Fall 2025

Electromagnetic Waves - ECE 331s

Propagation of a Gaussian Wave

Under Supervision of:

Dr. Hussein Kotb

Dr. Ahmed Sarhan

Prepared by:

Ahmed Magdy Roshdy	2300219
Karim Khaled Mohammed	2301018
Abdelrahman Ahmed Shaker	2301050

Gaussian Wave Project

Introduction

Can light be spatially confined and transported in free space without angular spread? Although the wave nature of light precludes the possibility of such idealized transport, light can, in fact, be confined in the form of beams that come as close as possible to waves that are spatially localized and nondiverging. The two extremes of angular and spatial confinement are the plane wave and the spherical wave, respectively. The wavefront normal (rays) of a plane wave coincide with the direction of travel of the wave so that there is no angular spread, but its energy extends spatially over all space. The spherical wave, in contrast, originates from a single spatial point, but its wavefront normals (rays) diverge in all angular directions. Waves whose wavefront normals make small angles with the z axis are called paraxial waves. They must satisfy the paraxial Helmholtz equation. The Gaussian beam is an important solution of this equation that exhibits the characteristics of an optical beam, as attested to by a number of its properties. The beam power is principally concentrated within a small cylinder that surrounds the beam axis. The intensity distribution in any transverse plane is a circularly symmetric Gaussian function centered about the beam axis. The width of this function is minimum at the beam waist and gradually becomes larger as the distance from the waist increases in both directions. The wavefronts are approximately planar near the beam waist, then gradually curve as the distance from the waist increases, and ultimately become approximately spherical far from the beam waist. The angular divergence of the wavefront normals assumes the minimum value permitted by the wave equation for a given beam width. The wavefront normals are therefore much like a thin pencil of rays. Under ideal conditions, the light from many types of lasers takes the form of a Gaussian beam.

Approximated Complex Amplitude of the Gaussian beam

The general expression for the complex amplitude U(r) of the Gaussian beam:

U(r) = Ao $\frac{Wo}{W(z)}$ $e^{\frac{-\rho^2}{w(z)^2}}$ $e^{(-jkz-jk\frac{\rho^2}{2R(z)}+j\zeta(z))}$

Where:

W(z) = Wo $\sqrt{1+(\frac{z}{zo})^2}$

R(z) = Z (1+$(\frac{zo}{z})^2$)

ζ(z) = $\tan^{-1}\frac{z}{zo}$

Wo = $\sqrt{\frac{\lambda zo}{\pi}}$

These are Beam Parameters.

The wavefronts are approximately planar near the beam waist, So at z = 0:

W(Z) = Wo

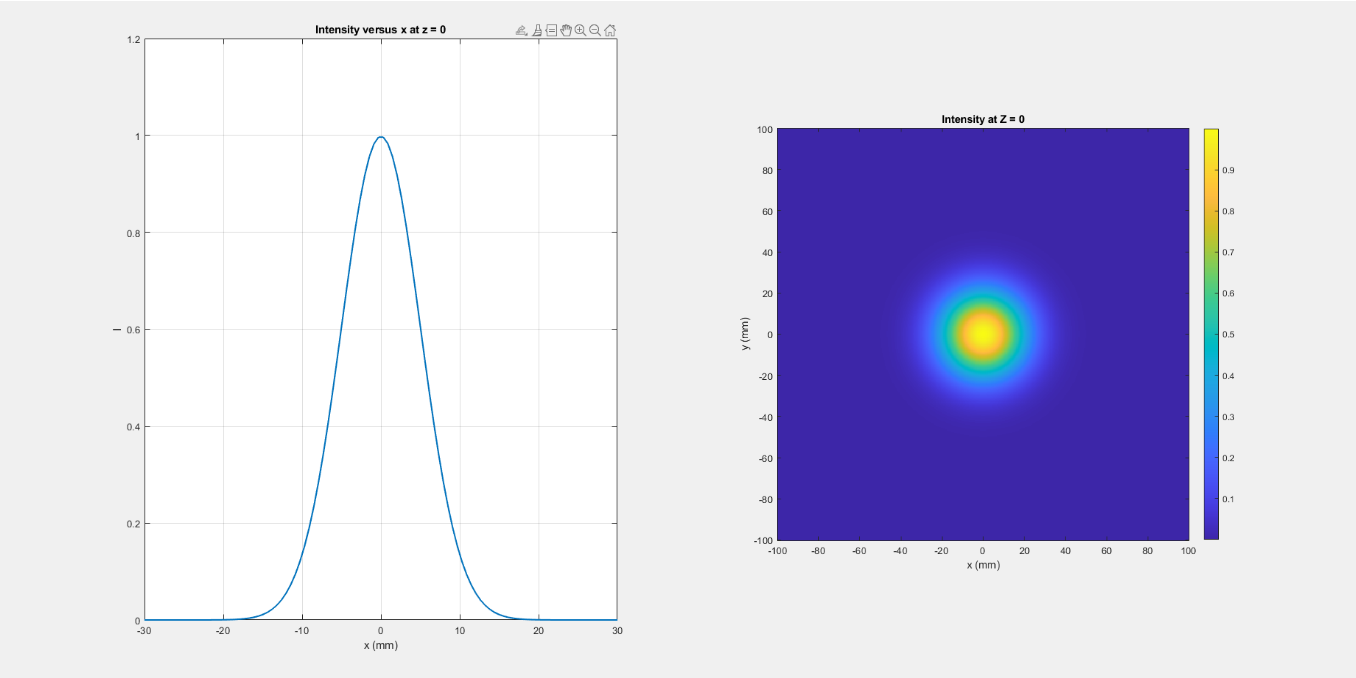
R(z) $\simeq \infty$

ζ(z) = 0

thus the complex amplitude becomes, U(r) = $Ao\ e^{\frac{-\rho^2}{wo^2}}$ at z = 0.

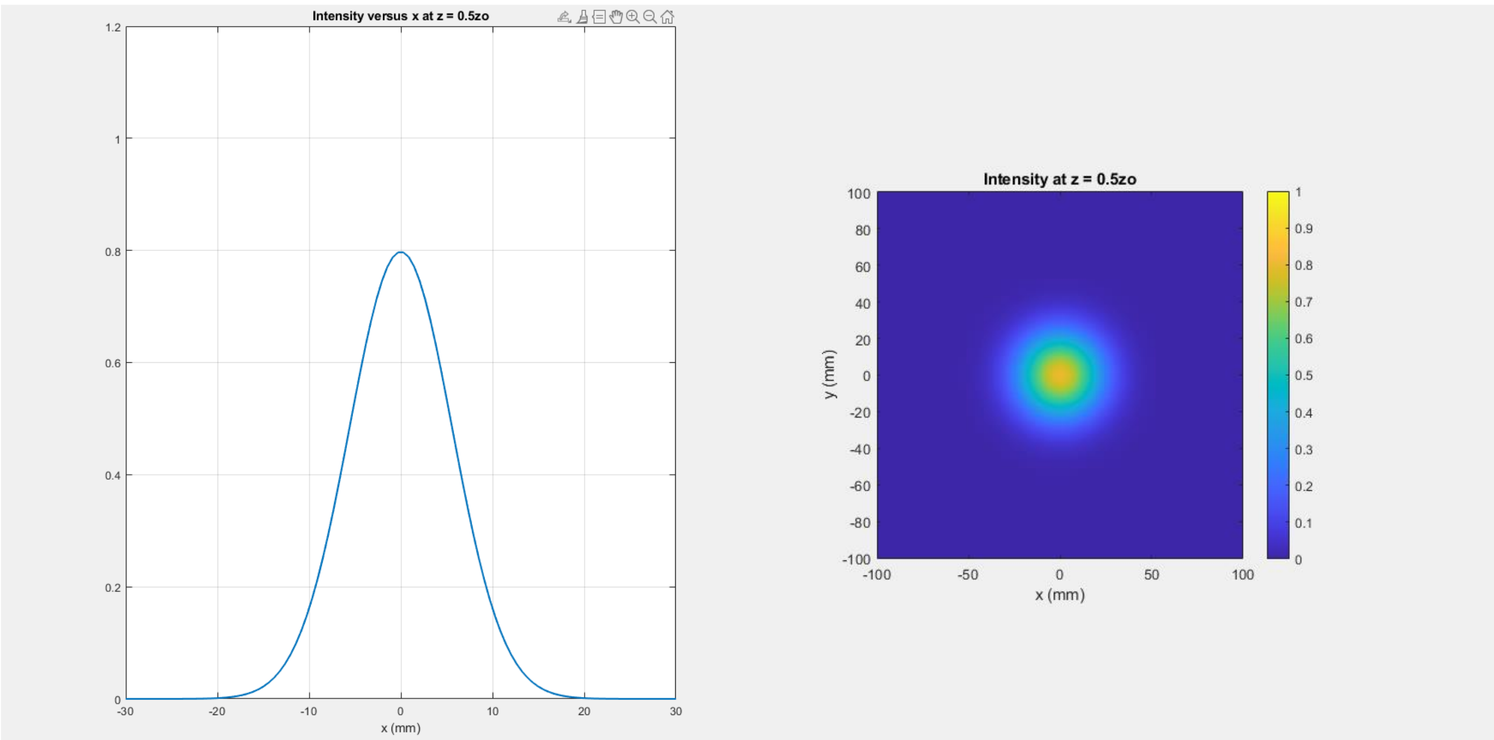
Intensity plots at different positions

- The intensity at z = 0:



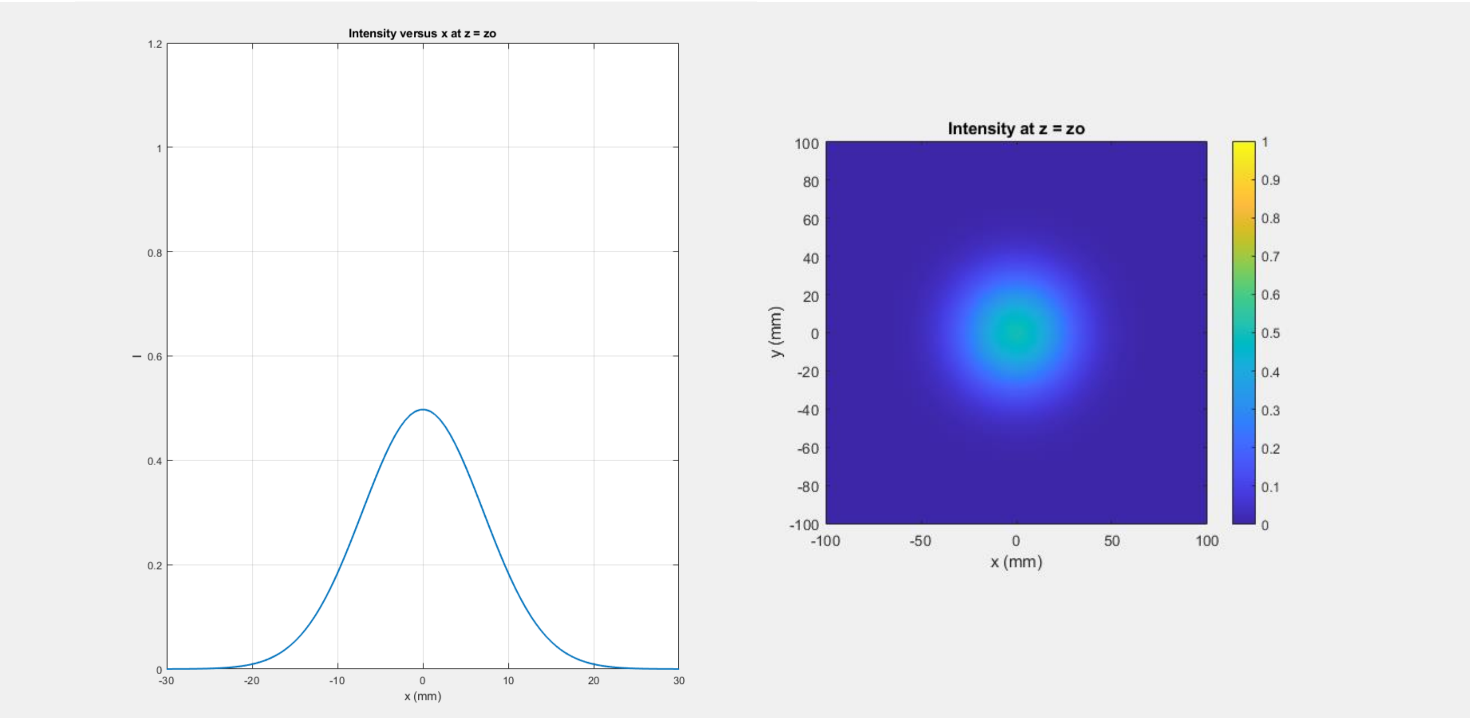
We can see that at the initial position The Gaussian function has its peak at z = 0. at x = 0, The Intensity is maximum and decreases monotonically as x increases.

- The intensity at z = 0.5zo:



Notice that the intensity peak has been reduced to 0.8 which was expected because the amplitude of the intensity = $(\frac{wo}{w(z)})^2$ and w(z) = $\frac{\sqrt{5}}{2}$ wo at z=0.5zo. The Spot diameter gets big and less bright as z increases.

- The intensity at z = zo:



At z = zo, the peak intensity has been reduced to 0.5 according to the expression $(\frac{wo}{w(z)})^2$. The spot's diameter is now bigger than the diameters of the previous ones, and it is also the least bright of them all.

- **MATLAB Simulation Code of Gaussian Beam Propagation and Intensity plots** (<u>The Value of z is changed after each plot</u>)

```
%%% Defining Parameters %%%
Wo     = 0.01;                % Beam Waist at z = 0
lambda = 0.0046;              % Wavelength
zo     = pi * Wo^2 / lambda;  % Rayleigh distance
k      = 2*pi/lambda;         % Wave number
N      = 512;                 % Number of sampling points
L      = 0.3;                 % Spatial domain size
z      = 0;                   % the distance the beam propagates

%%% 2D Spatial Coordinate System %%%
x = linspace(-L/2, L/2, N);
y = x;
[X, Y] = meshgrid(x, y);
rho_square = X.^2 + Y.^2;

%%% Spatial Frequency Grid for Kx and Ky %%%
dk = 2*pi/L;
kx = (-N/2:N/2-1) * dk;
ky = (-N/2:N/2-1) * dk;
[KX, KY] = meshgrid(kx, ky);

Uo = exp(-rho_square/(Wo^2));  % Complex Amplitude at z = 0

%%% Performing the spatial Fourier transform for Uo %%%
U_kx_ky = fftshift(fft2(ifftshift(Uo)));

kz = k - (KX.^2 + KY.^2)/(2*k);             % kz using paraxial approximation
H = exp(1j * kz * z);                       % The system transfer function
Uz_r = fftshift(ifft2(ifftshift(U_kx_ky .* H)));     % Field distribution at z

```

```
%%% Plotting Intensity versus x and spot intensity %%%

fixed_Intensity_x = [-30 30];  % Fixed x-axis for comparing peak intensities
fixed_Intensity_y = [0 1.2];  % Fixed y-axis for comparing peak intensities

figure(1);

%Intensity versus x
subplot(1,2,1);
plot(x*1000, abs(Uz_r(N/2, :)).^2, 'LineWidth', 1.7);    %Convert to mm for display  , %center row slice at  y= 0: Uz_r(N.2, :)
xlabel('x (mm)'); ylabel('I');
title('Intensity versus x at z');
grid on;
ylim(fixed_Intensity);
xlim(fixed_Intensity_x) ;

%Intensity Spot
subplot(1,2,2);
imagesc(x*1000, y*1000, abs(Uz_r).^2);
axis image;
axis xy;
xlabel('x (mm)'); ylabel('y (mm)');
title('Intensity spot at z');
colorbar;
```

Reflection from a parabolic reflector

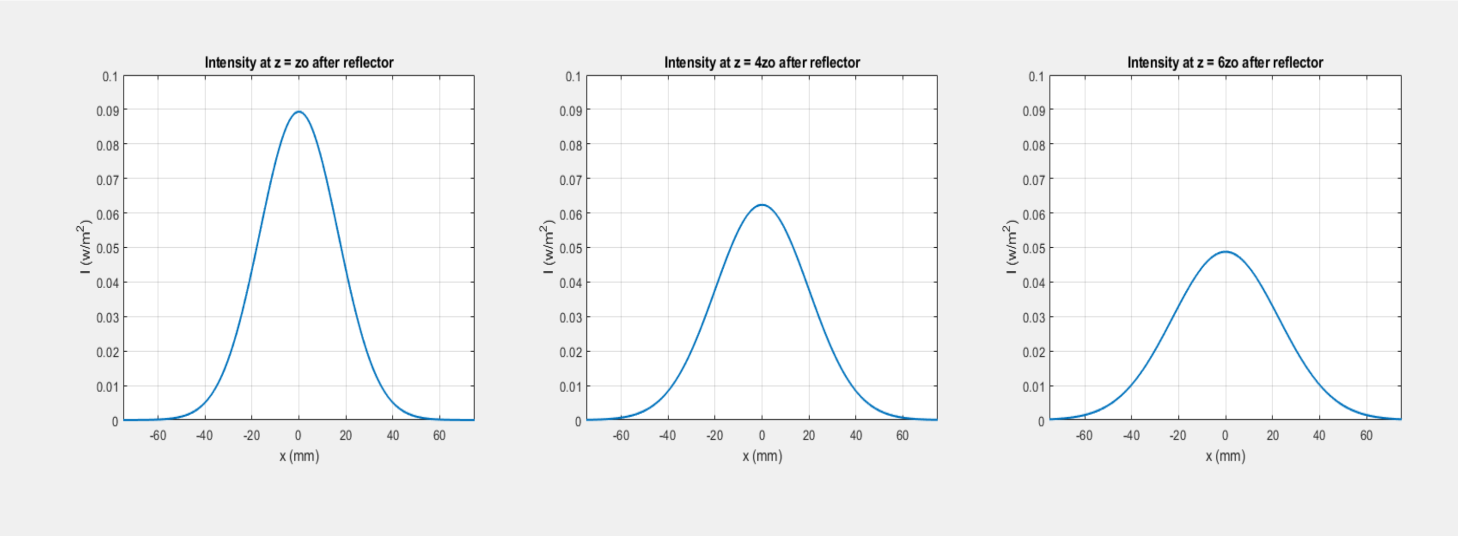
The Matlab code for reflection of the beam and propagating distances zo, 4zo and 6zo (<u>notice the value of z interface of Uz_r is changes after each plot)</u>

```
%%% Reflection from Parabolic Mirror %%%
f = -4*zo;                                  % Focus of the mirror
M = exp(1j * k * rho_square / (2*f));      % Transfer Function of the mirror
Uout_r = M .* Uz_r;
Uout_r = fftshift(fft2(ifftshift(Uout_r)));  % Output field after reflector

% Propagation after reflection
z1 = zo;
z2 = 4*zo;
z3 = 6*zo;

H1 = exp(1j * kz * z1);
Uz_zo = fftshift(ifft2(ifftshift(Uout_r.*H1)));     % Field at z = zo

H2 = exp(1j * kz * z2);
Uz_4zo = fftshift(ifft2(ifftshift(Uout_r.* H2)));  % Field at z = 4zo

H3 = exp(1j * kz * z3);
Uz_6zo = fftshift(ifft2(ifftshift(Uout_r.*H3)));   % Field at z = 6zo
```

The intensity plots for different propagation distances

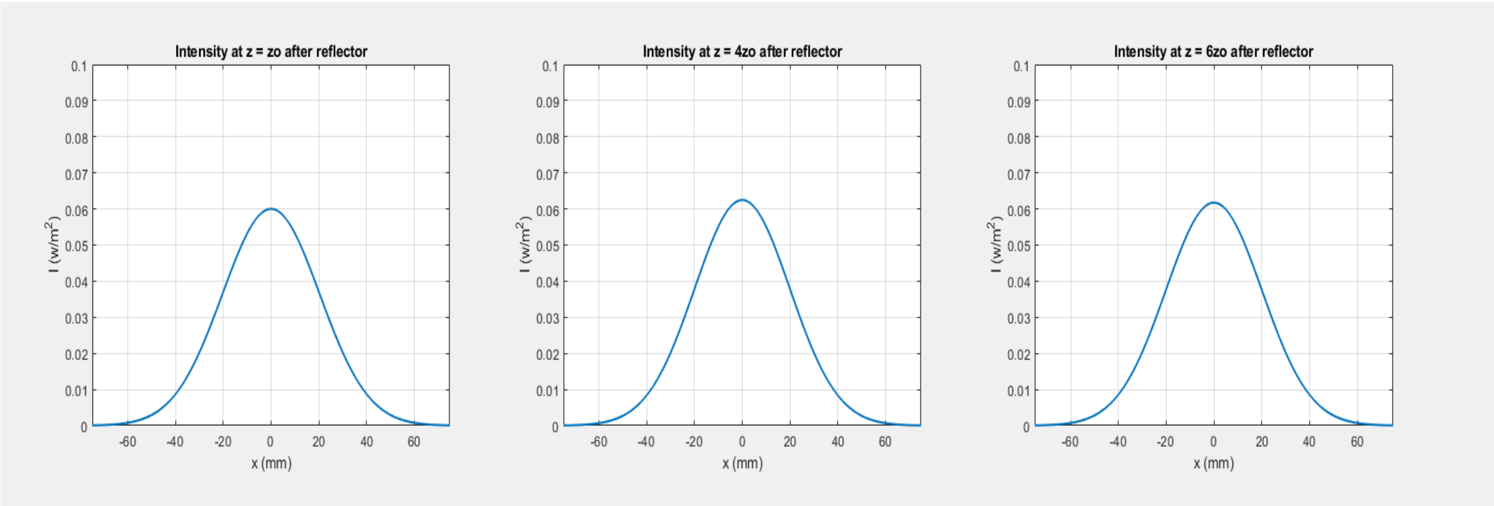
```
%%% Plotting beam after reflector at z = zo, 4zo, 6zo %%%
fixed_ylim = [0 0.1];       % Fixed y-axis for comparing peak intensities
fixed_xlim = [-75 75];      % Fixed x-axis for comparing peak intensities

figure(2);

% z = zo
subplot(2,3,1);
plot(x*1000, abs(Uz_zo(N/2,:)).^2,'LineWidth', 1.7);            %Convert to mm for display  , %center row slice at  y= 0: Uz_zo(N.2, :)
xlabel('x (mm)'); ylabel('I (w/m^2)');
title('Intensity at z = zo after reflector');
grid on;
ylim(fixed_ylim);
xlim(fixed_xlim);

% z = 4zo
subplot(2,3,2);
plot(x*1000, abs(Uz_4zo(N/2,:)).^2,'LineWidth', 1.7);           %Convert to mm for display  , %center row slice at  y= 0: Uz_4zo(N.2, :)
xlabel('x (mm)'); ylabel('I (w/m^2)');
title('Intensity at z = 4zo after reflector');
grid on;
ylim(fixed_ylim);
xlim(fixed_xlim);

% z = 6zo
subplot(2,3,3);
plot(x*1000, abs(Uz_6zo(N/2,:)).^2, 'LineWidth', 1.7);          %Convert to mm for display  , %center row slice at  y= 0: Uz_6zo(N.2, :)
xlabel('x (mm)'); ylabel('I (w/m^2)');
title('Intensity at z = 6zo after reflector');
grid on;
ylim(fixed_ylim);
xlim(fixed_xlim);
```

- For the interface at z = 3zo



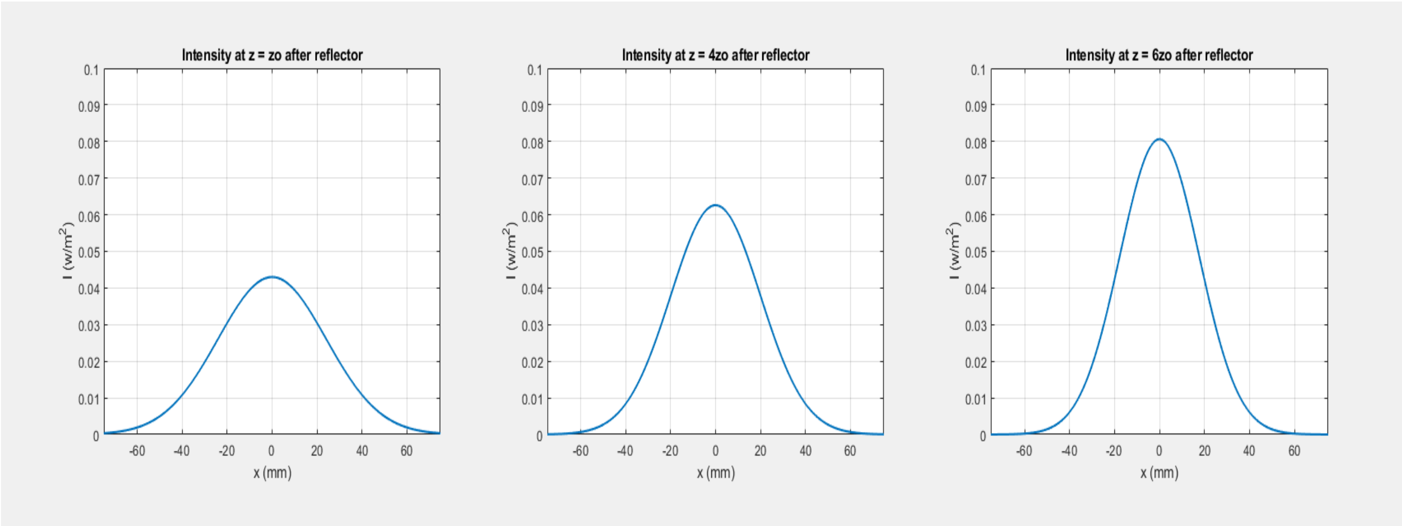
For the Interface at z = 3zo (before the focus), the beam is still converging toward the focus. When it reflects, the wavefront curvature is mismatched with the mirror's focal geometry so the reflected beam diverges and its intensity decreases with propagation.

- For the interface at z = 4zo



For the Interface at $4z_0$, the beam waist coincides with the mirror's focal point. Reflection produces a collimated beam (flat wavefront). intensity remains nearly constant with propagation — the mirror has "perfectly refocused" the beam.

• For the interface at z = 5zo



For the interface at $5z_0$ (after the focus), The beam is already diverging past its waist. Reflection flips the curvature, effectively turning a diverging beam into a converging one. The reflected beam intensity increases with propagation, because the mirror is now focusing it back down.

References

1- Bahaa E. A. Saleh, and Malvin Carl Teich, “Fundamentals of Photonics”, 2nd Edition, John Wiley and sons, Inc., ISBN 978-0-471-35832-9.

2- Lizuka, Elements of photonics, Vol I, Willey, 2002

Historical Notation

The Gaussian beam, named after the German mathematician Carl Friedrich Gauss (1777– 1855), is circularly symmetric and has a radial intensity that follows the form of a Gaussian distribution.