Motion Planning for Autonomous Vehicles

Timed Elastic Band

Geesara Kulathunga

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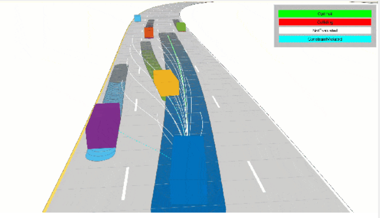

https://www.mathworks.com/help/nav/ug/highway-trajectory-planning-using-frenet.html

Timed Elastic Band

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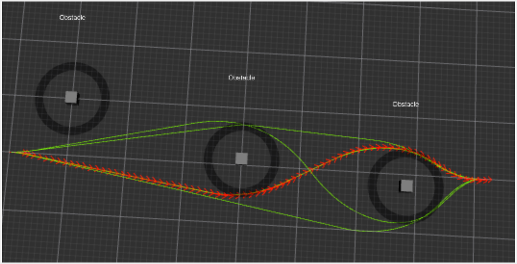
Timed Elastic Band

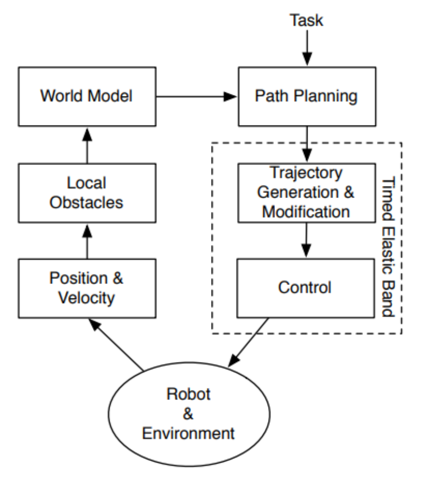
Elastic band **deforms** a path with respect to the **shortest path length** while **avoiding** contact with **obstacles**; does not take any **dynamic constraints** of the **robot**.



Timed elastic band explicitly considers **temporal aspects of the motion in terms of dynamic constraints** such as limited robot **velocities** and **accelerations**.

Rösmann, C., Feiten, W., Wösch, T., Hoffmann, F., Bertram, T. (2012, May). Trajectory modification considering dynamic constraints of autonomous robots. In ROBOTIK 2012; 7th German Conference on Robotics (pp. 1-6). VDE.

Architecture of a robot system with the **timed elastic band**



Rösmann, C., Feiten, W., Wösch, T., Hoffmann, F., Bertram, T. (2012, May). Trajectory modification considering dynamic constraints of autonomous robots. In ROBOTIK 2012; 7th German Conference on Robotics (pp. 1-6). VDE.

Elastic band is defined with a sequence of n intermediate robot poses $\mathbf{x}_i = (x_i, y_i, \beta_i)^\top \in \mathbb{R}^2 \times S^1$, where robot's position x_i, y_i and orientation β_i.

$$Q = \{\mathbf{x}_i\}_{i=0,\ldots,n} \quad N \in \mathbb{N}$$

Timed elastic band is augmented by the time intervals between ΔT two consecutive configurations

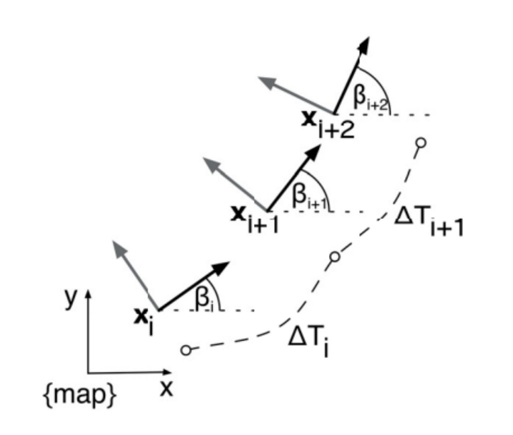
$$\tau = \{\Delta T_i\}_{i=0,..,n-1}$$

Hence, TEB is defined as

$$B := (Q, \tau)$$

TEB optimize both **configuration** and **time intervals**

$$f(B) = \Sigma_k \gamma f_k(B) \Rightarrow B* = \underset{B}{argmin}\, f(B)$$

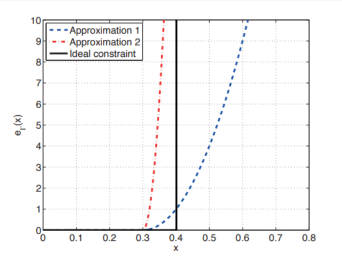


Timed Elastic Band: polynomial approximation of constraints

The TEB converts **obstacle avoidance hard** constraints into **soft constraints** by a **penalty** function

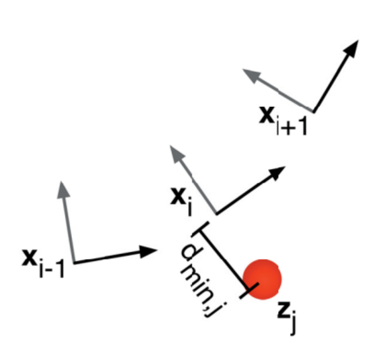
$$e_\Gamma(x, x_r, \epsilon, S, n) \simeq \begin{cases} \left(\frac{x-(x_r-\epsilon)}{S}\right)^n & \text{if} x > x_r - \epsilon \\ 0 & \text{otherwise} \end{cases}$$

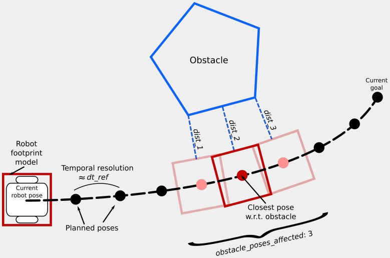
where term x_r, denoted the boundary, S is a scaling factor, n defines the order of the polynomial, and ϵ is a small translation



Timed Elastic Band : waypoints and obstacles

For **waypoints** ; $f_{path} = e_\Gamma(d_{min,j}, r_{p_{max}}, \epsilon, S, n)$
For **obstacles** ; $f_{obs} = e_\Gamma(-d_{min,j}, -r_{p_{max}}, \epsilon, S, n)$





Term $d_{min,j}$ minimum distance to a obstacle or a way point

Timed Elastic Band : Velocity and acceleration

Dynamic constraints on robot **velocity** and **acceleration** are also defined as a penalty functions

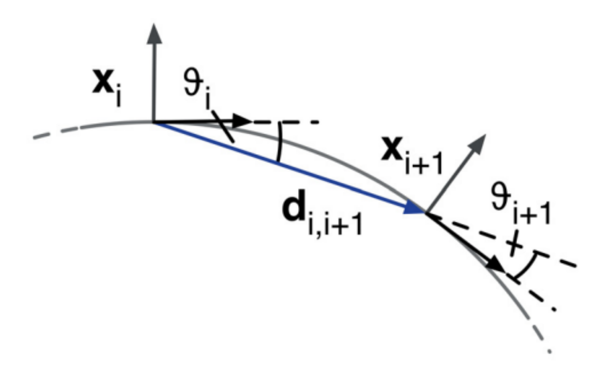
For consecutive robot poses $\mathbf{x}_i$, $\mathbf{x}_{i+1}$ and the time interval ΔT_i, then

$$v_i \simeq \frac{1}{\Delta T_i} \left\| \begin{pmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \end{pmatrix} \right\|$$

$$\omega_i \simeq \frac{\beta_{i+1} - \beta_i}{\Delta T_i}$$

$$a_i \simeq \frac{2(v_{i+1} - v_i)}{\Delta T_i + \Delta T_{i+1}}$$

Robot like a **differential drive** only possess two **local degrees of freedom**, i.e., only **execute motions** in the **direction of the robot's current heading**



Timed Elastic Band : Non-holonomic kinematics

$$\vartheta_i = \vartheta_{i+1} \Leftrightarrow \begin{pmatrix} cos(\beta_i) \\ sin(\beta_i) \\ 0 \end{pmatrix} \times \mathbf{d}_{i,i+1} = \mathbf{d}_{i,i+1} \times \begin{pmatrix} cos(\beta_{i+1}) \\ sin(\beta_{i+1}) \\ 0 \end{pmatrix}, \ \mathbf{d}_{i,i+1} = \begin{pmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \\ 0 \end{pmatrix}$$

The objective function:

$$f_k(\mathbf{x}_i, \mathbf{x}_{i+1}) = \left\| \left[\begin{pmatrix} cos(\beta_i) \\ sin(\beta_i) \\ 0 \end{pmatrix} + \begin{pmatrix} cos(\beta_{i+1}) \\ sin(\beta_{i+1}) \\ 0 \end{pmatrix} \right] \times \mathbf{d}_{i,i+1} \right\|^2$$

To obtain **the fastest path**

$$f_k = (\Sigma_{i=1}^{n} \Delta T_i)^2$$

Timed Elastic Band : Non-holonomic kinematics

Robot like a **carlike robots and Ackermann drives** only possess two **local degrees of freedom**, i.e., only **execute motions** in the **direction of the robot's current heading**. Also, **differential** drive robots are able to **rotate in place**. However, **carlike robots and Ackermann drives** can not rotate in place.
Hence, the **robot motion** is further restricted by a minimal turning radius

$$r_k(\mathbf{x}_i, \mathbf{x}_{i+1}) = \left(\left\| \frac{v_i}{\omega_i} \right\| \approx \frac{\|\mathbf{d}_{i,i+1}\|}{\|\beta_{i+1} - \beta_i\|} \right) - r_{min}$$

where r_{min} is the minimum allowable radius

Rösmann, C., Hoffmann, F., Bertram, T. (2017). Integrated online trajectory planning and optimization in distinctive topologies. Robotics and Autonomous Systems, 88, 142-153.

Timed Elastic Band: Pros and Cons

- Pros
  - Different types of constraints are satisfied, e.g., the shortest time, the shortest distance, away from obstacles, and kinematic dynamic constraints, e.g, fixed point and minimum turning radius
  - Handle static/dynamic obstacles
  - Works with various vehicle types: differential model and Ackerman model
- Cons
  - Based on **soft constraints**,not the best for real scenarios
  - A large number of adjustment parameters are required, i.e., if parameters and weights are set unreasonably or the environment is too dense, the planner fails
  - The solution is unstable because it is sensitive to the initial value, i.e., the same starting point and end point, different initial values would get different trajectories