INNOPOLIS UNIVERSITY

“Project Task1”

DYNAMICS OF NON-LINEAR ROBOTIC SYSTEMS

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Spherical Robot with XYX Wrist Configuration

Simple Graph of the Model:

Spherical robot is a 3 DOF robot with configuration of RRP. In this assignment, a spherical wrist is attached with YXY configurations for better position and orientation control. In the figure below, hand sketch model was drawn to define robot joints and their rotation axes. The overall configuration of the robot is RRPRRR.

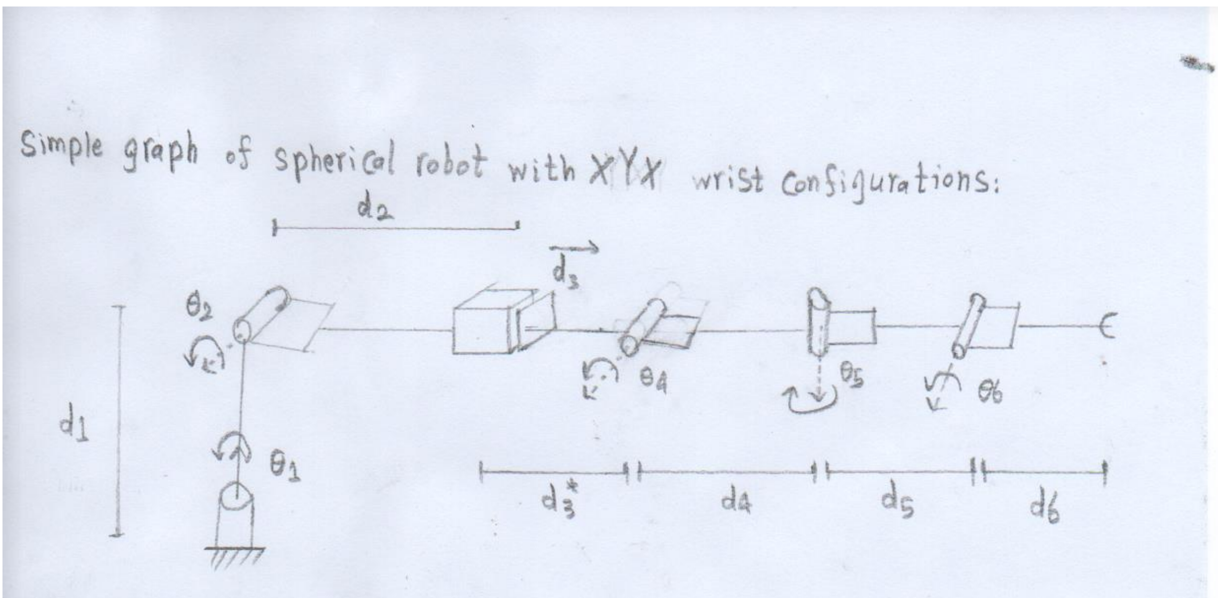

Figure 1; Simple graph of spherical robot

The robot links' length is defined as:

- d1 is distance between R_1 and R_2
- d2 is distance between R_2 and P_3
- d3* is distance between P_3 and R_4
- d4 is distance between R_4 and R_5
- d5 is distance between R_5 and R_6
- d6 is distance between R_6 and End Effector

Coordinates Assignment:

In order to apply Forward Kinematics (FK) and Inverse Kinematics (IK), Joint frames have to be specified with the following considerations:

- All the frames follow the right-hand rule.
- The z axis is chosen to be the axis of rotation and translation for all joints.

- Every frameO_{n-1} precedes link l_n.
- The last three frames are assigned to fulfill the XYX notation and have the same origin.
- All the frames follow DH rules as: x_n intersects and is perpendicular on z_{n-1}.

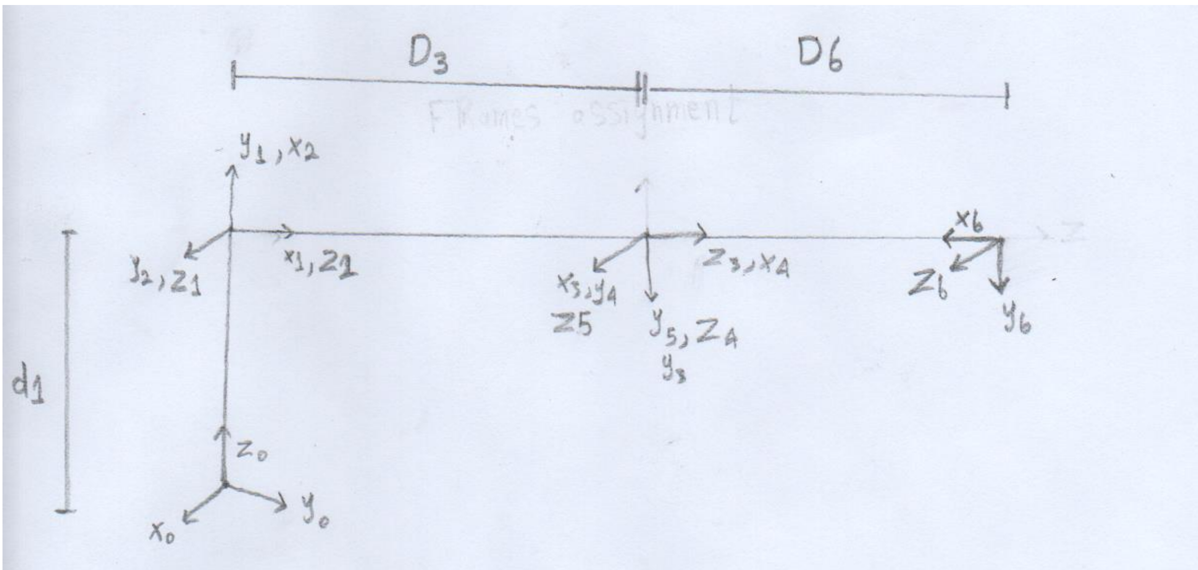

Figure 2; Frames Assignment

For simplification, the robot lengths were modified to represent distance between adjacent frames where:

- $D_3 = d_2 + d_3^* + d_4$
- $D_6 = d_5 + d_6$

Forward Kinematics:

Denavit Hatenberg (DH) was used to calculate the FK equations for the model. Based on the joint frames specified in fig(2), the DH parameters were calculated as follows:

Link	θ	d	a	α
1	$\theta_1^* + 90°$	d_1	0	90°
2	$\theta_2^* + 90°$	0	0	90°
3	90°	D_3^*	0	0
4	$\theta_4^* + 180°$	0	0	90°
5	$\theta_5^* + 180°$	0	0	90°
6	θ_6^*	0	$-D_6$	0

Note: The * in the table refers to the variable parameter associated with each joint.

Now, it is straightforward to compute the homogonous transformation between each two consecutive frames (A_i):

$$A_1 = R_z(\theta_1 + 90) \times T_z(d_1) \times R_x(90) = \begin{pmatrix} -S_1 & 0 & C_1 & 0 \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = R_z(\theta_2 + 90) \times R_x(90) = \begin{pmatrix} -S_2 & 0 & C_2 & 0 \\ C_2 & 0 & S_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_3 = R_z(90) \times T_z(D_3) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & D_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_4 = R_y(90) \times R_z(\theta_4 + 180) \times R_x(90) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -S_4 & 0 & C_4 & 0 \\ C_4 & 0 & S_4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_5 = R_z(\theta_5 + 180) \times R_x(90) = \begin{pmatrix} -C_5 & 0 & -S_5 & 0 \\ -S_5 & 0 & C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_6 = R_z(\theta_6) \times R_y(-90) = \begin{pmatrix} 0 & -S_6 & -C_6 & 0 \\ 0 & C_6 & -S_6 & 0 \\ 1 & 0 & 0 & D_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that A_4 is pre-multiplied by $R_y(90)$ and A_6 is post-multiplied by $R_y(-90)$ in order to attain the XYX notation.

Now, it will be easier to calculate the final FK matrix (T_6^0) as follow:

$$T_6^0 = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 = \begin{pmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where :

$$r_{11} = C_1C_5 - S_5(S_1S_2S_4 - S_1C_2C_4)$$

$$r_{21} = S_1C_5 + S_5(C_1S_2S_4 - C_1C_2C_4)$$

$$r_{31} = S_5(C_2 \quad S_4 + S_2C_4)$$

$$r_{12} = S_6(C_1 \quad S_5 + C_5(S_1S_2S_4 - S_1C_2C_4)) - C_6(C_2S_1S_4 + C_4S_2S_1)$$

$$r_{22} = S_6(S_1 \quad S_5 - C_5(C_1S_2S_4 - C_1C_2C_4)) + C_6(C_2C_1S_4 + C_4S_2C_1)$$

$$r_{32} = \left(C_5S_6(C_2S_4 + S_2C_4)\right) - C_6(C_2C_4 - S_2S_1)$$

$$r_{13} = C_6(C_1 \quad S_5 + C_5(S_1S_2S_4 - S_1C_2C_4)) + S_6(C_2S_1S_4 + C_4S_2S_1)$$

$$r_{23} = C_6(S_1 \quad S_5 - C_5(C_1S_2S_4 - C_1C_2C_4)) - S_6(C_2C_1S_4 + C_4S_2C_1)$$

$$r_{33} = S_6(C_2C_4 - S_2S_4) + C_5C_6(C_2S_4 + S_2C_4)$$

$$x = D_6S_6(C_2S_1S_4 + C_4S_1S_2) + D_6C_6(C_1S_5 + C_5(S_1S_2S_4) - S_1C_2C_4)) - D_3C_2S_1$$

$$y = D_6S_6(C_1C_2S_4 + C_1C_4S_2) - D_3C_2C_1 + D6 \quad C_6(S_1S_5 - C_5(C_1S_2S_4 - C_1C_2C_4)$$

$$z = D_1 + D_3S_2 - D_6S_6(C_2C_4 - S_2S_4) + D_6C_5C_6(C_2S_4 + S_2C_4)$$

XYX Validation:

In order to validate that the last three joints represent XYX notation, A comparison was made between XYX matrix and the rotation between last three joints R_3^6. The results were identical as shown in the two figures below:

```
>> A_36([1 2 3],[1 2 3])

ans =

[              cos(theta_5),                                       sin(theta_5)*sin(theta_6),                                       cos(theta_6)*sin(theta_5)]
[  sin(theta_4)*sin(theta_5), cos(theta_4)*cos(theta_6) - cos(theta_5)*sin(theta_4)*sin(theta_6), - cos(theta_4)*sin(theta_6) - cos(theta_5)*cos(theta_6)*sin(theta_4)]
[ -cos(theta_4)*sin(theta_5), cos(theta_6)*sin(theta_4) + cos(theta_4)*cos(theta_5)*sin(theta_6),   cos(theta_4)*cos(theta_5)*cos(theta_6) - sin(theta_4)*sin(theta_6)]

>> FK_DH_Parameters
>> rot_x4*rot_y5*rot_x6

ans =

[              cos(theta_5),                                       sin(theta_5)*sin(theta_6),                                       cos(theta_6)*sin(theta_5), 0]
[  sin(theta_4)*sin(theta_5), cos(theta_4)*cos(theta_6) - cos(theta_5)*sin(theta_4)*sin(theta_6), - cos(theta_4)*sin(theta_6) - cos(theta_5)*cos(theta_6)*sin(theta_4), 0]
[ -cos(theta_4)*sin(theta_5), cos(theta_6)*sin(theta_4) + cos(theta_4)*cos(theta_5)*sin(theta_6),   cos(theta_4)*cos(theta_5)*cos(theta_6) - sin(theta_4)*sin(theta_6), 0]
[                         0,                                                               0,                                                               0, 1]
```

Figure 3; Results for R_3^6 (above) and XYX Rotation matrix (below).

Inverse Kinematics (Kinematic Decoupling):

In this report, Kinematic Decoupling was used to deduce IK equations. The steps for IK were followed as explained in the lecture.

1. Tool Pose:

An assumption was made about symbolic final tool pose. The matrix for position and orientation of the tool pose was as follow:

```
  syms nx ny nz sx sy sz ax ay az x y z
A_06 =      [nx   sx ax x;
             ny   sy ay y;
             nz   sz az z;
             0    0  0  1];
```

Figure 4; Tool Pose matrix as an input to IK equations.

2. Solve FK to obtain A_0^3 and A_3^6:

Using the transformations obtained from the FK chapter, the following transformations were calculated:

$$A_0^3 = A_1 \times A_2 \times A_3 =$$

```
A_03 =

[ cos(theta_1), -sin(theta_1)*sin(theta_2), -cos(theta_2)*sin(theta_1), -d_3*cos(theta_2)*sin(theta_1)]
[ sin(theta_1),  cos(theta_1)*sin(theta_2),  cos(theta_1)*cos(theta_2),  d_3*cos(theta_1)*cos(theta_2)]
[            0,              -cos(theta_2),               sin(theta_2),         d_1 + d_3*sin(theta_2)]
[            0,                          0,                          0,                              1]
```

$$A_3^6 = A_4 \times A_5 \times A_6 =$$

```
A_36 =

[              cos(theta_5),                                          sin(theta_5)*sin(theta_6),                                          cos(theta_6)*sin(theta_5)
[  sin(theta_4)*sin(theta_5), cos(theta_4)*cos(theta_6) - cos(theta_5)*sin(theta_4)*sin(theta_6), - cos(theta_4)*sin(theta_6) - cos(theta_5)*cos(theta_6)*sin(theta_4)
[ -cos(theta_4)*sin(theta_5), cos(theta_6)*sin(theta_4) + cos(theta_4)*cos(theta_5)*sin(theta_6),   cos(theta_4)*cos(theta_5)*cos(theta_6) - sin(theta_4)*sin(theta_6)
[                          0,                                                                  0,                                                                  0
```

3. Equate last column of tool pose to the last column of A_0^3:

By equating the given coordinates of the toll pose to the last column of A_0^3, the following equations were calculated for θ_1 , θ_2 , d_3 :

$$x = -D_3 C_2 S_1$$

$$y = D_3 C_1 C_2$$

$$z = D_1 + D_3 S_2$$

From the previous three equations, the three joint parameters are obtained.

$$\theta_1 = atan2(-x, y)$$

$$\theta_2 = atan2(C_1(z - d_1), y)$$

$$D_3 = \frac{y}{C_1 C_2}$$

4. Solve R_6^3 as product of $(R_3^0)^{-1} \times R_6^0$:

R_6^3 were calculated as $(\boldsymbol{R_3^0})^{-1} \times \boldsymbol{R_6^0}$:

Where $(\boldsymbol{R_3^0})^{-1}$ is transpose of $\boldsymbol{R_3^0}$ and $\boldsymbol{R_6^0}$ is the input tool orientation, so $\boldsymbol{R_3^6}$ is calculated as follow:

```
ans =

[                                              nx*cos(theta_1) + ny*sin(theta_1),                                              sx*cos(theta_1) + sy*sin(theta_1),
[ ny*cos(theta_1)*sin(theta_2) - nz*cos(theta_2) - nx*sin(theta_1)*sin(theta_2), sy*cos(theta_1)*sin(theta_2) - sz*cos(theta_2) - sx*sin(theta_1)*sin(theta_2),
[ nz*sin(theta_2) + ny*cos(theta_1)*cos(theta_2) - nx*cos(theta_2)*sin(theta_1), sz*sin(theta_2) + sy*cos(theta_1)*cos(theta_2) - sx*cos(theta_2)*sin(theta_1),
```

Figure 5; R_3^6 (first two columns)

```
                                              ax*cos(theta_1) + ay*sin(theta_1)]
 ay*cos(theta_1)*sin(theta_2) - az*cos(theta_2) - ax*sin(theta_1)*sin(theta_2)]
 az*sin(theta_2) + ay*cos(theta_1)*cos(theta_2) - ax*cos(theta_2)*sin(theta_1)]
```

Figure 6;R_3^6 (last column)

5. Set R_6^3 equal to $R_6^3(\theta_4, \theta_5, \theta_6)$:

At final, the parameters for the last joints are obtained as:

$$R_6^3 = R_6^3(\theta_4 \ , \theta_5 \ , \theta_6 \ )$$

The following angles are obtained:

$$\theta_4 \ = \operatorname{atan} 2(ny, -nz) = atan2\left(\frac{ny\ C_1 S_2 - nzC_2 - nxS_2S_1}{-nzS_2 - nyC_1C_2 + nxC_2S_1}\right)$$

$$\theta_6 \ = \operatorname{atan} 2(sx, ax) = atan2\left(\frac{sx\ C_1 + syS_1}{axC_1 + ayS_1}\right)$$

$$\theta_5 \ = \operatorname{atan} 2\big(ax/\cos\,(\theta_6\ ), nx\big) = atan2\left(\frac{(sxC_1 + syS_1)/S_6}{nxC_1 + nyS_1}\right)$$