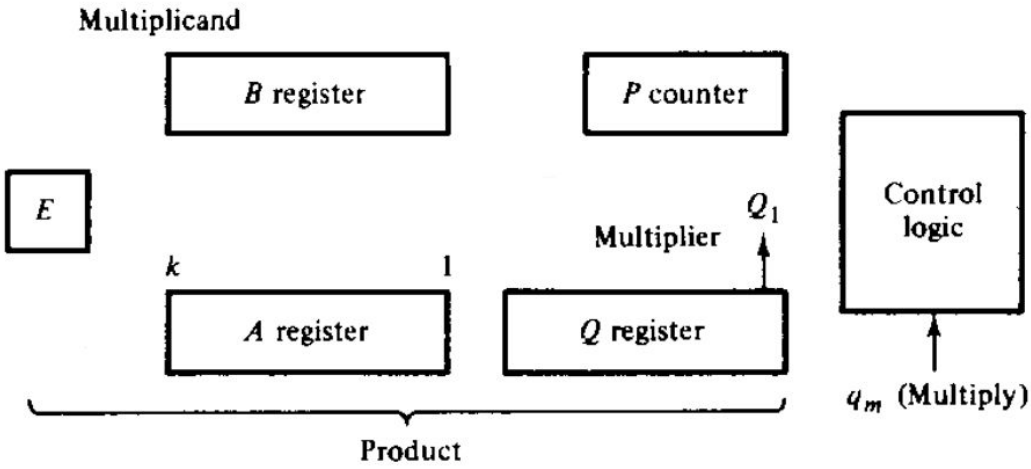
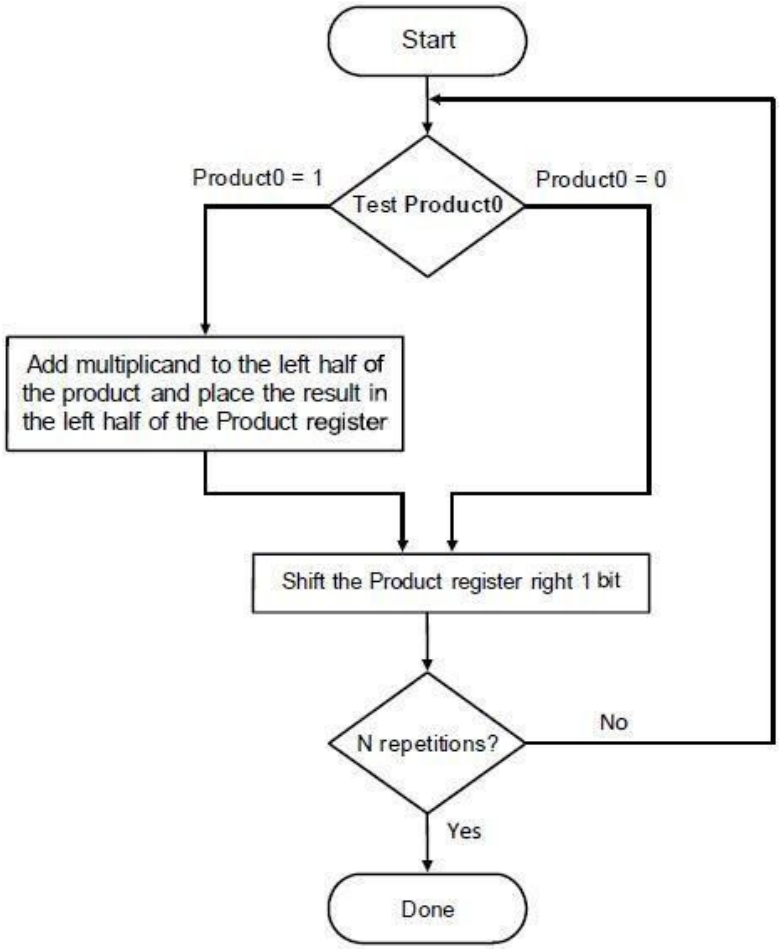
Problem statement:
Design a sequential shift-add multiplier for 4 bit unsigned binary numbers whose controller is designed using one flip flop per state.

Block diagram of equipment configuration:

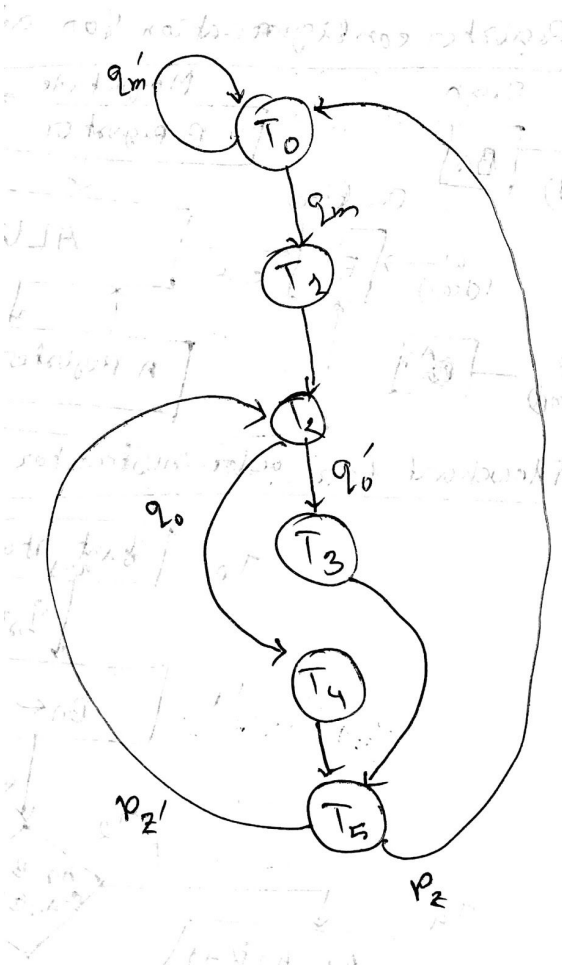


Register B holds the multiplicand, register A holds the partial multiplied answer and register Q holds the multiplier. The LSB of the Q register is used to decide if an add and shift operation will take place or only a shift operation will take place. Counter P keeps track of remaining digits on the multiplier. The pin qm on the control logic is used to indicate a multiplication operation. The register E is used to store the output carry of the add operations from the ALU.

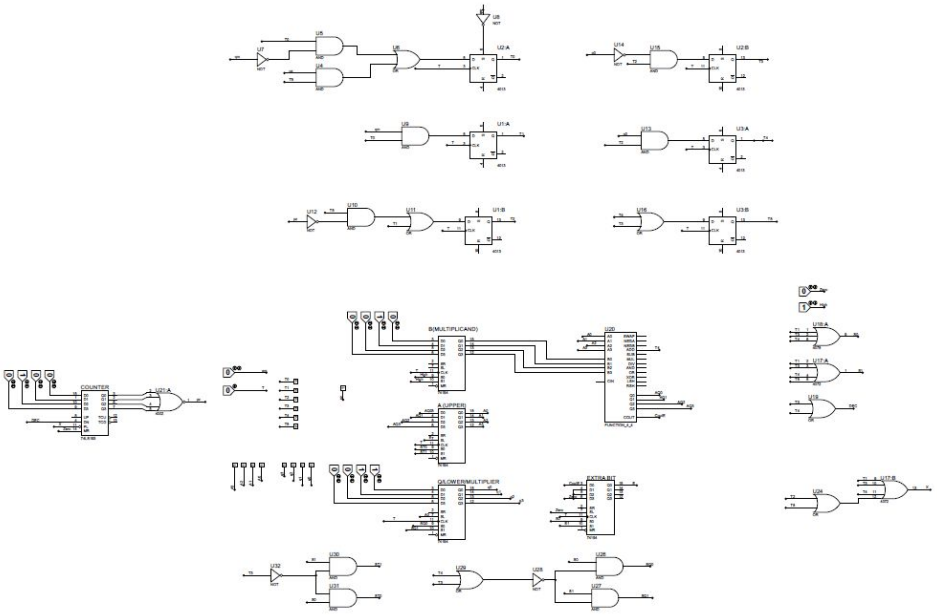
Flowchart of the algorithm:



State Diagram:



Circuit Diagram:



The states are on the top portion of the diagram. There are a total of 6 states.

The controls are on the right and lower portions.

The data processor is in the middle. For the register E we took one extra 4 bit shift register but only used one of the input and output.

The equations for the flip-flops representing states are,

$$DT_0=T_0.q'_m+P_z.T_5$$
$$DT_1=q_m.T_0$$
$$DT_2=P'_z.T_5+T_1$$
$$DT_3=q'_0.T_2$$
$$DT_4=q_0.T_2$$
$$DT_5=T_4+T_3$$

The equations for the control block output are,

$$S_0=T_1+T_3+T_4$$
$$S_1=T_1+T_3+T_4+T_5$$
$$SQ0=S_0.(T_4+T_3)'$$
$$SQ1=S_1.(T_4+T_3)'$$
$$DEC=T_3+T_4$$
$$x=T_1+T_2+T_3+T_4+T_5$$
$$ST_1=S1.T'_3$$
$$ST0=S0.T'_3$$

Discussions:

The circuit can be updated slightly for multiplying signed magnitude numbers. We only need to add another register that will hold the sign for the output and it will be calculated in state T1 by XORing the signs of the multiplier and the multiplicand. In that case we also must have extra 2 registers for storing the signs of the multiplier and the multiplicand.

Another change can be made, where we can eliminate the need of the shift register labeled extra bit. We can directly connect the cout of the alu to the SL pin of A shift register. However that would require some changes in the control logic.