MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS ENGINEERING DEPARTMENT
MTE 408 ROBOTICS

SESSION 8
INTRODUCTION TO ROBOTICS LAB

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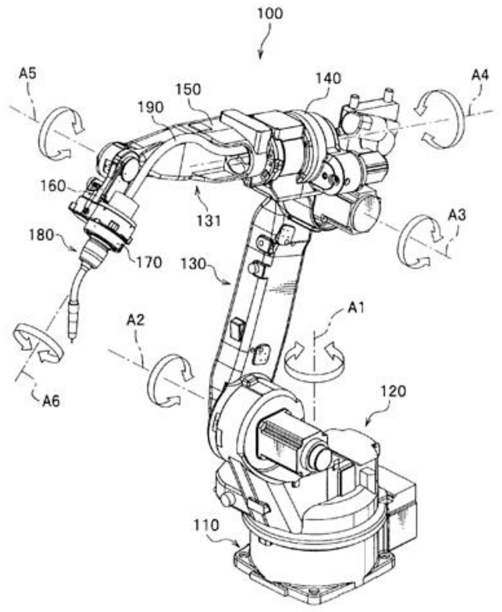


JACOBIAN

What is the Jacobian Matrix ($\boldsymbol{J}$)

$\dot{X}$ ... *The end effector velocity* $(m \; x \; 1)$

$\dot{q}$ ... *The joints variables angular velocity* $(n \; x \; 1)$

J ... *The jacobian matrix* $(m \; x \; n)$

n ... *The number of robot joints*

m ... *Robot location and orientation*

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6 \; x \; 1} = \begin{bmatrix} J_{11} & \cdots & J_{1n} \\ J_{21} & \cdots & J_{2n} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ J_{61} & \cdots & J_{6n} \end{bmatrix}_{m \; x \; n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n \; x \; 1}$$

Jacobian *is a matrix provides the relationship between the* ***end effector velocity*** $\dot{\boldsymbol{X}}$ *and the* ***joints angular velocities*** $\dot{\mathbf{q}}$

The number of rows is always 6 but the number of columns depneds on the number of joints

JACOBIAN

$$What\ is\ the\ Jacobian\ Matrix\ (\boldsymbol{J})$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \frac{\partial \theta_z}{\partial q_1} & \cdots & \frac{\partial \theta_z}{\partial q_1} \end{bmatrix}_{m\ x\ n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\ x\ 1}$$

$$\boldsymbol{Jacobian}\ is\ also\ the\ first\ derivative\ of\ the\ position\ and\ orientation$$

$$Recall\ that\ \boldsymbol{v} = \frac{dx}{dt} = \frac{x_{t+\Delta t} - x_t}{(t + \Delta t) - t}$$

JACOBIAN

What is the Jacobian Matrix (J)

We learned from the ***kinematics sections****, to transform from a frame to another we need to derive:*

The ***rotation*** *transformation*

The ***displacement*** *transformation*

$$ {}^{0}_{i-1}\boldsymbol{T} = \begin{bmatrix} R^{3\ x\ 3} & d^{3\ x\ 1} \\ 0 & 1 \end{bmatrix} $$

JACOBIAN

Jacobian transformation table

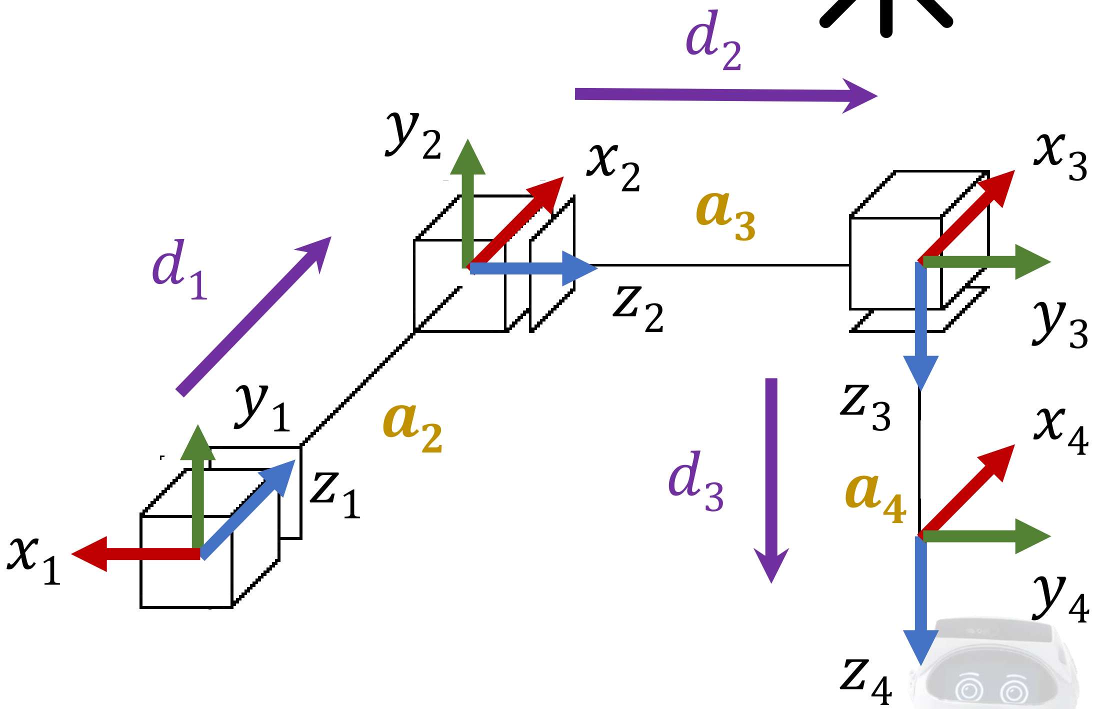
	Prismatic	Revolute
Linear	${}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ({}_{n}^{0}d - {}_{i-1}^{\ \ 0}d)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$J_{Rev} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} {}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \qquad J_{Pri} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} {}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ({}_{n}^{0}d - {}_{i-1}^{\ \ 0}d) \\ {}_{i-1}^{\ \ 0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

JACOBIAN

Cartesian robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \underbrace{\begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}_{6\ x\ 3}}_{3\ JOINTS\ JACOBIAN} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\ x\ 1}$$

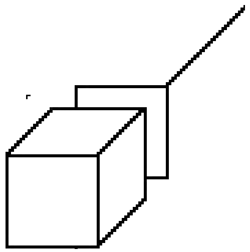


	Prismatic	*Revolute*
Linear	${}_{i-1}^{0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}_{i-1}^{0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ({}_{n}^{0}d - {}_{i-1}^{0}d)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}_{i-1}^{0}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

JACOBIAN

Cartesian robot

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix}_{6\ x\ 1} = \boxed{\begin{bmatrix} {}^{0}_{0}R\begin{bmatrix}0\\0\\1\end{bmatrix} & & \\ \begin{matrix}0\\0\\0\end{matrix} & & \end{bmatrix}}_{6\ x\ 3} \begin{bmatrix}\dot{d}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}_{3\ x\ 1}$$



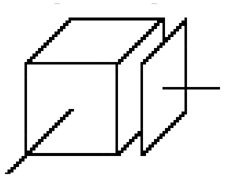
Base joint

No rotational axis

	Prismatic	***Revolute***
Linear	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}({}^{0}_{n}d - {}^{0}_{i-1}d)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$

JACOBIAN

Cartesian robot



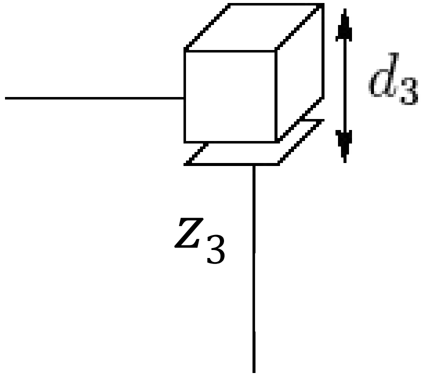
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\,x\,1} = \begin{bmatrix} {}^0_0R\begin{bmatrix}0\\0\\1\end{bmatrix} & \boxed{{}^0_1R\begin{bmatrix}0\\0\\1\end{bmatrix}} & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \end{bmatrix}_{6\,x\,3} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\,x\,1}$$

	Prismatic	***Revolute***
Linear	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}({}^0_nd - {}^0_{i-1}d)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	${}^{0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$

JACOBIAN

Cartesian robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} {}^0_0R\begin{bmatrix}0\\0\\1\end{bmatrix} & {}^0_1R\begin{bmatrix}0\\0\\1\end{bmatrix} & {}^0_2R\begin{bmatrix}0\\0\\1\end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\ x\ 3} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\ x\ 1}$$

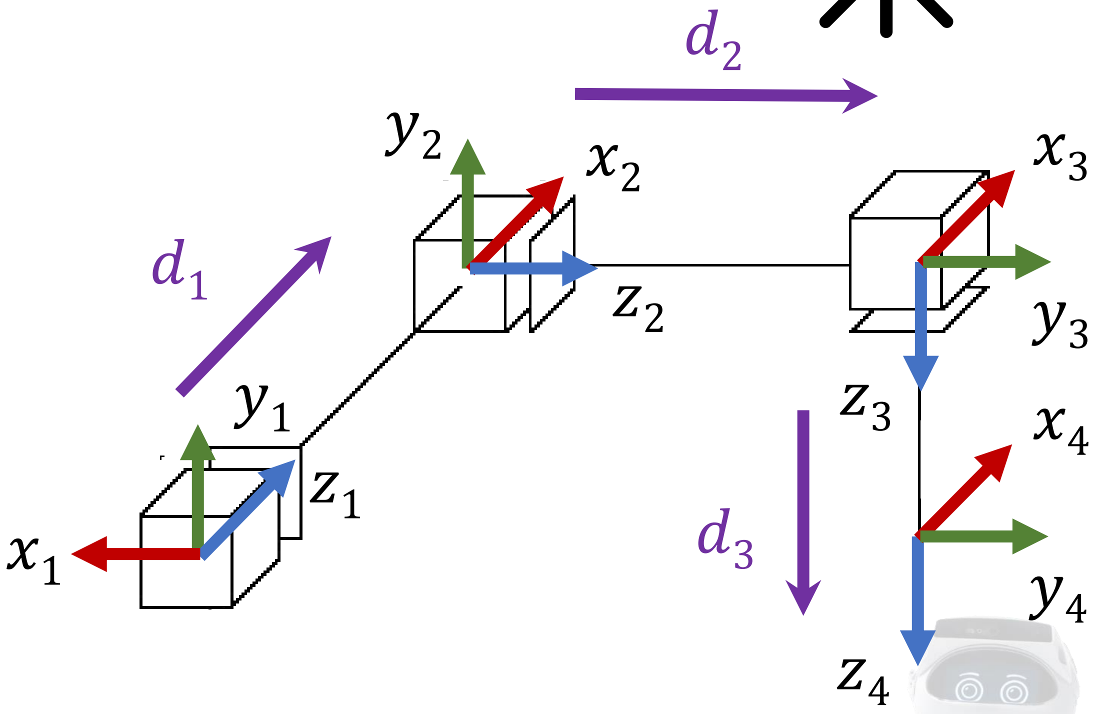


	Prismatic	***Revolute***
Linear	${}^{\ \ 0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$	${}^{\ \ 0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}({}^0_nd - {}^{\ \ 0}_{i-1}d)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	${}^{\ \ 0}_{i-1}R\begin{bmatrix}0\\0\\1\end{bmatrix}$

JACOBIAN

Cartesian robot

$$
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\,x\,1} = \begin{bmatrix} {}^0_0R\begin{bmatrix}0\\0\\1\end{bmatrix} & {}^0_1R\begin{bmatrix}0\\0\\1\end{bmatrix} & {}^0_2R\begin{bmatrix}0\\0\\1\end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\,x\,3} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\,x\,1}
$$



$$
{}^0_0\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^0_1\boldsymbol{R} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad {}^0_2\boldsymbol{R} = {}^0_1R\,{}^1_2R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}
$$

JACOBIAN

$\boldsymbol{Cartesian\ robot}$

$${}^{\mathbf{0}}_{\mathbf{0}}\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^{\mathbf{0}}_{\mathbf{1}}\boldsymbol{R} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad {}^{\mathbf{0}}_{\mathbf{2}}\boldsymbol{R} = {}^0_1R\,{}^1_2R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\ x\ 3} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\ x\ 1}$$

JACOBIAN

$\boldsymbol{Cartesian\ robot}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\ x\ 1} = \begin{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\ x\ 3} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}_{3\ x\ 1}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} & \begin{bmatrix} \mathbf{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{-1} \\ \mathbf{0} \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

JACOBIAN

$\boldsymbol{Cartesian\ robot}$

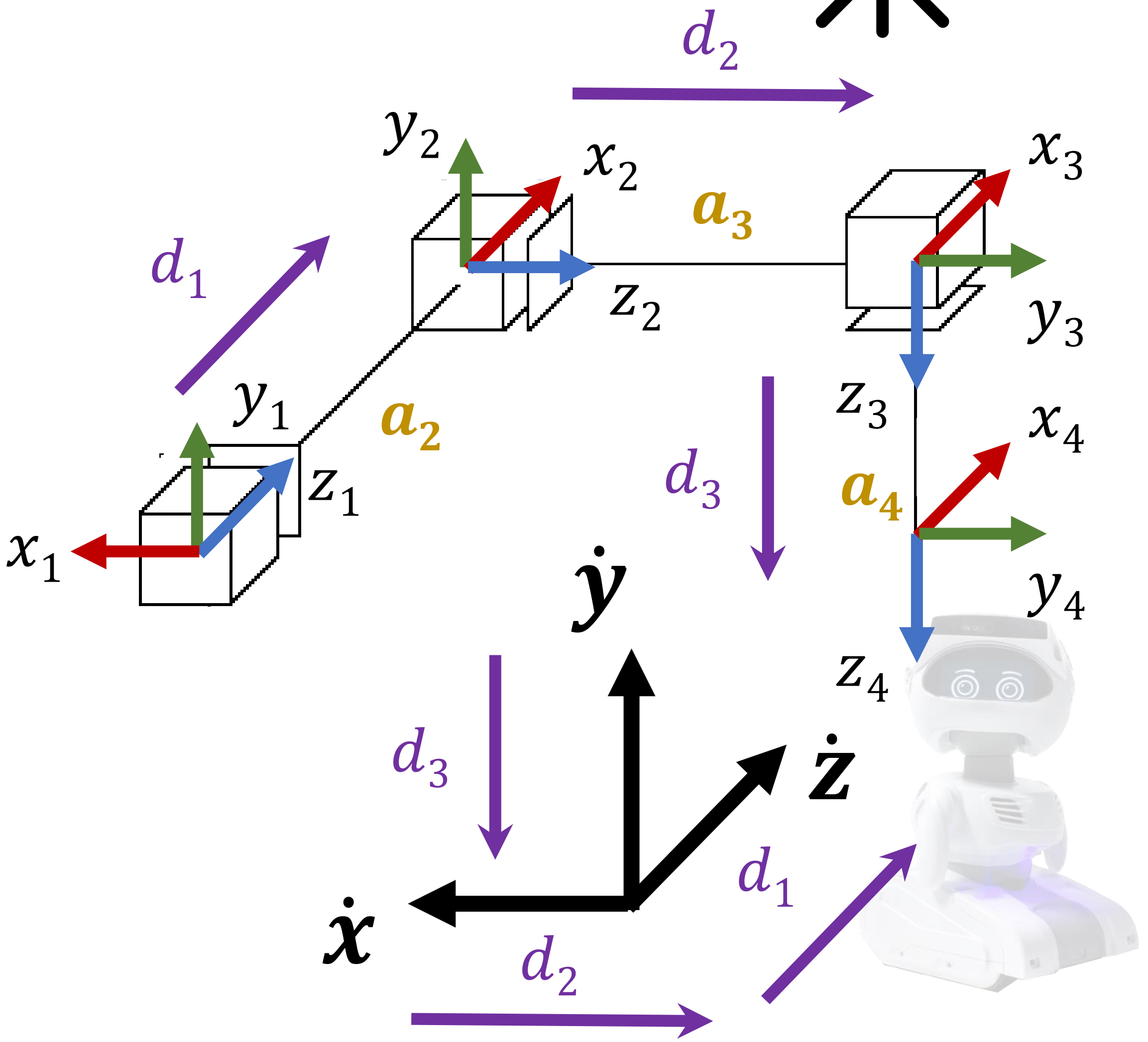
$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{1}\end{bmatrix} & \begin{bmatrix}\mathbf{-1}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} & \begin{bmatrix}\mathbf{0}\\ \mathbf{-1}\\ \mathbf{0}\end{bmatrix} \\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \begin{bmatrix}\dot{d}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix} = \begin{bmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \begin{bmatrix}\dot{d}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}$$

$$\dot{\boldsymbol{x}} = -\dot{d}_2 \quad \boldsymbol{\omega_x} = 0$$
$$\dot{\boldsymbol{y}} = -\dot{d}_3 \quad \boldsymbol{\omega_y} = 0$$
$$\dot{\boldsymbol{z}} = \dot{d}_1 \quad \boldsymbol{\omega_z} = 0$$

JACOBIAN

Cartesian robot

$$\dot{\boldsymbol{x}} = -\dot{d}_2 \quad \boldsymbol{\omega}_{\boldsymbol{x}} = 0$$
$$\dot{\boldsymbol{y}} = -\dot{d}_3 \quad \boldsymbol{\omega}_{\boldsymbol{y}} = 0$$
$$\dot{\boldsymbol{z}} = \dot{d}_1 \quad \boldsymbol{\omega}_{\boldsymbol{z}} = 0$$



INVERSE JACOBIAN

$\boldsymbol{Cartesian\ robot}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix} \rightarrow J^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J^{-1} J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix} \rightarrow J^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = I \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\begin{bmatrix} \dot{\boldsymbol{q}}_{\boldsymbol{1}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{2}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{3}} \\ \vdots \\ \dot{\boldsymbol{q}}_{\boldsymbol{n}} \end{bmatrix} = J^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

INVERSE JACOBIAN

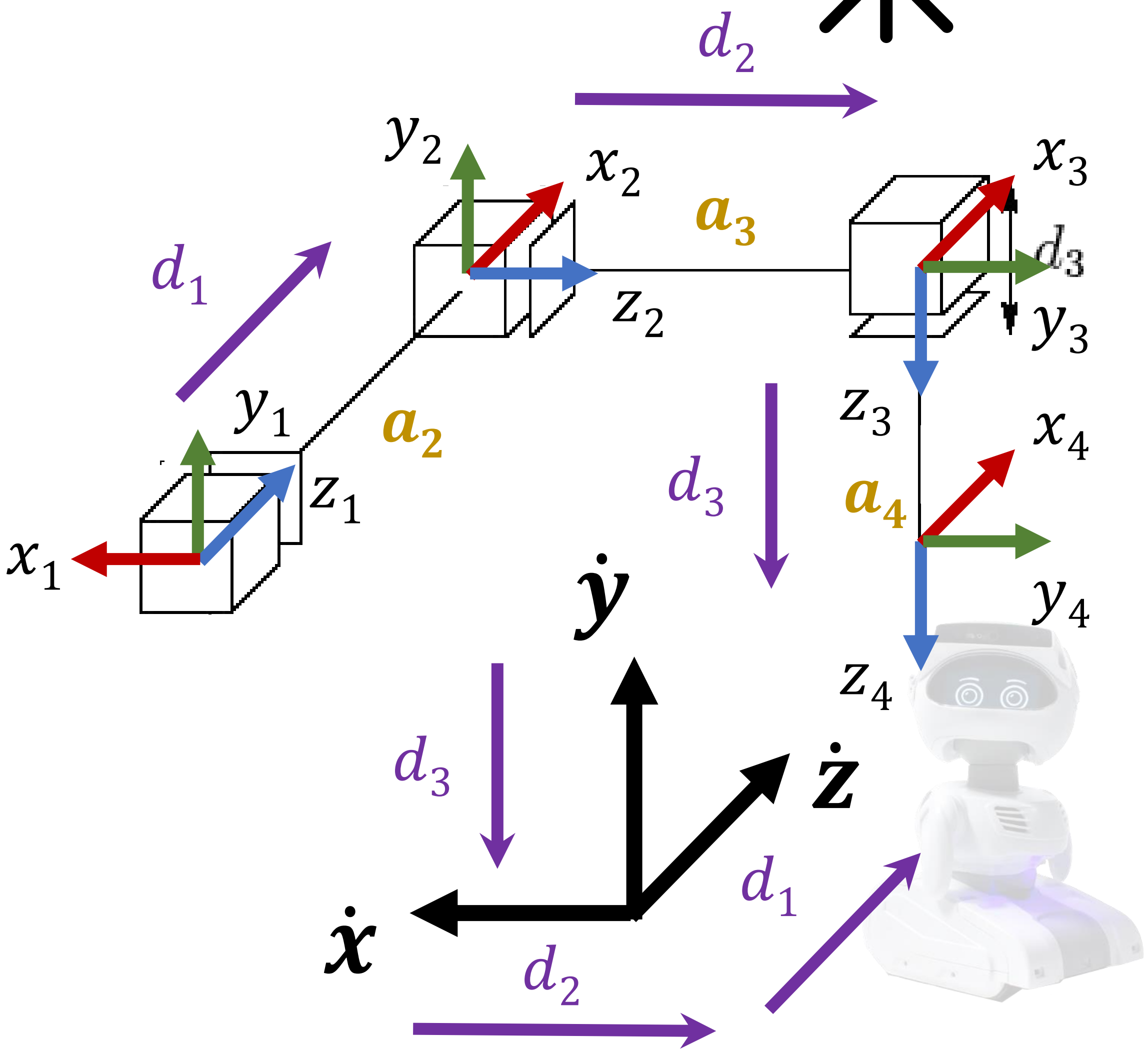
Assignment

Find using ***the Jacobian inverse***:

$$\dot{d}_1 = f(\dot{x}, \dot{y}, \dot{z})$$

$$\dot{d}_2 = f(\dot{x}, \dot{y}, \dot{z})$$

$$\dot{d}_3 = f(\dot{x}, \dot{y}, \dot{z})$$



$$\begin{aligned} \dot{d}_1 &= \dot{z} \\ \dot{d}_2 &= -\dot{x} \\ \dot{d}_3 &= -\dot{y} \end{aligned}$$

NEXT SECTION : *Jacobian of RRR robot*