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# PWM Inverter Drive for Modified Single Phase Induction Motors

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**Abstract**— This paper starts to look at the single phase induction machine which is a large consumer of domestic power supply with the goal of improving its operation. The existing capacitor-run single phase induction motor exhibits a significant level of torque pulsations during starting time and at steady state. This situation gives rise to noise and vibration in the machine. As part of efforts to mitigate these problems, an open loop two phase inverter drive strategy was developed and implemented in MATLAB/Simulink to reduce undesirable pulsations. Simulation results clearly show the superiority of motor performance under the proposed strategy when compared to motor performance under nominal power supply. Torque pulsations are eliminated at steady state with the consequence of reduced noise and heat loss to the environment. The motor is observed to develop significantly higher starting torque and its reliability is significantly increased as the strategy eliminates the need for capacitors and centrifugal switch associated with these motors.

**Keywords**— Capacitor-run, inverter, modulation, pulsations, torque, two phase

## 1 INTRODUCTION

Capacitor-run single phase induction motors (CRSPIM) find application in residential, commercial and industrial centers across the globe. They are used in washing machines, dish washers, fans, refrigerators and air-conditioners, e.t.c (Hosseini, 2016). This type of motor has a capacitor connected in series with the auxiliary winding and its purpose is to balance the currents in the main and auxiliary windings during running conditions thereby providing quiet operation and maximum efficiency. It is common to find another capacitor of higher capacitance provided in the auxiliary winding circuit to ensure high starting torque. This starting capacitor, however, is disconnected by a centrifugal switch after the motor attains about 75% of its rated speed (Baek et al, 1996; Krause et al, 2002). The problems with these motors are (i) high torque pulsations and (ii) low efficiency especially when used under non-rated conditions (Asghari and Fallah, 2012; Park, 2001; Hosseini, 2016).

A lot of work aimed at addressing these problems has been reported in literature. The aspiration in all the approaches used to address the problems of the single phase induction machine is the elimination of the unbalance in voltage and current of the two stator windings. Some works have proposed the continuous variation of the capacitance of the capacitor, connected to the auxiliary winding, throughout the entire motor speed range in order to reduce the unbalance (Muljadi, 1993; Hekmati et al, 2014). Others have developed vector controlled drives to try to balance the currents in the main and auxiliary winding in order to achieve the same objective (Azzolin et al, 2012; Caruso et al, 2012; Jang, 2007). A double unbalance in the supply currents and in the winding asymmetry was identified to be the cause of pulsations and low efficiency in the motor (Krause et al, 2013). It also complicates efforts to develop a vector-controlled drive for these motors in applications where high performance is an important requirement (Jang, 2013).

In this paper, effort is made to apply balanced currents to the windings of an existing CRSPIM using the sinusoidal pulse width modulation technique. Motor performance is examined under the proposed strategy. The performance of the motor under conditions of balanced currents and winding symmetry is also examined.

## 2 MODEL OF THE CRSPIM

The model equations of the capacitor-run single phase induction motor expressed in the stationary reference frame is given as; Krause *et al*, 2002; Carruso *et al*, 2012; Jannati *et al*, 2014):

$$V_{qs} = r_{as}i_{qs} + p\lambda_{qs} \quad (1)$$

$$V_{ds} = r_{bs}i_{ds} + p\lambda_{ds} \quad (2)$$

$$V_{ds} = -V_s + V_c \quad (3)$$

$$V'_{qr} = r'_r i'_{qr} - \omega_r \frac{N_q}{N_d} \lambda'_{dr} + p\lambda'_{qr} \quad (4)$$

$$V'_{dr} = r'_r i'_{dr} + \omega_r \frac{N_d}{N_q} \lambda'_{qr} + p\lambda'_{dr} \quad (5)$$

$$\lambda_{qs} = L_{las}i_{qs} + L_{mas}(i_{qs} + i'_{qr}) \quad (6)$$

$$\lambda'_{qr} = L'_{lr}i'_{qr} + L_{mas}(i_{qs} + i'_{qr}) \quad (7)$$

$$\lambda'_{ds} = L'_{lbs}i'_{ds} + L_{mas}(i'_{ds} + i'_{dr}) \quad (8)$$

$$\lambda'_{dr} = L'_{lr}i'_{dr} + L_{mas}(i'_{ds} + i'_{dr}) \quad (9)$$

$$T_e = \frac{p}{2} L_{mas}(i'_{dr}i_{qs} - i'_{ds}i'_{qr}) \quad (10)$$

The main or 'qs' winding has  $N_q$  equivalent turns with resistance  $r_{as}$  while the auxiliary winding or 'ds' winding has  $N_d$  equivalent turns with resistance  $r_{bs}$ . The rotor windings have resistance  $r'_r$ ;  $V_{qs}$  is the voltage applied across the main winding;  $V_{ds}$  is the voltage supplied to the auxiliary winding;  $i_{qs}$  is the current flowing in the main winding;  $i_{ds}$  is the current flowing in the auxiliary winding;  $V_{qr}$  is the voltage applied across the q-axis winding of the rotor;  $V_{dr}$  is the voltage applied across the d-axis winding of the rotor;  $i_{qr}$  is the current flowing in the q-axis winding of the rotor;  $i_{dr}$  is the current flowing in the d-axis winding of the rotor.  $\omega_r$  is the rotor speed;  $L_{mas}$  is the mutual inductance between the q – axis stator and rotor windings;  $L_{lqs}$  is the leakage inductance of the running winding.  $L_{lds}$  is the leakage inductance of the auxiliary winding referred to main winding;  $L'_{lr}$  - leakage inductance of the

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rotor winding referred to the main winding;  $T_e$  is the electromagnetic Torque.  $P$  is the number of poles on the machine;  $J$  is the total inertia of the machine and  $T_L$  is the load torque.  $p$  is the derivative operator given as  $\frac{d}{dt}$ . The application of the harmonic balance technique to the dynamic model yields the steady state model of the single phase induction machine (SPIM). The steady state voltage and torque equations (Omozusi, 1998) are provided as follows:

$$\begin{bmatrix} \bar{V}_{qs} \\ \bar{V}_{ds} \\ \bar{V}'_{qr} \\ \bar{V}'_{dr} \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b} X_s & 0 & j\frac{\omega_e}{\omega_b} X_{mas} & 0 \\ 0 & r_s + j(\frac{\omega_e}{\omega_b} X_s - \frac{\omega_b}{\omega_e} X'_c) & 0 & j\frac{\omega_e}{\omega_b} X_{mas} \\ j\frac{\omega_e}{\omega_b} X_{mas} & -\frac{\omega_r}{\omega_b} X_{mas} & r'_r + j\frac{\omega_e}{\omega_b} X'_r & -\frac{\omega_r}{\omega_b} X'_r \\ \frac{\omega_r}{\omega_b} X_{mas} & j\frac{\omega_e}{\omega_b} X_{mas} & \frac{\omega_r}{\omega_b} X'_r & r'_r + j\frac{\omega_e}{\omega_b} X'_r \end{bmatrix} \begin{bmatrix} \bar{i}_{qs} \\ \bar{i}_{ds} \\ \bar{i}'_{qr} \\ \bar{i}'_{dr} \end{bmatrix} \quad (11)$$

$$T_{av} = \frac{P}{4} L_{mas} \text{Re} [\bar{i}'_{dr} \bar{i}_{qs}^* - \bar{i}'_{ds} \bar{i}_{qr}^*] \quad (12)$$

Where  $*$  refers to complex conjugate.

$$T_{puls} = \frac{P}{4} L_{mas} \text{Re} [\bar{i}'_{dr} \bar{i}_{qs} - \bar{i}'_{ds} \bar{i}_{qr}] e^{j2\omega_e t} \quad (13)$$

Where  $T_{av}$  and  $T_{puls}$  are the average and pulsating torque components of the electromagnetic torque respectively. The magnitude of the torque pulsations is obtained from equation (13) and given as:

$$|T_{puls}| = \frac{P}{4} L_{mas} |\bar{i}'_{dr} \bar{i}_{qs} - \bar{i}'_{ds} \bar{i}_{qr}| e^{j2\omega_e t} \quad (14)$$

### 3 PROPOSED MODIFICATION TO THE EXISTING CRSPIM

It is proposed that the stator windings of the existing CRSPIM should have identical parameters. The parameters are the winding resistance, leakage inductance and the number of turns of each winding. The dynamic and steady state models, in the stationary reference frame, of the modified single phase induction motor are obtained if the following equations are imposed on the model equations (1) - (11).

$$N_{as} = N_{bs} = N_s \quad (15)$$

$$r_{as} = r'_{bs} = r_{bs} \frac{N_{as}}{N_{bs}} = r_s \quad (16)$$

$$l_{las} = l'_{lbs} = l_{lbs} \frac{N_{as}}{N_{bs}} = l_{ls} \quad (17)$$

where  $N_s$ ,  $r_s$  and  $l_{ls}$  are the number of turns, winding resistance and leakage inductance of each stator winding in the modified SPIM.

### 4 DEVELOPMENT OF A PWM OPEN LOOP INVERTER DRIVE STRATEGY

An open loop sinusoidal PWM control strategy is developed in which sinusoidal modulating signals,  $m_{ap}$  and  $m_{bp}$  are used to synthesize two independent voltages of fixed frequency and amplitude but phase-shifted by  $90^\circ$  from each other at the output of a two phase two leg inverter. The modulating signals are given as follows;

$$m_{ap} = m_a \cos(\omega_m t + \phi_m) \quad (18)$$

$$m_{bp} = m_a \cos(\omega_m t + \phi_m - \frac{\pi}{2}) \quad (19)$$

$m_{ap}$  and  $m_{bp}$  are the modulating signals for phases A and B respectively.  $\omega_m$  is the angular frequency,  $\phi_m$  is phase angle and  $m_a$  is the modulation index. The modulation index is chosen in such a way that the synthesized output voltage is equal to the rated voltage of the existing SPIM. For the purpose of analysis, the inverter is supplied with dc voltage and the output voltages are used to drive the modified SPIM as shown in the circuit arrangement of Figure 1.

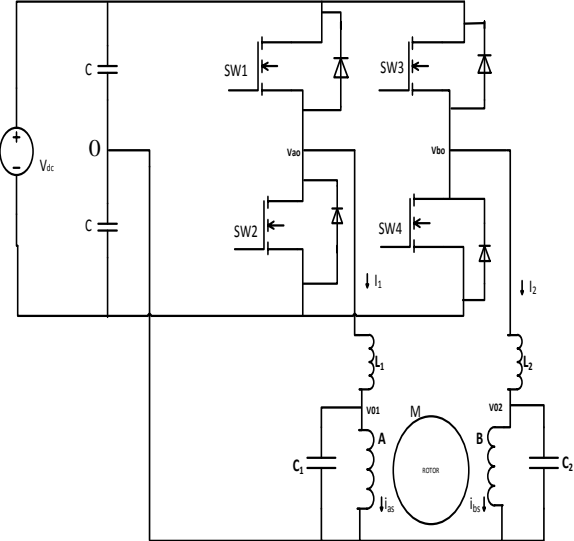


Fig 1: Two Phase Two Leg Inverter Topology

The topology of Figure 1 enables independent supply of voltage to each of the phase windings of the motor. The amplitude and frequency of the voltages applied to each winding may be controlled independently. In addition, the phase angle of the voltage applied to the auxiliary winding may be adjusted with respect to that of the main winding. This topology, especially when used in a way that ensures  $90^\circ$  phase difference between the supply currents and the right proportion of supply voltage amplitude, gives an excellent performance. (Ba-Thunya et al, 2001; Blaabjerg et al, 2004). The model equations of the inverter and the output LC filters are provided next.

### 5 MODEL EQUATIONS OF THE INVERTER AND ITS FILTERS

The model equations of the inverter and its filters can be obtained from Figure 1 and are given as:

$$V_{01} = V_{ao} - L_1 \frac{di_1}{dt} \quad (20)$$

$$C_1 \frac{dV_{01}}{dt} = I_1 - i_{as} \quad (21)$$

$$V_{02} = V_{bo} - L_2 \frac{di_2}{dt} \quad (22)$$

$$C_2 \frac{dV_{02}}{dt} = I_2 - i_{bs} \quad (23)$$

$V_{01}$  and  $V_{02}$  are the fundamental output voltages of phases A and B respectively.  $L_1$ ,  $C_1$  and  $L_2$ ,  $C_2$  are the inductance and capacitance of the output filter components of phases A and B respectively.  $V_{ao}$  and  $V_{bo}$  are the pole voltages of the inverter. The pole voltages with respect to the neutral

point 'o' are determined as follows; (Gholami, and Seifi, 2011; Holmes and Lipo, 2003)

$$V_{ao} = \frac{V_{dc}}{2}(2S_{ap} - 1) \quad (24)$$

$$V_{bo} = \frac{V_{dc}}{2}(2S_{bp} - 1) \quad (25)$$

The pole voltages are the high frequency output voltages of the inverter phases.  $V_{dc}$  is the dc input to the inverter. The inverter switches were pulse width modulated with the modulating signals of equations (18) and (19) for phases A and B respectively and a carrier triangular wave having a frequency of 1kHz. The frequency modulation ratio,  $m_f$  is as given in equation (26)

$$m_f = \frac{f_c}{f_m} \quad (26)$$

Where  $f_c$  is the frequency of the carrier triangular wave and  $f_m$  is the frequency of the modulating signal. The switching frequency of 1kHz was chosen because it helps, along with the filter components, to remove the harmonics present in the output voltage of the inverter phases while minimizing the switching losses in the inverter.  $V_{as}$  and  $V_{bs}$  being the voltages applied to the phase windings of the modified SPIM are the fundamental output voltages of the inverter phases A and B. Test motor data for two existing CRSPIMs designated as SPIM 1 and SPIM 2 were applied to the proposed model. The data are provided in Tables 1 and 2. The data given in Tables 3 and 4 are the modified data for motor 1 and motor 2 in respect of the implementation of their respective modified SPIM model. The values of the filter components and dc voltage for Motor 1 and Motor 2 are presented in Tables 5 and 6 respectively.

Table 1: Parameters of CRSPIM 1

Parameters	Values
Turn's ratio	1.18
Main winding resistance	2.02 $\Omega$
Auxiliary winding resistance	7.14 $\Omega$
Main winding leakage inductance	0.0074H
Aux. winding leakage inductance	0.0085H
Mutual Inductance	0.1772H
Rotor resistance	4.12 $\Omega$
Rotor leakage inductance	0.056H
Run capacitive Impedance	9 – j172 $\Omega$
Start capacitive Impedance	3 – j14.5 $\Omega$
Moment of Inertia	0.0146 $kgm^2$
Rated voltage	110V
Rated power	¼ hp
Rated speed	1728rpm
Rated Torque	1Nm
Rated frequency	60Hz

Table 2: Parameters of the CRSPIM 2

Parameters	Values
Turn's ratio	1.1293
Main winding resistance	4 $\Omega$
Auxiliary winding resistance	6.5 $\Omega$
Main winding leakage inductance	0.0203H
Aux. winding leakage inductance	0.0210H
Mutual Inductance	0.1954H
Rotor resistance	3.61 $\Omega$
Rotor leakage inductance	0.0304H
Run capacitance	40 $\mu$ F
Core loss resistance	910 $\Omega$
Rated voltage	220V
Rated power	1hp
Rated speed	1395rpm
Rated Torque	5.1Nm
Rated frequency	50Hz

Table 3: Parameters of the Modified SPIM 1

Parameters	Values
Turn's ratio	1
Main and Auxiliary winding resistance	2.02 $\Omega$
Main and Auxiliary winding leakage	0.056H
Rotor leakage inductance	0.0074mH
Rotor resistance	4.12 $\Omega$
Mutual Inductance	0.17724H

Table 4: Parameters of the Modified SPIM 2

Parameters	Values
Turn's ratio	1
Main and Auxiliary winding resistance	4 $\Omega$
Main and Auxiliary winding leakage	0.0203mH
Rotor leakage inductance	0.0304mH
Rotor resistance	3.61 $\Omega$
Mutual Inductance	0.1954H
Core loss resistance	910 $\Omega$

Table 5: DC Voltage and Filter Component Values for the Drive of Modified SPIM 1

Parameters	Values
$V_{dc}$	350V
$m_A$	0.8
$L_1 = L_2$	0.003mH
$C_1 = C_2$	0.18F
$f_c$	700Hz
$f_s$	1kHz

Table 6: DC Voltage and Filter Component Values for the Drive of Modified SPIM 2

Parameters	Values
$V_{dc}$	650V
$m_a$	0.8
$L_1 = L_2$	0.003mH
$C_1 = C_2$	0.18F
$f_c$	700Hz
$f_s$	1kHz

## 6 SIMULATION RESULTS & DISCUSSION

The dynamic torque performance results for the modified SPIM driven with the PWM Inverter Drive is discussed in this section. Figure 2 gives the electromagnetic torque produced in the modified SPIM 1 when driven from the two phase two leg inverter under sinusoidal PWM open loop control in the MATLAB/Simulink environment. The simulation run time was 3s and a rated load of 1Nm was applied at a time of 2s. It can be seen that the electromagnetic torque produced in the inverter-driven modified SPIM 1 exhibits zero torque pulsations in the steady state when the machine is not loaded and when it is loaded to its rated load of 1Nm.

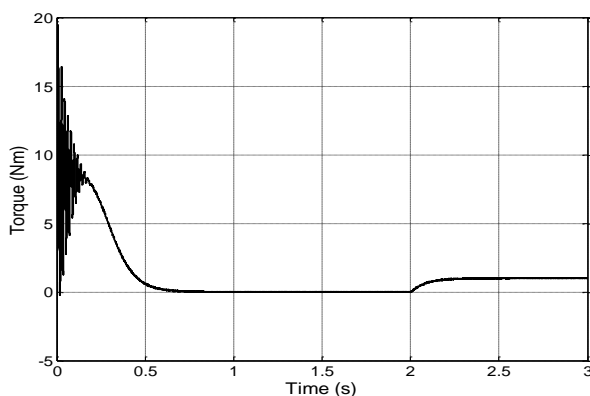


Fig 2: Dynamic Torque of the Inverter-Driven Modified SPIM 1 with Open Loop Control

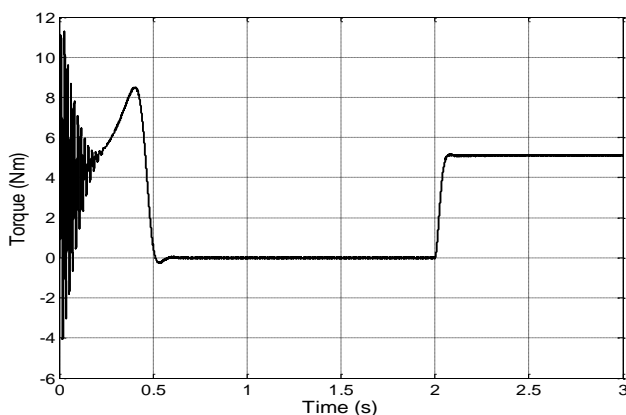


Fig 3: Dynamic Torque Production in the CRSPIM 1

The case of the existing CRSPIM 1 driven conventionally with a fixed voltage source and series-connected

capacitors is presented in Figure 3. It can be observed that the torque pulsations are significant. Moreover, the starting torque is seen to be 4Nm which is 75% lower than the starting torque in Figure 2. Figure 4 gives the electromagnetic torque produced in the modified SPIM 2 when driven from a two phase two leg inverter under the PWM open loop control in the MATLAB/Simulink environment. The simulation run time was 3s and a rated load of 5.1Nm was applied at a time of 2s. It can be seen that the electromagnetic torque produced in the modified motor driven from a two phase two leg inverter exhibits zero torque pulsations in the steady state as well.

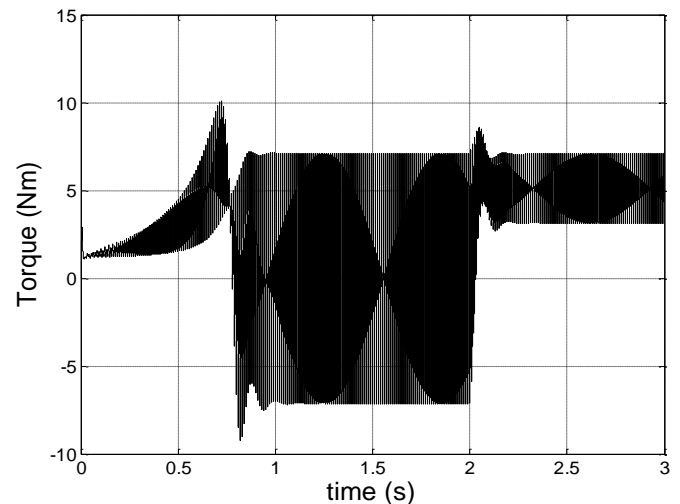


Fig 4: Dynamic Torque of the Inverter-Driven Modified SPIM 2 with Open Loop Control

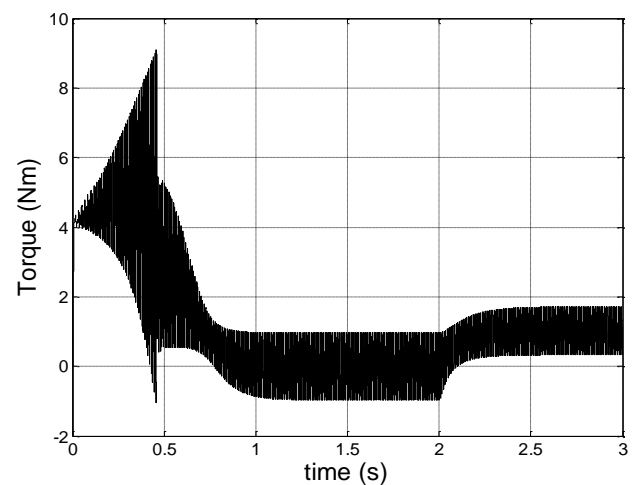


Fig 5: Dynamic Torque Production in the CRSPIM 2

Figure 5 gives the torque production in the existing CRSPIM 2 driven conventionally with a fixed voltage source and series-connected capacitors. Like CRSPIM 1, the torque pulsations are significant. The peak to peak magnitude of the torque pulsations is 3.4Nm at rated load. It can be observed that the torque pulsations are as high as 14Nm peak to peak when the motor is not loaded at steady state. In addition, the starting torque is 1.2Nm. The starting torque of the same machine with its windings modified and driven by a two phase inverter drive is approximately 6Nm as seen in Figure 4. This represents a 400% increase. This wide margin is due, in part, to the action of the inverter and also due to the lack of an



optimal starting capacitor. It is clear from Figures 2 - 5 that the modified SPIMs driven from a two leg two phase inverter under the PWM open loop control exhibit significantly improved torque performance over those of the existing CRSPIMs. Figure 6 shows the sinusoidal modulating or control signals and the triangular carrier wave for one period of the control signal. The point of intersection between the sinusoidal signals and the triangular carrier wave determine the switching instants of the semiconductor devices of the inverter.

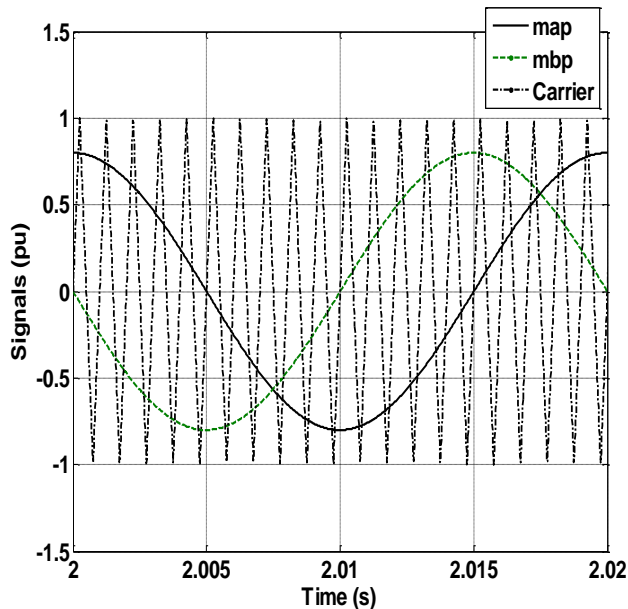


Fig 6: Sine-Triangular Wave Comparison of the Inverter Drive of the Modified SPIM 2

The result of the sine – triangular comparison is the set of pulses shown in Figure 7 which control the switching of the devices in such a manner that the inverter synthesizes an output similar to the control signal. The switching functions are as obtained from the PWM algorithm in a period of the modulating signal. It can be observed that the pulses have varying width and this variation is in accordance with the amplitude of the control signal. Figure 8 shows the stator winding voltages  $V_{as}$  and  $V_{bs}$  supplied by the two phase inverter for the period between 2s and 2.04s. It can be seen that the winding voltages are 90° apart in time and that their magnitudes are equal. In other words, the voltages are balanced. Figure 9 shows the corresponding currents in the two stator windings. It can be observed that the stator winding currents are balanced as well. The supply of balanced voltages and currents to symmetrical or balanced windings on the stator explains why the torque pulsations are eliminated in steady state. The torque profiles in steady state for the CRSPIMs with and without winding modifications are shown in Figures 10 – 13.

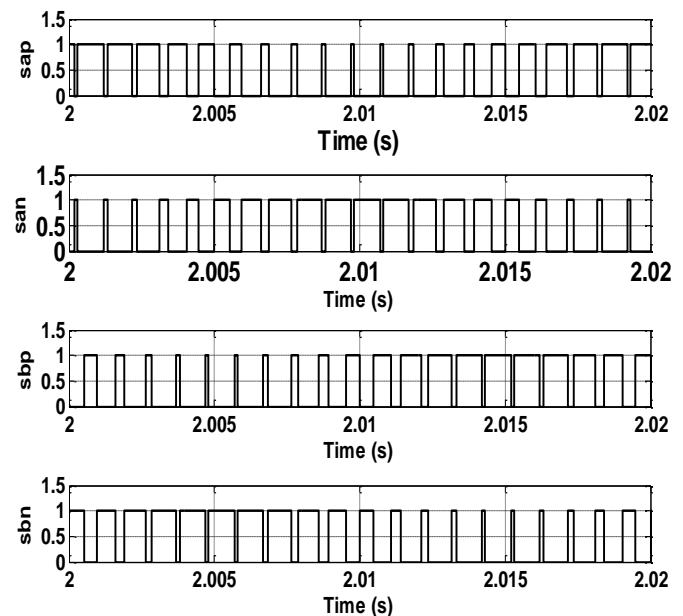


Fig 7: Switching Functions of the Inverter Drive of Modified SPIM 2

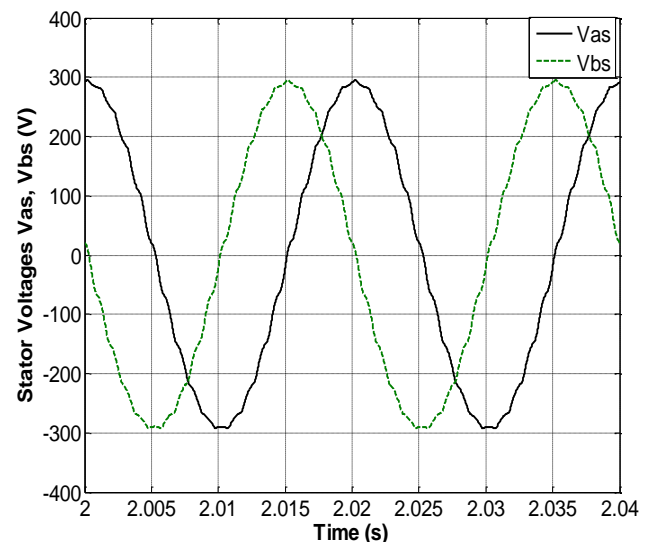


Fig 8: Stator Winding Voltages over 2 Periods

The result of the sine – triangular comparison is the set of pulses shown in Figure 7 which control the switching of the devices in such a manner that the inverter synthesizes an output similar to the control signal. The switching functions are as obtained from the PWM algorithm in a period of the modulating signal. It can be observed that the pulses have varying width and this variation is in accordance with the amplitude of the control signal. Figure 8 shows the stator winding voltages  $V_{as}$  and  $V_{bs}$  supplied by the two phase inverter for the period between 2s and 2.04s. It can be seen that the winding voltages are 90° apart in time and that their magnitudes are equal. In other words, the voltages are balanced. Figure 9 shows the corresponding currents in the two stator windings. It can be observed that the stator winding currents are balanced as well. The supply of balanced voltages and currents to symmetrical or balanced windings on the stator explains why the torque pulsations are eliminated in steady state. The torque profiles in steady state for the CRSPIMs with and without winding modifications are shown in Figures 10 – 13.

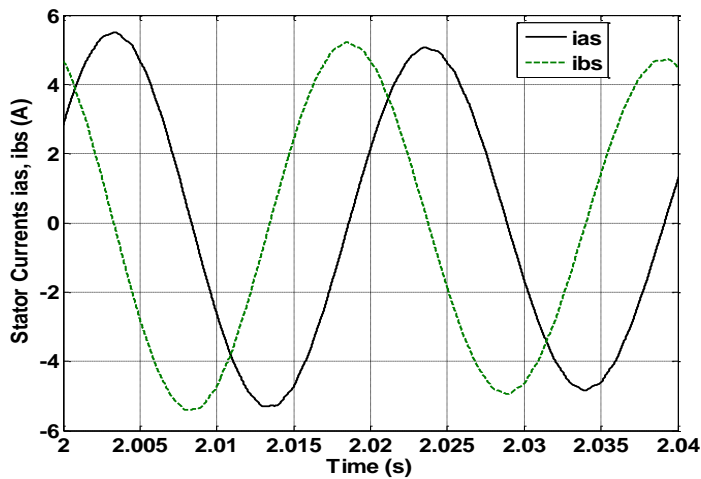


Fig 9: Stator Winding Currents in the Modified SPIM 2 for 2 Periods.

The average torque and the magnitude of torque pulsations are plotted together for Motor 1 in Figure 10 and for its modified counterpart in Figure 11. It can be observed that the average torque is higher across the motor speed range in the modified SPIM than it is in the original motor. The magnitude of torque pulsations, on the other hand, is zero across entire speed range in the modified SPIM while it exhibits significant values in the original motor across the same speed range and reaching a minimum value of 0.68Nm at rated operating point. These observations are valid for the Motor 2 and its modified counterpart as evident in Figures 12 and 13. The magnitude of torque pulsations reaches a value of 1.4Nm at rated operating point.

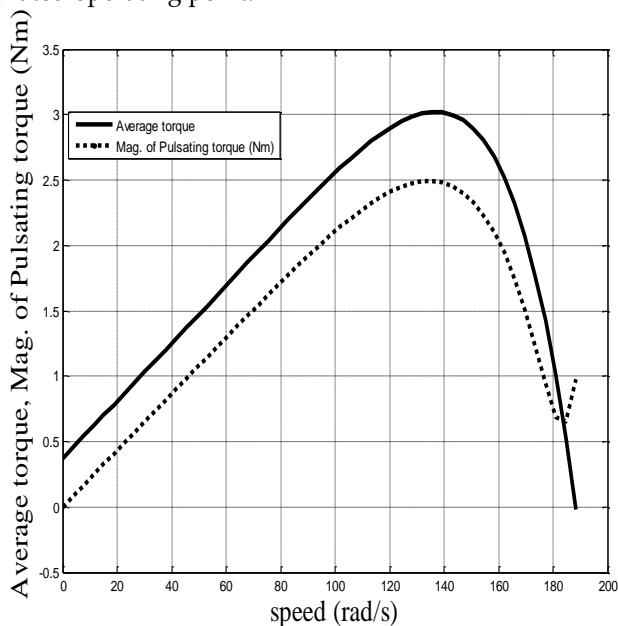


Fig 10 Average torque and Magnitude of torque pulsation of the CRSPIM 1

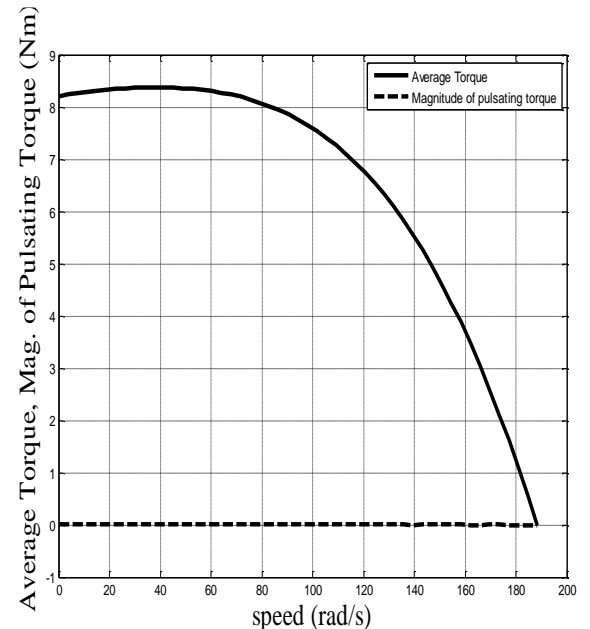


Fig 11 Average torque and Magnitude of torque pulsation of the Modified SPIM 1 driven from a simulated two phase source

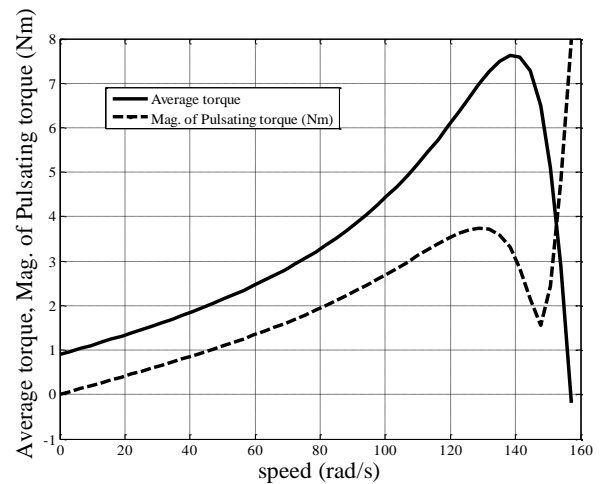


Fig 12 Average torque and Magnitude of torque pulsation of the CRSPIM 2

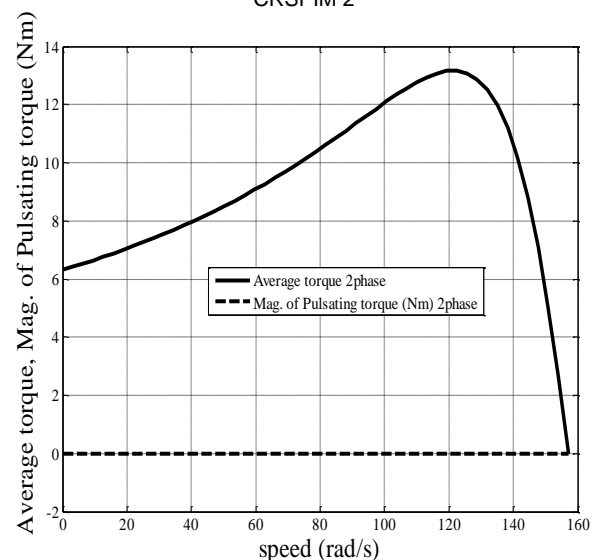


Fig 13 Average torque and Magnitude of torque pulsation of the Modified SPIM 2 driven from a simulated two phase source

## 7 CONCLUSION

The proposed PWM inverter-driven modified single phase induction machine is superior to the existing line-operated capacitor-run motor in terms of the potentials for delivering better efficiency, obtaining higher average torque and zero torque pulsations. This point has been demonstrated in the simulation results. The challenge going forward is that of designing a new auxiliary winding for the existing CRSPIM, the parameters of which must be the same as those of the main winding of the existing CRSPIM as proposed in this paper. A vector-control strategy is recommended for the purpose of generating the necessary modulating signals required for comprehensive motor control. Furthermore, a performance/cost analysis of the proposed drive scheme may be necessary in order to weigh it against the existing line-operated scheme for good decision making as to the deployment of such a scheme.

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