Misr University for Science & Technology
College of Engineering
Mechatronics Department
MTE 408 Robotics
Quiz 1
Time allowed: 30 min

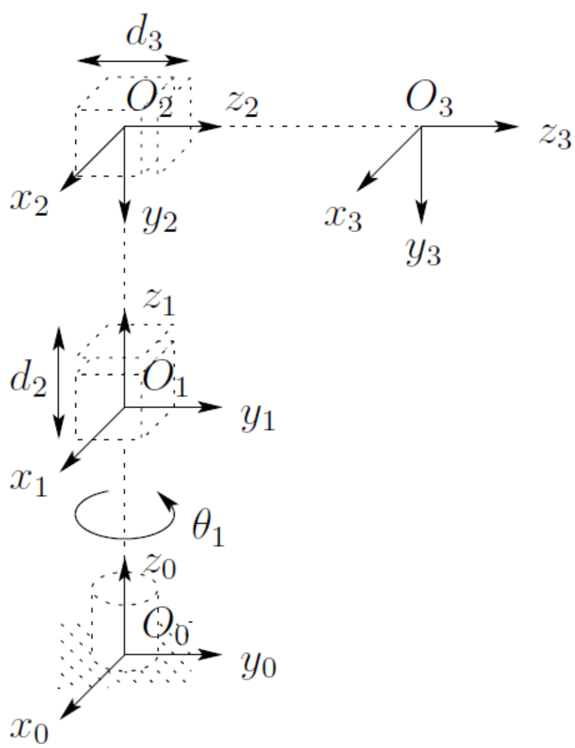
NAME:______________________________ **ID**:______________

Given this **CYLINDERICAL manipulator**:

I. Find the forward kinematic transformation matrix ${}^{0}_{3}T$ using the **complete Homogeneous Transform** method (axis rotation and translation matrix).

II.

DON'T BREAK THE RIGHT-HAND RULE!



- Use a pencil in drawing the kinematic diagram. Pen or ink used in drawing would reduce your mark.
- Pay attention to the time allowed.

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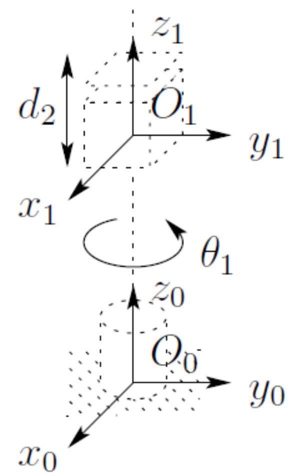
MODEL ANSWER

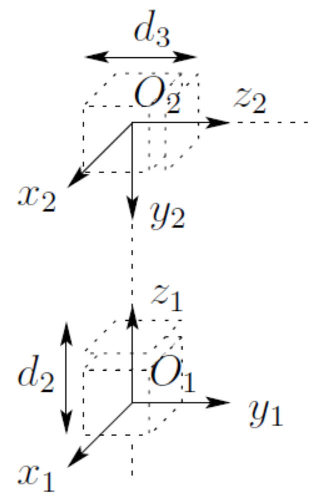


To commence, we may observe that Y-axis is the axis of rotation from frame 0 to frame 1, which shows no axis rotation from z_1 to z_2, **therefore**:

$${}^{0}_{1}T = \ R_z {}^{0}_{1}R + {}^{0}_{1}t$$

$$\therefore\ R_z = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{0}_{1}R = \begin{bmatrix} c(0) & 0 & s(0) \\ 0 & 1 & 0 \\ -s(0) & 0 & c(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{0}_{1}t = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix}$$

$${}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



For the transformation from frame 1 to frame 2, the x-axis was selected

$$= \ R_z {}^{1}_{2}R + {}^{1}_{2}t$$

$$\therefore\ R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{1}_{2}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -s(-90) \\ 0 & s(-90) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},$$

$${}^{1}_{2}t = \begin{bmatrix} 0 \\ 0 \\ a_2 + d_2 \end{bmatrix}$$

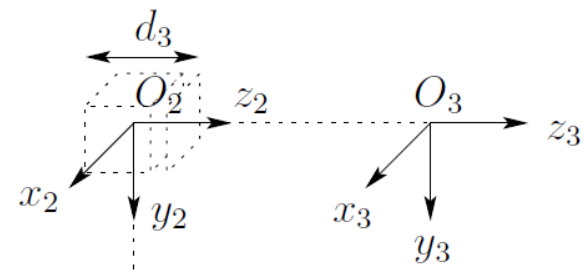
$${}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, the transformation to the end effector

$$= R_z {}^{2}_{3}R + {}^{2}_{3}t$$

$$\therefore\ R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{2}_{3}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{2}_{3}t = \begin{bmatrix} 0 \\ 0 \\ a_3 + d_3 \end{bmatrix}$$

$${}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



The overall transformation ${}^{0}_{3}T$

$${}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T$$

$${}^{0}_{3}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & a_1 + a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\boldsymbol{{}^{0}_{3}T = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & -(a_3 + d_3)s\theta_1 \\ s\theta_1 & 0 & c\theta_1 & (a_3 + d_3)c\theta_1 \\ 0 & -1 & 0 & a_1 + a_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Grading

$${}^{0}_{1}T \rightarrow 3\ Marks$$
$${}^{1}_{2}T \rightarrow 3\ Marks$$
$${}^{2}_{3}T \rightarrow 2\ Marks$$
$${}^{0}_{3}T \rightarrow 2\ Marks$$
$$-------$$
$$10\ Marks$$