Motion Planning for Autonomous Vehicles

Hamiltonian (Optimal control theory)

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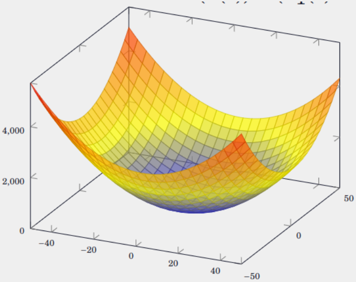

Hamiltonian (Optimal control theory)

Contents

Find the point on the line $y_1 + y_2 = 5$ that is nearest the origin.

Elimination method (direct method)

- $$\begin{aligned} \underset{y_1, y_2 \in \mathbb{R}}{\text{minimize}} \quad & f(y_1, y_2) = y_1^2 + y_2^2, \quad \text{square distance} \\ \text{subject to} \quad & y_1 + y_2 = 5 \end{aligned}$$

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- The differential

$$df(y_1, y_2) = \left(\frac{\partial f(\cdot)}{\partial y_1}\right)\Delta y_1 + \left(\frac{\partial f(\cdot)}{\partial y_2}\right)\Delta y_2 \tag{1}$$

where $f(\cdot) = f(y_1, y_2)$.

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- If $f(y_1^*, y_2^*)$ is the extreme point,

$$df(y_1^*, y_2^*) = \left(\frac{\partial f(y_1^*, y_2^*))}{\partial y_1}\right)\Delta y_1 + \left(\frac{\partial f(y_1^*, y_2^*))}{\partial y_2}\right)\Delta y_2 \tag{2}$$

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- However, in **this example**, y_1 and y_2 are **dependent**.
- Hence, considering $f(y_1, y_2)$ only function of y_2

$$
\begin{aligned}
df(y_2^*) &= \left(-10 + 4y_2^*\right)\Delta y_2 = 0 \\
&\Rightarrow y_2^* = 2.5,\ y_1^* = 2.5
\end{aligned}
\tag{3}
$$