1 Problem formulation

Assume that you have a walking robot standing on 4 feet. We model the robot as a point-mass with four reaction forces acting on it:

$$\begin{cases} m\ddot{\mathbf{r}}_c = \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \mathbf{f}_4 + m\mathbf{g} \\ 0 = \sum_{i=1}^{4} (\mathbf{r}_i - \mathbf{r}_C) \times \mathbf{f}_i \end{cases} \tag{1}$$

where $\mathbf{r}_C \in \mathbb{R}^3$ is the position of the center of mass of the robot, $\mathbf{f}_i \in \mathbb{R}^3$ are the reaction forces, $\mathbf{r}_i \in \mathbb{R}^3$ are position of the feet, m is the mass of the robot, and $\mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ -9.8 \end{bmatrix}$ is gravitational vector.

We additionally assume that the reaction forces lie in the friction cone, with friction coefficient μ.

2 Tasks

1. Given $\mathbf{r}_C$, and $\mathbf{r}_i$ find if there exit reaction forces that allow the robot to maintain static stability (1) with $\ddot{\mathbf{r}}_c = 0$, given that the robot stands on horizontal ground.
2. Solve the previous problem, but assume that the robot stands on tilted ground.
3. Solve the previous problem, but assume that of of the feet pushes against a vertical wall instead of standing on the ground.
4. Solve the previous problem, but assume that one of the feet is nailed to the floor.
5. Assume a constant external force $\mathbf{f}_e$ acts on the robot, $||\mathbf{M}\mathbf{f}_e + \mathbf{f}_{e,0}|| \leq 1$, can you guarantee that the robot will remain stable?