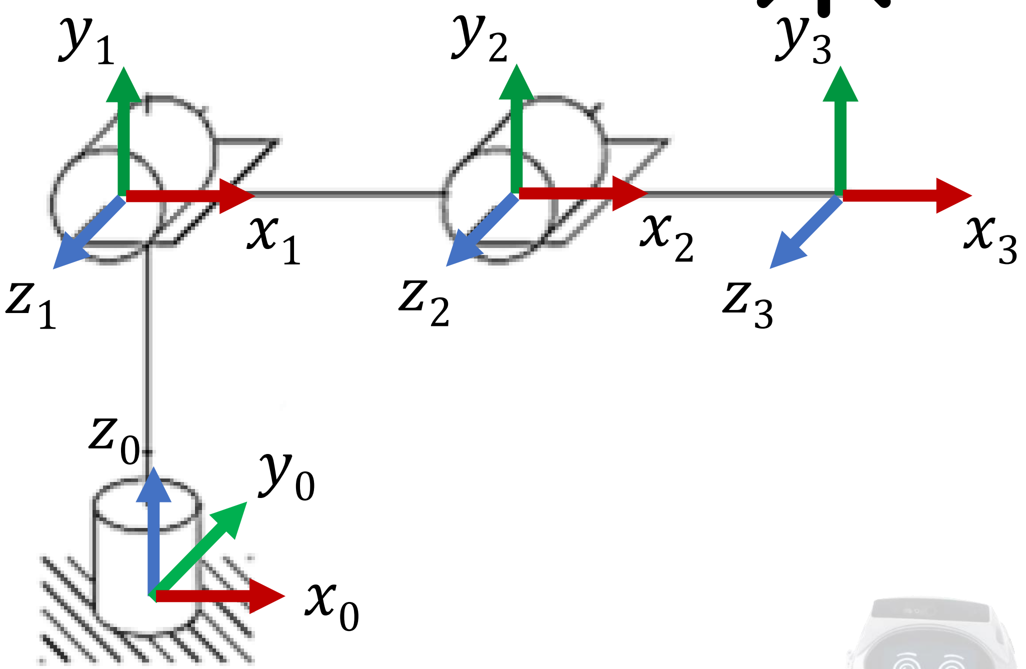


	Prismatic	***Revolute***
Linear	${}_{i-1}^{0}R \begin{bmatrix}0\\0\\1\end{bmatrix}$	${}_{i-1}^{0}R \begin{bmatrix}0\\0\\1\end{bmatrix} \times ({}_{n}^{0}d - {}_{i-1}^{0}d)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	${}_{i-1}^{0}R \begin{bmatrix}0\\0\\1\end{bmatrix}$

JACOBIAN

Articulated robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} {}_{i-1}^{\;\;0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left({}_{n}^{0}d - {}_{i-1}^{\;\;0}d \right) \\ {}_{i-1}^{\;\;0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

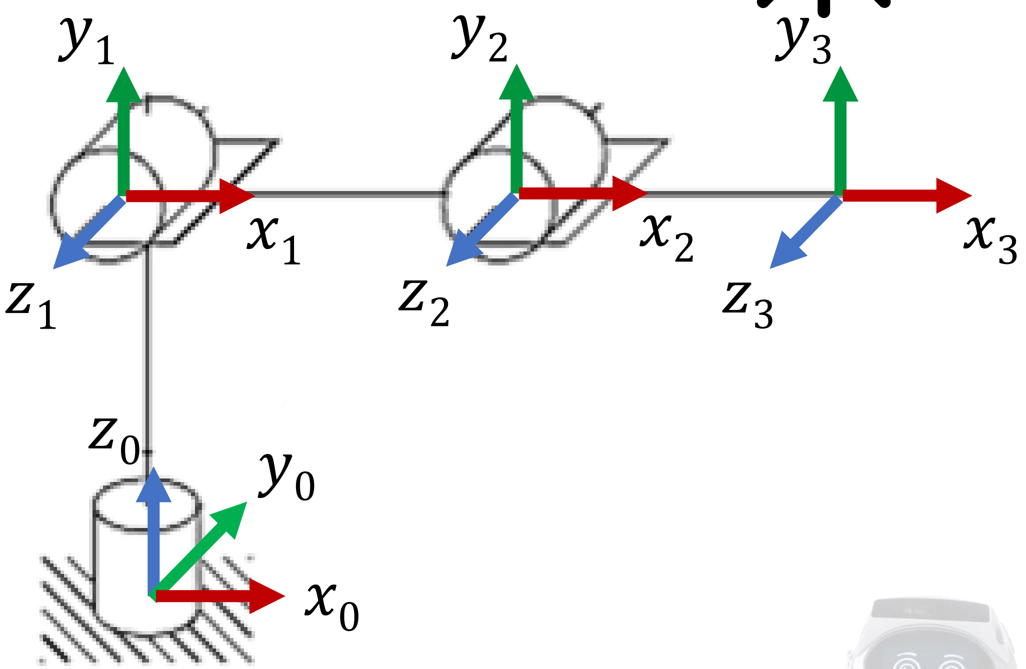


	Prismatic	***Revolute***
Linear	${}_{i-1}^{\;\;0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}_{i-1}^{\;\;0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left({}_{n}^{0}d - {}_{i-1}^{\;\;0}d \right)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}_{i-1}^{\;\;0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

JACOBIAN

Articulated robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} {}^0_0R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^0_3d - {}^0_0d\right) & {}^0_1R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^0_3d - {}^0_1d\right) & {}^0_2R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^0_3d - {}^0_2d\right) \\ {}^0_0R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & {}^0_1R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & {}^0_2R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

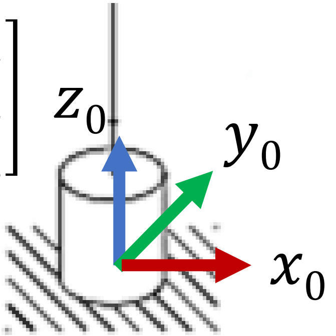


	Prismatic	*Revolute*
Linear	${}^0_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}^0_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left({}^0_nd - {}^0_{i-1}d\right)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^0_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

JACOBIAN

Articulated robot

$$
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^{0}_{3}d - {}^{0}_{0}d \right) & {}^{0}_{1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^{0}_{3}d - {}^{0}_{1}d \right) & {}^{0}_{2}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^{0}_{3}d - {}^{0}_{2}d \right) \\
\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & {}^{0}_{1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & {}^{0}_{2}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{bmatrix}
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}
$$

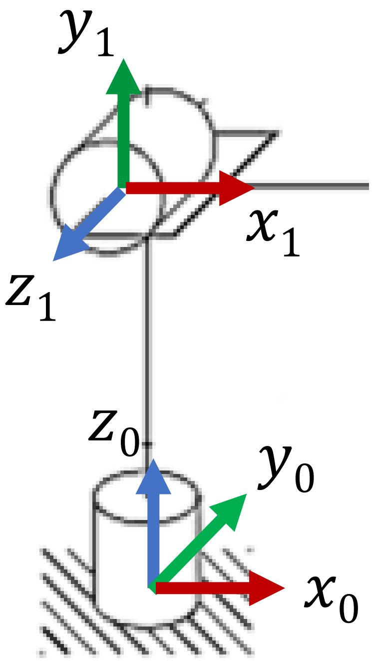


$$
{}^{0}_{0}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

	Prismatic	***Revolute***
Linear	${}^{0}_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}^{0}_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left({}^{0}_{n}d - {}_{i-1}^{\ \ 0}d \right)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^{0}_{i-1}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

JACOBIAN

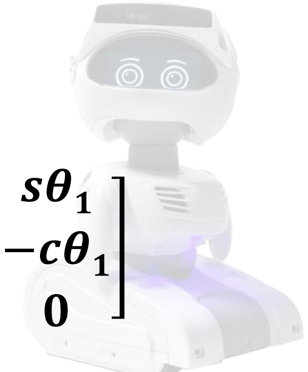
Articulated robot



$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix}\left({}^0_3d - {}^0_0d\right) & \begin{bmatrix}\boldsymbol{c\theta_1} & \mathbf{0} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1} & \mathbf{0} & \boldsymbol{-c\theta_1}\\ \mathbf{0} & \mathbf{1} & \mathbf{0}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\left({}^0_3d - {}^0_1d\right) & {}^0_2R\begin{bmatrix}0\\0\\1\end{bmatrix}\left({}^0_3d - {}^0_2d\right) \\ \begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}\boldsymbol{c\theta_1} & \mathbf{0} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1} & \mathbf{0} & \boldsymbol{-c\theta_1}\\ \mathbf{0} & \mathbf{1} & \mathbf{0}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} & {}^0_2R\begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$

$${}^0_1T = \boldsymbol{R_z}\,{}^{\mathbf{0}}_{\mathbf{1}}\boldsymbol{R} + {}^0_1t$$

$${}^0_1R = \boldsymbol{R_z}\,{}^{\mathbf{0}}_{\mathbf{1}}\boldsymbol{R} = \begin{bmatrix}c\theta_1 & -s\theta_1 & 0\\ s\theta_1 & c\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2})\\ 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2})\end{bmatrix} = \begin{bmatrix}c\theta_1 & -s\theta_1 & 0\\ s\theta_1 & c\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{1} & 0 & 0\\ 0 & 0 & \mathbf{-1}\\ 0 & \mathbf{1} & 0\end{bmatrix} = \begin{bmatrix}\boldsymbol{c\theta_1} & \mathbf{0} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1} & \mathbf{0} & \boldsymbol{-c\theta_1}\\ \mathbf{0} & \mathbf{1} & \mathbf{0}\end{bmatrix}$$



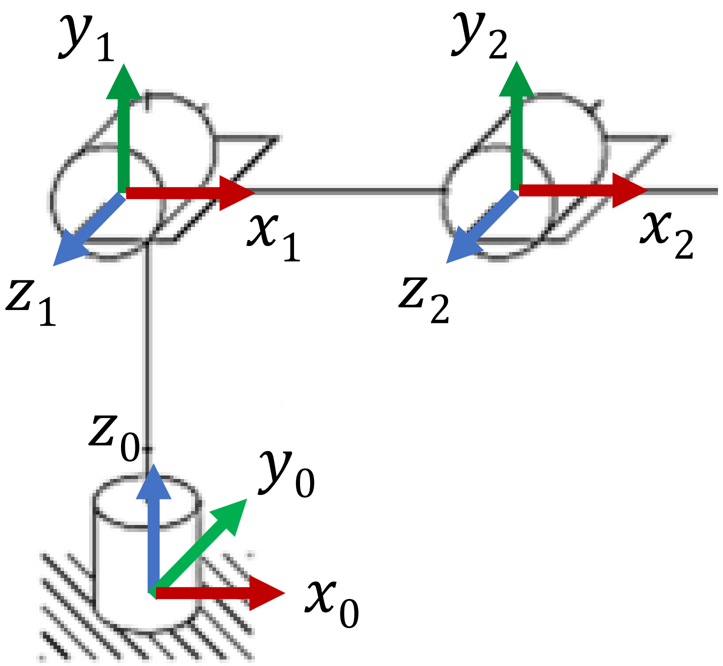
JACOBIAN

Articulated robot

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix}\left({}^0_3d - {}^0_0d\right) & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix}\left({}^0_3d - {}^0_1d\right) & \begin{bmatrix}\boldsymbol{c\theta_1 c\theta_2} & \boldsymbol{-c\theta_1 s\theta_2} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1 c\theta_2} & \boldsymbol{-s\theta_1 s\theta_2} & \boldsymbol{-c\theta_1}\\ \boldsymbol{s\theta_2} & \boldsymbol{c\theta_2} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\left({}^0_3d - {}^0_2d\right) \\ \begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix} & \begin{bmatrix}\boldsymbol{c\theta_1 c\theta_2} & \boldsymbol{-c\theta_1 s\theta_2} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1 c\theta_2} & \boldsymbol{-s\theta_1 s\theta_2} & \boldsymbol{-c\theta_1}\\ \boldsymbol{s\theta_2} & \boldsymbol{c\theta_2} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$

$$ {}^1_2R = \boldsymbol{R_z}\,{}^1_2\boldsymbol{R} = \begin{bmatrix}c\theta_2 & -s\theta_2 & 0\\ s\theta_2 & c\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{1} & 0 & 0\\ 0 & \boldsymbol{1} & 0\\ 0 & 0 & \boldsymbol{1}\end{bmatrix} = \begin{bmatrix}c\theta_2 & -s\theta_2 & 0\\ s\theta_2 & c\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$ {}^0_2R = {}^0_1\boldsymbol{R}\,{}^1_2\boldsymbol{R} = \begin{bmatrix}c\theta_1 & 0 & s\theta_1\\ s\theta_1 & 0 & -c\theta_1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}c\theta_2 & -s\theta_2 & 0\\ s\theta_2 & c\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\boldsymbol{c\theta_1 c\theta_2} & \boldsymbol{-c\theta_1 s\theta_2} & \boldsymbol{s\theta_1}\\ \boldsymbol{s\theta_1 c\theta_2} & \boldsymbol{-s\theta_1 s\theta_2} & \boldsymbol{-c\theta_1}\\ \boldsymbol{s\theta_2} & \boldsymbol{c\theta_2} & \boldsymbol{0}\end{bmatrix}$$

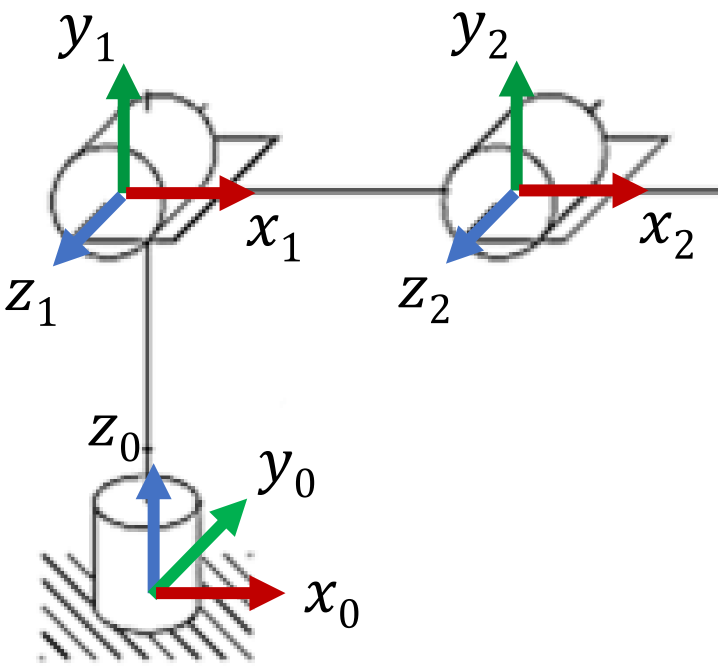


JACOBIAN

Articulated robot

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \times \left({}^0_3d - {}^0_0d\right) & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix} \times \left({}^0_3d - {}^0_1d\right) & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix} \times \left({}^0_3d - {}^0_2d\right) \\ \begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix} & \begin{bmatrix}s\theta_1\\-c\theta_1\\0\end{bmatrix} \end{bmatrix} \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$

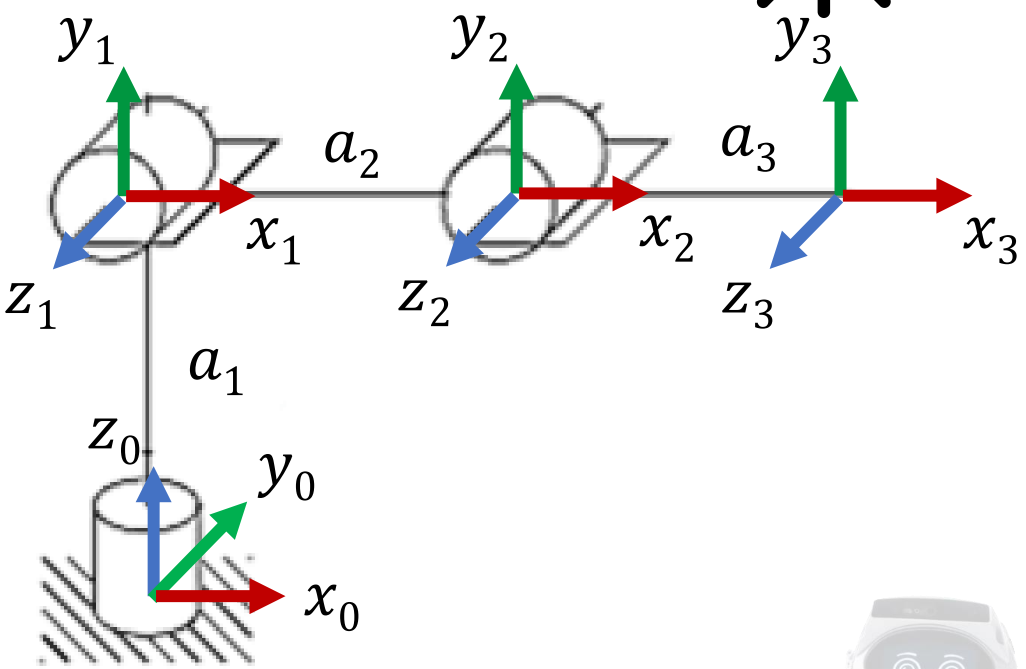
$\times \cdots$ *cross product*



JACOBIAN

Articulated robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left({}^0_3d - {}^0_0d\right) & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \times \left({}^0_3d - {}^0_1d\right) & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \times \left({}^0_3d - {}^0_2d\right) \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$



n	**θ**	**d**	**α**	**a**
1	θ_1	a_1	90^o	0
2	θ_2	0	0^o	a_2
3	θ_3	0	0^o	a_3

$D - H\ parameters$

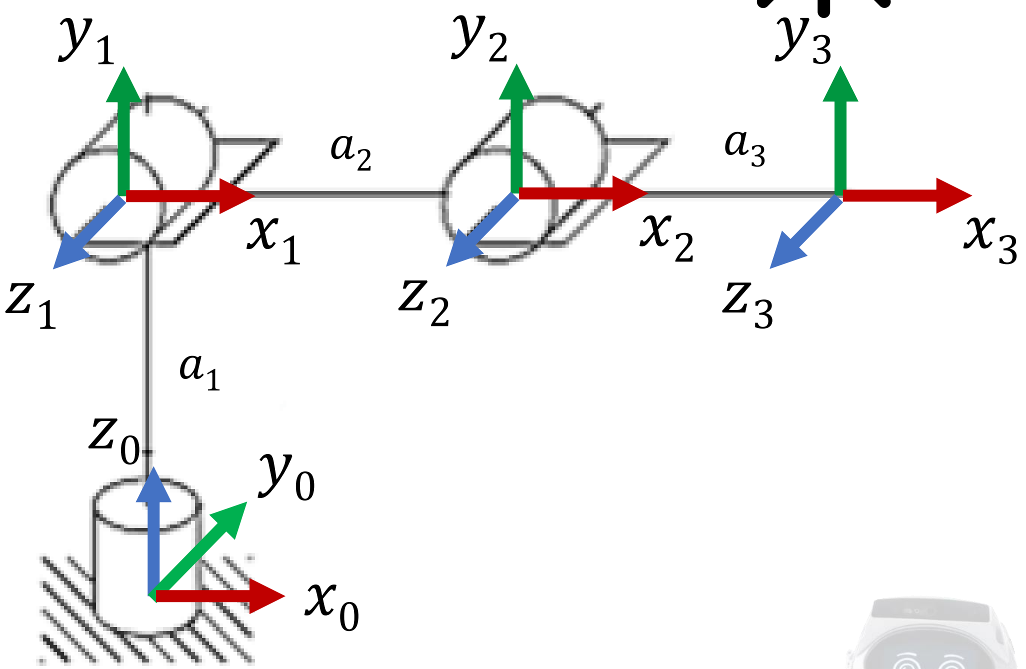
JACOBIAN

Articulated robot

n	θ	d	α	a
1	θ_1	a_1	90^o	0
2	θ_2	0	0^o	a_2
3	θ_3	0	0^o	a_3

$D - H\ parameters$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



JACOBIAN

Articulated robot

n	θ	d	α	a
1	θ_1	a_1	90^o	0
2	θ_2	0	0^o	a_2
3	θ_3	0	0^o	a_3

$D - H\ parameters$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

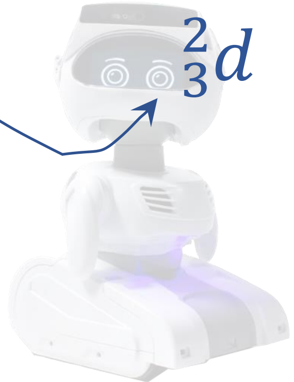
$ {}^0_1d $

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$ {}^1_2d $

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$ {}^2_3d $

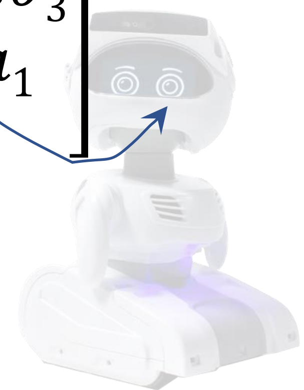


JACOBIAN

Articulated robot

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_1d = \begin{bmatrix} 0 \\ 0 \\ a_1 \\ 1 \end{bmatrix}
\quad
{}^0_2d = \begin{bmatrix} a_2c\theta_1c\theta_2 \\ a_2s\theta_1c\theta_2 \\ a_1s\theta_2 + a_1 \\ 1 \end{bmatrix}
\quad
{}^0_3d = \begin{bmatrix} a_3c\theta_1c^2\theta_2 + a_2c\theta_1c\theta_2 - a_3c\theta_1s\theta_2s\theta_3 \\ a_3s\theta_1c^2\theta_2 + a_2s\theta_1c\theta_2 - a_3s\theta_1s\theta_2s\theta_3 \\ a_3s\theta_2c\theta_2 + a_3c\theta_2s\theta_3 + a_2s\theta_2 + a_1 \\ 1 \end{bmatrix}
$$

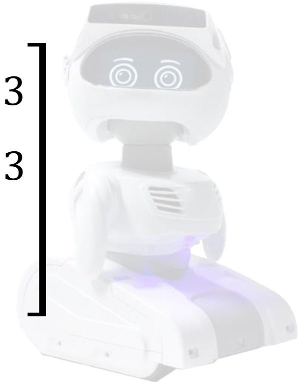


JACOBIAN

Articulated robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left({}^{0}_{\mathbf{3}}d\right) & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \times \left({}^{0}_{\mathbf{3}}d - {}^{0}_{\mathbf{1}}d\right) & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \times \left({}^{0}_{\mathbf{3}}d - {}^{0}_{\mathbf{2}}d\right) \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} & \begin{bmatrix} s\theta_1 \\ -c\theta_1 \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$${}^{0}_{1}d = \begin{bmatrix} 0 \\ 0 \\ a_1 \\ 1 \end{bmatrix} \qquad {}^{0}_{2}d = \begin{bmatrix} a_2 c\theta_1 c\theta_2 \\ a_2 s\theta_1 c\theta_2 \\ a_1 s\theta_2 + a_1 \\ 1 \end{bmatrix} \qquad {}^{0}_{3}d = \begin{bmatrix} a_3 c\theta_1 c^2\theta_2 + a_2 c\theta_1 c\theta_2 - a_3 c\theta_1 s\theta_2 s\theta_3 \\ a_3 s\theta_1 c^2\theta_2 + a_2 s\theta_1 c\theta_2 - a_3 s\theta_1 s\theta_2 s\theta_3 \\ a_3 s\theta_2 c\theta_2 + a_3 c\theta_2 s\theta_3 + a_2 s\theta_2 + a_1 \\ 1 \end{bmatrix}$$



JACOBIAN

Cross Product

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad A \times B = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

JACOBIAN (Linear Velocity)