

Expo Lab Partitions

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1 Findings

We started our exploration with compositions. We used recursion, stars and bars, bijections, Pascal's identity and various other combinatorial methods to find our results, many of which centered around the Fibonacci sequence.

Then we turned to partitions, which presented a whole new sea to explore, rich with many interesting bijections!

Some interesting examples: In compositions, we found that taking the sum of the product of all parts of all compositions with k parts yielded an interesting bijection to bars and stars, since the number of ways to select k balls, one from each of k groups of balls, is the product we were looking for!

In partitions, we found a bijection between the number of partitions of n in which no part is divisible by d to the number of partitions of n in which no part appears more than d times by considering pairing up the converses: partitions of n in which at least one part is divisible by d and partitions of n in which at least one part appears more than d times. We paired these up by collapsing the d elements which are the same into one element divisible by d .

Some things we are looking to explore are possible closed forms for compositions and partitions, as well as interesting patterns in partitions similar to the patterns Ramanujan discovered. We are also hopeful that Simon will give us some more intriguing questions to think about pertaining to partitions.