Motion Planning for Autonomous Vehicles

Frenet frame trajectory planning

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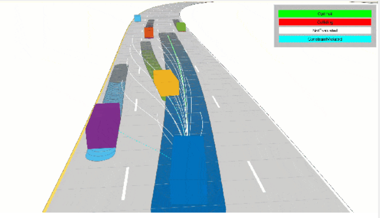
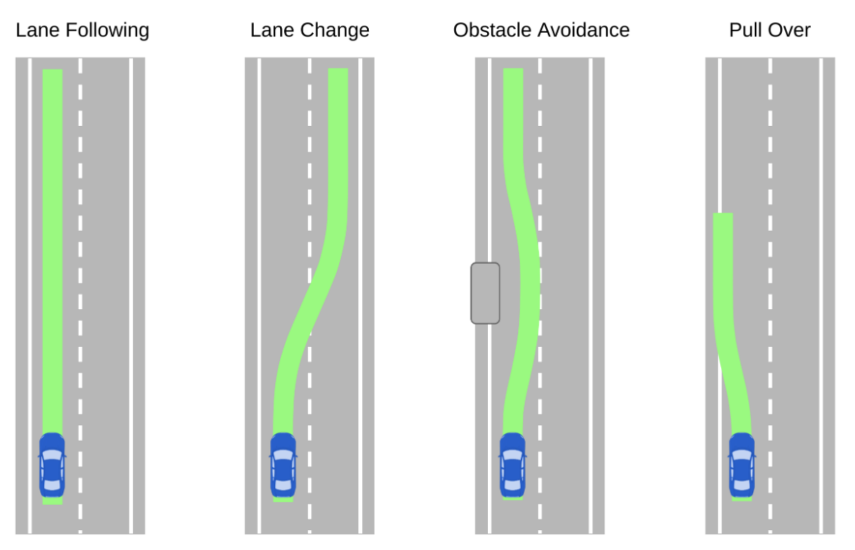


https://www.mathworks.com/help/nav/ug/highway-trajectory-planning-using-frenet.html

Frenet frame trajectory planning

Contents

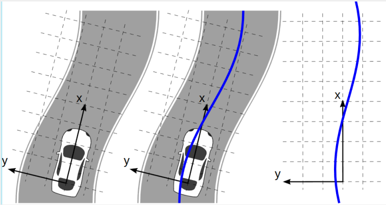
Different scenarios



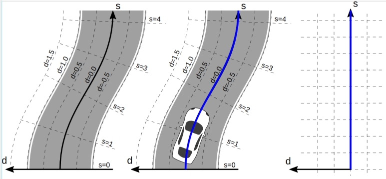
https:
//autowarefoundation.github.io/autoware.universe/main/planning/behavior_path_planner/

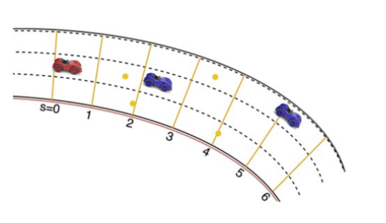
Frenet frame

World frame W



Frenent frame F



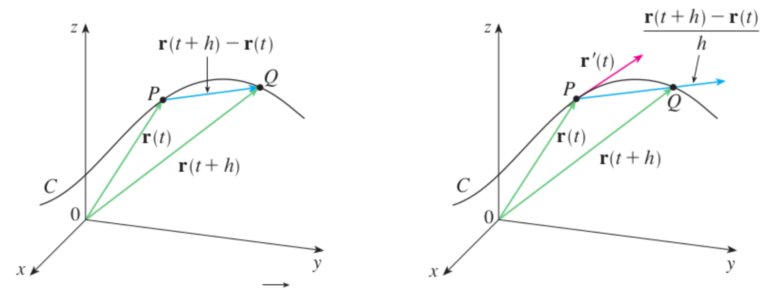


https://raw.githubusercontent.com/fjp/frenet/master/docs/images/cart_refpath.svg?sanitize=true, https://caseypen.github.io/posts/2021/01/FrenetFrame/

Curvature

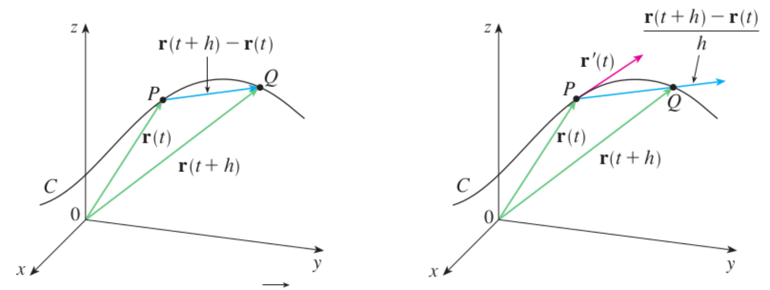
Let $\mathbf{r}(t) = x(t)i + y(t)j + z(t)k$ be a vector-valued function. That is, for every t, there is unique vector in $\mathbf{V}_3$ denoted by $\mathbf{r}(t)$ whose components are $x(t), y(t)$, and $z(t)$.
The derivative $\dot{\mathbf{r}}(t)$

$$\frac{d\mathbf{r}}{dt} = \lim_{h\to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h}$$



Multivariable Calculus: Stewart, James

The vector $\dot{\mathbf{r}}(t)$ is called tangent line to the defined curve $\mathbf{r}$ at point P, provided that $\dot{\mathbf{r}}(t)$ exists and $\dot{\mathbf{r}}(t) \neq 0$



Unit tangent vector

$$T(t) = \mathbf{t}' = \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$$

Example 01

Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\dot{\mathbf{r}}(t)$ is orthogonal to $\mathbf{r}$ for all t.

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$$\mathbf{r}(t) \cdot \mathbf{r}(t) = |\mathbf{r}(t)|^2 = c^2$$

$$0 = \frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{r}(t)] = \dot{\mathbf{r}}(t) \cdot \mathbf{r}(t) + \mathbf{r}(t) \cdot \dot{\mathbf{r}}(t) = 2\dot{\mathbf{r}}(t)\mathbf{r}(t)$$