Compressible and Viscous Aerodynamics

Kármán–Pohlhausen Approach of solving the boundary layer over an arbitrary airfoil

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SEC: 1 BN:11

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Abstract

The previous part of the project discussed the potential flow over the arbitrary airfoils; NACA 0006, 0012, 0018. Using the Source/Vortex Panel Method the surface velocities at different locations on the airfoil were calculated. A free stream velocity of 50 m/s along with an angle of attack of 0^o were used.

In this part of the project, using the previously calculated velocities at different locations, the Kármán–Pohlhausen (KP) Method will be implied to analyse the boundary layer over the three arbitrary airfoils. This method will allow us to describe the following: The distribution of the boundary layer thickness, δ, The distribution of the boundary layer displacement thickness, δ_1, The distribution of the boundary layer momentum thickness, δ_2, The distribution of shear stress, τ_w, (Local) Skin Friction Stress Coefficient, C_{f_x}. This study will take into consideration two analyses; one without transition taken into consideration (the whole airfoil assumed to be laminar), and the second one taking the transition effect into consideration using the smith model. Studying the boundary layer is important since it plays key roles in drag and lift analysis and it's impossible to understand a stalling airfoil without understanding how boundary layers evolve. One main drawback of using KP approach after calculating the surface velocities using the Source/Vortex Panel method is that it leaves some coordinates with un-identified values and causes discontinuities at transition. Therefore, Real surface velocities of the NACA0012 airfoil will be gathered from an experimental method and analysed after presenting the actual results.

Introduction

Following Prandtl's pioneering treatment of boundary layers and the shape-preserving similarity solution reported by Blasius, the momentum-integral approach, introduced through two sequential papers by Kármán and Pohlhausen, is widely regarded as the staple in boundary-layer analysis. This is mainly due to its effectiveness at providing a wealth of useful details for boundary layers in both low and high Reynolds number flows. Despite its simplicity, the KP approach continues to play a pivotal role in several monographs that devote themselves to boundary-layer theory. The KP approach has also been employed at the basis of predicting the stability of laminar boundary layers at separation, although solutions of the differential equations of motion, often manifested in the similarity-preserving Blasius and Falkner–Skan type expressions, have gradually superseded integral approximations. The resulting methodological shift may be attributed, in part, to the increased precision often associated with the differential approach and, in part, to the substantial advancements made in computational tools.

The Implementation of the KP approach

Starting from the Kármán Moment-Integral Equation:

$$\frac{\tau_w}{\rho U^2} = \frac{d\delta_2}{dx} + \frac{2\delta_2 + \delta_1}{U}\frac{dU}{dx}$$

Ignoring the Suction Velocity term. We can realize the following definitions using the Pohlhausen approximation:

Dimensional Analysis

Defining a new variable, $Z = \frac{\delta_2^2}{\nu_\infty}$ by saying that $\tau_w = \nu * \frac{du}{dy}\Big|_{y=0}$

The approximation is $\bar{u} = 2\bar{y} - 2\bar{y}^3 + \bar{y}^4 + \frac{\Lambda}{6}(\bar{y} - 3\bar{y}^2 + 3\bar{y}^3 - \bar{y}^4)$

$$\Lambda = \frac{U'}{\nu_\infty}\delta^2 = Re_L \bar{U}'\bar{\delta}^2 \equiv Pohlhausen\ Parameter$$

Using the new variable, the MIE take the form: $\frac{dZ}{dx} = \left\{\begin{matrix} \frac{1}{U}F(K) \\ \frac{1}{U}f(\Lambda) \end{matrix}\right\}, K = U'Z$

Where we can state that:

$$K(\Lambda) = (\frac{37}{315} - \frac{1}{945}\Lambda - \frac{1}{9072}\Lambda^2)\Lambda$$

$$f_1(\Lambda) \equiv \frac{\delta_1}{\delta_2} = \frac{\frac{3}{10} - \frac{\Lambda}{120}}{\frac{37}{315} - \frac{1}{945}\Lambda - \frac{1}{9072}\Lambda^2}$$

$$f_2(\Lambda) \equiv \frac{\tau_w \delta_2}{\mu_\infty U} = (2 + \frac{\Lambda}{6})(\frac{37}{315} - \frac{1}{945}\Lambda - \frac{1}{9072}\Lambda^2)$$

$$F(\Lambda) = 2f_2(\Lambda) - 4K(\Lambda) - 2K(\Lambda)F_1(\Lambda)$$

Using the forward Euler Scheme:

$Z_{i+1} = Z_i + \left[\frac{F(\Lambda)}{U}\right]_i \Delta x$ where $Z_0 = \frac{K_0}{U_0''}$, $\left[\frac{F(\Lambda)}{U}\right]_0 = -0.0652857 * \frac{U_0''}{U_0'^2}$

Nondimensionalization

$\bar{Z} = \frac{Z}{L/U_\infty}, \bar{x} = \frac{x}{L}, \bar{U} = \frac{U}{U_\infty}, \bar{U}' = \frac{U'}{U_\infty/L}, \bar{U}'' = \frac{U''}{U_\infty/L^2}, \bar{\delta} \equiv \frac{\delta}{L} = \sqrt{\frac{\Lambda}{Re_L \bar{U}'}}, \bar{\delta_1} \equiv \frac{\delta_1}{L}, \delta_2 \equiv \frac{\delta_2}{L} = \sqrt{\frac{\bar{Z}}{Re_L}}$

$$C_{f_x} = \frac{\tau_w}{\rho_\infty U^2/2} = \frac{\mu_\infty f_2 U/\delta_2}{\rho_\infty U^2/2} = \frac{2}{\sqrt{Re_L}} * \frac{f_2}{\bar{U}\sqrt{\bar{Z}}}$$

Smith's Model

$$\log_{10}(Re_{x_{tr}}) = -40.4557 + 64.8066H - 26.7538H^2 + 3.3819H^3$$

$$where\ 2.1 < H \equiv \frac{\delta_1}{\delta_2} \equiv f_1(\Lambda) < 2.8$$

Starting the turbulent flow calculation:

$x = x_{tr}$ $let\ \delta_{2_{T_{tr}}} = \delta_{2_{L_{tr}}}$ and $\delta_{T_{tr}} = 1.4 * \delta_{T_{tr}}$

Then solving this equation for the smallest root:

$$\frac{\delta}{\delta_2} - H = 3.3 + 1.5501(H - 0.6778)^{-3.064}$$

Obtaining C_{f_x} from:

$$C_{f_x} = 0.246 \times 10^{-0.678H} Re_{\delta_2}^{-0.268} \qquad where\ Re_{\delta_2} = \frac{\rho_\infty U \delta_2}{\mu_\infty}$$

Obtaining $\bar{\delta}_2$ from:

$$\frac{d\bar{\delta}_2}{d\bar{x}} = \frac{C_{f_x}}{2} - \frac{\bar{\delta}_2}{\bar{U}}\frac{d\bar{U}}{d\bar{x}}(H + 2)$$

Therefore:

$$\overline{\delta_{2new}} = \overline{\delta_{2old}} + \frac{d\bar{\delta}_2}{d\bar{x}}(x_{new} - x_{old})$$

Obtaining H_1 numerically from:

$$\frac{d}{d\bar{x}}\left(\bar{U}\bar{\delta}_2 H_1\right) = \bar{U}F(H_1) = \bar{U} * 0.0306(H_1 - 3)^{-0.6169}$$

Finally calculating H from:

$$H = \begin{cases} 3 & , H_1 \le 3.3 \\ 0.6778 + 1.1536(H_1 - 3.3)^{-0.326} & , 3.3 < H_1 \le 3.3 \\ 1.1 + 0.86(H_1 - 3.3)^{-0.777} & , 5.3 < H_1 \end{cases}$$

Therefore, if $1.8 < H < 2.8$ then it can be considered turbulent, separation

Application of the Kármán and Pohlhausen Approach

Applying this method to calculate the different characteristics of the boundary layer of the 3 airfoils, NACA 0006, NACA 0012, NACA 0018 and a validated NACA0012. We will compute the distribution of the boundary layer thickness, δ, The distribution of the boundary layer displacement thickness, δ_1, The distribution of the boundary layer momentum thickness, δ_2, The distribution of shear stress, τ_w, (Local) Skin Friction Stress Coefficient, C_{f_x}. It was implemented using MATLab.

Using 400 Panels, with uniform flow velocity of 50 m/s and a chord of 1 meter.

Results of the Application

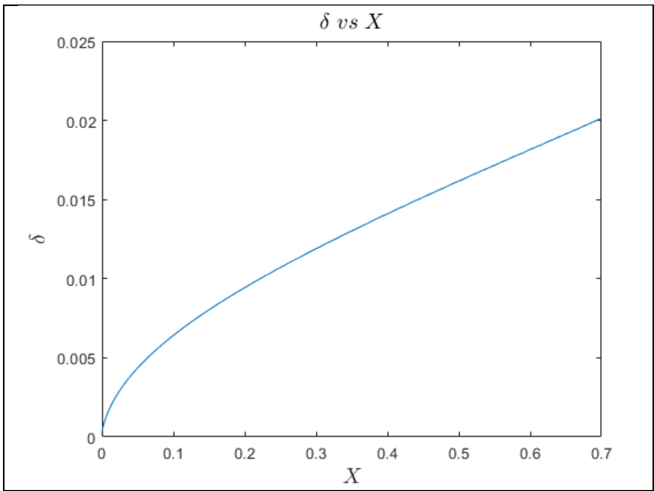
NACA 0006

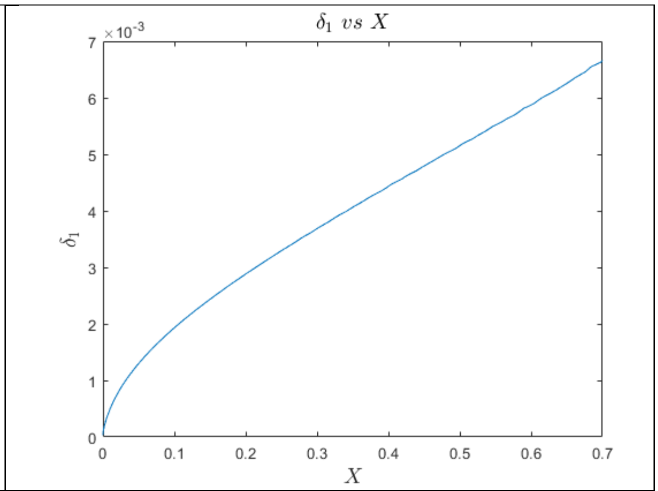
Boundary Layer Characteristics

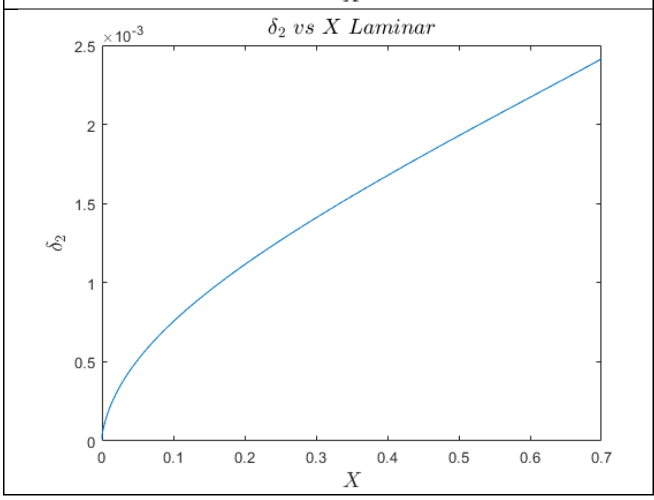
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
i	x	Δx	U	U'	Z	$K = U'Z$	Λ	f_1	f_2	F	$\delta = \sqrt{\frac{\nu_\infty \Lambda}{U'}}$	$\delta_1 = f_1 \delta_2$	$\delta_2 = \sqrt{\nu_\infty Z}$	$\tau_w = \frac{\mu_\infty U}{\delta_2} f_2$	$C_{f_x} = \frac{2\tau_w}{\rho_\infty U^2}$
0	0	0.2	0	285980	0.00067	0.07703	7.0523	2.308	0.331	0	0.000134	0	0	4.363	3.35273
1	0.2	0.2	54.48	-5.56	4.3874	-0.0097	-0.699	2.588	0.222	0.53	0.009582	0.0029	0.0011	1.353	0.00074
2	0.4	0.2	53.26	-6.632	9.728	-0.0258	-1.822	2.648	0.201	0.64	0.014163	0.0044	0.0016	0.806	0.00046
3	0.6	0.2	52.02	-6.15	16.298	-0.0401	-2.80	2.704	0.183	0.74	0.018248	0.0059	0.0022	0.552	0.00033
4	0.8	0.2	50.63	-8.489	24.786	-0.0841	-5.856	2.909	0.122	1.07	0.022445	0.0078	0.0026	0.291	0.00018
5	1.0	N/A	42.68	0	0	0	0	0	0	0	0	0	0	0	0

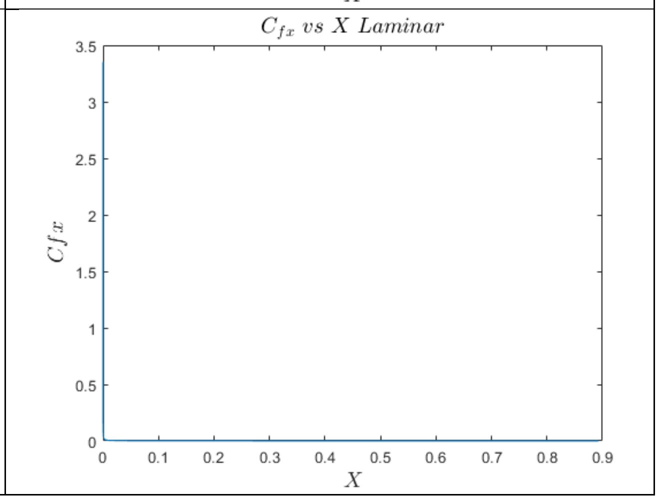
Graphical Representations

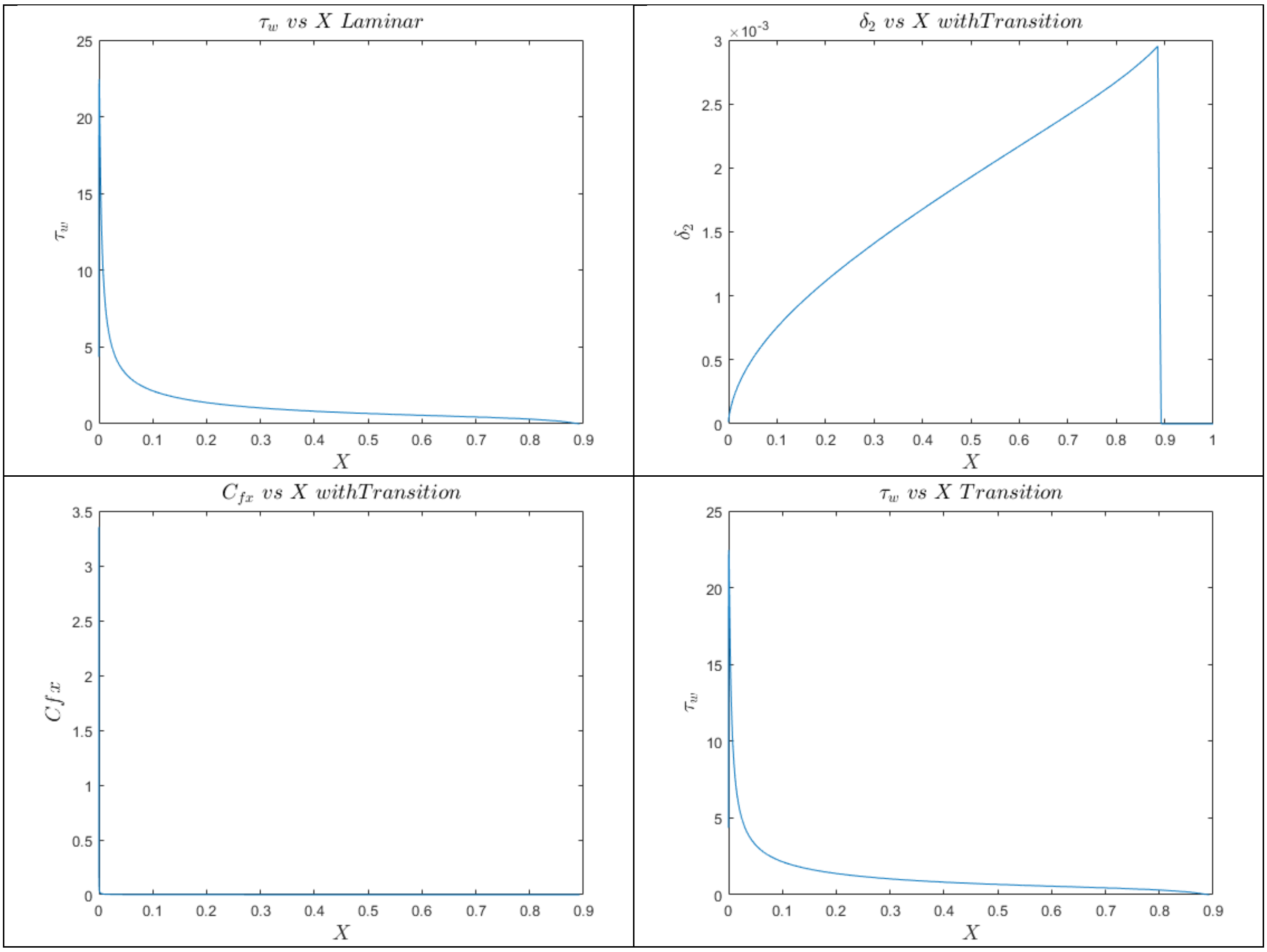
Here the graphical representations of different variables will be presented. Transition will also be taken into consideration to show the difference.











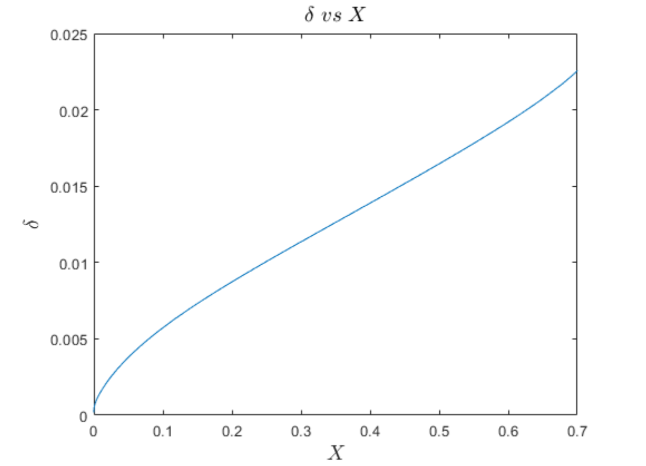
It shows that the transition point is at about x=0.9. The C_{f_x} graphs are quite unacceptable and wouldn't probably represent the airfoil due to discontinuities.

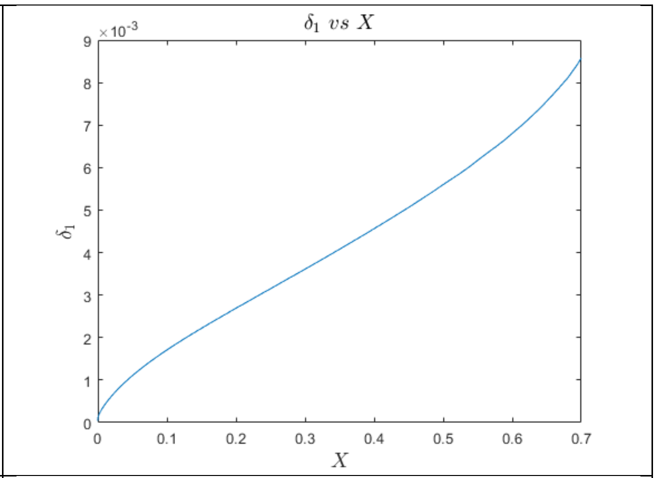
NACA 0012

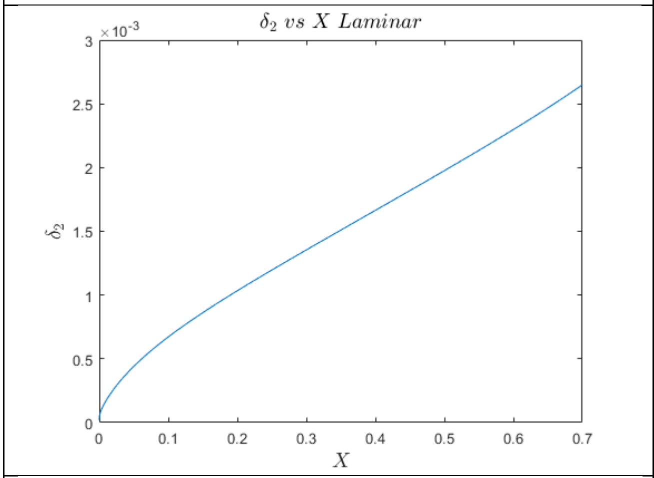
Boundary Layer Characteristics

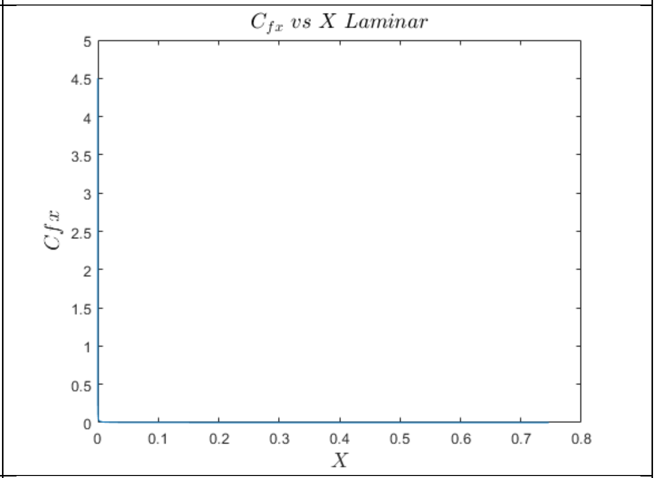
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
i	x	Δx	U	U'	Z	$K = U'Z$	Λ	f_1	f_2	F	$\delta = \sqrt{\frac{\nu_\infty \Lambda}{U'}}$	$\delta_1 = f_1 \delta_2$	$\delta_2 = \sqrt{\nu_\infty Z}$	$\tau_w = \frac{\mu_\infty U}{\delta_2} f_2$	$C_{f_x} = \frac{2\tau_w}{\rho_\infty U^2}$
0	0	0.2	1.4577	285980	0.0007	0.0770	7.0523	2.3081	0.3319	0.0000	0.0001	0.0000	0.0000	3.3527	4.3635
1	0.2	0.2	54.447	-5.5628	4.3874	-0.0098	-0.6994	2.5886	0.2225	0.5346	0.0096	0.0029	0.0011	0.0007	1.3539
2	0.4	0.2	53.252	-6.6329	9.7289	-0.0258	-1.8221	2.6481	0.2019	0.6438	0.0142	0.0045	0.0017	0.0005	0.8068
3	0.6	0.2	51.989	-6.1593	16.2982	-0.0402	-2.8089	2.7049	0.1832	0.7441	0.0182	0.0059	0.0022	0.0003	0.5521
4	0.8	0.2	50.578	-8.4891	24.7868	-0.0842	-5.8570	2.9097	0.1227	1.0719	0.0224	0.0078	0.0027	0.0002	0.2919
5	1	N/A	42.686	N/A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	N/A	N/A

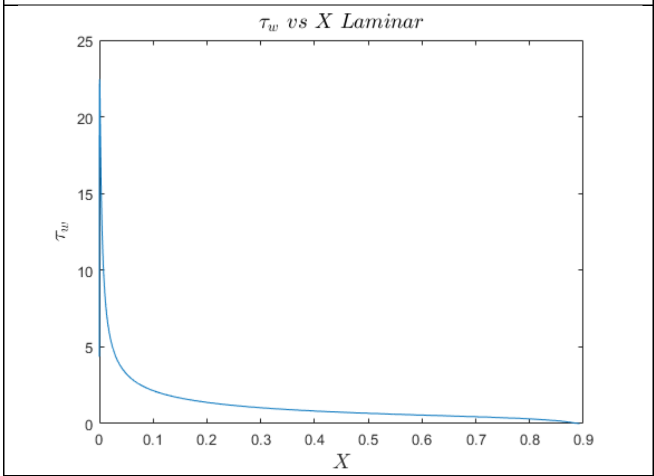
Graphical Representation

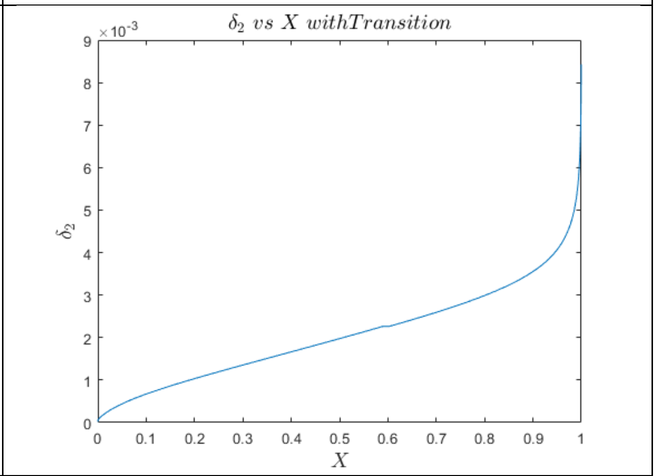


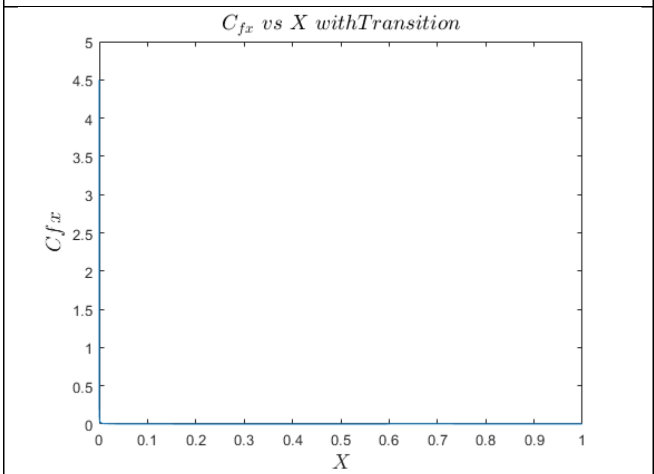


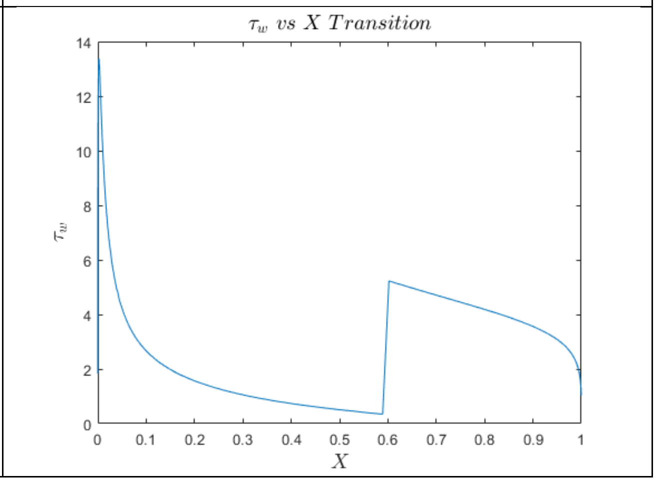










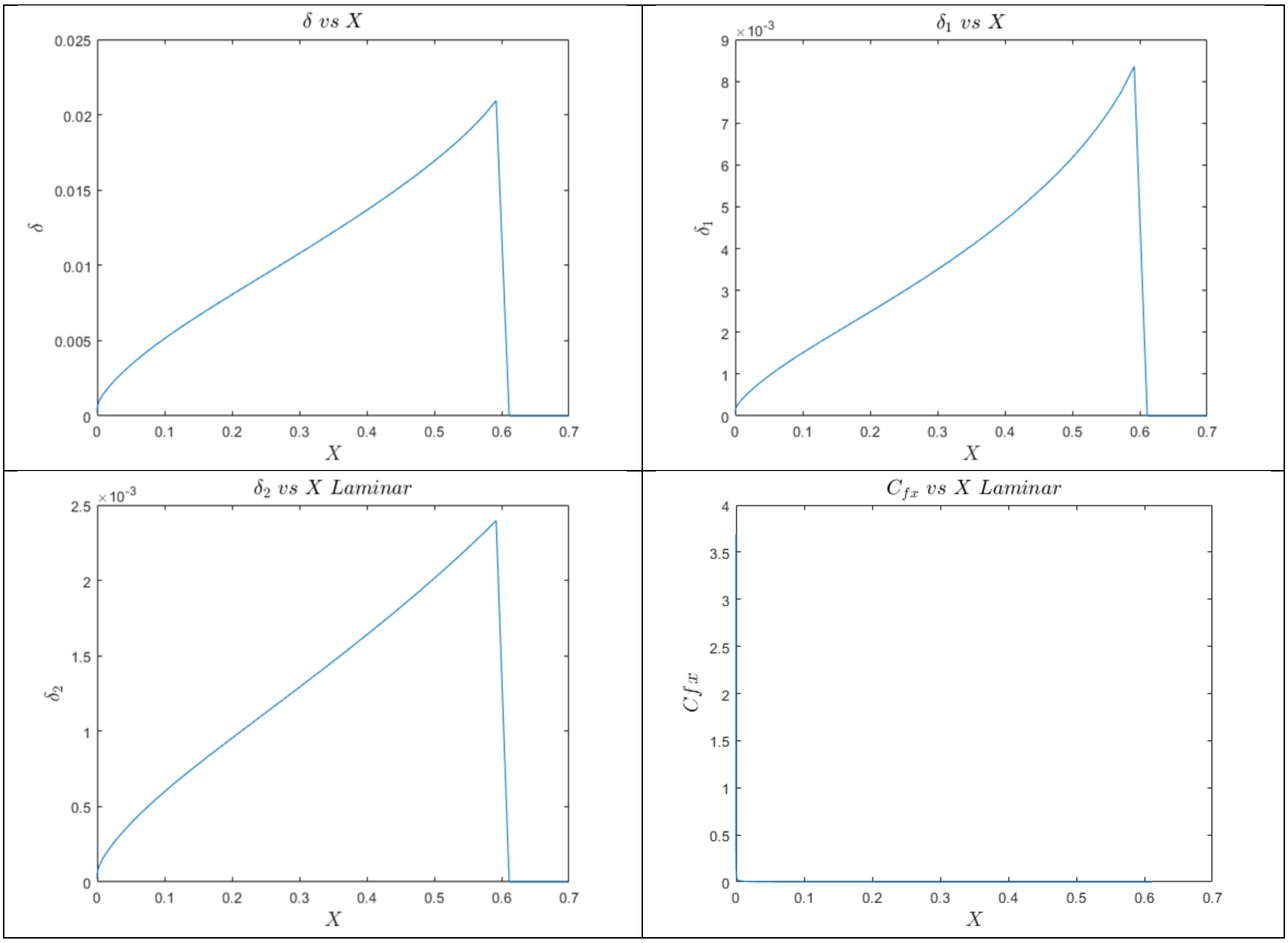


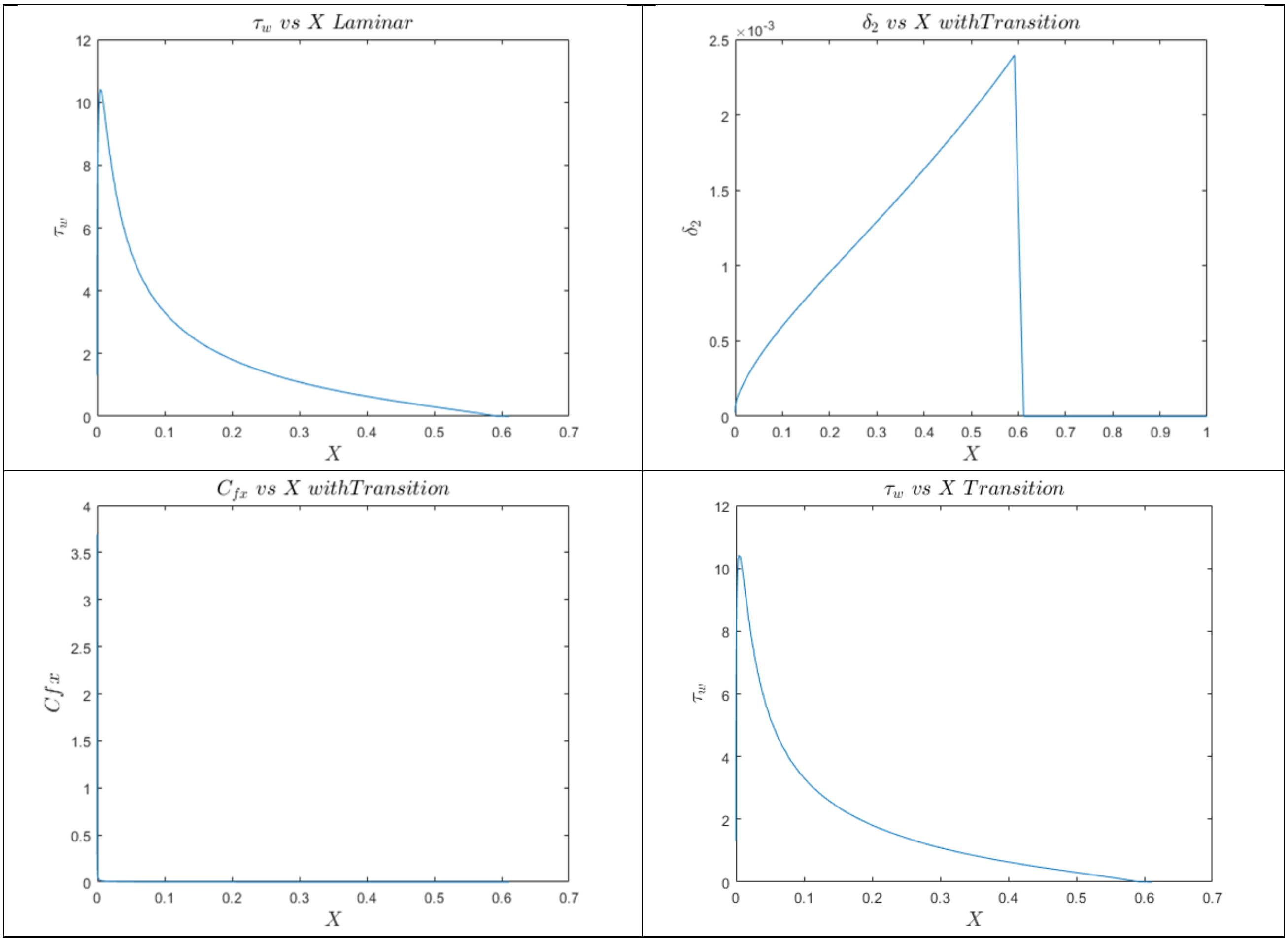
NACA 0018

Boundary Layer Characteristics

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
i	x	Δx	U	U'	Z	$K = U'Z$	Λ	f_1	f_2	F	$\delta = \sqrt{\frac{\nu_\infty \Lambda}{U'}}$	$\delta_1 = f_1 \delta_2$	$\delta_2 = \sqrt{\nu_\infty Z}$	$\tau_w = \frac{\mu_\infty U}{\delta_2} f_2$	$C_{f_x} = \frac{2\tau_w}{\rho_\infty U^2}$
0	0	0.2	0.7626	95053.2264	0.0020	0.0770	7.0523	2.3081	0.3319	0.0000	0.0002	0.0001	0.0000	1.3161	3.6947
1	0.2	0.2	63.2152	-12.5302	3.2201	-0.0161	-1.1488	2.6118	0.2144	0.5776	0.0082	0.0025	0.0010	1.7676	0.0007
2	0.4	0.2	59.5146	-19.8344	9.7031	-0.0770	-5.3490	2.8723	0.1330	1.0161	0.0140	0.0048	0.0017	0.5947	0.0003
3	0.6	0.2	55.5627	-19.8575	21.1455	-0.1680	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8	0.2	51.1168	-26.4198	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	N/A	N/A
5	1	N/A	32.8747	N/A	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	N/A	N/A

Graphical Representation





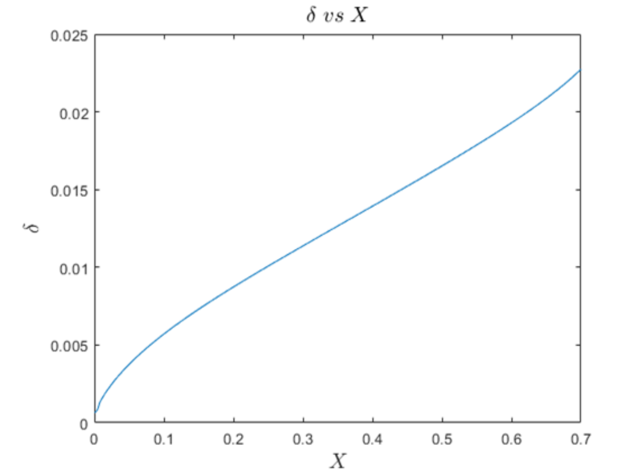
The graphs of the NACA0018 show the most uncertainty and discontinuous.

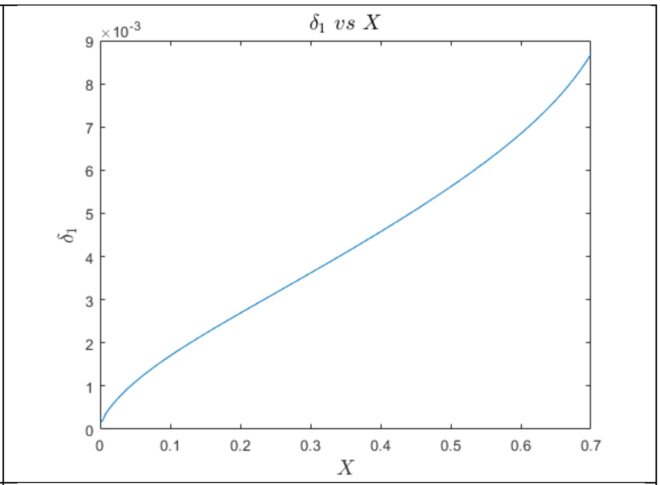
Validated NACA0012 Analysis

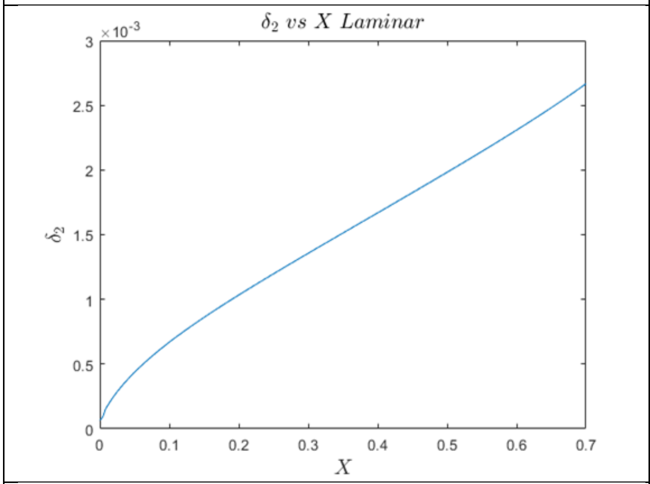
Boundary Layer Characteristics

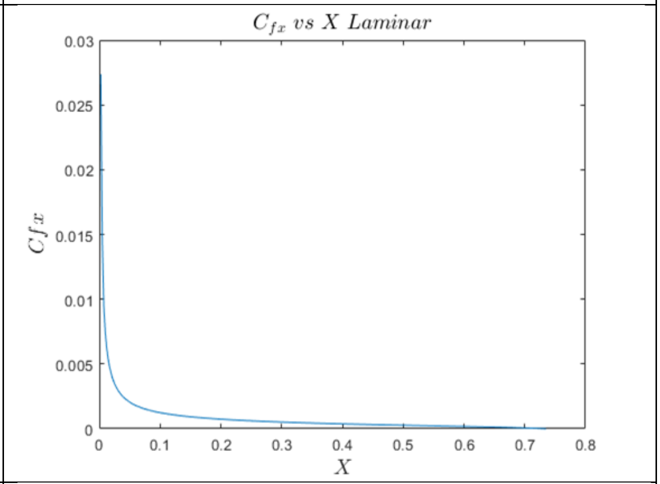
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
i	x	Δx	U	U'	Z	$K = U'Z$	Λ	f_1	f_2	F	$\delta = \sqrt{\frac{\nu_\infty \Lambda}{U'}}$	$\delta_1 = f_1 \delta_2$	$\delta_2 = \sqrt{\nu_\infty Z}$	$\tau_w = \frac{\mu_\infty U}{\delta_2} f_2$	$C_{f_x} = \frac{2\tau_w}{\rho_\infty U^2}$
0	0	0.2	0.0000	12406.3	0.0155	0.0770	7.0523	2.3081	0.3319	0.0000	0.0006	0.0002	0.0001	N/A	Inf
1	0.2	0.2	58.890	-9.2391	3.6580	-0.0135	-0.9647	2.6022	0.2177	0.5599	0.0087	0.0027	0.0010	1.5692	0.0007
2	0.4	0.2	56.530	-12.831	9.5294	-0.0489	-3.4087	2.7416	0.1715	0.8069	0.0139	0.0046	0.0017	0.7352	0.0004
3	0.6	0.2	53.960	-12.923	18.2413	-0.0943	-6.5868	2.9661	0.1079	1.1524	0.0269	0.0068	0.0023	0.3192	0.0029
4	0.8	0.2	51.115	-17.116	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0031	N/A	0.0026
5	1	N/A	38.169	-979.82	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0078	N/A	N/A

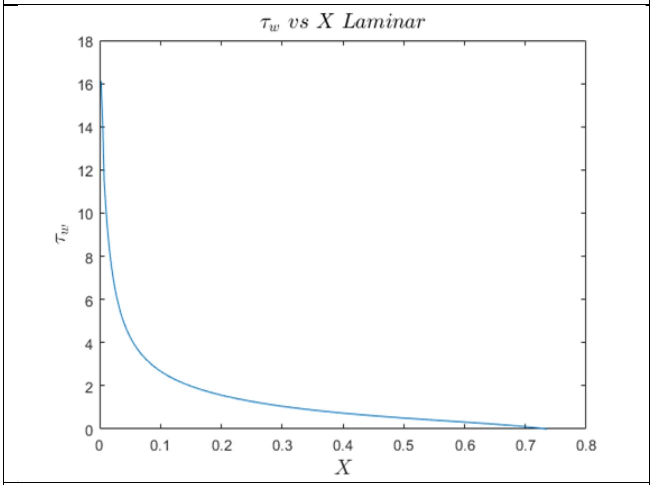
Graphical Representation

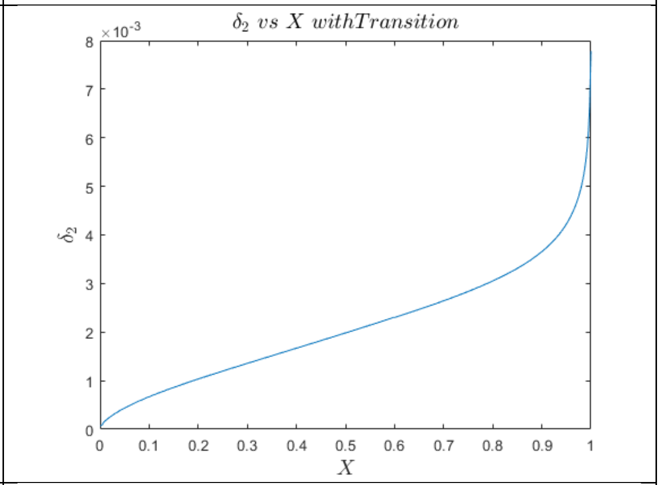


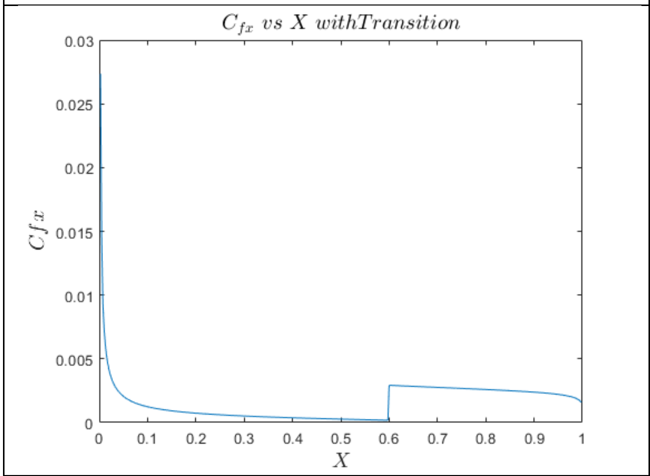


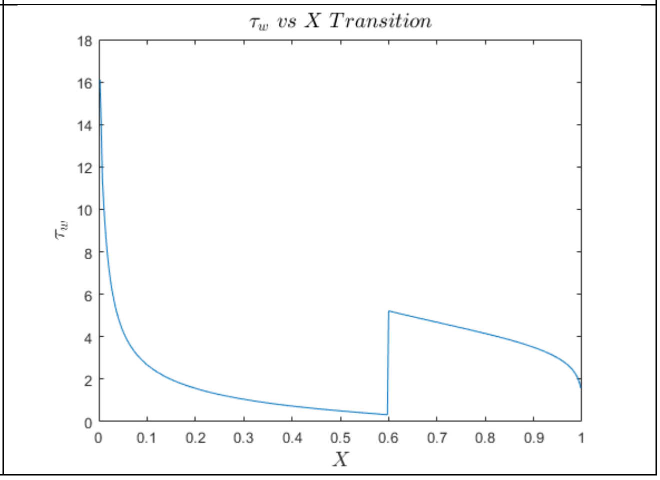












Conclusion

The Pohlhausen Approach proves to be quite simple and easy to apply, but it had many drawbacks regarding the Pohlhausen parameter. Many of the times, the Pohlhausen parameter were outside the range -12<$\Lambda < 12$ due to the velocities not being continuous, which in turn made finding the roots impossible and was in turn left by the code to determine it as zero. This left some regions of the analysis unsolved and a conclusion can be made that the Pohlhausen method is unreliable when using the source/ vortex method to find the surface velocities. The Validated results were quite acceptable compared to the results that were based upon the velocities calculated from the source/vortex method. The general validated results showed that at the transition, a spike in shear stress and coefficient of local shear, meanwhile no drastic change in the momentum thickness.

Future Work

Further work on this approach must be made to validate the results and make sure that this approximation can be used for the wide variety of arbitrary shapes and the results stay consistent to reality, since this approach is mostly used on flat plates and very thin symmetric airfoils.

References

[1] [Online]. Available: https://www.youtube.com/watch?v=u3qz2-676pk.

[2] [Online]. Available: http://web.mit.edu/fluids-modules/www/highspeed_flows/ver2/bl_Chap2/node12.html.

[3] [Online]. Available: https://farside.ph.utexas.edu/teaching/336L/Fluidhtml/node118.html.

[4] *Dr. Khalil's Notes.*

[5] *Dr. Hesham's Handouts.*

[6] E. Houghton, Aerodynamics for Engineering Students.

Appendix

Appendix A: Script

```
clc
clear
close all
```

Importing the airfoil characteristics

```
path="NACA0018_Location_Velocity.xlsx";
[X,Udim]=Import_Script(path);
Uinf=50;
U=Udim/Uinf;
U_d = zeros(1,length(U));                                          %U' intialization
length_U = length(U);

U_d=gradient(U,X);                                                 %U' calculation
U_dd=gradient(U_d,X);                                              %U'' calculation

del1=[-120^-1 3/10];                                               %delta1 polynomial
representation in pohlhausen parameter *1st degree*
del2=[-1/9072 -1/945 37/315];                                      %delta2 polynomial
representation in pohlhausen parameter *2nd degree*
k_p=conv(conv(del2,del2),[1 0]);                                   %K=delta2^2*(Pohlhausen
parameter) polynomial  *5th order*
f2_p=conv([1/6 2],del2);                                           %f2 in polynomial form =
delta2*(2+(pohl parameter)/6) *3rd degree*
F_del2=(conv((2*[0 0 f2_p]-4*k_p),del2)-[0 conv(2*k_p,del1)]);     %F*delta2=7agat
(polynomial) %zeros are to adjust to 5th degree, first term will be a 7th degree,
lamda_s=roots(F_del2);                                             %roots of the polynomial
F*delta2
a=real(lamda_s)<12 & real(lamda_s)>-12;                            %checking which root is
within the range -12<lambda<12

lamda_itr = zeros(1, length_U);                                    %Lambda for each position/
velocity on the airfoil
lamda_itr(1) = lamda_s(a);                                         %

mu = 1.789*10^-5;
rho = 1.225;

delta1_ = zeros(1, length_U);
delta2_ = zeros(1, length_U);
delta1_(1) = 0.3-lamda_itr(1)/120;                                 %Delta 1
(non_dimensionalized)
delta2_(1) = 37/315-lamda_itr(1)/945-lamda_itr(1)^2/9072;          %Delta 2
(non_dimensionalized)

K = zeros(1, length_U);
K(1) = delta2_(1)^2*lamda_itr(1);

z = zeros(1, length_U);
z(1) = K(1)/U_d(1);


F = zeros(1, length_U);
f1 = zeros(1, length_U);
f2 = zeros(1, length_U);
```

```
f1(1) = delta1_(1)/delta2_(1);
f2(1) = (2+lamda_itr(1)/6)*delta2_(1);
F(1) = 2*f2(1)-4*K(1)-2*K(1)*f1(1);


F0_U0 = -0.0652*U_dd(1)/U_d(1)^2;
z(2) = z(1)+F0_U0*( X(2) - X(1) );

delta = zeros(1, length_U);
delta1 = zeros(1, length_U);
delta2 = zeros(1, length_U);

delta(1) = sqrt(lamda_itr(1)*mu/rho/U_d(1));
delta1(1) = delta(1)*delta1_(1);
delta2(1) = delta(1)*delta2_(1);
%



for i = 2:length(U_d)
    K(i) = z(i)*U_d(i);

     P1=[0 0 0 0 0 -K(i)]+k_p;
     Lamda_itr_s=roots(P1);
    j=abs(imag(Lamda_itr_s))>0;
    Lamda_itr_s(j)=[];
    b=real(Lamda_itr_s)<12 & real(Lamda_itr_s)>-12;
    if b==0
        break
    end
    lamda_itr(i) = Lamda_itr_s(b);

    delta1_(i) = 0.3-lamda_itr(i)/120;
    delta2_(i) = 37/315-lamda_itr(i)/945-lamda_itr(i)^2/9072;

    delta(i) = sqrt(lamda_itr(i)*mu/rho/U_d(i));
    delta1(i) = delta(i)*delta1_(i);
    delta2(i) = delta(i)*delta2_(i);



    f1(i) = delta1_(i)/delta2_(i);
    f2(i) = (2+lamda_itr(i)/6)*delta2_(i);

    F(i) = 2*f2(i)-4*K(i)-2*K(i)*f1(i);

    z(i+1) = z(i)+F(i)/U(i)*(X(i+1)-X(i));
end
```

Plotting Without Transition

```
% Delta vs X
figure(1)
plot(X, delta)
xlim([0 0.7])
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\delta$','interpreter','latex','FontSize',14);
title('$\delta$ $vs$ $X$','interpreter','latex','FontSize',14);
```

```
% Delta1 vs X
figure(2)
plot(X, delta1)
xlim([0 0.7])
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\delta_{1}$','interpreter','latex','FontSize',14);
title('$\delta_{1}$ $vs$ $X$','interpreter','latex','FontSize',14);


% Delta2 vs X
figure(3)
plot(X, delta2)
xlim([0 0.7])
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\delta_{2}$','interpreter','latex','FontSize',14);
title('$\delta_{2}$ $vs$ $X$ $Laminar$','interpreter','latex','FontSize',14);


% Cfx vs X
figure(4)
cfx = ( 2.*f2 )./ (sqrt(rho*Uinf./mu).*U.* sqrt(z));

plot(X,cfx)
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$C{fx}$','interpreter','latex','FontSize',14);
title('$C_{fx}$ $vs$ $X$ $Laminar$','interpreter','latex','FontSize',14);

% tau_w vs x
tau_w_notbar = 0.5*rho*cfx.*U.^2*Uinf^2;
figure(5)
plot(X,tau_w_notbar)
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\tau_{w}$','interpreter','latex','FontSize',14);
title('$\tau_{w}$ $vs$ $X$ $Laminar$','interpreter','latex','FontSize',14);
```

Accounting for Transition

```
H=f1;
log_Rex=log10(rho.*U.*X./mu);
F_H=-40.4557+64.8066*H-26.7538*H.^2+3.3819*H.^3;
d=find(abs(log_Rex-F_H)<0.01);
x_s=X(d);
f=f1(d);
delta2(d) = delta2(d-1);
delta(d) = 1.4*delta(d-1);

syms x
eqn= (-delta(d)/delta2(d)+x+3.3+1.5501*(x-0.6778)^-3.064==0);
H_(d) = double(vpasolve(eqn,x,1));
H_11 = double(vpasolve(1.1+0.86*(x-3.3)^-0.777==H_(d),x,1));


H_1(d)=H_11;
for i = d+1:length(U_d)
    if H_(i-1) > 1.8 && H_(i-1) < 2.8
       break
    end

    Re_delta2_(i-1) = Uinf*U(i-1) .* delta2(i-1) * (rho/mu);
```

```
    cfx(i-1) = 0.246*10^(-0.678*H_(i-1)) * (Re_delta2_(i-1))^-0.268;
    delta2_by_dx = cfx(i-1)/2 - ((delta2(i-1) / U(i-1)) * U_d(i-1))*(H_(i-1)+2);
    delta2(i) = delta2(i-1) + delta2_by_dx * (X(i) - X(i-1));

    H_1(i) = (0.306e-1 * U(i) * ((H_1(i - 1) - 3) ^ (-0.6169e0)) + (U(i) * delta2(i) * H_1(i - 1) /
(X(i) - X(i - 1)))) / ((U(i) * delta2(i) - U(i - 1) * delta2(i - 1)) / (X(i) - X(i - 1)) + U(i) *
delta2(i) / (X(i) - X(i - 1)));

    if (H_1(i) <= 3.3)
        H_(i) = 3;
    elseif (H_1(i) > 5.3)
        H_(i) = 1.1+0.86*(H_1(i)-3.3)^-0.777;
    else
        H_(i) = 0.6778 + 1.1536*(H_1(i) - 3.3 )^-0.326;
    end
end
```

Plotting with Transition

```
% Delta2 vs X
figure(6)
plot(X, delta2)
xlim([0 1])
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\delta_{2}$','interpreter','latex','FontSize',14);
title('$\delta_{2}$ $vs$ $X$ $with Transition$','interpreter','latex','FontSize',14);


% Cfx vs X
figure(7)
plot(X,cfx)
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$C{fx}$','interpreter','latex','FontSize',14);
title('$C_{fx}$ $vs$ $X$ $with Transition$','interpreter','latex','FontSize',14);

% tau_w vs x
tau_w = cfx * 0.5 * rho.*U.^2*Uinf^2;
figure(8)
plot(X,tau_w)
xlabel('$X$','interpreter','latex','FontSize',14);
ylabel('$\tau_{w}$','interpreter','latex','FontSize',14);
title('$\tau_{w}$ $vs$ $X$ $Transition$','interpreter','latex','FontSize',14);
```