Computational intelligence
Homework 2

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April 2022

Task 1

This task can be solved as feasibility problem. We will check if the robot can stand on horizontal ground while maintaining certain constraints. The problem will be formulated as follows:

$$\min_{x} \ 0 \tag{1}$$

$$\text{subject to } Ax = b \tag{2}$$

$$||E_t^T f_i|| \leq \mu e_n^T f_i \qquad i = 1, 2, 3, 4 \tag{3}$$

- The first constraint (2) refers to static equilibrium condition in which summation of all forces ($f_i \in \mathbb{R}^3$) is zero as $\ddot{r} = 0$ and sum of moments is zeros around robot center of mass. r_i is the position of each leg and r_c is position of robot center of mass..
  where $A = \begin{bmatrix} I_{3x3} & I_{3x3} & I_{3x3} & I_{3x3} \\ [r_1 - r_c] & [r_2 - r_c] & [r_3 - r_c] & [r_4 - r_c] \end{bmatrix}_{6x12}$, $x = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}_{12x1}$, $b = \begin{bmatrix} -mg \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6x1}$
- Second constraint (3) imposes that each reaction force should lie within the friction cone where $E_t^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3x2}$ is basis for tangential plane and $e_n = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3x1}$ is orthonormal vector to that plane. μ is a scalar value of friction coefficient.

Task 2

This task can be solved as feasibility problem as previous one. The difference here is that the robot is on tilted ground .The tilted plane will be defined by a random normal vector (e_n) and the plane orthonormal basis E_t. Thus, the problem will be formulated as follows:

$$\min_{x} \ 0 \tag{4}$$

$$\text{subject to } Ax = b \tag{5}$$

$$||E_t^T f_i|| \leq \mu e_n^T f_i \qquad i = 1, 2, 3, 4 \tag{6}$$

- All the matrices are the same as previous task except for E_t^T, e_n which define the new titled plane. E_t^T will be the left null space of e_n which is a randomly generated normal vector. Also the same coordinate system of task 1 is used here to avoid any use of trigonometric functions.

Task 3

This task can be solved as feasibility problem as previous one. The difference here is that one of the robot legs (f_4) on vertical wall. Thus, the problem will be formulated as follows:

$$\min_x \quad 0 \tag{7}$$

$$\text{subject to } Ax = b \tag{8}$$

$$||E_t^T f_i|| \leq \mu e_n^T f_i \qquad i = 1, 2, 3 \tag{9}$$

$$||e_n^T f_i|| \leq \mu E_t^T f_i \qquad i = 4 \tag{10}$$

- All the matrices are the same as previous task except for the friction cone constrain. It will differ due to the new vertical plane as seen in eq (9) and eq(10). Assuming the two planes are perpendicular to each other.

Task 4

This task can be solved as feasibility problem as previous one. The difference here is that one of the robot's legs (f_4) is nailed to the ground. Thus, the problem will have the same formulations as follows:

$$\min_x \quad 0 \tag{11}$$

$$\text{subject to } Ax = b \tag{12}$$

$$||E_t^T f_i|| \leq \mu e_n^T f_i \qquad i = 1, 2, 3 \tag{13}$$

- All the matrices are the same as previous task except for (13) which excludes the nailed leg from the friction cone constraint.

Task 5

In this task, we will assume that the external forces are applied in the center of mass r_c (no moment associated with external forces). So the static equilibrium equations will be:

$$f_1 + f_2 + f_3 + f_4 = mg + f_e \tag{14}$$

$$\sum_{i=0}^{4} [r_i - r_c] \times f = 0 \tag{15}$$

The previous equations can be formulated in matrix format as:

$$Af = b \tag{16}$$

where $A = \begin{bmatrix} I_{3X3} & I_{3X3} & I_{3X3} & I_{3X3} \\ (r_1 - r_c) & (r_2 - r_c) & (r_3 - r_c) & (r_4 - r_c) \end{bmatrix}_{6x6}, f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}_{6x1}, b = \begin{bmatrix} mg + f \\ 0_{3x3} \end{bmatrix}_{6x1}$

The solution of Eq.16 will be:

$$f = A^{+}b + Nz \tag{17}$$

However, we need to eliminate the zero rows of vector b so we will reformulate 17 as:

$$f = QA^{+}(mg + f_e + QNz) \tag{18}$$

Eq 18 contains the constraints regarding static equilbirum in presence of external forces. Now, we will discuss the friction cone constraint as follows:

$$||E_t^T f_i|| \leq \mu e_n^T f_i \tag{19}$$

Since this inequality is hard to solve, we will convert the cone to polytope approximation so the previous constraint will be:

$$Cf \leq \mu D \tag{20}$$

Substituting 18 in 20 we yield:

$$c_i^T(QA^{+}(mg + f_e)) + QNz \leq d_i \tag{21}$$

$$c_i^T QNz + c_i^T QA^{+} f_e \leq d_i - c_i^T QA^{+} mg \tag{22}$$

Furthermore, the external forces are constrained to lie within ellipse defined as:

$$||Mf_e + f_{e,0}|| \leq 1 \tag{23}$$

Where M is positive definite matrix. Assume $y = Mf_e + f_{e,0}$ where $||y|| \leq 1$ so :

$$f_e = M^{-1}(y - f_{e,0}) \tag{24}$$

substituting 24 in 22 will result in:

$$c_i^T QNz + c_i^T QA^{+} M^{-1} y \leq d_i - c_i^T QA^{+} mg + c_i^T QA^{+} M^{-1} f_{e,0} \tag{25}$$

This problem are not converted to min max problem. So to find the reaction forces, we should account for worst scenario. In this case, y should have maximum value. This will occur when y has a norm of 1 and is aligned in the same direction as $c_i^T QA^{+} M^{-1}$ so:

$$y = \frac{c_i^T QA^{+} M^{-1}}{||c_i^T QA^{+} M^{-1}||} \tag{26}$$

now substitute in 25:

$$c_i^T QNz \leq d_i - c_i^T QA^{+} mg + c_i^T QA^{+} M^{-1} f_{e,0} - ||c_i^T QA^{+} M^{-1}|| \tag{27}$$

Now, this inequality will ensure static equilibrium in case of external forces.