MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 1
INTRODUCTION TO ROBOTICS LAB

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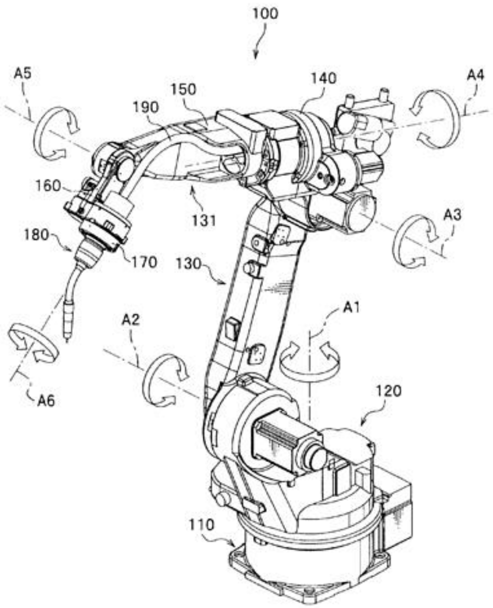
MARCH 2022

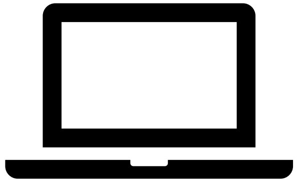


AGENDA

COURSE FLOW

MATRIX ALGEBRA USING MATLAB

LAB SOFTWARE REQUIREMENTS



- Laptop with dual boot (Windows/MAC) and Linux
- **Windows** 10 64-bit

  MATLAB R2021a or higher (64-bit)
  Peter Croke toolbox (**RTB10.4**.mltbx)
  Python 3.8 (optional) for bonus assignments
  VS Code with Python Extension Pack for bonus assignments

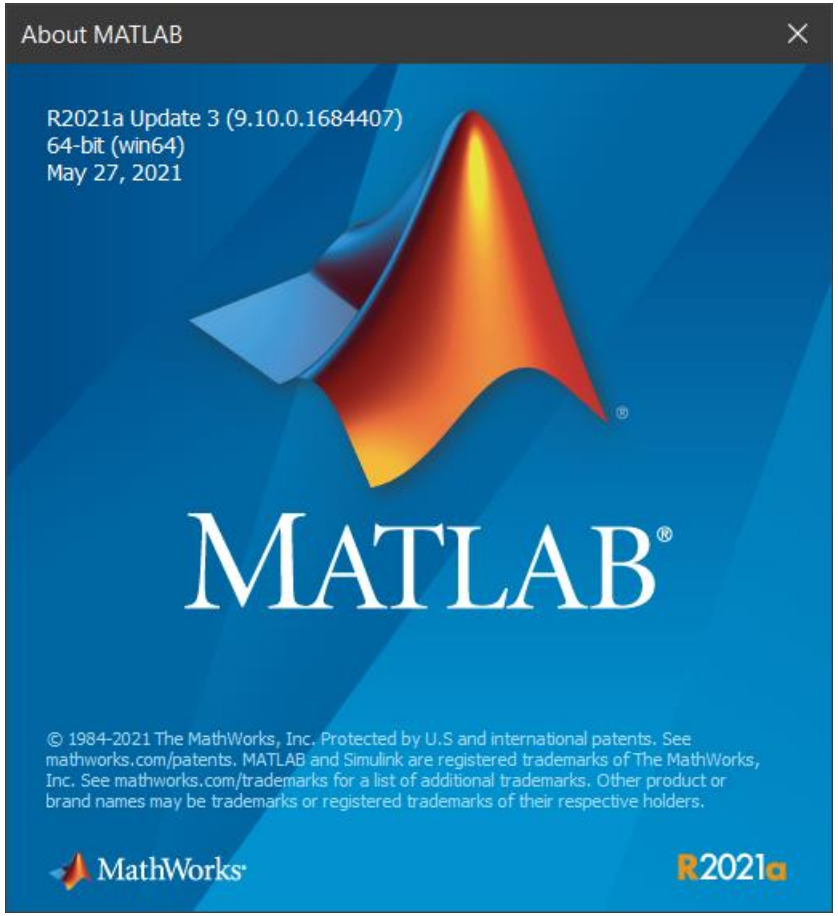
- **Linux** UBUNTU 20.04

  ROS Noetic
  Python 3.8 or higher and **C++** compiler chain
  VS Code – ROS extension, Python Extension, C++ extension

LAB SOFTWARE REQUIREMENTS

MATLAB R2021a or Higher

This is a mandatory requirements for the course. Previous releases may work but you won’t be able to use any new features in your project



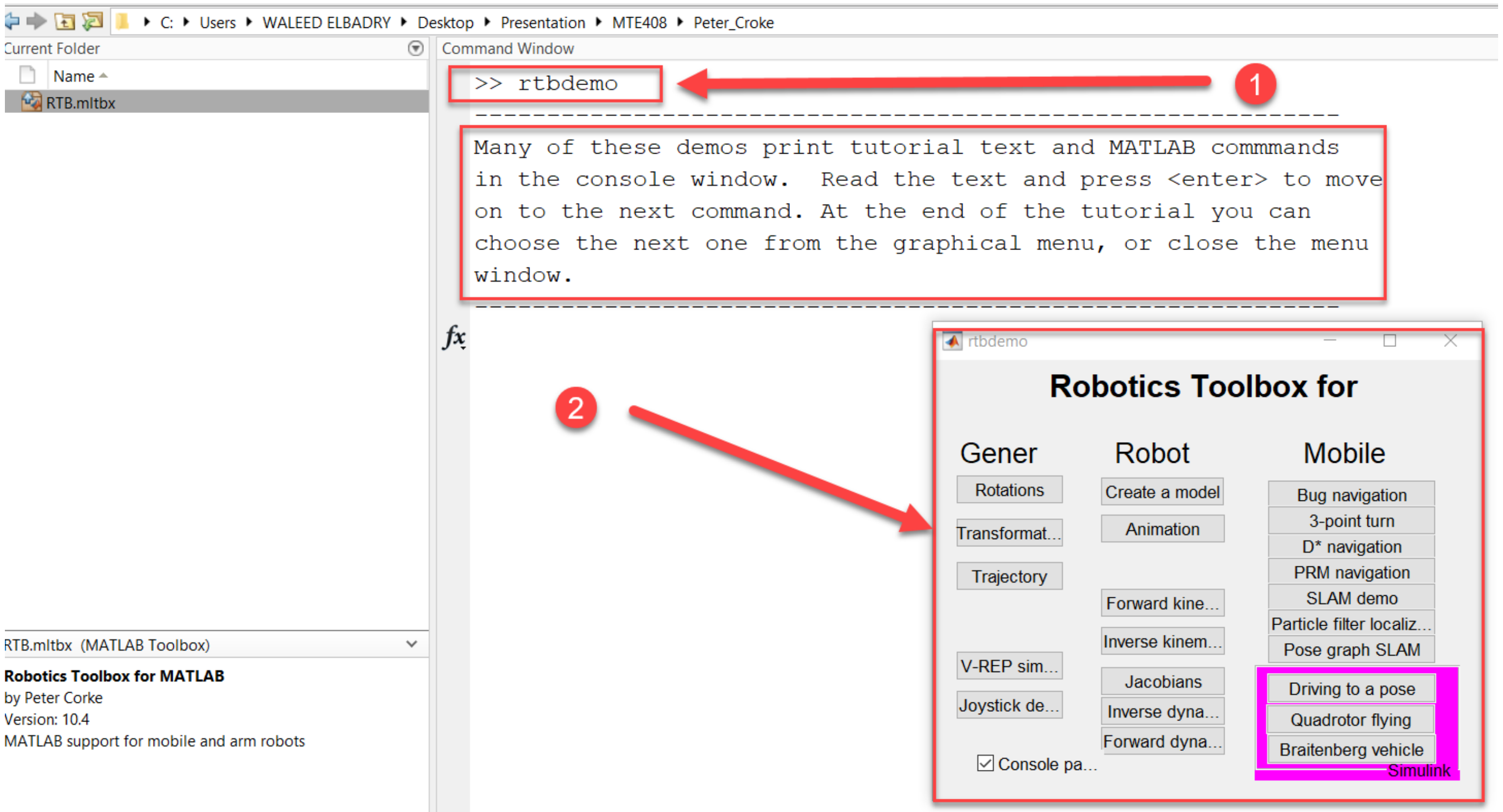
LAB SOFTWARE REQUIREMENTS

Peter Croke Toolbox

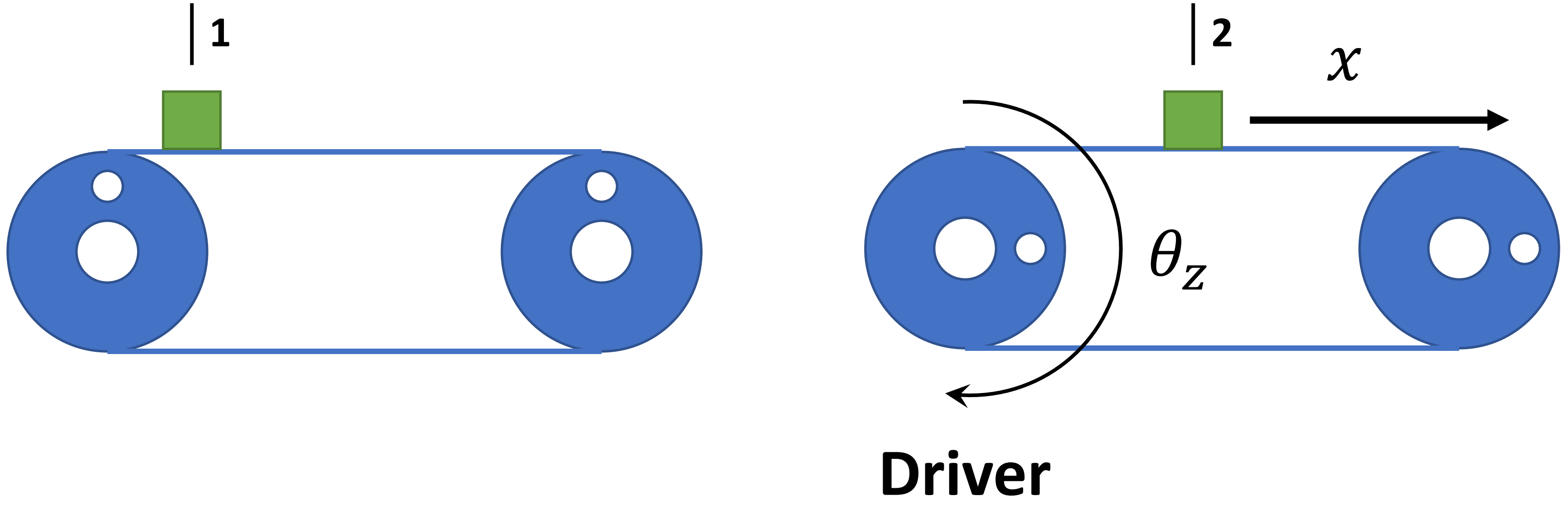
1. Go to : https://petercorke.com/toolboxes/robotics-toolbox/
2. Scroll down to section "*install from mltbx*" (MATLAB toolbox plugin).
3. Download "**RTB10.4.mltbx**" to your MTE408 course folder.

LAB SOFTWARE REQUIREMENTS

Peter Croke Toolbox verification

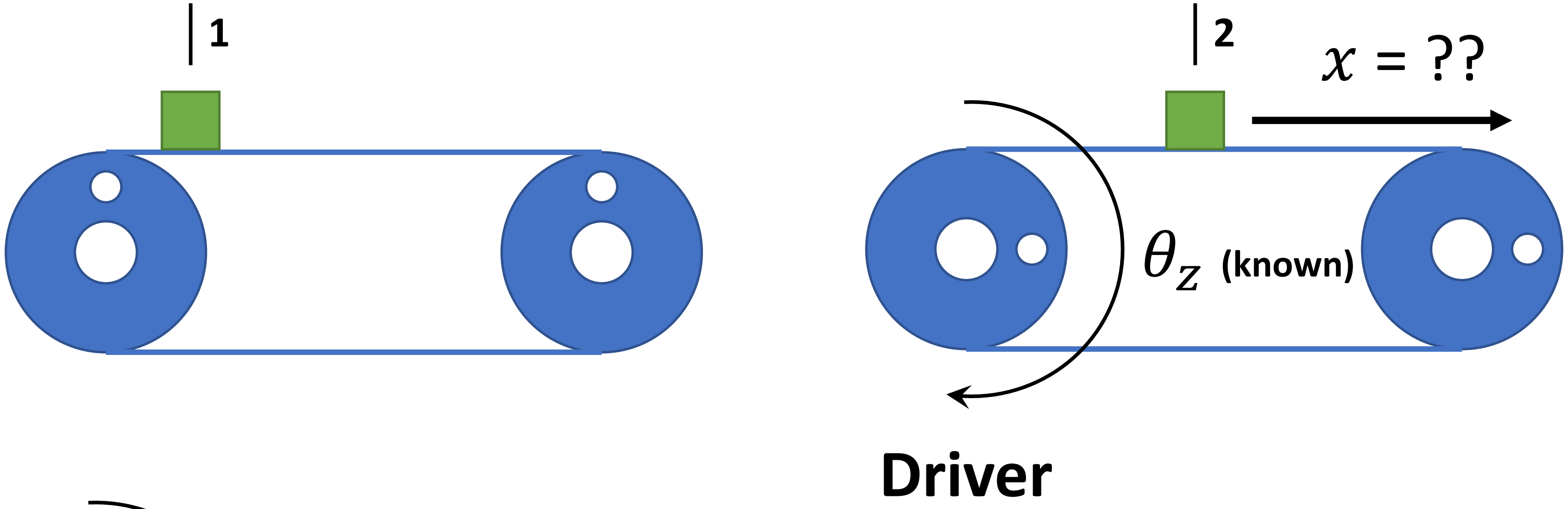


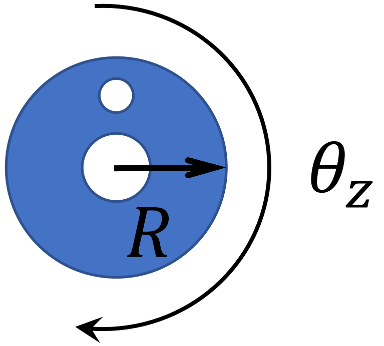
ROBOTICS MAIN COMPUTATUON



If you move the motor with an angle of $\boldsymbol{\theta_z}$, what is the object displacement $\boldsymbol{X}$?

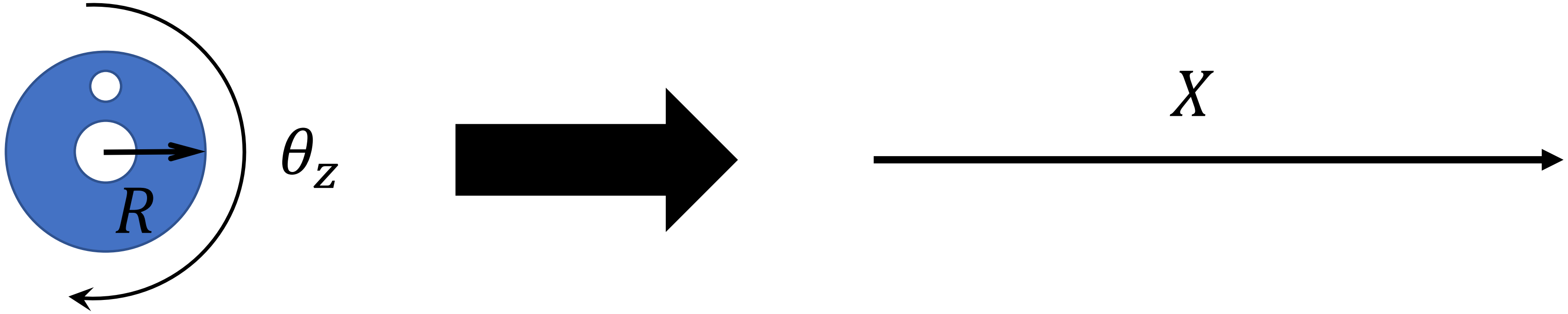
ROBOTICS MAIN COMPUTATUON





$$\boldsymbol{X}(m) = \frac{2\pi R\ (m)}{360\ (deg)}\ \boldsymbol{\theta_z}(deg)$$

ROBOTICS MAIN COMPUTATUON



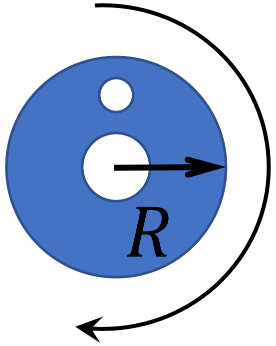
ROTATION

LINEAR

$$\boldsymbol{X}(m) = \frac{2\pi R\ (m)}{360\ (deg)}\ \boldsymbol{\theta_z}(deg) \rightarrow \boldsymbol{Foward}\ Kinematics$$

$$\boldsymbol{\theta_z}(deg) = \frac{360\ (deg)}{2\pi R\ (m)}\ \boldsymbol{X}(m) \rightarrow \boldsymbol{Inverse}\ Kinematics$$

ROBOTICS MAIN COMPUTATUON



$\theta_z, \dot{\theta}_z, \ddot{\theta}_z$

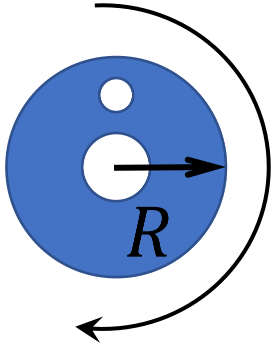
$X, \dot{X}, \ddot{X}$

ROTATION

LINEAR

$\boldsymbol{Forward\ Kinemtaics}$

ROBOTICS MAIN COMPUTATUON



$\theta_z, \dot{\theta}_z, \ddot{\theta}_z$

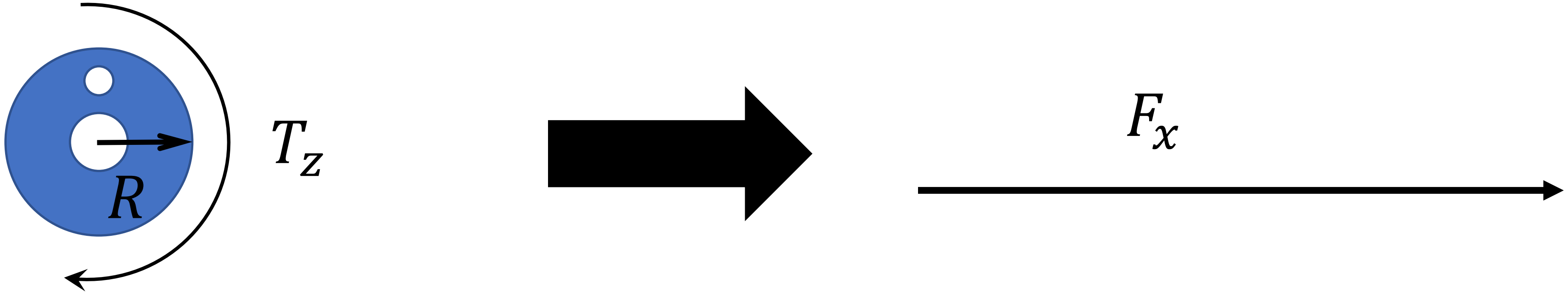
$X, \dot{X}, \ddot{X}$

ROTATION

LINEAR

$$\boldsymbol{Inverse\ Kinemtaics}$$

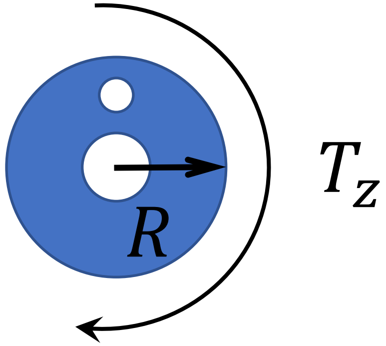
ROBOTICS MAIN COMPUTATUON



ROTATION **LINEAR**

Forward Dynamics

ROBOTICS MAIN COMPUTATUON

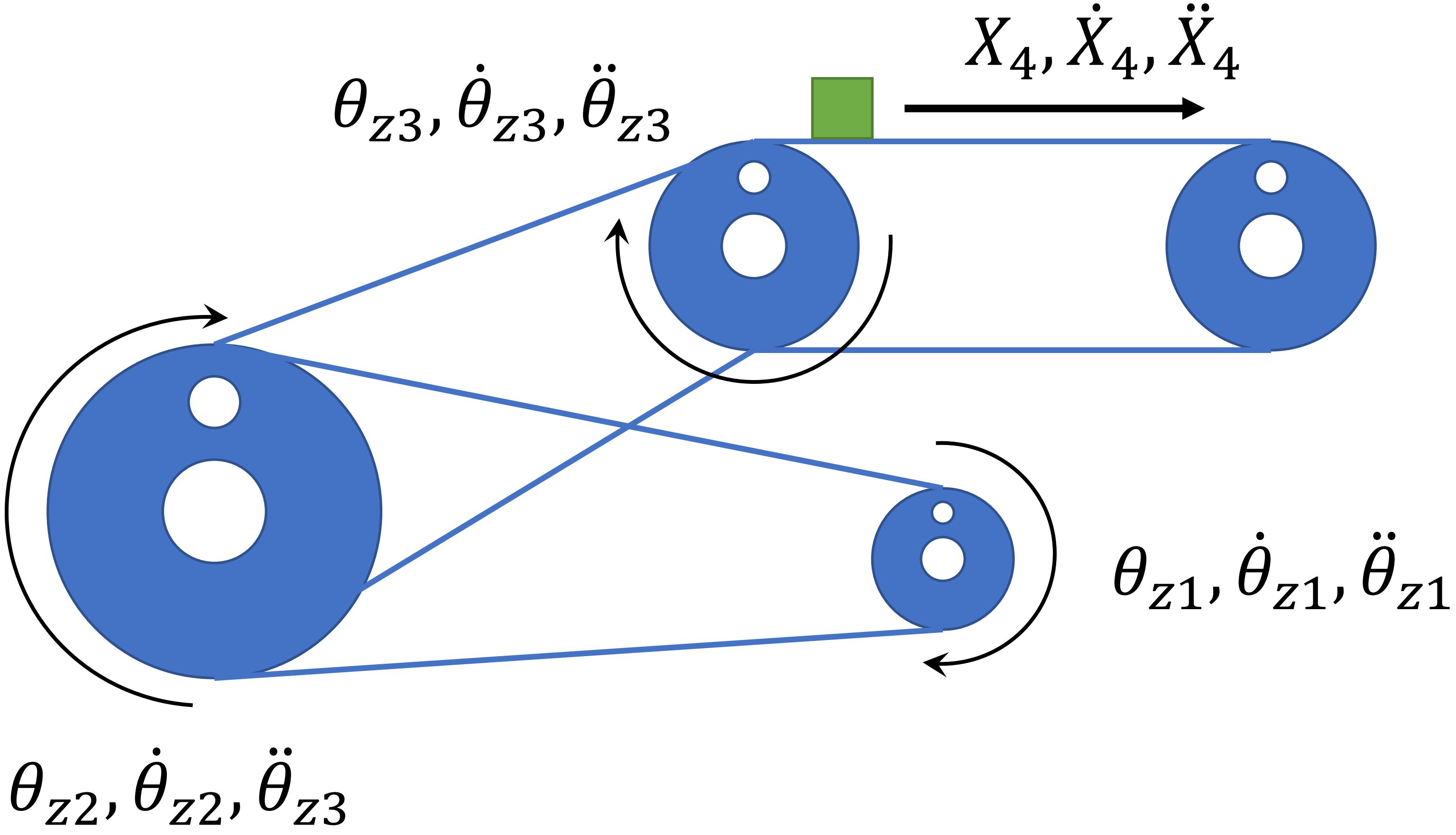


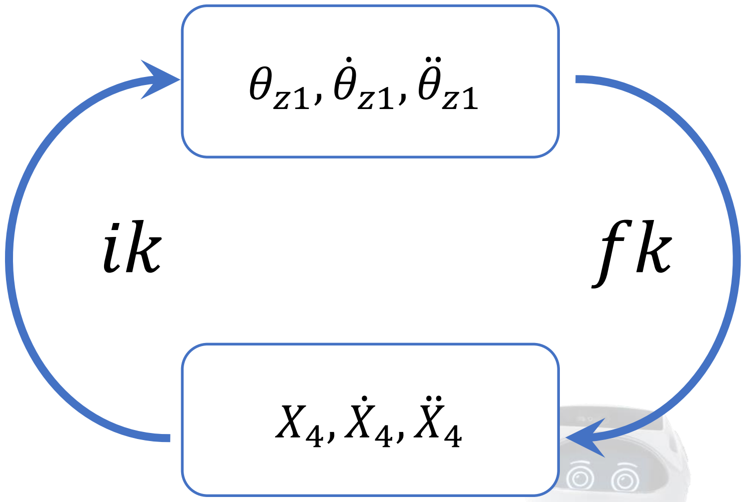
ROTATION

LINEAR

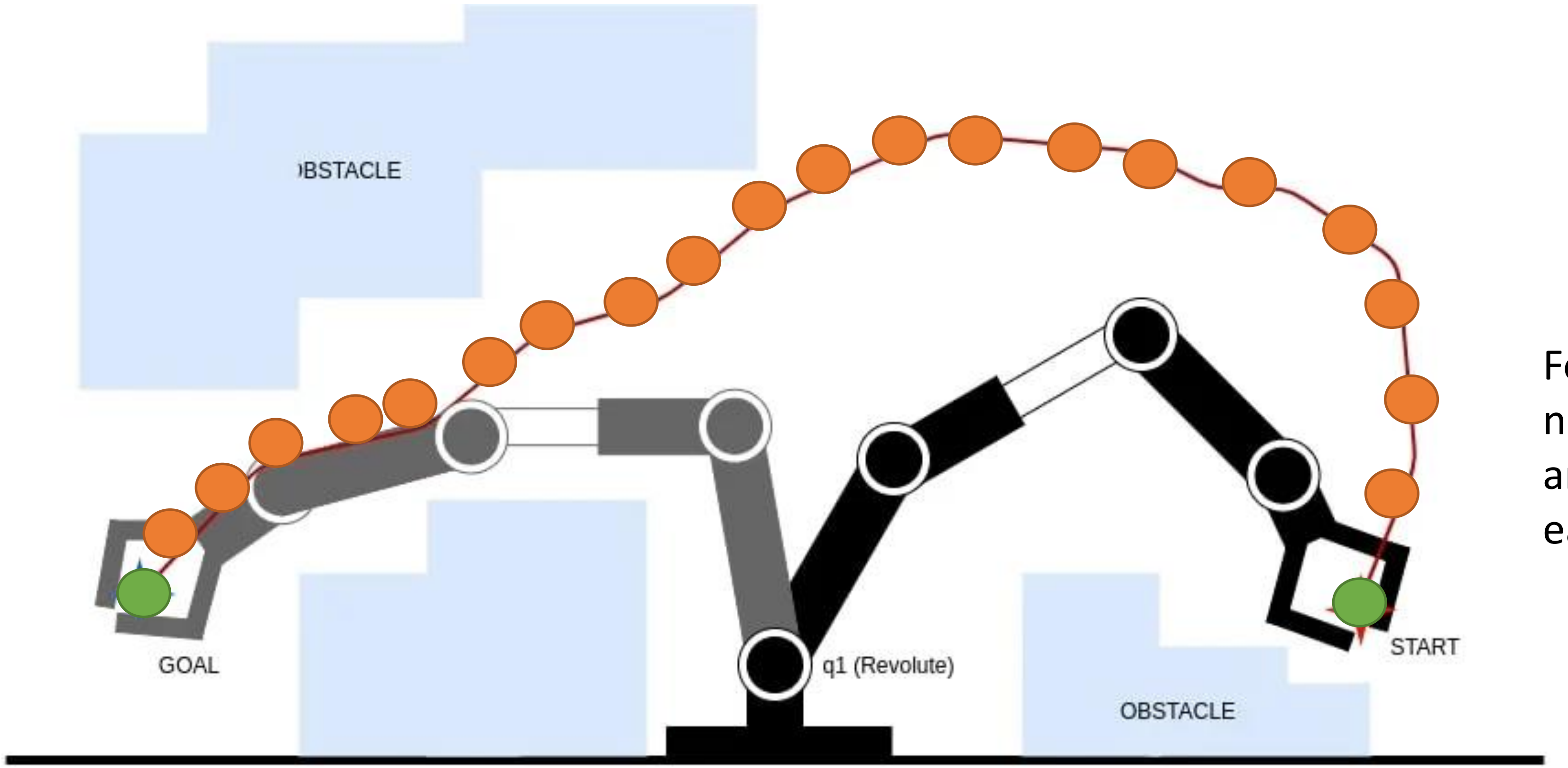
$$\boldsymbol{Inverse\ Dynamics}$$

ROBOTICS MAIN COMPUTATUON





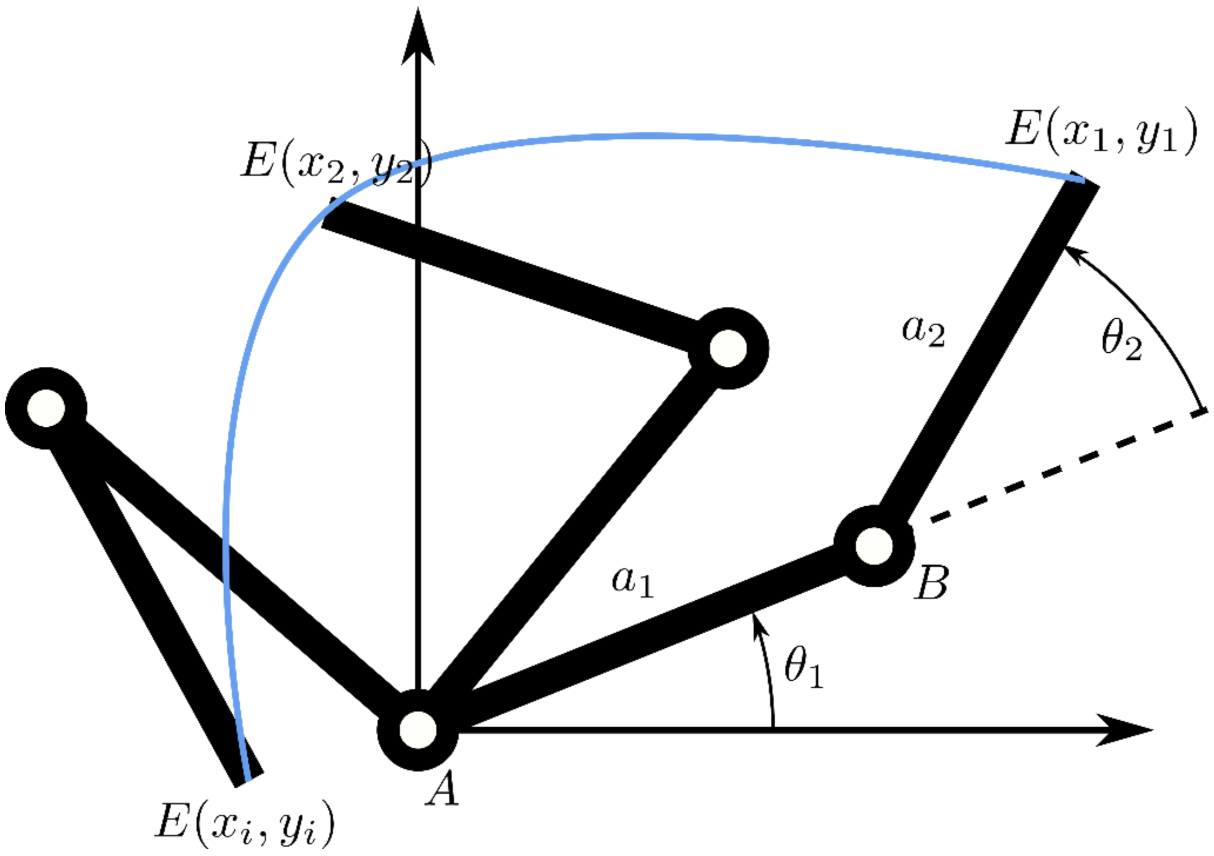
ROBOTICS MAIN COMPUTATUON



For each point on the path, you need to find the appropriate angle, speed and acceleration for each joint.

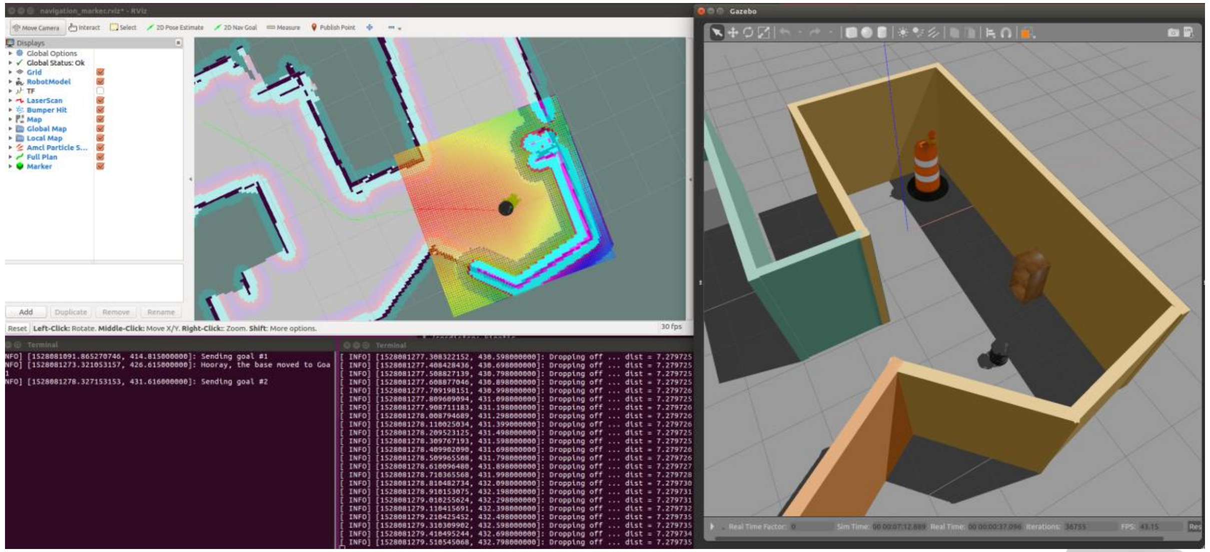
LEARNING PATH

1. Coordinate system
2. Pose
3. Translation and Rotation
4. Forward Kinematics
5. Inverse Kinematics
6. Forward Dynamics
7. Inverse Dynamics
8. Path Planning
9. Trajectory Generation



LEARNING PATH (LAB)

1. Matrix Algebra
2. Coordinate Systems
3. Homogeneous Transform
4. Forward Kinematics
5. Inverse Kinematics
6. Forward Dynamics
7. Inverse Dynamics
8. Path planning
9. ROS navigation stack (AMCL + Dijkstra)



MATRIX ALGEBRA WITH MATLAB

ROW VECTOR

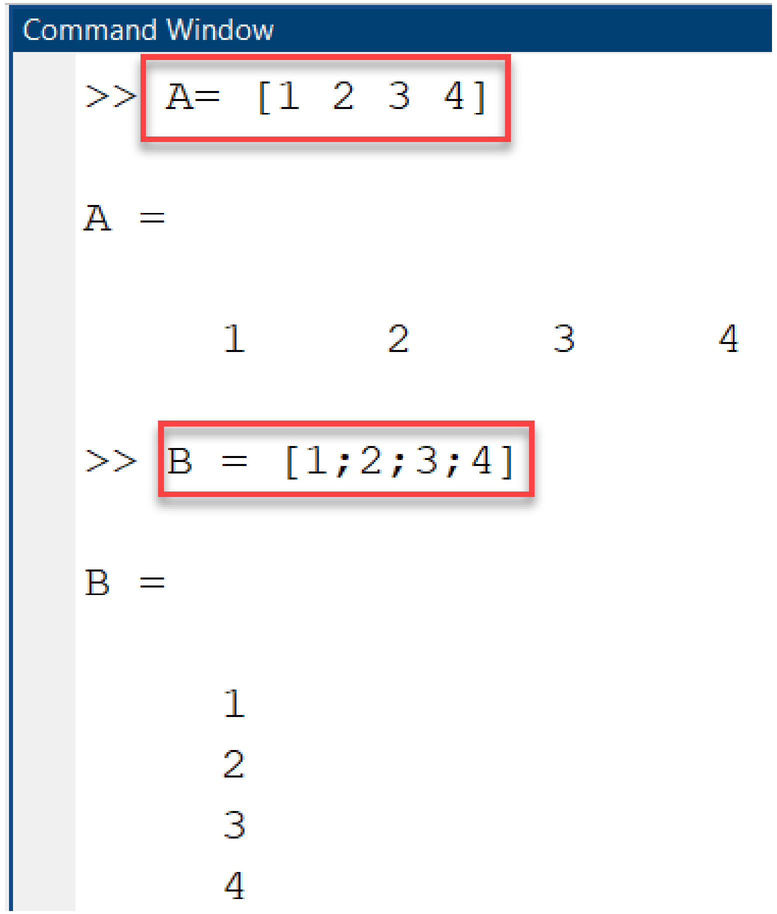
$\gg \; A = [\,1\;2\;3\;4]$

$\gg \; A = [\,1,2,3,4]$

COLUMN VECTOR

$\gg B = [\,1;2\,;3\,;4]$

$\gg B = [\,1$
2
3
$4]$



MATRIX ALGEBRA WITH MATLAB

RECTAGULAR MATRIX

$$\gg \; A = [\,1\ 2\ 3\ 4;5\ 6\ 7\ 8]$$

SQUARE VECTOR

$$\gg B = [\,1\ 2\ 3;4\ 5\ 6;7\ 8\ 9]$$

Command Window

```
>> A = [1 2 3 4;5,6,7,8]

A =

     1     2     3     4
     5     6     7     8

>> B = [1 2 3;4 5 6;7,8,9]

B =

     1     2     3
     4     5     6
     7     8     9
```

MATRIX ALGEBRA WITH MATLAB

DIAGONAL MATRIX

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\gg d = [\,1\;2\;1];$$

$$\gg A = diag(d)$$

```
Command Window
>> d = [1 2 1];
>> A = diag(d)

A =

     1     0     0
     0     2     0
     0     0     1
```

IDENTITY MATRIX - I

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \gg A = eye(3)$$

```
Command Window
>> A = eye(3)

A =

     1     0     0
     0     1     0
     0     0     1
```

MATRIX ALGEBRA WITH MATLAB

NULL MATRIX

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\gg A = zeros(3)$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\gg B = zeros(2,3)$$

Command Window

```
>> A = zeros(3)

A =

     0     0     0
     0     0     0
     0     0     0

>> B = zeros(2,3)

B =

     0     0     0
     0     0     0
```

MATRIX ALGEBRA WITH MATLAB

SCALAR MATRIX

$$A = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$\gg c = 6;$

$\gg d = ones(1,4);$

$\gg A = diag(d) * c$

```
Command Window
>> c = 6;
>> d = ones(1,4);
>> A = diag(d) * c

A =

     6     0     0     0
     0     6     0     0
     0     0     6     0
     0     0     0     6
```

MATRIX ALGEBRA WITH MATLAB

ADDITION

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix}$$

$$\gg [7\ 3\ -1; 2\ -5\ \ 6] + [1{,}5{,}6; -4\ -2\ 3]$$

```
Command Window
>> [7 3 -1;2 -5  6]+[1,5,6;-4 -2 3]

ans =

     8     8     5
    -2    -7     9
```

MATRIX ALGEBRA WITH MATLAB

SCALAR MULTIPLICATION

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} \quad k = 4$$

$$\gg A = [3\ -1; 2,1; 2\ -3; 4\ 1];$$

$$\gg k = 4;$$

Command Window

```
>> A=[3 -1;2,1;2 -3;4 1]

A =

     3    -1
     2     1
     2    -3
     4     1

>> k =4

k =

     4

>> A * k

ans =

    12    -4
     8     4
     8   -12
    16     4
```

MATRIX ALGEBRA WITH MATLAB

MULTIPLICATION OF MATRICES

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & 4 \\ 6 & -1 & 0 \end{bmatrix}$$

$\gg A = [3\ -1; 2{,}1; 2\ -3; 4\ 1];$

$\gg size(A)$

$\gg B = [2\ 3\ 4;\ 6\ -1\ 0];$

$\gg size(B)$

$\gg A\ *B$

Command Window

```
>> A=[3 -1;2,1;2 -3;4 1];
>> size(A)

ans =

     4     2

>> B = [2 3 4; 6 -1 0];
>> size(B)

ans =

     2     3

>> A * B

ans =

     0    10    12
    10     5     8
   -14     9     8
    14    11    16
```

MATRIX ALGEBRA WITH MATLAB

MULTIPLICATION OF MATRICES

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

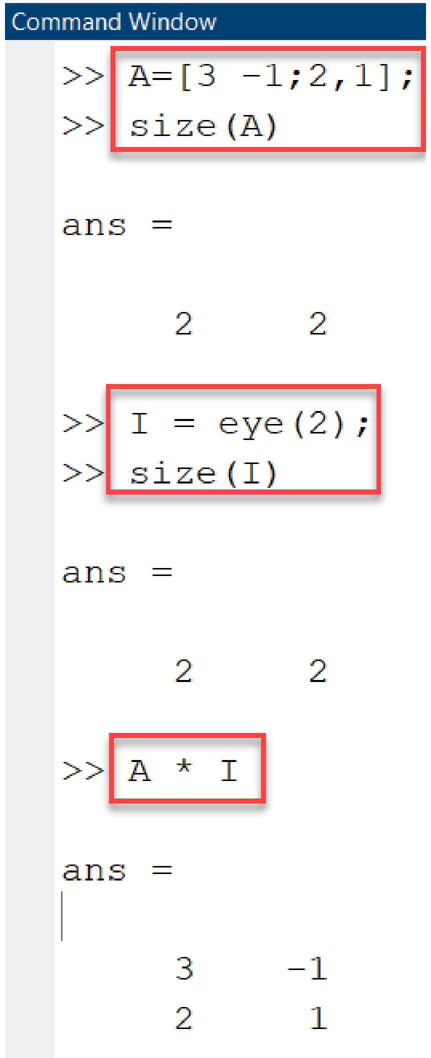
$\gg A = [3\ -1; 2,1];$

$\gg size(A)$

$\gg I = eye(2);$

$\gg size(I)$

$\gg A\ * I$



MATRIX ALGEBRA WITH MATLAB

MATRIX TRANSPOSE

$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ B= $\begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$

$\gg A = [3\ \ -1; 2{,}1];$

$\gg B = A'$

$\gg B = transpose(A)$

Command Window

```
>> A= [3 -1;2,1]

A =

     3    -1
     2     1

>> B = A'

B =

     3     2
    -1     1

>> B = transpose(A)

B =

     3     2
    -1     1
```

MATRIX ALGEBRA WITH MATLAB

EXERCISE

$Prove\ using\ \boldsymbol{MATLAB}$

$(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$$A = \begin{bmatrix} 1 & 7 & 13 \\ 3 & 9 & 15 \\ 5 & 11 & 17 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 8 & 14 \\ 4 & 10 & 16 \\ 6 & 12 & 18 \end{bmatrix}$$

```
>> (A+B)'

ans =

     3     7    11
    15    19    23
    27    31    35
```

```
>> (A*B)'

ans =

   108   132   156
   234   294   354
   360   456   552
```

MATRIX ALGEBRA WITH MATLAB

MATRIX INVERSE

$An\ \boldsymbol{invertible\ matrix}\ must\ be$:

$-\ Square\ M_{nxn}$

$-\ Linearly\ independant\ (no\ column\ is\ related\ to\ other\ columns$

$\boldsymbol{singularity}$

$-\ The\ result\ of\ M_{nxn}\,.\,M^{-1}\ = I\ (Identity)$

MATRIX ALGEBRA WITH MATLAB

MATRIX INVERSE

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$\gg inv(A)$

Command Window

```
>> A =[3 1;2 1]

A =

     3     1
     2     1

>> inv(A)

ans =

    1.0000   -1.0000
   -2.0000    3.0000
```

MATRIX INVERSE

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad \textit{Cofactors and Adjoint method}$$

$1.\ \textit{Matrix of minors}\ (\boldsymbol{1^{st}row})$

$$\begin{bmatrix} \cancel{3} & \cancel{0} & \cancel{2} \\ \cancel{2} & 0 & -2 \\ \cancel{0} & 1 & 1 \end{bmatrix}, \begin{bmatrix} \cancel{3} & \cancel{0} & \cancel{2} \\ 2 & \cancel{0} & -2 \\ 0 & \cancel{1} & 1 \end{bmatrix}, \begin{bmatrix} \cancel{3} & \cancel{0} & \cancel{2} \\ 2 & 0 & \cancel{-2} \\ 0 & 1 & \cancel{1} \end{bmatrix}$$

$$\begin{bmatrix} (0*1)-(-2*1) & (2*1)-(-2*0) & (2*1)-(0*0) \\ (0*1)-(2*1) & (3*1)-(2*0) & (3*1)-(0*0) \\ (0*-2)-(2*0) & (3*-2)-(2*2) & (3*0)-(0*2) \end{bmatrix}$$

MATRIX INVERSE

Cofactors and Adjoint method

1. *Matrix of minors*

$$M = \begin{bmatrix} 2 & 2 & 2 \\ -2 & 3 & 3 \\ 0 & -10 & 0 \end{bmatrix} \rightarrow \{1\}$$

2. *Matrix of cofactors (Chekboard method)*

$$Sign = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow M = \begin{bmatrix} 2 & 2 & 2 \\ -2 & 3 & 3 \\ 0 & -10 & 0 \end{bmatrix} \rightarrow \mathbf{C} = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & -3 \\ 0 & 10 & 0 \end{bmatrix}$$

MATRIX INVERSE

Cofactors and Adjoint method

3. *Adjoint matrix (Transpose cofactor matrix)*

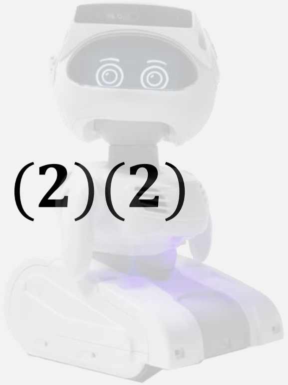
$$\mathbf{Adj} = \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix} \rightarrow \{3\}$$

4. *Find the determinant*

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & -3 \\ 0 & 10 & 0 \end{bmatrix}$$

$$\mathbf{det(A)} = \mathbf{(3)(2) + (0)(-2) + (2)(2)}$$

$$\mathbf{det(A)} = \mathbf{6 + 0 + 4 = 10}$$



MATRIX INVERSE

Cofactors and Adjoint method

4. *Find the determinant*

$$\mathbf{det(A) = 6 + 0 + 4 = 10}$$

5. *Find the inverse*

$$\boldsymbol{A^{-1}} = \frac{\mathbf{1}}{\boldsymbol{det}}[Adj] = \frac{1}{10}\begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}$$

$$\boldsymbol{Check} \rightarrow \boldsymbol{AA^{-1}} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

MATRIX ALGEBRA WITH MATLAB

MATRIX INVERSE

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

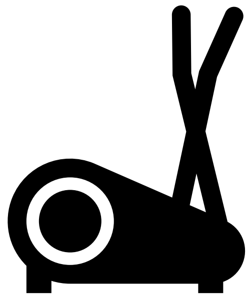
$$\gg A = [3\;0\;2;2\;0\;-2;0\;1\;1];$$

$$\gg inv(A)$$

$$\gg A * inv(A)$$

Command Window

```
>> A= [3 0 2;2 0 -2;0 1 1]

A =

     3     0     2
     2     0    -2
     0     1     1

>> inv(A)

ans =

    0.2000    0.2000         0
   -0.2000    0.3000    1.0000
    0.2000   -0.3000         0

>> A * inv(A)

ans =

    1.0000         0         0
   -0.0000    1.0000         0
         0         0    1.0000
```



SHEET WILL BE SOLVED NEXT SECTION