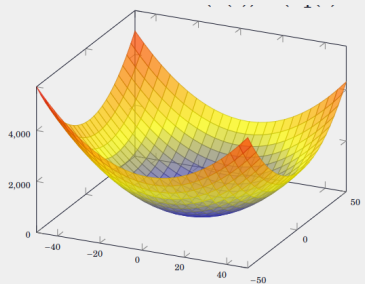


MOTION PLANNING FOR AUTONOMOUS VEHICLES

HAMILTONIAN (OPTIMAL CONTROL THEORY)

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HAMILTONIAN (OPTIMAL CONTROL THEORY)

- Constrained Minimization of functions
 - ▶ Elimination method (direct method)
 - ▶ The Lagrange multiplier method: examples, general formulation
- Constrained Minimization of functional: Point constraints, differential equation constraints
- Hamiltonian
- The necessary condition for optimal control
- Boundary conditions for optimal control: with the fixed final time and the final state specified or free
- Boundary conditions for optimal control: with the free final time and the final state specified, free, lies on the moving point $x_f = \theta(t_f)$, or lies on a moving surface $m(x(t))$

CONSTRAINED MINIMIZATION OF FUNCTIONS

Find the point on the line $y_1 + y_2 = 5$ that is nearest the origin.



ELIMINATION METHOD (DIRECT METHOD)



$$\begin{array}{ll} \underset{y_1, y_2 \in \mathbb{R}}{\text{minimize}} & f(y_1, y_2) = y_1^2 + y_2^2, \quad \text{square distance} \\ \text{subject to} & y_1 + y_2 = 5 \end{array}$$

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■ The differential

$$df(y_1, y_2) = \left(\frac{\partial f(\cdot)}{\partial y_1} \right) \Delta y_1 + \left(\frac{\partial f(\cdot)}{\partial y_2} \right) \Delta y_2 \quad (1)$$

where $f(\cdot) = f(y_1, y_2)$.

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■ If $f(y_1^*, y_2^*)$ is the extreme point,

$$df(y_1^*, y_2^*) = \left(\frac{\partial f(y_1^*, y_2^*)}{\partial y_1} \right) \Delta y_1 + \left(\frac{\partial f(y_1^*, y_2^*)}{\partial y_2} \right) \Delta y_2 \quad (2)$$

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- However, in **this example**, y_1 and y_2 are **dependent**.
- Hence, considering $f(y_1, y_2)$ only function of y_2

$$\begin{aligned}df(y_2^*) &= (-10 + 4y_2^*)\Delta y_2 = 0 \\ \Rightarrow y_2^* &= 2.5, y_1^* = 2.5\end{aligned}\tag{3}$$