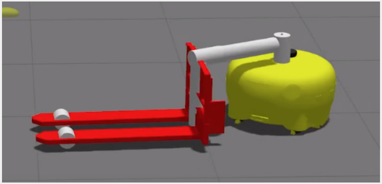
Autonomous Mobile Robotics

Motion Planning and Control

Geesara Kulathunga



September 5, 2022

Control of Mobile Robots

Contents

- Kinematics of wheeled mobile robots: internal, external, direct, and inverse
  - Differential drive kinematics
  - Bicycle drive kinematics
  - Rear-wheel bicycle drive kinematics
  - Car(Ackermann) drive kinematics
- Wheel kinematics constraints: rolling contact and lateral slippage
- Wheeled Mobile System Control: pose and orientation
  - Control to reference pose
  - Control to reference pose via an intermediate point
  - Control to reference pose via an intermediate direction
  - Control by a straight line and a circular arc
  - Reference path control
- Smooth path planning in a given 2-D space for vehicles with nonholonomic constraints using Hybrid A*

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- For mobile robotics kinematic model is sufficient

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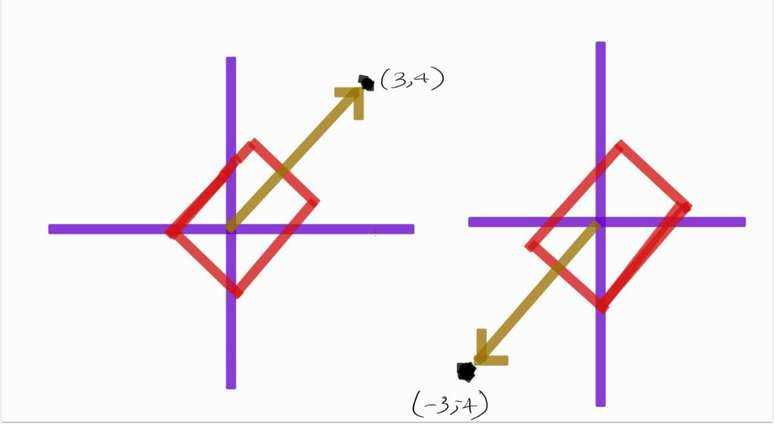
- Inverse kinematics: robot inputs as a function of desired robot pose

The difference between atan and atan2



The difference between atan and atan2

Quadrant	Angle	sin	cos	tan
I	0 < α < π/2	+	+	+
II	π/2 < α < π	+	-	-
III	π < α < 3π/2	-	-	+
IV	3π/2 < α < 2π	-	+	-

- Given that the value of $tan(\alpha)$ is positive, we cannot distinguish, whether the angle was from the first or third quadrant and if it is negative, it could come from the second or fourth quadrant. So by convention, atan() returns an angle from the first or fourth quadrant (i.e. $-\pi/2 <= atan() <= \pi/2$), regardless of the original input to the tangent

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- To get full information, we must not use the result of the division $sin(\alpha)/cos(\alpha)$ but we have to look at the values of the sine and cosine separately. And this is what atan2() does. It takes both, the $sin(\alpha)$ and $cos(\alpha)$ and resolves all four quadrants by adding π to the result of atan() whenever the cosine is negative

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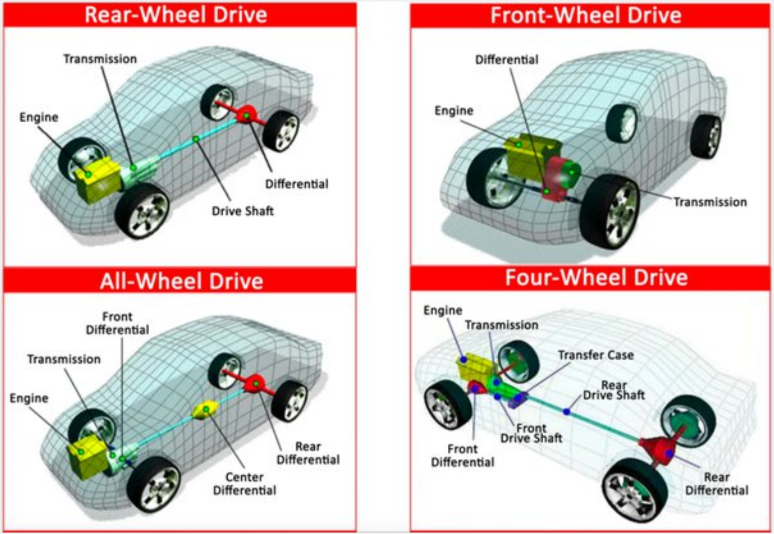
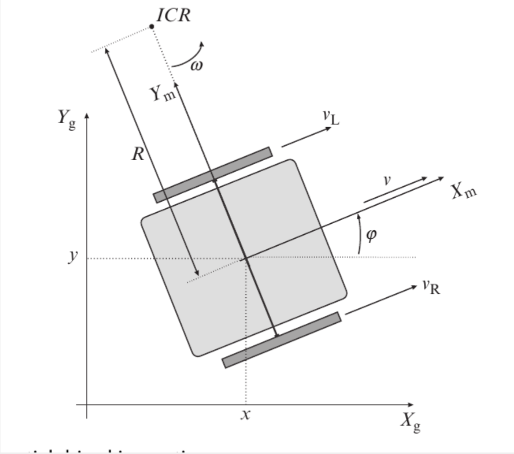
6 tan2(y,x) <pi and atan: -pi/2 < atan(y/x) < pi/2

Differential Drive Kinematics



https://cartreatments.com/types-of-differentials-how-they-work/


ICR
ω
Y_m
Y_g
v_L
R
v
X_m
φ
y
v_R
x
X_g

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- Well-fit for smaller mobile robots
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- According to Fig. 9,
  - Terms $\mathbf{v}_R(t), \mathbf{v}_L(t)$ denoted velocity of right and left wheels, respectively
  - Wheel radius r , distance between wheels L, and term R(t) depicts the instantaneous radios (ICR) of the vehicle. Angular velocity is same for both left and right wheels around the ICR.

- Tangential velocity

$$\mathbf{v}(t) = \omega(t)R(t) = \frac{\mathbf{v}_R(t) + \mathbf{v}_L(t)}{2} \tag{1}$$

, where $\omega = \mathbf{v}_L(t)/(R(t) - L/2) = \mathbf{v}_R(t)/(R(t) + L/2)$. Hence, ω and $R(t)$ can be determined as follows:

$$\begin{aligned} \omega(t) &= \frac{\mathbf{v}_R(t) - \mathbf{v}_L(t)}{L} \\ R(t) &= \frac{L}{2}\frac{\mathbf{v}_R(t) + \mathbf{v}_L(t)}{\mathbf{v}_R(t) - \mathbf{v}_L(t)} \end{aligned} \tag{2}$$

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- Wheels tangential velocities

$$\mathbf{v}_L = r\omega_L(t), \quad \mathbf{v}_R = r\omega_R(t) \quad (3)$$

- Internal robot kinematics

$$\begin{bmatrix} \dot{x}_m(t) \\ \dot{y}_m(t) \\ \dot{\Phi}(t) \end{bmatrix} = \begin{bmatrix} v_{X_m}(t) \\ v_{Y_m} \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} \omega_L(t) \\ \omega_R(t) \end{bmatrix} \tag{4}$$

, where $\omega(t)$ and $\mathbf{v}(t)$ are the control variables

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Differential Drive Kinematics

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- Discrete time dynamics using Euler integration

$$\begin{aligned} x(k+1) &= x(k) + v(k)T_s cos(\Phi(k)) \\ y(k+1) &= y(k) + v(k)T_s sin(\Phi(k)) \\ \Phi(k+1) &= \Phi(k) + \omega(k)T_s \end{aligned} \tag{6}$$

, where discrete time instance $t = kT_s$, k=0,1,2,.., for T_s

Differential Drive Kinematics

- Forward robot kinematics (given a set of wheel speeds, determine robot velocity)

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- We can also try trapezoidal numerical integration for better approximation

$$\begin{aligned} x(k+1) &= x(k) + v(k)T_s cos(\Phi(k) + \omega(k)T_s/2) \\ y(k+1) &= y(k) + v(k)T_s sin(\Phi(k) + \omega(k)T_s/2) \\ \Phi(k+1) &= \Phi(k) + \omega(k)T_s \end{aligned} \tag{8}$$

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  - the most challenging case compared to direct or forward kinematics
  - given target pose how many possible ways to get there?
  - What if robot goes can perform only two type of motions: forward and rotations

$$\begin{aligned} \mathbf{v}_R = \mathbf{v}_L = \mathbf{v}_R, \omega(t) = 0, \mathbf{v}(t) = \mathbf{v}_R // \mathit{forward} \\ \mathbf{v}_R = -\mathbf{v}_L = \mathbf{v}_R, \omega(t) = 2\mathbf{v}_R/L, \mathbf{v}(t) = 0 // \mathit{rotation} \end{aligned} \tag{9}$$

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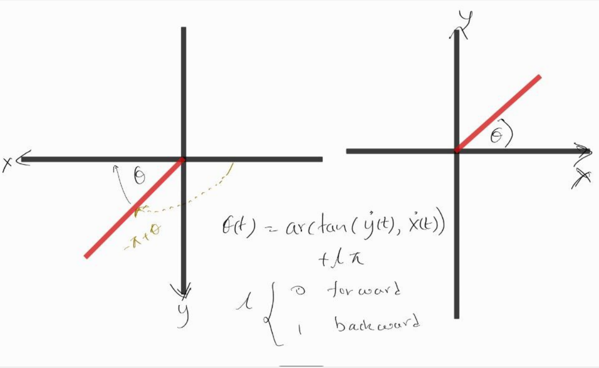
  - If there is disturbance in the trajectory and know the desired pose at time t, i.e., $x(t), y(t)$

$$\begin{aligned}
\mathbf{v}(t) &= \pm\sqrt{\dot{x}^2(t) + \dot{y}^2(t)} // \text{+ forward and - reverse} \\
\Phi(t) &= arctan2(\dot{y}(t), \dot{x}(t)) + l\pi, \quad l \in \{0, 1\} \\
&\quad // \text{ 0 forward and 1 reverse} \\
\omega(t) &= \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{\dot{x}^2(t) + \dot{y}^2(t)} = v(t)k(t)
\end{aligned} \tag{10}$$

, where k(t) is the path curvature and $\omega(t) = \dot{\Phi}(t)$


x
θ
$-\pi+\theta$
y
$\theta(t) = \arctan(\dot{y}(t), \dot{x}(t))$
$+ l\pi$
l
0 forward
1 backward
y
θ
x

Motion control of Bicycle mobile robots

https://helpfulcolin.com/bike-riding-robots-are-helped-by-gyroscopes-cameras/

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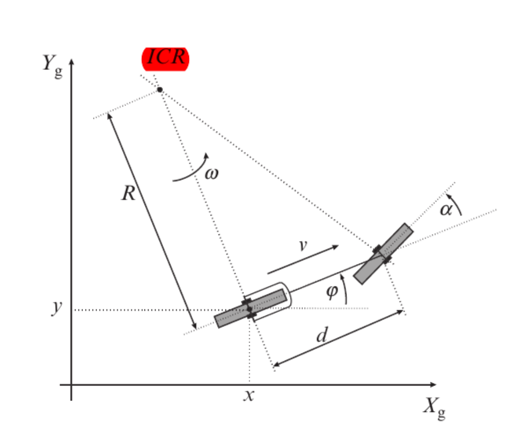

Figure: Caption

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- Angular velocity ω around ICR

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- Steering wheel velocity

$$\mathbf{v}_S(t) = \omega_S(t) r \tag{13}$$

- Internal robot kinematics

$$\begin{aligned} \dot{x}_m(t) &= \mathbf{v}_S(t)cos(\alpha(t)) \\ \dot{y}_m(t) &= 0 \\ \Phi(\dot{t}) &= \frac{\mathbf{v}_S(t)}{d}sin(\alpha(t) \end{aligned} \tag{14}$$

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$$\begin{bmatrix}\dot{x}(t)\\ \dot{y}(t)\\ \dot{\Phi(t)}\end{bmatrix} = \begin{bmatrix}cos(\Phi(t)) & 0\\ sin(\Phi(t)) & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{v}(t)\\ \omega(t)\end{bmatrix} \tag{16}$$

, where $\mathbf{v}(t) = \mathbf{v}_S(t)cos(\alpha(t))$ and $\omega(t) = \frac{\mathbf{v}_S}{d}sin(\alpha(t))$

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