# A USER GUIDE FOR TRIVAC VERSION5

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#### SUMMARY

TRIVAC is a computer code intended to compute the neutron flux in a fractional or in a full core representation of a nuclear reactor. Interested readers can obtain fundamental informations about full-core calculations in Chapter 5 of Ref. 1. The multigroup and multidimensional form of the diffusion equation or simplified  $P_n$  equation is first discretized to produce a consistent matrix system. This matrix system is subsequently solved using iterative techniques (inverse or preconditioned power method with ADI preconditioning) and sparse matrix algebra techniques (triangular factorization). The actual implementation of TRIVAC allows the discretization of 1-D geometries (slab and cylindrical), 2-D geometries (Cartesian, cylindrical and hexagonal) and 3-D geometries (Cartesian and hexagonal). Many discretization techniques are available, including mesh corner or mesh centered finite difference methods, collocation techniques of various order and finite element methods based on a primal or dual functional formulation. TRIVAC also permits the equations of the generalized perturbation theory (GPT) to be solved as fixed source eigenvalue problems. Finally, several implicit numerical schemes are available for the solving of space-time neutron kinetics problems.

The execution of TRIVAC is controlled by the generalized GAN driver. [2] It is modular and can be interfaced easily with other production codes.

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## 1 INPUT DATA SPECIFICATIONS

## 1.1 Syntactic rules for input data specifications

The input data to any module is read in free format using the subroutine REDGET. The rules for specifying the input data are therefore given in this section. The users guide was written using the following conventions:

- the parameters surrounded by single square brackets '[]' denote an optional input;
- the parameters surrounded by double square brackets '[[ ]]' denote an optional input which may be repeated as many times as desired;
- the parameters in braces separated by vertical bars '{ | | }' denote a choice of input where (one and only one is mandatory);
- the parameters in **bold face** and in brackets '( )' denote an input structure;
- the parameters in italics and in brackets with an index '(data(i), i=1,n)' denote a set of n inputs;
- the words using the typewriter font are character constants keywordS used as keywords;
- the words in italics are user defined variables, they should be lower case and are of type integer (starting with i to n) and real (starting with a to h or o to z) or of type character in uppercase CHARACTER.

# 1.2 The global input structure

TRIVAC is built around the GAN generalized driver. [2] Input data must therefore follow the calling specifications given below:

Table 1: Structure (TRIVAC)

```
[LINKED_LIST [[ NAME1 ]] ; ]
[XSM_FILE [[ NAME2 ]] ; ]
[SEQ_BINARY [[ NAME3 ]] ; ]
[SEQ_ASCII [[ NAME4 ]] ; ]
[MODULE [[ NAME5 ]] ; ]
[[ (specif) ]]
END: ;
```

```
where
```

```
NAME1 Character*12 name of a LCM object.

NAME2 Character*12 name of an XSM file.

NAME3 Character*12 name of a sequential binary file.

NAME4 Character*12 name of a sequential ASCII file.

NAME5 Character*12 name of a module.

(specif) Input specifications for a single module. Specifications for TRIVAC modules will be given in the following sections.
```

The input data always begin with the declaration of each LCM object, XSM file, sequential (binary or ASCII) file that will be required by the following modules. This is followed by the declaration of the modules actually used in the input data deck. The following data describe a sequence of module calls, in the format of the GAN generalized driver. As indicated in Fig. 1, the modules communicate with each other throught LCM objects or XSM files whose specifications are given in section 2. The TRIVAC user generally have the choice to declare its data structures as LINKED\_LIST to reduce CPU time resources or as XSM\_FILE to reduce CPU memory resources.

The input data always end with a call to the END: module.

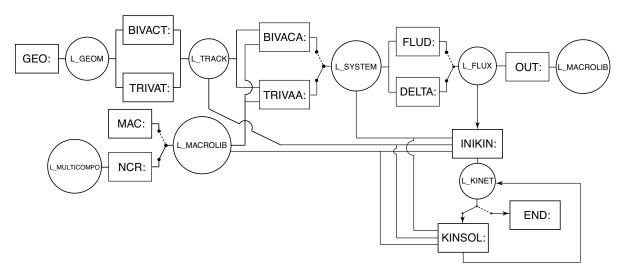


Figure 1: The TRIVAC modular approach.

#### 1.3 The GEO: module

The GEO: module is used to create or modify a geometry. The geometry definition module in TRIVAC permits all the characteristics (coordinates, material mixture type indices and boundary conditions) of a simple or complex geometry to be specified. The method used to specify the geometry is independent of the discretization module to be used subsequently. Each geometry is represented by a name (character\*12) and is saved in a LCM object or an XSM file under its given name. It is always possible to modify a given existing geometry or copy it into a neighbouring LCM object under a new name. The calling specifications are:

Table 2: Structure (GEO:)

```
{ GEOM1 := GEO: :: (geo_data1) | GEOM1 := GEO: { GEOM1 | GEOM2 } :: (geo_data2) }
```

where

GEOM1 character\*12 name of the LCM object (type L\_GEOM) that will contain the geometry.

GEOM2 character\*12 name of a LCM object (type L\_GEOM) containing the existing geometry. The type and all the characteristics of GEOM2 will be copied onto GEOM1.

```
(geo_data1) structure describing the characteristics of a new geometry (see Sect. 1.3.1).
(geo_data2) structure describing the change to the characteristics of an existing geometry (see Sect. 1.3.1).
```

### 1.3.1 Data input for module GEO:

where

lx

Structures (geo\_data1) and (geo\_data2) serve to define the principle components of a geometry (dimensions, materials, boundary conditions):

Table 3: Structure (geo\_data1)

Table 4: Structure (geo\_data2)

```
[ EDIT iprint ]
(descBC)
(descMC)
(descPOS)
;
```

number of subdivisions along the X axis (before mesh-splitting).

```
HOMOGE
             infinite homogeneous geometry.
             one dimensional plane geometry (infinite slabs).
CAR1D
             cylindrical geometry (infinite tubes or cylinders).
TUBE
SPHERE
             spherical geometry (concentric spheres).
CAR2D
             two-dimensional cartesian geometry.
             polar geometry (R-Z).
TUBEZ
CAR3D
             three-dimensional cartesian geometry.
HEX
              two-dimensional hexagonal geometry.
HEXZ
             three-dimensional hexagonal geometry.
```

```
lv
             number of subdivisions along the Y axis (before mesh-splitting).
              number of subdivisions along the Z axis (before mesh-splitting).
12
lr
              number of cylinders or spherical shells (before mesh-splitting).
lh
             number of hexagons in an axial plane (including the virtual hexagons).
EDIT
              keyword used to set iprint.
             index used to control the printing in module GEO:. =0 for no print; =1 for minimum
iprint
             printing (default value); =2 for printing the geometry state vector.
(descBC)
             structure allowing the boundary conditions surrounding the geometry to be treated.
(descMC)
             structure allowing material mixtures to be associated with a geometry.
(descPOS) structure allowing the coordinates of a geometry to be described.
```

The inputs corresponding to the (descBC) structure are the following:

Table 5: Structure (descBC)

```
where
X-
              negative X side.
Y-
              negative Y side.
Z-
              negative Z side.
              positive X side.
χ+
              positive Y side.
Y+
Z+
              positive Z side.
              side surrounding cylinders or spheres.
R+
HBC
              side surrounding a hexagonal geometry.
VOID
              the side under consideration has a zero incoming current boundary condition.
```

REFL the side under consideration has a reflective boundary condition.

DIAG the side under consideration is external to a diagonal axis of symmetry.

TRAN the side under consideration is connected to the opposite side of the domain. This option

permits a translation condition to be treated.

SYME the side under consideration is next to an axial axis of symmetry. (symmetric with respect

to the central axis of the last row of volumes). The SYME condition can also be used in

hexagonal geometry, but only with \$30 and \$A60 symmetries.

ALBE the side under consideration has an arbitrary albedo to be specified.

albedo geometrical albedo corresponding to the boundary condition ALBE (albedo  $\geq 0.0$ ).

icode index of a physical albedo corresponding to the boundary condition ALBE. The numerical

values of the physical albedo are supplied by the module MAC:.

ZERO the side under consideration has a zero flux boundary condition.

CYLI the side under consideration has a zero incoming current boundary condition with a circular correction applied on the Cartesian boundary. This option is only available in the X-Y

plane for CAR2D and CAR3D geometries defined for TRIVAC full-core calculations.

ACYL the side under consideration has an arbitrary albedo with a circular correction applied on

the Cartesian boundary. This option is only available in the X-Y plane for CAR2D and

CAR3D geometries defined for TRIVAC full-core calculations.

hexagonal symmetry of one twelfth of an assembly (see Fig. 2).

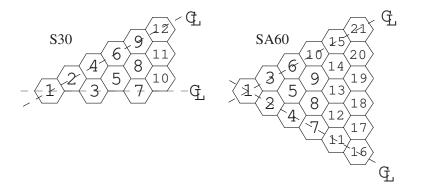


Figure 2: Hexagonal geometries of type S30 and SA60

SA60	hexagona	l symmetry c	of one	sixth	of ar	ı assemb	ly of	i type A	. (se	e Fig	g. $^{2}$	).
------	----------	--------------	--------	-------	-------	----------	-------	----------	-------	-------	-----------	----

SB60 hexagonal symmetry of one sixth of an assembly of type B (see Fig. 3).

because hexagonal symmetry of one quarter of an assembly (see Fig. 3).

R120 hexagonal symmetry of one third of an assembly (rotational symmetry) (see Fig. 4).

R180 rotational symmetry of a half assembly (see Fig. 4).

SA180 hexagonal symmetry of half a type A assembly (see Fig. 5).

SB180 hexagonal symmetry of half a type B assembly (see Fig. 6).

COMPLETE complete hexagonal assembly (see Fig. 7).

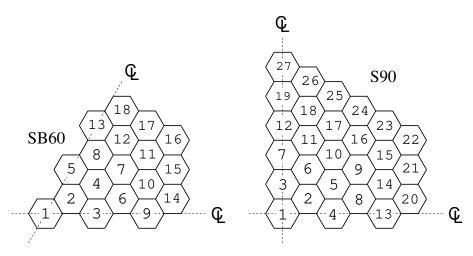


Figure 3: Hexagonal geometries of type SB60 and S90  $\,$ 

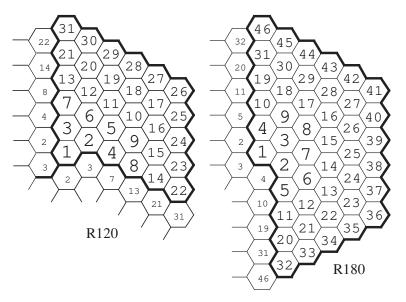


Figure 4: Hexagonal geometries of type R120 and R180  $\,$ 

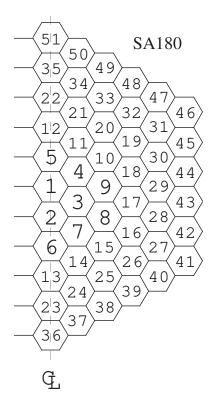


Figure 5: Hexagonal geometry of type SA180

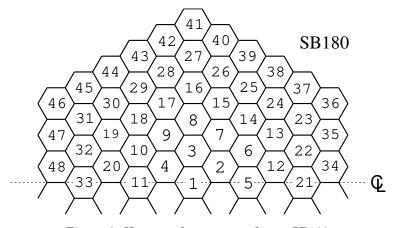


Figure 6: Hexagonal geometry of type SB180

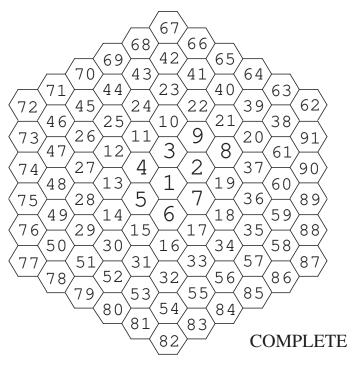


Figure 7: Hexagonal geometry of type COMPLETE

RADS This keyword is used to specify the cylindrical correction applied in the X-Y plane for CAR2D and CAR3D geometries. [13]

This keyword allows the angle (see Fig. 8) of the cylindrical notch to be set. By default, no notch is present.

nrads Number of different corrections along the cylinder main axis (i.e. the Z axis).

xrad(ir) Coordinate of the Z axis from which the correction is applied.

rrad(ir) Radius of the real cylindrical boundary.

ANG

ang(ir) Angle of the cylindrical notch. This data is given if and only if the keyword ANG is present.  $ang(ir) = \frac{\pi}{2}$  by default (i.e. the correction is applied at every angle).

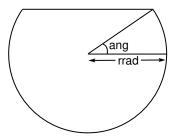


Figure 8: Cylindrical correction in Cartesian geometry

The only combinations of diagonal symmetry permitted are: X+DIAG Y-DIAG and X-DIAG Y+DIAG. In these cases the geometry must be a square. The only combinations of translational symmetry permitted are: X-TRAN X+TRAN, Y-TRAN Y+TRAN and Z-TRAN Z+TRAN.

The input corresponding to the (descMC) structure are the following:

Table 6: Structure (descMC)

where

MIX

keyword to attribute an material mixture number to each volume inside the axes of symmetry. When a volume is located inside the axes of symmetry but outside the calculation region it must be declared 'virtual' (for example, the corners of a nuclear reactor). The material mixture number should be specified for each volume before mesh-splitting.

imix

type of material mixture associated with a region. It is important that  $imix \le nmixt$  where nmixt is defined in the module. If imix=0, the corresponding volume is replaced by a VOID boundary condition. In this case the volume is considered to be virtual and the flux is not calculated. In the case of a diagonal symmetry, the type indicator must not be specified for the volumes outside the axis of symmetry. These values must be specified in the following order: from X- to X+, from Y- to Y+, from Z- to Z+ and finally radially from the inside out.

PLANE

keyword to attribute mixture numbers to each volume inside a single 2D plane. This option is valid only for 3D geometries, Cartesian or hexagonal.

iplan

plane number for which material mixture are input.

SAME

keyword to attribute the same material mixture numbers of the *iplan1* plane to the *iplan* plane. In hexagonal geometry, it can indicate that the mixture numbers of the current crown of the *iplan1*th plane will be identical to those of the same crown of the *iplan1*th plane.

iplan1

plane number used as reference to input the current plane or crown(s).

lр

number of volumes in a plane. In Cartesian geometry, lp = lx \* ly and in hexagonal geometry, lp = lh.

CROWN

keyword to attribute mixture numbers to each hexagon of a single crown. This option is only valid for COMPLETE hexagonal geometry definition. Each use of the keyword CROWN increases the crown number by 1. So it is not required to give its number, but crowns must be defined from the center to the peripherical regions of a plane.

lc

number of hexagons in the current crown. For the *i*th crown of a compelete hexagonal plane, lc = (i-1) \* 6. The first crown is composed of only one hexagon.

ALL

keyword to specify that the lc material mixture number of the current crown have the same value jmix.

UPTO

keyword to attribute material mixture numbers of the current crown up to the ic one.

ic

number of the last crown in UPTO option. Its value must be greater than equal to the current crown number.

Here we will assume that lreg is the exact number of cells or elementary cases to be considered. For example, if we had used the DIAG option with a geometry of type CAR3D (lx=ly), we would have: lreg=(lx+1)\*ly\*lz/2.

The following dimensional constraints must also be respected:

- nmerge=number of merged cells (with  $nmerge \ge lreg.$ ),
- ngen=number of generation cells (with  $ngen \ge nmerge$ .),

The inputs corresponding to the (descPOS) structure are the following:

Table 7: Structure (descPOS)

where keyword for the mesh of the geometry along the X axis. **MESHX MESHY** keyword for the mesh of the geometry along the Y axis. MESHZ keyword for the mesh of the geometry along the Z axis. RADIUS keyword for the mesh of the geometry in the radial direction. SIDE keyword for the length of a side of a hexagon. abscissa, corresponding to the limits of the regions making up the geometry. These values XXXmust be given in order, from X- to X+. If the geometry presents a diagonal symmetry this data will also be used for the ordinate. ordinate, corresponding to the limits of the regions making up the geometry. These values ууу must be given in order, from Y- to Y+. height, corresponding to the limits of the regions making up the geometry. These values ZZZmust be given in order, from Z- to Z+. rrrRadii in the cases of cylindrical (TUBE or TUBEZ), spherical (SPHERE). It is important to note that we must have rrr(1)=0.0. sidhex length of a side of a hexagon. keyword for mesh splitting of the geometry along the X axis. SPLITX SPLITY keyword for mesh splitting of the geometry along the Y axis. SPLITZ keyword for mesh splitting of the geometry along the Z axis.

SPLITR keyword for mesh splitting of the geometry in the radial direction.

ispltx number of sub-volumes that will be defined for each row of the volume along the X-axis. If the geometry presents a diagonal symmetry this input will also be used for the splitting along the Y-axis. By default, ispltx=1.

isplty number of sub-volumes that will be defined for each row of the volume along the Y-axis. If the geometry presents a diagonal symmetry this input will also be used for the splitting along the X-axis. By default, isplty=1.

ispltz number of sub-volumes that will be defined for each row of the volume along the Z-axis. By default, ispltz(i)=1.

ispltr the value of ispltr gives the number of sub-volumes that will be defined for each tube or each spherical shell. A negative value permits a splitting into equal sub-volumes; a positive value permits a splitting into equal sub-radius spacings. By default, ispltr=1.

SPLITH keyword to specify that a triangular mesh splitting of the hexagonal geometry is to be performed – for HEX and HEXZ type geometries.

isplth value of the triangular mesh splitting. The number of triangles per hexagon is given by  $6 \times isplth^2$ . isplth = 0 is used for full hexagon discretization.

SPLITL keyword to specify that a lozenge mesh splitting of the hexagonal geometry is to be performed – for HEX and HEXZ type geometries.

ispltl value of the lozenge splitting. The number of lozenges per hexagon is given by  $3 \times ispltl^2$ .

The user of the options described above should take care not to exceed the limits imposed by the amount of dynamically allocated memory available. For a pure geometry, let us define the variables lxp, lyp, lzp and lrp as:

$$\begin{array}{rcl} lxp & = & \displaystyle\sum_{i=1}^{lx} ispltx(i) \\ \\ lyp & = & \displaystyle\sum_{i=1}^{ly} isplty(i) \\ \\ lzp & = & \displaystyle\sum_{i=1}^{lz} ispltz(i) \\ \\ lrp & = & \displaystyle\sum_{i=1}^{lr} ispltr(i) \end{array}$$

thus, the limits that must be respected are the following:

- $lxp \ge maxpts$  for a CAR1D geometry.
- *lh*≥*maxpts* for a HEX geometry.
- $lrp \ge maxpts$  for the TUBE and SPHERE geometries.
- lxp \* lyp > maxpts for the CAR2D geometry without diagonal symmetry.
- $lxp*(lyp+1)/2 \ge maxpts$  for the CAR2D geometry with diagonal symmetry.
- $lrp * lzp \ge maxpts$  for the TUBEZ geometry.
- $lxp*lyp*lzp \ge maxpts$  for the CAR3D geometry without diagonal symmetry.

- $lxp*(lyp+1)*lzp/2 \ge maxpts$  for the CAR3D geometry with diagonal symmetry.
- $lh*lzp \ge maxpts$  for the HEXZ geometry.

## 1.3.2 Examples of geometries

We will now give a few examples which will permit users to better understand the procedure used to define the geometries in TRIVAC.

1. Slab geometry (see Fig. 9):

```
GEOMETRY1 := GEO: :: CAR1D 6
X- VOID X+ ALBE 1.2
MESHX 0.0 0.1 0.3 0.5 0.6 0.8 1.0
SPLITX 2 2 2 1 2 1
MIX 1 2 3 4 5 6
;
```



Figure 9: Slab geometry with mesh-splitting

2. Two-dimensional hexagonal geometry (see Fig. 10):

```
GEOMETRY4 := GEO: :: HEX 12

HBC S30 ALBE 1.6

SIDE 1.3

MIX 1 1 1 2 2 2 3 3 3 4 5 6

.
```

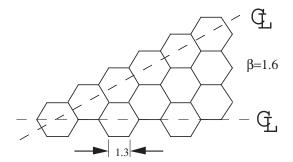


Figure 10: Two-dimensional hexagonal geometry

#### 1.4 The MAC: module

In TRIVAC the macroscopic cross sections and diffusion coefficients are read from the input data file using REDLEC. The general format of the data for the MAC: module in TRIVAC is the following:

Table 8: Structure (MAC:)

```
MACR1 := MAC: [ \{ MACR1 \mid MACR2 \} ] :: (mac_data)
```

where

MACR1

character\*12 name of the LCM object (type L\_MACROLIB) containing the new Macrolib produced by the module. A Macrolib contains macroscopic cross sections and diffusion coefficients. If *MACR1* appears on both LHS and RHS, it is updated; otherwise, it is created. If *MACR1* is created, all macroscopic cross sections and diffusion coefficients are first initialized to zero.

MACR2

character\*12 name of the LCM object (type L\_MACROLIB) containing a read-only Macrolib. The information existing in MACR2 is copied into MACR1, but MACR2 is not modified.

(mac\_data) structure containing the data to module MAC: (see Sect. 1.4.1).

#### 1.4.1 Data input for module MAC:

Table 9: Structure (mac\_data)

```
[ EDIT iprint ]
[ NGRO ngroup ]
[ NIFI nifiss ]
[ DELP ndel ]
[ ANIS naniso ]
[ NMIX nmixt ]
[ DELP ndg ]
[ ANIS naniso ]
[ ANIS naniso ]
[ ALBP nalbp ((albedp(ig,ia),ig=1,ngroup),ia=1,nalbp) ]
[ READ INPUT { [[ (macxs) ]] | OLD (triv2) | DOLD (trip2) } ]
[[ STEP istep READ INPUT [[ (macxs) ]] ]
```

where

EDIT keyword used to set iprint.

iprint

index used to control the printing in module MAC:. =0 for no print. The macroscopic cross sections will be printed if the parameter iprint is greater than or equal to 2. The transfer cross sections will be printed if this parameter is greater than or equal to 3.

NGRO keyword used to define the number of energy groups. This data is given if and only if MACR1 is created.

ngroup the number of energy groups used for the calculations in TRIVAC.

NIFI keyword used to specify the maximum number of fissile spectrum associated with each mixture. Each fission spectrum generally represents a fissile isotope. This information is required only if *MACLIB* is created and the cross sections are taken directly from the input data stream.

nifiss the maximum number of fissile isotopes per mixture. The default value is nifiss=1.

DELP keyword used to specify the number of delayed neutron groups.

ndel the number of delayed neutron groups. The default value is ndel=0.

ANIS keyword used to specify the maximum level of anisotropy permitted in the scattering cross sections. This information is required only if *MACLIB* is created and the cross sections are taken directly from the input data stream.

naniso number of Legendre orders for the representation of the scattering cross sections. The default value is naniso=1 corresponding to the use of isotropic scattering cross sections.

NMIX keyword used to define the number of material mixtures. This data is given if and only if *MACR1* is created.

nmixt the maximum number of material mixtures (a material mixture is characterized by a distinct set of macroscopic cross sections).

DELP keyword used to set ndg. This data is used only if the fission spectrum  $\vec{\chi}_p$  is different from the delayed neutron spectrum  $\vec{\chi}_i$  for each precursor group i.

ndg number of delayed neutron groups.

ANIS keyword used to specify the maximum level of anisotropy permitted in the diffusion cross sections. This data is given only if *MACR1* is created.

naniso the maximum level of anisotropy. The default value is naniso=1.

ALBP keyword used for the input of the physical albedos.

nalbp the number of physical albedos per energy group.

albedp multigroup physical albedo array (real numbers).

STEP keyword used to create a perturbation directory.

istep the index of the perturbation directory.

READ keyword used to specify input of the cross section information from default input by REDLEC.

(macxs) structure describing the format used for reading the mixture cross sections and diffusion coefficients (or perturbation values of the cross sections and diffusion coefficients) from the input data file.

OLD keyword used to specify input of the cross section information from default input by REDLEC in the TRIVAC-2 format. The nuclear data will be translated into TRIVAC format and printed on the listing.

(triv2) structure describing the format used for reading the mixture cross sections and diffusion coefficients from the input data file in TRIVAC-2 format.

DOLD keyword used to specify perturbed input of the cross section information from default input by REDLEC in the TRIVAC-2 format. The perturbed nuclear data will be translated into TRIVAC format and printed on the listing.

structure describing the format used for reading the mixture values of the perturbed cross (trip2) sections and diffusion coefficients from the input data file in TRIVAC-2 format.

# 1.4.2 Description of the nuclear data

Table 10: Structure (macxs)

```
MIX matnum
   \{ \text{ NTOTO} \mid \text{TOTAL } \} (xssigt(jg), jg=1,ngroup) ]
   NTOT1 (xssig1(jg), jg=1,ngroup)
   TRANC (xsstra(jg), jg=1, ngroup)
   NUSIGF ((xssigf(jf,jg), jg=1,ngroup), jf=1,nifiss)]
   CHI ((xschi(jf,jg), jg=1,ngroup), jf=1,nifiss)]
   FIXE (xsfixe(jg), jg=1, ngroup)
   DIFF (diff(jg), jg=1, ngroup)
   DIFFX (xdiffx(jg), jg=1, ngroup)
   DIFFY (xdiffy(jg), jg=1, ngroup)
   DIFFZ (xdiffz(jg), jg=1, ngroup)
   NUSIGD (((xssigd(jf,idel,jg), jg=1,ngroup), idel=1,ndel), jf=1,nifiss)
   CHDL (((xschid(jf,idel,jg), jg=1,ngroup), idel=1,ndel), jf=1,nifiss)]
  OVERV (overv(jg), jg=1,ngroup) ]
   H-FACTOR (xhfact(jg), jg=1, ngroup)
   SCAT ((nbscat(jl,jg), ilastg(jl,jg), (scat(jl,jg,ig), ig=1,nbscat(jl,jg)), jg=1,ngroup), jl=1,naniso)
```

where

keyword to specify that the macroscopic cross sections associated with a new mixture are MIX

to be read.

matnum identifier for the next mixture to be read. The maximum value permitted for this identifier is nmixt. When matnum is absent, the mixtures are numbered consecutively starting with

1 or with the last mixture number read either on the GOXS or the input stream.

NTOTO keyword to specify that the total macroscopic cross sections for this mixture follows.

TOTAL alias keyword for NTOTO.

array representing the multigroup total macroscopic cross section ( $\Sigma^g$  in cm<sup>-1</sup>) associated xssigt

with this mixture.

NTOT1 keyword to specify that the  $P_1$ -weighted total macroscopic cross sections for this mixture

follows.

array representing the multigroup  $P_1$ -weighted total macroscopic cross section ( $\Sigma_1^g$  in xssig1

cm<sup>-1</sup>) associated with this mixture.

TRANC keyword to specify that the transport correction macroscopic cross sections for this mixture

follows.

16 IGE-369array representing the multigroup transport correction macroscopic cross section ( $\Sigma_{\mathrm{tc}}^{g}$  in xsstra cm<sup>-1</sup>) associated with this mixture. NUSIGF keyword to specify that the macroscopic fission cross section multiplied by the average number of neutrons per fission for this mixture follows. xssigf array representing the multigroup macroscopic fission cross section multiplied by the average number of neutrons per fission  $(\nu \Sigma_f^g$  in cm<sup>-1</sup>) for all the fissile isotopes associated with this mixture. CHI keyword to specify that the fission spectrum for this mixture follows. By default, if CHI is not provided, all fission neutrons are emitted in group index 1 (fast group). array representing the multigroup fission spectrum  $(\chi^g)$  for all the fissile isotopes associated xschi with this mixture. FIXE keyword to specify that the fixed neutron source density for this mixture follows. array representing the multigroup fixed neutron source density for this mixture  $(S^g)$  in xsfixe  $s^{-1}cm^{-3}$ ). DIFF keyword to specify that the isotropic diffusion coefficient for this mixture follows.

diff array representing the multigroup isotropic diffusion coefficient for this mixture  $(D^g \text{ in } cm)$ .

DIFFX keyword for input of the X-directed diffusion coefficient.

xdiffx array representing the multigroup X-directed diffusion coefficient  $(D_x^g \text{ in cm})$  for the mixture matnum.

DIFFY keyword for input of the Y-directed diffusion coefficient.

xdiffy array representing the multigroup Y-directed diffusion coefficient  $(D_y^g \text{ in cm})$  for the mixture matnum.

DIFFZ keyword for input of the Z-directed diffusion coefficient.

xdiffz array representing the multigroup Z-directed diffusion coefficient ( $D_z^g$  in cm) for the mixture matnum.

NUSIGD keyword to specify that the delayed macroscopic fission cross section multiplied by the average number of neutrons per fission for this mixture follows.

xssigd array representing the delayed multigroup macroscopic fission cross section multiplied by the average number of neutrons per fission  $(\nu \Sigma_f^{g,idel})$  in cm<sup>-1</sup> for all the fissile isotopes associated with this mixture.

CHDL keyword to specify that the delayed fission spectrum for this mixture follows.

xschid array representing the delayed multigroup fission spectrum  $(\chi^{g,idel})$  for all the fissile isotopes associated with this mixture.

OVERV keyword for input of the multigroup average of the inverse neutron velocity.

overv array representing the multigroup average of the inverse neutron velocity  $(<1/v>_m^g)$  for the mixture matnum.

H-FACTOR keyword to specify that the power factor for this mixture follows.

hfact array representing the multigroup power factor for this mixture  $(H^g \text{ in } MeV \text{ } cm^{-1}).$ 

SCAT keyword to specify that the macroscopic scattering cross section matrix for this mixture follows.

nbscat array representing the number of secondary groups ig with non vanishing macroscopic scattering cross section towards the primary group jg considered for each anisotropy level associated with this mixture.

ilastg array representing the group index of the most thermal group with non-vanishing macroscopic scattering cross section towards the primary group jg considered for each anisotropy level associated with this mixture.

array representing the multigroup macroscopic scattering cross section  $(\Sigma_{sl}^{ig \to jg})$  in cm<sup>-1</sup> from the secondary group ig towards the primary group jg considered for each anisotropy level associated with this mixture. The elements are ordered using decreasing secondary group number ig, from ilastg to (ilastg-nbscat+1), and an increasing primary group number jg.

For example, the two group isotropic and linearly anisotropic scattering cross sections (ngroup=2, naniso=2) given by:

$$\begin{array}{ccccccc} L & \Sigma_{s,l}^{1\to 1} & \Sigma_{s,l}^{1\to 2} & \Sigma_{s,l}^{2\to 1} & \Sigma_{s,l}^{2\to 2} \\ 0 & 0.50 \ \mathrm{cm}^{-1} & 0.20 \ \mathrm{cm}^{-1} & 0.03 \ \mathrm{cm}^{-1} & 0.40 \ \mathrm{cm}^{-1} \\ 1 & 0.05 \ \mathrm{cm}^{-1} & 0.00 \ \mathrm{cm}^{-1} & 0.00 \ \mathrm{cm}^{-1} & 0.04 \ \mathrm{cm}^{-1} \end{array}$$

must be entered as:

#### 1.5 The BIVACT: module

The BIVACT: module is used to perform a BIVAC-type TRACKING on a 1D/2D geometry. [3,4,15] The geometry is analyzed and a LCM object with signature L\_BIVAC is created with the following information:

- Diagonal and hexagonal symmetries are unfolded and the mesh-splitting operations are performed.
   Volumes, material mixture and averaged flux recovery indices are computed on the resulting geometry.
- A finite element discretization is performed and the corresponding numbering is saved.
- The unit finite element matrices (mass, stiffness, etc.) are recovered.

The calling specifications are:

Table 11: Structure (BIVACT:)

```
TRACK := BIVACT: [ TRACK ] GEOM :: (bivact_data)
```

where

TRACK character\*12 name of the LCM object (type L\_BIVAC) containing the TRACKING information. If TRACK appears on the RHS, the previous settings will be applied by default.

GEOM character\*12 name of the LCM object (type L\_GEOM) containing the geometry.

(bivact\_data) structure containing the data to module BIVACT: (see Sect. 1.5.1).

#### 1.5.1 Data input for module BIVACT:

Table 12: Structure (bivact\_data)

where

EDIT keyword used to set *iprint*.

iprint index used to control the printing in module BIVACT:. =0 for no print; =1 for minimum printing (default value); Larger values produce increasing amounts of output.

TITL keyword which allows the run title to be set.

TITLE the title associated with a TRIVAC run. This title may contain up to 72 characters.

The default when TITL is not specified is no title.

MAXR keyword which permits the maximum number of regions to be considered during a

TRIVAC run to be specified.

maxpts maximum dimensions of the problem to be considered. The default value is set to the

number of regions previously computed by the GEO: module but this value is insufficient

if symmetries or mesh-splitting are specified.

PRIM keyword to set a primal finite element (classical) discretization.

DUAL keyword to set a mixed-dual finite element discretization. If the geometry is hexagonal,

a Thomas-Raviart-Schneider method is used.<sup>[16]</sup>

MCFD keyword to set a mesh-centered finite difference discretization in hexagonal geometry.

ielem order of the finite element representation. The values permitted are 1 (linear polyno-

mials), 2 (parabolic polynomials), 3 (cubic polynomials) or 4 (quartic polynomials). By

default ielem=1.

icol type of quadrature used to integrate the mass matrices. The values permitted are 1 (an-

alytical integration), 2 (Gauss-Lobatto quadrature) or 3 (Gauss-Legendre quadrature). By default icol=2. The analytical integration corresponds to classical finite elements; the Gauss-Lobatto quadrature corresponds to a variational or nodal type collocation and

the Gauss-Legendre quadrature corresponds to superconvergent finite elements.  $\,$ 

PN keyword to set a spherical harmonics  $(P_n)$  expansion of the flux.<sup>[9]</sup>

SPN keyword to set a simplified spherical harmonics  $(SP_n)$  expansion of the flux. [9,10] This

option is currently available with 1D and 2D Cartesian geometries and with 2D hexagonal

geometries.

n order of the  $P_n$  or  $SP_n$  expansion (odd number). Set to zero for diffusion theory (default

value).

SCAT keyword to limit the anisotropy of scattering sources.

DIFF keyword to force using  $1/3D^g$  as  $\Sigma_1^g$  cross sections. A  $P_1$  or  $SP_1$  method will therefore

behave as diffusion theory.

iscat number of terms in the scattering sources. iscat = 1 is used for isotropic scattering in the

laboratory system. iscat = 2 is used for linearly anisotropic scattering in the laboratory

system. The default value is set to n+1 in  $P_n$  or  $SP_n$  case.

VOID keyword to set the number of base points in the Gauss-Legendre quadrature used to

integrate void boundary conditions if icol = 3 and  $n \neq 0$ .

type of quadrature. The values permitted are: 0 (use a (n+2)-point quadrature consistent with  $P_n$  theory), 1 (use a (n+1)-point quadrature consistent with  $S_{n+1}$  theory), 2

(use an analytical integration of the void boundary conditions). By default nvd=0.

Various finite element approximations can be obtained by combining different values of ielem and icol:

• PRIM 1 1 : Linear finite elements;

nvd

• PRIM 1 2 : Mesh corner finite differences;

• PRIM 1 3 : Linear superconvergent finite elements;

• PRIM 2 1 : Quadratic finite elements;

- PRIM 2 2 : Quadratic variational collocation method;
- PRIM 2 3 : Quadratic superconvergent finite elements;
- PRIM 3 1 : Cubic finite elements;
- PRIM 3 2 : Cubic variational collocation method;
- PRIM 3 3 : Cubic superconvergent finite elements;
- PRIM 4 2 : Quartic variational collocation method;
- DUAL 1 1 : Mixed-dual linear finite elements;
- DUAL 1 2 : Mesh centered finite differences;
- DUAL 1 3: Mixed-dual linear superconvergent finite elements (numerically equivalent to PRIM 1 3);
- DUAL 2 1 : Mixed-dual quadratic finite elements;
- DUAL 2 2 : Quadratic nodal collocation method;
- DUAL 2 3: Mixed-dual quadratic superconvergent finite elements (numerically equivalent to PRIM 2 3);
- DUAL 3 1 : Mixed-dual cubic finite elements;
- DUAL 3 2 : Cubic nodal collocation method;
- DUAL 3 3 : Mixed-dual cubic superconvergent finite elements (numerically equivalent to PRIM 3 3);
- DUAL 4 2 : Quartic nodal collocation method;

#### 1.6 The TRIVAT: module

The TRIVAT: module is used to perform a TRIVAC-type TRACKING on a 1D/2D/3D geometry. [4–8, 15] The geometry is analyzed and a LCM object with signature L\_TRIVAC is created with the following information:

- Diagonal and hexagonal symmetries are unfolded and the mesh-splitting operations are performed.
   Volumes, material mixture and averaged flux recovery indices are computed on the resulting geometry.
- A finite element discretization is performed and the corresponding numbering is saved.
- The unit finite element matrices (mass, stiffness, etc.) are recovered.
- Indices related to an ADI preconditioning with or without supervectorization are saved.

The calling specifications are:

Table 13: Structure (TRIVAT:)

```
TRACK := TRIVAT: [ TRACK ] GEOM :: (trivat_data)
```

where

TRACK character\*12 of the LCM object (type L\_TRIVAC) containing the TRACKING information. If TRACK appears on the RHS, the previous settings will be applied by default.

GEOM character\*12 of the LCM object (type L\_GEOM) containing the geometry.

(trivat\_data) structure containing the data to module TRIVAT: (see Sect. 1.6.1).

1.6.1 Data input for module TRIVAT:

Table 14: Structure (trivat\_data)

```
[ EDIT iprint ]
[ TITL TITLE ]
[ MAXR maxpts ]
[ { PRIM [ ielem ] | DUAL [ ielem icol ] | MCFD [ ielem ] | LUMP [ ielem ] } ]
[ SPN n [ SCAT [ DIFF ] iscat ] [ VOID nvd ] ]
[ ADI nadi ]
[ VECT [ iseg ] [ PRTV impv ] ]
;
```

where

EDIT keyword used to set *iprint*.

iprint index used to control the printing in module TRIVAT:. =0 for no print; =1 for minimum printing (default value); Larger values produce increasing amounts of output.

TITL keyword which allows the run title to be set.

TITLE the title associated with a TRIVAC run. This title may contain up to 72 characters. The default when TITL is not specified is no title.

MAXR keyword which permits the maximum number of regions to be considered during a TRIVAC run to be specified.

maxpts maximum dimensions of the problem to be considered. The default value is set to the number of regions previously computed by the GEO: module but this value is insufficient if symmetries or mesh-splitting are specified.

PRIM keyword to set a discretization based on the variational collocation method.

DUAL keyword to set a mixed-dual finite element discretization. If the geometry is hexagonal, a Thomas-Raviart-Schneider method is used. [16]

MCFD keyword to set a discretization based on the nodal collocation method. The mesh centered finite difference approximation is the default option and is generally set using MCFD 1. The MCFD approximations are numerically equivalent to the DUAL approximations with *icol*=2; however, the MCFD approximations are less expensive.

LUMP keyword to set a discretization based on the nodal collocation method with serendipity approximation. The serendipity approximation is different from the MCFD option in cases with *ielem*≥2. This option is not available for hexagonal geometries.

ielem order of the finite element representation. The values permitted are: 1 (linear polynomials), 2 (parabolic polynomials), 3 (cubic polynomials) or 4 (quartic polynomials). By default ielem=1.

icol type of quadrature used to integrate the mass matrices. The values permitted are: 1 (analytical integration), 2 (Gauss-Lobatto quadrature) or 3 (Gauss-Legendre quadrature). By default icol=2. The analytical integration corresponds to classical finite elements; the Gauss-Lobatto quadrature corresponds to a variational or nodal type collocation and the Gauss-Legendre quadrature corresponds to superconvergent finite elements.

SPN keyword to set a simplified spherical harmonics  $(SP_n)$  expansion of the flux.<sup>[9,10]</sup> This option is available with 1D, 2D and 3D Cartesian geometries and with 2D and 3D hexagonal geometries.

n order of the  $P_n$  or  $SP_n$  expansion (odd number). Set to zero for diffusion theory (default value).

SCAT keyword to limit the anisotropy of scattering sources.

DIFF keyword to force using  $1/3D^g$  as  $\Sigma_1^g$  cross sections. A  $P_1$  or  $SP_1$  method will therefore behave as diffusion theory.

iscat number of terms in the scattering sources. iscat = 1 is used for isotropic scattering in the laboratory system. iscat = 2 is used for linearly anisotropic scattering in the laboratory system. The default value is set to n + 1 in  $P_n$  or  $SP_n$  case.

VOID keyword to set the number of base points in the Gauss-Legendre quadrature used to integrate void boundary conditions if icol = 3 and  $n \neq 0$ .

nvd type of quadrature. The values permitted are: 0 (use a (n+2)-point quadrature consistent with  $P_n$  theory), 1 (use a (n+1)-point quadrature consistent with  $S_{n+1}$  theory), 2 (use an analytical integration of the void boundary conditions). By default nvd=0.

ADI keyword to set the number of ADI iterations at the inner iterative level.

nadi number of ADI iterations (default: nadi = 2).

VECT keyword to set an ADI preconditionning with supervectorization. By default, TRIVAC

uses an ADI preconditionning without supervectorization.

iseg width of a vectorial register. iseg is generally a multiple of 64. By default, iseg=64.

PRTV keyword used to set impv.

impv index used to control the printing in supervectorization subroutines. =0 for no print; =1

for minimum printing (default value); Larger values produce increasing amounts of output.

Various finite element approximations can be obtained with different values of ielem (see Sect. 1.5).

#### 1.7 The BIVACA: module

The BIVACA: module is used to compute the finite element system matrices (type L\_SYSTEM) corresponding to a BIVAC TRACKING (type L\_BIVAC) and to a set of nuclear properties (type L\_MACROLIB). The calling specifications are:

Table 15: Structure (BIVACA:)

```
SYST := BIVACA: [ SYST ] MACRO TRACK :: (bivaca_data)
```

where

SYST character\*12 name of the LCM object (type L\_SYSTEM) containing the system matrices. If SYST appears on the RHS, the system matrices previously stored in SYST are kept.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the macroscopic cross sections and diffusion coefficients.

TRACK character\*12 name of the LCM object (type L\_BIVAC) containing the BIVAC TRACKING.

(bivaca\_data) structure containing the data to module BIVACA: (see Sect. 1.7.1).

#### 1.7.1 Data input for module BIVACA:

Table 16: Structure (bivaca\_data)

```
[ EDIT iprint ] [ UNIT ] ;
```

where

EDIT keyword used to set iprint.

iprint index used to control the printing in module BIVACA:. =0 for no print; =1 for minimum printing (default value); Larger values produce increasing amounts of output.

UNIT A system matrix corresponding to cross sections all set to 1.0 is computed. This keyword is mandatory if the system matrices in *SYST* are going to be used by INIKIN: or KINSOL: modules (see Sects. 1.14 and 1.15).

#### 1.8 The TRIVAA: module

The TRIVAA: module is used to compute the finite element system matrices (type L\_SYSTEM) corresponding to a TRIVAC TRACKING (type L\_TRIVAC) and to a set of nuclear properties (type L\_MACROLIB). The calling specifications are:

Table 17: Structure (TRIVAA:)

```
SYST := TRIVAA: [ SYST ] MACRO TRACK [ DMACRO ] :: (trivaa_data)
```

where

SYST character\*12 name of the LCM object (type L\_SYSTEM) containing the system matrices. If SYST appears on the RHS, the system matrices previously stored in SYST are kept.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the macroscopic cross sections and diffusion coefficients.

TRACK character\*12 name of the LCM object (type L\_TRIVAC) containing the TRIVAC TRACKING.

DMACRO character\*12 name of the LCM object (type L\_MACROLIB) containing derivatives or perturbations of the macroscopic cross sections and diffusion coefficients. If DMACRO is given,

only the derivatives or perturbations of the system matrices are computed.

(trivaa\_data) structure containing the data to module TRIVAA: (see Sect. 1.8.1).

# 1.8.1 Data input for module TRIVAA:

Table 18: Structure (trivaa\_data)

```
[ EDIT iprint ]
[ SKIP ] [{ DERI | PERT }] [ UNIT ] [ OVEL ]
;
```

where

EDIT keyword used to set *iprint*.

index used to control the printing in module TRIVAA:. =0 for no print; =1 for minimum

printing (default value); Larger values produce increasing amounts of output.

SKIP keyword used to skip the system matrix assembly but to perform the  $L-D-L^T$  factor-

ization. Use the system matrices already present in SYST.

DERI The information recovered from DMACRO is used as derivatives of nuclear properties with

respect to a state variable. Derivatives of system matrices with respect to the same state

variable are computed.

PERT The information recovered from DMACRO is used as the perturbation of the nuclear properties. Perturbations of the system matrices are computed.

UNIT A system matrix corresponding to cross sections all set to 1.0 is computed. This keyword is mandatory if the system matrices in *SYST* are going to be used by INIKIN: or KINSOL: modules (see Sects. 1.14 and 1.15).

OVEL The reciprocal neutron velocities for each material mixture are recovered from the input MACROLIB MACRO and used to compute the corresponding system matrices. This capability is deprecated.

#### 1.9 The FLUD: module

The FLUD: module is used to compute the solution to an eigenvalue problem corresponding to a set of system matrices (type L\_SYSTEM). The calling specifications are:

Table 19: Structure (FLUD:)

```
FLUX := FLUD: [ FLUX ] SYST TRACK [ MACRO ] :: (flud_data)
```

where

FLUX character\*12 name of the LCM object (type L\_FLUX) containing the solution. If FLUX

appears on the RHS, the solution previously stored in FLUX is used to initialize the new

iterative process; otherwise, a uniform unknown vector is used.

SYST character\*12 name of the LCM object (type L\_SYSTEM) containing the system matrices.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING.

MACRO character\*12 name of the optional LCM object (type L\_MACROLIB) containing the cross

sections. This object is only used to set a link to the MACROLIB name inside the FLUX object. By default, the name of the MACROLIB is recovered from the link in the SYSTEM

object.

(flud\_data) structure containing the data to module FLUD: (see Sect. 1.18.1).

## 1.9.1 Data input for module FLUD:

Table 20: Structure (flud\_data)

```
[ EDIT iprint ]
[ { VAR1 | ACCE } icl1 icl2 ] [ IRAM blsz korg [ nstard [ EPSG epsmsr ] ] ]
[ EXTE [ maxout ] [ epsout ] ]
[ THER [ maxthr ] [ epsthr ] ]
[ ADI nadi ]
[ ADJ ]
[ MONI lmod [ RAND ] ]
[ RELAX relax ]
;
```

where

EDIT keyword used to set *iprint*.

iprint index used to control the printing in module FLUD:. =0 for no print; =1 for minimum printing (default value); =2 iteration history is printed; =3 the solution is printed; =4 at each iteration, the new solution is compared to a reference solution previously stored in FLUX under name REF; =5 the convergence histogram is stored in FLUX.

VAR1 keyword used to set the parameters *icl1* and *icl2*. These parameter are used with the symmetrical variational acceleration technique (SVAT) for convergence of the generalized eigenvalue problem (default option) and to accelerate up-scattering iterations.

ACCE alias keyword for VAR1.

icl1 number of free outer iterations in a cycle of the variational acceleration technique. The default value is icl1 = 3.

icl2 number of accelerated outer iterations in a cycle of the variational acceleration technique. The default value is icl2 = 3. A convergence in free iterations is obtained by setting icl1 = 200 (or icl1 = maxout) and icl2 = 0.

IRAM keyword used to to switch on the implicit restarted Arnoldi method (IRAM) and to set the parameters blsz, korg and nstard. [11] By default, the symmetrical variational acceleration technique (SVAT) is used.

blsz block size of the Arnoldi Hessenberg matrix. blsz is the number of fixed-source Boltzmann transport equations solved similtaneously at each iteration of the implicit restarted Arnoldi method. The recommended value is blsz = 3.

korg number of desired eigenvalues with  $korg \ge blsz$ .

number of iterations before restarting with the GMRES(m) acceleration method for solving the ADI-preconditionned linear systems in Trivac. The maximum number of GMRES iterations is set to *nadi*. By default, GMRES(m) acceleration is not used and *nadi* free iterations are performed.

EPSG keyword to specify the inner iteration GMRES epsilon.

epsmsr convergence criterion for the inner iteration GMRES iterations. The fixed default value is epsmsr =  $1.0 \times 10^{-6}$ .

EXTE keyword to specify that the control parameters for the external iteration are to be modified.

maxout maximum number of external iterations. The fixed default value is maxout = 200.

epsout convergence criterion for the external iterations. The fixed default value is  $epsout = 1.0 \times 10^{-4}$ . The outer iterations are stopped when the following criteria is reached:

$$\max_i |\Phi_i^{(k-1)} - \Phi_i^{(k)}| \ \leq \ epsout \times \max_i |\Phi_i^{(k)}|$$

where  $\vec{\Phi}^{(k)} = \text{col}\{\Phi_i^{(k)}; i = 1, I\}$  is the product of the B matrix times the unknown vector at the k-th outer iteration.

THER keyword to specify that the control parameters for the thermal iterations are to be modified.

maxthr maximum number of thermal iterations. The fixed default value is maxthr = 0 corresponding to no thermal iterations.

epsthr convergence criterion for the thermal iterations. The fixed default value is epsthr =  $1.0 \times 10^{-5}$ .

ADI keyword used to set the number of alternating direction implicit (ADI) inner iterations in cases where Trivac is used. This keyword is also used to set the number of flux iterations over Legendre orders with  $SP_n$  Bivac and Trivac cases if  $n \geq 3$ .

number of ADI inner or Legendre order iterations per outer iteration. The default value is nadi = 1. If this value causes a failure of the acceleration process, it is recommended that a larger value be tried. The optimal choice is generally the minimum value of nadi which allows a convergence in less than 75 outer iterations. nadi = 1 or nadi = 2 is generally

the best choice for production-type calculations. The greater nadi is, the smaller the asymptotic convergence constant (ACC) becomes. Taking an arbitrary large value (e.g., nadi = 20) leads to numerical results identical to those of the inverse power method where the system matrices are accurately inverted at each outer iteration (at a prohibitive CPU cost). In this case, the ACC is almost equal to the dominance ratio of the iterative matrix. The default value is recovered in the state vector of the TRACKING object TRACK.

ADJ

keyword used to obtain the solution to both the direct and adjoint eigenvalue problems. The adjoint solution is required if we subsequently want to perform a perturbation calculation. This option is limited to Trivac.

MONI

keyword used to obtain the first harmonics of the solution and to set *lmod*. A full core representation of the reactor should be used to compute its harmonics. If symmetries are set in the geometry, some harmonics may be skipped. If the reactor is symmetric, a uniform initial estimate of the harmonics may cause some harmonics to be skipped; the keyword RAND should therefore be used.

lmod

the lmod first bi-orthonormalized harmonics of the solution are computed using the SVAT-accelerated preconditioned power method with a Hotelling deflation procedure.<sup>[12]</sup>

RAND

keyword used to initialize the harmonics calculations (option MONI) with a random estimate rather than a uniform estimate. This option has no effect if *FLUX* appears on the RHS.

RELAX

keyword used to set the relaxation parameter. This keyword must be specified each time a relaxation is required.

relax

relaxation parameter selected in the interval  $0 < relax \le 1.0$  and used to update the flux information in the FLUX object. The updated value is taken equal to (1.0-relax) times the previous value (given in the RHS FLUX object) plus relax times the value computed within current FLUD: call. The default value is relax = 1.0.

# 1.10 The DELTA: module

The DELTA: module is used to compute the source components of a fixed source eigenvalue problem corresponding to a set of unperturbed and perturbation system matrices (type L\_SYSTEM).

In the direct case, the fixed source is computed as:

$$\vec{S} = (\delta \, \mathbb{A} - \lambda_o \, \delta \mathbb{B}) \, \vec{\Phi} - \delta \lambda \, \mathbb{B}_o \, \vec{\Phi} \tag{1.1}$$

where the direct source vector  $\vec{S}$  is orthogonal to the unperturbed adjoint flux  $\Phi^*$ .

In the adjoint case, the fixed source is computed as:

$$\vec{S}^* = \left(\delta \,\mathbb{A}^\top - \lambda_o \,\delta \mathbb{B}^\top\right) \vec{\Phi}^* - \delta \lambda \,\,\mathbb{B}_o^\top \,\vec{\Phi}^* \tag{1.2}$$

where the adjoint source vector  $\vec{S}^*$  is orthogonal to the unperturbed direct flux  $\Phi$  and where  $\delta\lambda$  is the perturbation of the eigenvalue, as computed from the Rayleigh ratio.

The calling specifications are:

Table 21: Structure (DELTA:)

```
GPT := DELTA: [ GPT ] FLUX0 SYST0 DSYST TRACK :: (delta_data)
```

where

GPT character\*12 name of the LCM object (type L\_SOURCE) containing the fixed source. If GPT appears on the RHS, this information is used to initialize the state vector.

FLUX0 character\*12 name of the LCM object (type L\_FLUX) containing the unperturbed flux.

SYST0 character\*12 name of the LCM object (type L\_SYSTEM) containing the unperturbed system matrices.

DSYST character\*12 name of the LCM object (type L\_SYSTEM) containing a perturbation to the system matrices.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING.

(delta\_data) structure containing the data to module DELTA: (see Sect. 1.10.1).

# 1.10.1 Data input for module DELTA:

Table 22: Structure (delta\_data)

```
[ EDIT iprint ]
[ ADJ ]
;
```

EDIT keyword used to set *iprint*.

iprint index used to control the printing in module DELTA:.

ADJ keyword used to set the source on an adjoint fixed source eigenvalue problem.

#### 1.11 The GPTFLU: module

The GPTFLU: module is used to compute the solution to a fixed source eigenvalue problem corresponding to a set of unperturbed system matrices and sources vectors.

If  $\vec{S}$  is the source term of the explicit generalized adjoint equation, this module will solve:

$$(\mathbb{A}_o - \lambda_o \, \mathbb{B}_o) \, \vec{\Gamma}_i = \vec{S}_i \tag{1.3}$$

where the direct source vector  $\vec{S}_i$  is orthogonal to the adjoint flux.

If  $\hat{S}$  is the source term of the implicit generalized adjoint equation, this module will solve:

$$\left(\mathbb{A}_o^{\top} - \lambda_o \, \mathbb{B}_o^{\top}\right) \, \vec{\Gamma}_j^* = \vec{S}_j^* \tag{1.4}$$

where the adjoint source vector  $\vec{S}_j^*$  is orthogonal to the direct flux.

The calling specifications are:

Table 23: Structure (GPTFLU:)

```
FLUX\_GPT := \texttt{GPTFLU}: [ FLUX\_GPT ] GPT FLUX0 SYST TRACK :: (gptflu\_data)
```

where

FLUX\_GPT character\*12 name of the LCM object (type L\_FLUX) containing the GPT solution. If FLUX\_GPT appears on the RHS, the solution previously stored in FLUX\_GPT is used to initialize the new iterative process; otherwise, a uniform unknown vector is used.

GPT character\*12 name of the LCM object (type L\_SOURCE) containing the fixed sources.

FLUX0 character\*12 name of the LCM object (type L\_FLUX) containing the unperturbed flux used to decontaminate the GPT solution.

SYST character\*12 name of the LCM object (type L\_SYSTEM) containing the unperturbed system matrices.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING.

(gptflu\_data) structure containing the data to module GPTFLU:.

### 1.11.1 Data input for module GPTFLU:

Table 24: Structure (gptflu\_data)

where

EDIT keyword used to set iprint.

index used to control the printing in module GPTFLU:. =0 for no print; =1 for minimum printing (default value); =2 iteration history is printed; =3 the solution is printed; =4 at each iteration, the new solution is compared to a reference solution previously stored in FLUX\_GPT under the name REF; =5 the convergence histogram is stored in FLUX\_GPT.

VAR1 keyword used to set the parameters *icl1* and *icl2*. These parameter are used with the variational acceleration technique for convergence of the fixed-source iterations (default option) and to accelerate up-scattering iterations.

ACCE alias keyword for VAR1.

icl1 number of free outer iterations in a cycle of the variational acceleration technique. The default value is icl1 = 3.

icl2 number of accelerated outer iterations in a cycle of the variational acceleration technique. The default value is icl2 = 3. A convergence in free iterations is obtained by setting icl1 = 200 (or icl1 = maxout) and icl2 = 0.

GMRES keyword to switch on the GMRES(m) acceleration of the fixed-source iterations. By default, the variational acceleration technique is used.

nstart restarts the GMRES method every nstart outer iterations. The recommended value is nstart = 10.

EXTE keyword to specify that the control parameters for the external iteration are to be modified.

maxout maximum number of external iterations. The fixed default value is maxout = 200.

epsout convergence criterion for the external iterations. The fixed default value is  $epsout = 1.0 \times 10^{-4}$ . The outer iterations are stopped when the following criteria is reached:

$$\max_i |\Gamma_i^{(k-1)} - \Gamma_i^{(k)}| \ \leq \ epsout \times \max_i |\Gamma_i^{(k)}|$$

where  $\vec{\Gamma}^{(k)} = \text{col}\{\Gamma_i^{(k)} ; i = 1, I\}$  is the product of the  $\mathbb B$  matrix times the unknown vector at the k-th outer iteration.

THER keyword to specify that the control parameters for the thermal iterations are to be modified.

maxthr maximum number of thermal iterations. The fixed default value is maxthr = 0 corresponding to no thermal iterations.

epsthr convergence criterion for the thermal iterations. The fixed default value is  $epsthr = 1.0 \times 10^{-2}$ .

ADI keyword used to set *nadi* in cases where Trivac is used.

number of alternating direction implicit (ADI) inner iterations per outer iteration. The default value is nadi = 1. If this value causes a failure of the acceleration process, it is recommended that a larger value be tried. The optimal choice is generally the minimum value of nadi which allows a convergence in less than 75 outer iterations. nadi = 1 or nadi = 2 is generally the best choice for production-type calculations. The greater nadi is, the smaller the asymptotic convergence constant (ACC) becomes. Taking an arbitrary large value (e.g., nadi = 20) leads to numerical results identical to those obtained by inverting the system matrices at each outer iteration (at a prohibitive CPU cost). In this case, the ACC is almost equal to the dominance ratio of the iterative matrix.

EXPLICIT keyword used to obtain the solution of an direct fixed source eigenvalue problem.

IMPLICIT keyword used to obtain the solution of an adjoint fixed source eigenvalue problem. If neither 'EXPLICIT' nor 'IMPLICIT' are provided the default value will be chosen as a function of

 $n_{var}$  and  $n_{cst} + 1$ .

FROM-TO keyword used to specify the numbers of the sources for which a generalized adjoint will be

calculated.

ALL keyword used to recover all sources available in GPT.

 $i_{src1}$  number of the first source.

 $i_{src1}$  number of the last source.

#### 1.12 The OUT: module

The OUT: module is used to compute the reaction rates and to store them in an extended MACROLIB (type L\_MACROLIB) corresponding to a solution (type L\_FLUX) of the matrix system. The calling specifications are:

Table 25: Structure (OUT:)

```
MACRO2 := OUT: FLUX TRACK MACRO GEOM :: (out_data)
```

where

MACRO2 character\*12 name of the LCM object (type L\_MACROLIB) containing the extended MACROLIB.

FLUX character\*12 name of the LCM object (type L\_FLUX) containing a solution.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing a TRACKING.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the reference MACROLIB.

GEOM character\*12 name of the LCM object (type L\_GEOM) containing the reference GEOMETRY.

(out\_data) structure containing the data to module OUT:.

# 1.12.1 Data input for module OUT:

Table 26: Structure (out\_data)

```
[ EDIT iprint ]
[ MODE imode ]
[ { DIRE | PROD } ]
[ { POWR power | SOUR snumb } ]
[ COND [ { NONE | (icond(i), i=1,ngcond) } ] ]
[ INTG { NONE | IN | MIX | (ihom(i), i=1,nreg) } ]
;
```

where

EDIT keyword used to set iprint.

iprint index used to control the printing in module OUT:. =0 for no print; =1 for minimum

printing (default value).

MODE keyword to specify the flux harmonic index imode.

imode index of the flux harmonic recovered by the OUT: module if the MONI keyword was set in

module FLUD: (see Sect. 1.18.1). By default, it is assumed that the MONI keyword was not

used.

DIRE use the direct flux to perform homogenization and/or condensation (default option).

PROD use the product of adjoint and direct fluxes to perform homogenization and/or condensa-

tion.

POWR keyword used to set power.

power value of the power in MW used to normalize the flux. By default, the flux is not normalized.

SOUR keyword used to set snumb.

snumb number of source particles used to normalize the flux. By default, the flux is not normalized.

COND keyword to specify that a group condensation of the flux is to be performed. By default,

no group condensation of the flux is to be performed, so that ngcond = ngroup.

icond array of increasing energy group limits that will be associated with each of the ngcond

condensed groups. We must have ngcond≤ngroup. By default, if COND is set and icond is

not set, all energy groups are condensed together.

NONE keyword to specify that no group condensation of the flux is to be performed, so that

ngcond=ngroup (default option).

INTG keyword used to compute the reaction rates.

NONE keyword for computing the reaction rates on the geometry mesh (see Sect. 1.3.1) after

mesh-splitting.

IN keyword for computing the reaction rates on the geometry mesh (see Sect. 1.3.1) before

mesh-splitting.

MIX keyword for computing the reaction rates on the mixture mesh previously used to define

the geometry (see Sect. 1.3.1) before mesh-splitting.

ihom index of the homogenized region corresponding to the each region of the geometry (see

Sect. 1.3.1) before mesh-splitting.

#### 1.13 The ERROR: module

The ERROR: module is used to compare reaction rates contained into two extended MACROLIBS and to print statistics regarding the comparison.

The QUANDRY-type power densities are first compared. These power densities are defined by the following relation:

$$P_i^{\text{quandry}} = \frac{\sum_i V_i}{V_i} \frac{P_i}{\sum_i P_i}$$

where  $P_i$  is the total power and  $V_i$  is the volume of the region i. The maximum and averaged errors are respectively defined by:

$$\epsilon_{\max} = \max_{i} \frac{|P_{i}^{\text{quandry}} - P_{i}^{\text{quandry*}}|}{P_{i}^{\text{quandry*}}}$$

and

$$\bar{\epsilon} = \frac{1}{V_{\text{core}}} \sum_{i} \left\lceil \frac{|P_{i}^{\text{quandry}} - P_{i}^{\text{quandry*}}|}{P_{i}^{\text{quandry*}}} \right\rceil V_{i}$$

where  $P_i^{\text{quandry}*}$  is computed using the reference powers (stored in MACRO1) and  $V_{\text{core}}$  is the total volume of the regions where the power density is not equal to zero.

The normalized removal rates  $T_{i,g}^{\text{norm}}$  in each region i and energy group g are next computed using the following formula:

$$T_{i,g} = (\Sigma_{i,g} - \Sigma_{wi,g}) \ \phi_{i,g} V_i$$

$$T_{i,g}^{\text{norm}} = \frac{1}{\sum_{i} \sum_{g} T_{i,g}} T_{i,g}$$

where  $\Sigma_{i,g}$  is the total macroscopic cross section,  $\Sigma_{\text{w}i,g}$  is the within-group scattering cross section and  $\phi_{i,g}$  is the neutron flux. The maximum and averaged errors are respectively defined by:

$$\epsilon_{\text{max }g} = \max_{i} \frac{|T_{i,g}^{\text{norm}} - T_{i,g}^{\text{norm*}}|}{T_{i,g}^{\text{norm*}}}$$

and

$$\bar{\epsilon}_g = \frac{1}{N} \sum_i \left[ \frac{|T_{i,g}^{\text{norm}} - T_{i,g}^{\text{norm*}}|}{T_{i,g}^{\text{norm*}}} \right]$$

where  $T_{i,g}^{\text{norm}*}$  is computed using the reference values (stored in MACRO1) and N is the total number of regions in the MACROLIB.

The calling specifications are:

Table 27: Structure (ERROR:)

ERROR: MACRO1 MACRO2 :: [HREA hname] [NREG nreg];

where

 $MACRO1 \qquad \texttt{character*12} \, \texttt{name} \, \texttt{of} \, \texttt{the} \, \texttt{LCM} \, \texttt{object} \, (\texttt{type} \, \texttt{L\_MACROLIB}) \, \texttt{containing} \, \texttt{the} \, \texttt{extended} \, \texttt{MACROLIB}$ 

used to compute the reference reaction rates.

MACRO2 character\*12 name of the LCM object (type L\_MACROLIB) containing the extended MACROLIB

used to compute the approximate reaction rates.

HREA keyword used to set the character name hname.

hname character\*8 name of the nuclear reaction used to compute the power map. By default,

reaction H-FACTOR is used.

NREG keyword used to set the nreg number.

nreg integer number set to the number of regions used in statistics. By default, all available

regions are used.

#### 1.14 The INIKIN: module

The INIKIN: module is used to recover the steady-state solution and to initialize the kinetics parameters. The delayed neutron information can be provided directly from the input file or recovered from the MACROLIB data structure.

The initial presursor concentrations are obtained as a function of the strady-state solution. If  $\phi_g(\mathbf{r}, t_0)$  is the initial flux in energy group g divided by  $k_{\text{eff}}$ , the corresponding initial conditions of the precursors are obtained as

$$c_{\ell}(\boldsymbol{r}, t_0) = \frac{1}{\lambda_{\ell}} \sum_{h=1}^{G} \nu \Sigma_{\mathrm{f}\ell, h}^{\mathrm{del}}(\boldsymbol{r}) \, \phi_h(\boldsymbol{r}, t_0); \quad \ell = 1, N_d.$$

$$(1.5)$$

where  $\nu \Sigma_{\mathrm{f}\ell,h}^{\mathrm{del}}(\boldsymbol{r})$  is  $\nu$  times the delayed macroscopic fission cross section in energy group h for precursor group  $\ell$ .

The calling specifications are:

Table 28: Structure (INIKIN:)

```
KINET := INIKIN: MACRO TRACK SYST FLUX :: (inikin_data)
```

where

KINET character\*12 name of the LCM object (type L\_KINET) to be created by the module.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the MACROLIB information.

madon.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING information

SYST character\*12 name of the LCM object (type L\_SYSTEM) corresponding to MACROLIB MACRO and TRACKING TRACK.

FLUX character\*12 name of the LCM object (type L\_FLUX) containing the initial steady-state solution.

(inikin\_data) structure containing the data to module INIKIN: (see Sect. 1.14.1).

## 1.14.1 Data input for module INIKIN:

Table 29: Structure (inikin\_data)

# Structure (inikin\_data)

continued from last page

```
[ NORM { fnorm | MAX | POWER-INI power } ];
```

where

EDIT keyword used to set iprint index.

integer index used to control the printing in module INIKIN:. =0 for no print; =1 for

minimum printing (default value); larger values of iprint will produce increasing amounts

of output.

NGRP keyword used to set the ngrp number. By default, this information is recovered from the

solution object FLUX.

ngrp integer total number of energy groups.

NDEL keyword used to set the ndg number.

ndg integer total number of the delayed neutron groups.

BETA keyword used to indicate the reading of beta values from the input file. If these values are

not provided, they should be recorded in the MACROLIB data structure.

beta real array containing the delayed neutron fractions for each delayed group.

LAMBDA keyword used to indicate the reading of lambda values from the input file. If these values

are not provided, they should be recorded in the MACROLIB data structure.

lambda real array containing the precursors decay constants for each delayed group.

CHID keyword used to indicate the reading of *chid* values from the input file. If these values are

not provided, they should be recorded in the MACROLIB data structure.

chid real array representing the delayed multigroup fission spectrum.

NORM keyword used to normalize the initial flux. By default, the flux is not normalized.

fnorm real normalization factor.

MAX keyword used to set the flux normalization factor to  $1/f_{\text{max}}$  where  $f_{\text{max}}$  is the maximum

flux in the core.

POWER-INI keyword used to set the flux normalization factor to a given value of the initial power.

power real initial power in MW.

#### 1.15 The KINSOL: module

The KINSOL: module is used to solve the space-time neutron kinetics equations at current time step of transient.

### 1.15.1 The direct (forward) solution

We first consider the discretization of the legacy forward space-time kinetics equation. Several implicit numerical schemes are available for this purpose. Consider first the differential equation for precursor concentrations:

$$\frac{\partial c_{\ell}(\boldsymbol{r},t)}{\partial t} + \lambda_{\ell} c_{\ell}(\boldsymbol{r},t) = \sum_{h=1}^{G} \nu \Sigma_{\mathrm{f}\ell,h}^{\mathrm{del}}(\boldsymbol{r}) \phi_{h}(\boldsymbol{r},t); \quad \ell = 1, N_{d}.$$
(1.6)

Consider a solution between times  $t_{n-1}$  and  $t_n = t_{n-1} + \Delta t_n$ . First, an analytic solution can be obtained by assuming a ramp variation of the fission reaction rates over time step  $\Delta t_n$ . This solution is written

$$c_{\ell}(\boldsymbol{r}, t_{n}) = c_{\ell}(\boldsymbol{r}, t_{n-1}) e^{-\lambda_{\ell} \Delta t_{n}} + \frac{F_{\ell}(\boldsymbol{r}, t_{n-1})}{\lambda_{\ell}} \left[ \frac{1}{\lambda_{\ell} \Delta t_{n}} \left( 1 - e^{-\lambda_{\ell} \Delta t_{n}} \right) - e^{-\lambda_{\ell} \Delta t_{n}} \right] + \frac{F_{\ell}(\boldsymbol{r}, t_{n})}{\lambda_{\ell}} \left[ 1 - \frac{1}{\lambda_{\ell} \Delta t_{n}} \left( 1 - e^{-\lambda_{\ell} \Delta t_{n}} \right) \right]$$

$$(1.7)$$

where the delayed fission reaction rates are defined as

$$F_{\ell}(\boldsymbol{r}, t_n) = \sum_{h=1}^{G} \nu \Sigma_{f\ell, h}^{\text{del}}(\boldsymbol{r}) \, \phi_h(\boldsymbol{r}, t_n) = \beta_{\ell} \sum_{h=1}^{G} \nu \Sigma_{fh}(\boldsymbol{r}) \, \phi_h(\boldsymbol{r}, t_n). \tag{1.8}$$

An implicit theta solution is presented in Chapter 5 of Ref. 1. This solution is written

$$c_{\ell}(\mathbf{r}, t_{n}) = \left[ \frac{1 - (1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] c_{\ell}(\mathbf{r}, t_{n-1}) + \frac{F_{\ell}(\mathbf{r}, t_{n-1})}{\lambda_{\ell}} \left[ \frac{(1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] + \frac{F_{\ell}(\mathbf{r}, t_{n})}{\lambda_{\ell}} \left[ \frac{\Theta_{p} \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right]$$

$$(1.9)$$

where  $\Theta_{\rm p}$  is the theta-factor for precursors.

The fixed-source corresponding to the analytic solution for precursors is written

$$S_g^{\text{exact}}(\boldsymbol{r}, t_n) = \frac{1}{V_{n,g} \Delta t_n} \phi_g(\boldsymbol{r}, t_{n-1}) + \sum_{\ell} \lambda_{\ell} \left[ 1 - \Theta_f + \Theta_f e^{-\lambda_{\ell} \Delta t_n} \right] \chi_{\ell,g}^{\text{del}}(\boldsymbol{r}) c_{\ell}(\boldsymbol{r}, t_{n-1})$$

$$+ (1 - \Theta_f) \left\{ \nabla \cdot \mathbb{D}_g(\boldsymbol{r}) \nabla \phi_g(\boldsymbol{r}, t_{n-1}) - \Sigma_{rg}(\boldsymbol{r}) \phi_g(\boldsymbol{r}, t_{n-1}) \right\}$$

$$+ \sum_{h=1 \atop h \neq g}^{G} \Sigma_{g \leftarrow h}(\boldsymbol{r}) \phi_h(\boldsymbol{r}, t_{n-1}) + \chi_g^{\text{ss}}(\boldsymbol{r}) F(\boldsymbol{r}, t_{n-1}) \right\}$$

$$- \sum_{\ell} \left[ 1 - \Theta_f - \Theta_f \left( \frac{1}{\lambda_{\ell} \Delta t_n} \left( 1 - e^{-\lambda_{\ell} \Delta t_n} \right) - e^{-\lambda_{\ell} \Delta t_n} \right) \right] \chi_{\ell,g}^{\text{del}}(\boldsymbol{r}) F_{\ell}(\boldsymbol{r}, t_{n-1}) \quad (1.10)$$

where the steady-state fission reaction rates are defined as

$$F(\mathbf{r}, t_n) = \sum_{h=1}^{G} \nu \Sigma_{fh}(\mathbf{r}) \, \phi_h(\mathbf{r}, t_n). \tag{1.11}$$

The fixed-source corresponding to the implicit theta solution is presented in Chapter 5 of Ref. 1 and is written

$$S_{g}^{\Theta}(\boldsymbol{r},t_{n}) = \frac{1}{V_{n,g} \Delta t_{n}} \phi_{g}(\boldsymbol{r},t_{n-1}) + \sum_{\ell} \lambda_{\ell} \left[ 1 - \Theta_{f} + \Theta_{f} \frac{1 - (1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] \chi_{\ell,g}^{\text{del}}(\boldsymbol{r}) c_{\ell}(\boldsymbol{r},t_{n-1})$$

$$+ (1 - \Theta_{f}) \left\{ \nabla \cdot \mathbb{D}_{g}(\boldsymbol{r}) \nabla \phi_{g}(\boldsymbol{r},t_{n-1}) - \Sigma_{rg}(\boldsymbol{r}) \phi_{g}(\boldsymbol{r},t_{n-1}) \right\}$$

$$+ \sum_{h=1 \atop h \neq g}^{G} \Sigma_{g \leftarrow h}(\boldsymbol{r}) \phi_{h}(\boldsymbol{r},t_{n-1}) + \chi_{g}^{\text{ss}}(\boldsymbol{r}) F(\boldsymbol{r},t_{n-1}) \right\}$$

$$- \sum_{\ell} \left[ 1 - \Theta_{f} - \Theta_{f} \frac{(1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] \chi_{\ell,g}^{\text{del}}(\boldsymbol{r}) F_{\ell}(\boldsymbol{r},t_{n-1}). \tag{1.12}$$

The flux equation at end-of-step is now presented. The equation corresponding to the analytic solution for precursors is written

$$\frac{1}{V_{n,g} \Delta t_n} \phi_g(\boldsymbol{r}, t_n) - \Theta_f \boldsymbol{\nabla} \cdot \mathbb{D}_g(\boldsymbol{r}) \boldsymbol{\nabla} \phi_g(\boldsymbol{r}, t_n) + \Theta_f \Sigma_{rg}(\boldsymbol{r}) \phi_g(\boldsymbol{r}, t_n) 
= S_g^{\text{exact}}(\boldsymbol{r}, t_n) + \Theta_f \sum_{h=1 \atop h \neq g}^{G} \Sigma_{g \leftarrow h}(\boldsymbol{r}) \phi_h(\boldsymbol{r}, t_n) 
+ \Theta_f \chi_g^{\text{ss}}(\boldsymbol{r}) F(\boldsymbol{r}, t_n) - \Theta_f \sum_{\ell} \chi_{\ell,g}^{\text{del}}(\boldsymbol{r}) \frac{1}{\lambda_{\ell} \Delta t_n} \left(1 - e^{-\lambda_{\ell} \Delta t_n}\right) F_{\ell}(\boldsymbol{r}, t_n).$$
(1.13)

The equation corresponding to the implicit theta solution is presented in Chapter 5 of Ref. 1 and is written

$$\frac{1}{V_{n,g} \Delta t_n} \phi_g(\boldsymbol{r}, t_n) - \Theta_f \nabla \cdot \mathbb{D}_g(\boldsymbol{r}) \nabla \phi_g(\boldsymbol{r}, t_n) + \Theta_f \Sigma_{rg}(\boldsymbol{r}) \phi_g(\boldsymbol{r}, t_n) 
= S_g^{\Theta}(\boldsymbol{r}, t_n) + \Theta_f \sum_{\substack{h=1 \ h \neq g}}^{G} \Sigma_{g \leftarrow h}(\boldsymbol{r}) \phi_h(\boldsymbol{r}, t_n) 
+ \Theta_f \chi_g^{ss}(\boldsymbol{r}) F(\boldsymbol{r}, t_n) - \Theta_f \sum_{\ell} \chi_{\ell,g}^{del}(\boldsymbol{r}) \frac{1}{1 + \Theta_p \lambda_{\ell} \Delta t_n} F_{\ell}(\boldsymbol{r}, t_n).$$
(1.14)

# 1.15.2 The adjoint (backward) solution

The negative sign in front of the term  $(1/v)\partial\phi^*/\partial t$  suggest some sort of backward approach to compute the importance (as opposed to the direct or forward approach for the direct neutron flux). Hence, while it is necessary to define an initial state of the system to solve the direct equations, solving the importance equations requires final conditions and to proceed backward with respect to time.

Discretization of the adjoint space-time kinetics equations using the *implicit theta solution* leads to the following equations. The solution for precursors is written

$$c_{\ell}^{*}(\boldsymbol{r}, t_{n-1}) = \left[\frac{1 - (1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}}\right] c_{\ell}^{*}(\boldsymbol{r}, t_{n}) + \left[\frac{(1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}}\right] \sum_{h=1}^{G} \chi_{\ell,h}^{\text{del}}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r}, t_{n}) + \left[\frac{\Theta_{p} \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}}\right] \sum_{h=1}^{G} \chi_{\ell,h}^{\text{del}}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r}, t_{n-1}).$$

$$(1.15)$$

The flux equation at beginning-of-step is written

$$\frac{1}{V_{n,g} \Delta t_{n}} \phi_{g}^{*}(\boldsymbol{r}, t_{n-1}) - \Theta_{f} \nabla \cdot \mathbb{D}_{g}(\boldsymbol{r}) \nabla \phi_{g}^{*}(\boldsymbol{r}, t_{n-1}) + \Theta_{f} \Sigma_{rg}(\boldsymbol{r}) \phi_{g}^{*}(\boldsymbol{r}, t_{n-1})$$

$$= S_{g}^{*\Theta}(\boldsymbol{r}, t_{n-1}) + \Theta_{f} \sum_{h=1 \atop h \neq g}^{G} \Sigma_{h \leftarrow g}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r}, t_{n-1})$$

$$+ \Theta_{f} \sum_{h=1}^{G} \left[ \nu \Sigma_{fg}(\boldsymbol{r}) \chi_{h}^{ss}(\boldsymbol{r}) - \sum_{\ell} \nu \Sigma_{f\ell,g}^{del}(\boldsymbol{r}) \chi_{\ell,h}^{del}(\boldsymbol{r}) \frac{1}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n-1}} \right] \phi_{h}^{*}(\boldsymbol{r}, t_{n-1}) \tag{1.16}$$

where the fixed-source  $S_q^{*\Theta}(\boldsymbol{r},t_{n-1})$  is written

$$S_{g}^{*\Theta}(\boldsymbol{r},t_{n-1}) = \frac{1}{V_{n,g} \Delta t_{n}} \phi_{g}(\boldsymbol{r},t_{n}) + \sum_{\ell} \nu \Sigma_{f\ell,g}^{del}(\boldsymbol{r}) \left[ 1 - \Theta_{f} + \Theta_{f} \frac{1 - (1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] c_{\ell}^{*}(\boldsymbol{r},t_{n})$$

$$+ (1 - \Theta_{f}) \left\{ \boldsymbol{\nabla} \cdot \mathbb{D}_{g}(\boldsymbol{r}) \boldsymbol{\nabla} \phi_{g}^{*}(\boldsymbol{r},t_{n}) - \Sigma_{rg}(\boldsymbol{r}) \phi_{g}^{*}(\boldsymbol{r},t_{n}) \right\}$$

$$+ \sum_{h=1 \atop h \neq g}^{G} \Sigma_{h \leftarrow g}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r},t_{n}) + \sum_{h=1}^{G} \nu \Sigma_{fg}(\boldsymbol{r}) \chi_{h}^{ss}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r},t_{n}) \right\}$$

$$- \sum_{\ell} \left[ 1 - \Theta_{f} - \Theta_{f} \frac{(1 - \Theta_{p}) \lambda_{\ell} \Delta t_{n}}{1 + \Theta_{p} \lambda_{\ell} \Delta t_{n}} \right] \sum_{h=1}^{G} \nu \Sigma_{f\ell,g}^{del}(\boldsymbol{r}) \chi_{\ell,h}^{del}(\boldsymbol{r}) \phi_{h}^{*}(\boldsymbol{r},t_{n}). \tag{1.17}$$

The equations corresponding to the analytic solution for precursors are obtained by replacing the following terms in Eqs. (1.15) to (1.17):

$$\frac{1}{1 + \Theta_{\rm p} \lambda_{\ell} \Delta t_n} \Rightarrow \frac{1 - \exp^{-\lambda_{\ell} \Delta t_n}}{\lambda_{\ell} \Delta t_n} \quad \text{and} \quad \frac{1 - (1 - \Theta_{\rm p} \lambda_{\ell} \Delta t_n)}{1 + \Theta_{\rm p} \lambda_{\ell} \Delta t_n} \Rightarrow \exp^{-\lambda_{\ell} \Delta t_n}. \tag{1.18}$$

#### 1.15.3 The calling specifications

The calling specifications are:

Table 30: Structure (KINSOL:)

KINET := KINSOL: KINET MACRO TRACK SYST [ MACRO\_0 SYST\_0 ] :: (kinsol\_data)

where

KINET character\*12 name of the LCM object (type L\_KINET) in modification mode.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the MACROLIB information corresponding to the current time step of a transient.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING information.

SYST character\*12 name of the LCM object (type L\_SYSTEM) corresponding to MACROLIB MACRO and TRACKING TRACK.

MACRO\_0 character\*12 name of the LCM object (type L\_MACROLIB) containing the MACROLIB information corresponding to the beginning-of-step conditions in case a ramp variation of the cross sections in set. Beginning-of-step conditions should not be confused with beginning-of-transient or initial conditions. By default, a step variation is set where cross sections are assumed constant and given by MACRO.

SYST\_0 character\*12 name of the LCM object (type L\_SYSTEM) corresponding to MACROLIB MACRO\_0 and TRACKING TRACK.

(kinsol\_data) structure containing the data to module KINSOL: (see Sect. 1.15.4).

# 1.15.4 Data input for module KINSOL:

Table 31: Structure (kinsol\_data)

```
[ EDIT iprint ]
DELTA delta
SCHEME FLUX [ TEXP ] { IMPLIC | CRANK | THETA ttflx }
PREC { IMPLIC | CRANK | EXPON | THETA ttprc }
[ { VAR1 | ACCE } icl1 icl2 ]
[ EXTE [ maxout ] [ epsout ] ]
[ THER [ maxthr ] [ epsthr ] ]
[ ADI nadi ]
[ ADJ ]
[ PICK >> power_out << ]
;</pre>
```

where

EDIT keyword used to set *iprint* index.

iprint integer index used to control the printing in module KINSOL:. =0 for no print; =1 for minimum printing (default value); larger values of iprint will produce increasing amounts of output.

DELTA keyword used to set the delta value.

delta current time increment  $\Delta t_n$  of transient.

SCHEME keyword used to indicate the temporal numerical schemes.

TEXP keyword used to enable the exponential transformation procedure on transient flux. Mixtureand group-dependent factors  $\omega_{m,g}$  are set such that the flux at point r is defined as

$$\phi_a(\mathbf{r},t) = e^{\omega_{m,g}t} \tilde{\phi}_a(\mathbf{r},t) \tag{1.19}$$

where m is the mixture index corresponding to point r. Factors  $\omega_{m,g}$  are initialized to zero by module INIKIN: and are recomputed at the end of each time step.

FLUX keyword used to select the temporal scheme for the fluxes equations.

PREC keyword used to select the temporal scheme for the precursors equations.

IMPLIC keyword used to indicate the full implicit temporal scheme.

CRANK keyword used to indicate the Crank-Nicholson temporal scheme.

EXPON keyword used to indicate the analytical integration scheme for precursors equations.

THETA keyword used to indicate the general temporal scheme according to the theta method.

ttflx value of theta parameter  $\Theta_f$  for the flux equations. This value should be greater than 0.5 and less than 1.0.

ttprc value of theta parameter  $\Theta_p$  for the precursors equations. This value should be greater than 0.5 and less than 1.0.

VAR1 keyword used to switch on the variational acceleration technique and to set the parameters icl1 and icl2.

ACCE alias keyword for VAR1.

icl1 number of free outer iterations in a cycle of the variational acceleration technique. The default value is icl1 = 3.

icl2 number of accelerated outer iterations in a cycle of the variational acceleration technique. The default value is icl2 = 3. A convergence in free iterations is obtained by setting icl1 = 200 (or icl1 = maxout) and icl2 = 0.

EXTE keyword to specify that the control parameters for the external iteration are to be modified.

maxout maximum number of external iterations. The fixed default value is maxout = 200.

epsout convergence criterion for the external iterations. The fixed default value is epsout =  $1.0 \times 10^{-4}$ . The outer iterations are stopped when the following criteria is reached:

$$\max_{i} |\Phi_{i}^{(k-1)} - \Phi_{i}^{(k)}| \leq epsout \times \max_{i} |\Phi_{i}^{(k)}|$$

where  $\vec{\Phi}^{(k)} = \text{col}\{\Phi_i^{(k)}; i = 1, I\}$  is the product of the B matrix times the unknown vector at the k-th outer iteration.

THER keyword to specify that the control parameters for the thermal iterations are to be modified.

maxthr maximum number of thermal iterations. The fixed default value is maxthr = 0 corresponding to no thermal iterations.

epsthr convergence criterion for the thermal iterations. The fixed default value is  $epsthr = 1.0 \times 10^{-2}$ .

ADI keyword used to set nadi in cases where Trivac is used.

number of alternating direction implicit (ADI) inner iterations per outer iteration. The default value is nadi = 1. If this value causes a failure of the acceleration process, it is recommended that a larger value be tried. The optimal choice is generally the minimum value of nadi which allows a convergence in less than 75 outer iterations. nadi = 1 or nadi = 2 is generally the best choice for production-type calculations. The greater nadi is, the smaller the asymptotic convergence constant (ACC) becomes. Taking an arbitrary large value (e.g., nadi = 20) leads to numerical results identical to those obtained by inverting the system matrices at each outer iteration (at a prohibitive CPU cost). In this case, the ACC is almost equal to the dominance ratio of the iterative matrix. The default value is recovered in the state vector of the TRACKING object TRACK.

ADJ keyword used to perform an adjoint (backward) space-time kinetics calculation. By default, a direct (forward) space-time kinetics calculation is performed.

PICK keyword used to recover the end-of-stage power (in MW) in a CLE-2000 variable.

power\_out character\*12 CLE-2000 variable name in which the extracted power value will be placed.

#### 1.16 The VAL: module

The VAL: module supplies an interpolation of the flux in diffusion calculations for Cartesian geometries. The calling specifications are:

Table 32: Structure (VAL:)

```
IFLU := VAL: TRKNAM FLUNAM :: (descval)
```

where

IFLU character\*12 name of the INTERPFLUX data structure (L\_FVIEW signature) where the

interpolated flux distribution will be stored.

TRKNAM character\*12 name of the read-only TRACKING data structure (L\_TRACK signature)

containing the tracking.

FLUNAM character\*12 name of the read-only FLUXUNK data structure (L\_FLUX signature) con-

taining a transport solution.

(descval) structure containing the input data to this module to compute interpolated flux (see

Section 1.16.1).

# 1.16.1 Data input for module VAL:

Table 33: Structure (descval)

where

EDIT keyword used to modify the print level *iprint*.

iprint integer index used to control the printing in module VAL:. =0 for no print; =1 for min-

imum printing (default value); larger values of iprint will produce increasing amounts

of output.

MODE keyword to specify the flux harmonic index imode.

imode index of the flux harmonic recovered by the VAL: module if the MONI keyword was set

in module FLUD: (see Sect. 1.18.1). By default, it is assumed that the MONI keyword

was not used.

POWR keyword used to set power.

power value of the power in MW used to normalize the flux. By default, the flux is not

normalized.

DIM keyword to specify the number dim.

dim number of dimension of the geometry.

dxyz mesh interval along each direction which is used to define the grid where the flux is

interpolated.

## 1.17 The NSST: module

The NSST: module is used to perform a TRACKING for the nodal expansion method (NEM). [19]

The nodal expansion method is based on an expansion of the transverse integrated flux in terms of polynomials defined over the (-0.5, 0.5) interval:

$$P_{0}(u) = 1$$

$$P_{1}(u) = u$$

$$P_{2}(u) = u^{2} - \frac{1}{12}$$

$$P_{3}(u) = \left(u^{2} - \frac{1}{4}\right)u$$

$$P_{4}(u) = \left(u^{2} - \frac{1}{4}\right)\left(u^{2} - \frac{1}{20}\right)$$
(1.20)

There is the option of using hyperbolic functions in some energy groups:

$$P_3(u) = \sinh(\zeta_g u)$$

$$P_4(u) = \cosh(\zeta_g u) - \frac{2}{\zeta} \sinh(\zeta_g/2)$$
(1.21)

where

$$\zeta_g = \Delta x \sqrt{\frac{\Sigma_{r,g}}{D_g}} \tag{1.22}$$

where  $\Delta x$ ,  $\Sigma_{r,g}$  and  $D_g$  are the node width (cm), the macroscopic removal cross section (cm<sup>-1</sup>) and the diffusion coefficient (cm) in group g, respectively.

The calling specifications are:

Table 34: Structure (NSST:)

```
TRACK := NSST: [TRACK] GEOM :: (NSST_data)
```

where

TRACK character\*12 of the LCM object (type L\_TRIVAC) containing the TRACKING information. If TRACK appears on the RHS, the previous settings will be applied by default.

GEOM character\*12 of the LCM object (type L\_GEOM) containing the geometry.

(NSST\_data) structure containing the data to module NSST: (see Sect. 1.17.1).

# 1.17.1 Data input for module NSST:

Table 35: Structure (NSST\_data)

```
[ EDIT iprint ]
[ TITL TITLE ]
[ HYPE igmax ]
[ LUMP ]
;
```

where

EDIT keyword used to set *iprint*.

iprint index used to control the printing in module NSST:. =0 for no print; =1 for minimum

printing (default value); Larger values produce increasing amounts of output.

TITL keyword which allows the run title to be set.

TITLE the title associated with a nodal expansion method run. This title may contain up to 72

characters. The default when TITL is not specified is no title.

HYPE keyword used to specify the type of nodal expansion base functions used to represent the

transverse integrated flux. methods. By default, the polynomial base (1.20) is used.

igmax hyperbolic base functions (1.21) are used for energy groups with indices  $\geq$  igmax.

LUMP keyword used to include only the averaged nodal fluxes in the unknown vector. By default,

the nodal expansion coefficients are included.

# 1.18 The NSSF: module

The NSSF: module is used to compute the solution to an eigenvalue problem corresponding to a nodal expansion method (NEM) discretization. The actual implementation is limited to 1D Cartesian geometries. The calling specifications are:

Table 36: Structure (NSSF:)

```
FLUX := NSSF: TRACK MACRO :: (NSSF_data)
```

where

FLUX character\*12 name of the LCM object (type L\_FLUX) containing the solution.

TRACK character\*12 name of the LCM object (type L\_TRACK) containing the TRACKING.

MACRO character\*12 name of the LCM object (type L\_MACROLIB) containing the cross sections,

diffusion coefficients and discontinuity factors.

(NSSF\_data) structure containing the data to module NSSF: (see Sect. 1.18.1).

#### 1.18.1 Data input for module NSSF:

Table 37: Structure (NSSF\_data)

```
[EDIT iprint]
[EXTE [maxout] [epsout]]
[NODF]
[BUCK valb2];
```

where

 ${\tt EDIT} \qquad \qquad {\tt keyword} \ {\tt used} \ {\tt to} \ {\tt set} \ {\it iprint}.$ 

iprint index used to control the printing in module NSSF:. =0 for no print; =1 for minimum

printing (default value).

EXTE keyword to specify that the control parameters for the external iteration are to be modified.

maxout maximum number of external iterations. The fixed default value is maxout = 1000.

epsout convergence criterion for the external iterations. The fixed default value is epsout =  $1.0 \times$ 

 $10^{-6}$ .

NODF keyword used to force discontinuity factors to one.

BUCK keyword used to specify the fixed buckling. By default,  $valb2 = 0 \text{ cm}^{-2}$ 

valb2 value of the fixed total buckling in  $cm^{-2}$ .

# 2 EXAMPLES OF INPUT DATA FILES

# 2.1 IAEA-2D benchmark

The IAEA-2D benchmark is defined in Refs. 3,20 and its geometry is represented in Fig. 11. Here, it is solved using a parabolic variational collocation method without mesh splitting of the elements:

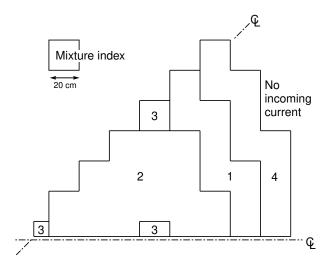


Figure 11: Description of the IAEA-2D benchmark.

```
LINKED_LIST IAEA MACRO TRACK SYSTEM FLUX EDIT ;
MODULE GEO: MAC: TRIVAT: TRIVAA: FLUD: OUT: END: ;
IAEA := GEO: :: CAR2D 9 9
           EDIT 2
           X- DIAG X+ VOID
           Y- SYME Y+ DIAG
           MIX 3 2 2 2 3 2 2 1 4
                  2 2 2 2 2 2 1 4
                    2 2 2 2 1 1 4
                      2 2 2 1 4 4
                        3 1 1 4 0
                          1 4 4 0
                            4 0 0
                              0 0
           MESHX 0.0 20.0 40.0 60.0 80.0 100.0 120.0 140.0 160.0 180.0
MACRO := MAC: ::
 EDIT 2 NGRO 2 NMIX 4
 READ INPUT
 MIX
         1
      DIFF 1.500E+00 4.0000E-01
     TOTAL 3.012E-02 8.0032E-02
    NUSIGF
           0.000E+00 1.3500E-01
 H-FACTOR 0.000E+00 1.3500E-01
      SCAT
           1 1 0.0 2 2 0.0 0.2E-01
 XIM
         2
      DIFF 1.500E+00 4.0000E-01
```

```
TOTAL 3.012E-02 8.5032E-02
   NUSIGF 0.000E+00 1.3500E-01
 H-FACTOR 0.000E+00 1.3500E-01
     SCAT 1 1 0.0 2 2 0.0 0.2E-01
XIM
        3
     DIFF 1.500E+00 4.00000E-01
    TOTAL 3.012E-02 1.30032E-01
   NUSIGF 0.000E+00 1.35000E-01
 H-FACTOR 0.000E+00 1.35000E-01
     SCAT 1 1 0.0 2 2 0.0 0.2E-01
XIM
     DIFF 2.000E+00 3.0000E-01
    TOTAL 4.016E-02 1.0024E-02
     SCAT 1 1 0.0 2 2 0.0 0.4E-01
TRACK := TRIVAT: IAEA ::
     TITLE 'IAEA-2D BENCHMARK'
     MAXR 81 PRIM 2;
SYSTEM := TRIVAA: MACRO TRACK :: ;
FLUX := FLUD: SYSTEM ::
     EDIT 2 ;
EDIT := OUT: FLUX ::
      EDIT 2 INTG
      1 2 3 4 5 6 7 8
         9 10 11 12 13 14 15 0
           16 17 18 19 20 21 0
              22 23 24 25 0
                 26 27 28 0
                             0
                    29 0 0 0
                          0 0
                        0
                             0
                             0
END: ;
```

# 2.2 Biblis-2D benchmark

The rods-withdrawn configuration of the Biblis-2D benchmark is defined in Ref. 3 and its geometry is represented in Fig. 12. Here, it is solved using a parabolic variational collocation method without mesh splitting of the elements:

```
LINKED_LIST BIBLIS MACRO TRACK SYSTEM FLUX EDIT;

MODULE GEO: MAC: TRIVAT: TRIVAA: FLUD: OUT: END:;

*

BIBLIS := GEO: :: CAR2D 9 9

EDIT 2

X- DIAG X+ VOID

Y- SYME Y+ DIAG

MIX 1 8 2 6 1 7 1 4 3

1 8 2 8 1 1 4 3

1 8 2 7 1 4 3

2 8 1 8 4 3

2 5 4 3 3

4 4 3 0

3 3 0

0 0
```

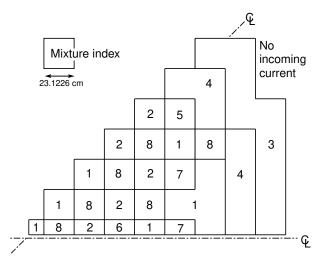


Figure 12: Description of the Biblis-2D benchmark, rods-withdrawn configuration.

```
0
      MESHX 0.0 23.1226 46.2452 69.3678 92.4904 115.613 138.7356
            161.8582 184.9808 208.1034
      ;
MACRO := MAC: ::
EDIT 2 NGRO 2 NMIX 8
READ INPUT
MIX
        1
     DIFF
           1.436000E+00 3.635000E-01
    TOTAL
          2.725820E-02 7.505800E-02
   NUSIGF
           5.870800E-03 9.606700E-02
 H-FACTOR
           2.376800E-03
                         3.889400E-02
           1 1 0.0 2 2 0.0 1.775400E-02
     SCAT
MIX
        2
     DIFF
           1.436600E+00 3.636000E-01
    TOTAL
           2.729950E-02 7.843600E-02
   NUSIGF
           6.190800E-03 1.035800E-01
 H-FACTOR
           2.506400E-03 4.193500E-02
     SCAT
           1 1 0.0 2 2 0.0 1.762100E-02
MIX
        3
     DIFF
           1.320000E+00 2.772000E-01
    TOTAL
           2.576220E-02 7.159600E-02
     SCAT
          1 1 0.0 2 2 0.0 2.310600E-02
MIX
        4
     DIFF
           1.438900E+00 3.638000E-01
    TOTAL
           2.746400E-02 9.140800E-02
   NUSIGF
           7.452700E-03
                         1.323600E-01
 H-FACTOR 3.017300E-03 5.358700E-02
     SCAT
           1 1 0.0 2 2 0.0 1.710100E-02
XIM
        5
     DIFF
           1.438100E+00 3.665000E-01
    TOTAL
          2.729300E-02 8.482800E-02
   NUSIGF
           6.190800E-03 1.035800E-01
 H-FACTOR 2.506400E-03 4.193500E-02
     SCAT
           1 1 0.0 2 2 0.0 1.729000E-02
MIX
        6
```

```
DIFF 1.438500E+00 3.665000E-01
    TOTAL 2.732400E-02 8.731400E-02
   NUSIGF 6.428500E-03 1.091100E-01
 H-FACTOR 2.602600E-03 4.417400E-02
    SCAT 1 1 0.0 2 2 0.0 1.719200E-02
MIX 7
    DIFF 1.438900E+00 3.679000E-01
    TOTAL 2.729000E-02 8.802400E-02
   NUSIGF 6.190800E-03 1.035800E-01
 H-FACTOR 2.506400E-03 4.193500E-02
    SCAT 1 1 0.0 2 2 0.0 1.712500E-02
MIX 8
    DIFF 1.439300E+00 3.680000E-01
    TOTAL 2.732100E-02 9.051000E-02
   NUSIGF 6.428500E-03 1.091100E-01
 H-FACTOR 2.602600E-03 4.417400E-02
     SCAT 1 1 0.0 2 2 0.0 1.702700E-02
TRACK := TRIVAT: BIBLIS ::
     TITLE 'BIBLIS BENCHMARK'
     EDIT 5 MAXR 81 PRIM 2;
SYSTEM := TRIVAA: MACRO TRACK ::
     EDIT 5;
FLUX := FLUD: SYSTEM ::
     EDIT 2 ;
EDIT := OUT: FLUX ::
      EDIT 2 INTG
      1 2 3 4 5 6 7 8 0
         9 10 11 12 13 14 15 0
          16 17 18 19 20 21 0
              22 23 24 25 26 0
                27 28 29 0 0
                   30 31 0 0
                       0 0 0
                         0 0
                            0
END: ;
```

#### 2.3 IAEA-3D benchmark

The IAEA-3D benchmark is defined in Ref. 20 and its geometry is represented in Fig. 13. Here, it is solved using a cubic mixed-dual method with mesh splitting of the second axial plane:

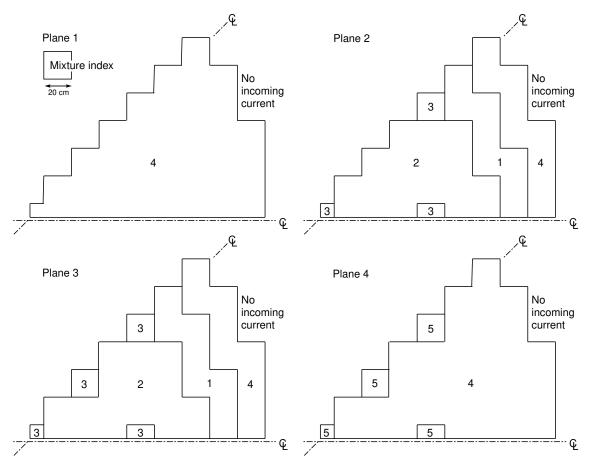


Figure 13: Description of the IAEA-3D benchmark.

```
LINKED_LIST IAEA3D MACRO TRACK SYSTEM FLUX EDIT ;
MODULE GEO: MAC: TRIVAT: TRIVAA: FLUD: OUT: END: ;
IAEA3D := GEO: :: CAR3D 9 9 4
          EDIT 2
          X- DIAG
                  X+ VOID
          Y- SYME
                  Y+ DIAG
          Z- VOID
                  Z+ VOID
          MESHX 0.0 20.0 40.0 60.0 80.0 100.0 120.0 140.0 160.0 180.0
          MESHZ 0.0 20.0 280.0 360.0 380.0
          SPLITZ 1 2 1 1
          (* PLANE NB 1 *)
          MIX 4 4 4 4 4 4 4 4 4
                4 4 4 4 4 4 4 4
                  4 4 4 4 4 4 4
                    4 4 4 4 4 4
                      4 4 4 4 0
                        4 4 4 0
                          4 0 0
```

```
0 0
                            0
             (* PLANE NB 2 *)
             3 2 2 2 3 2 2 1 4
               2 2 2 2 2 2 1 4
                 2 2 2 2 1 1 4
                   2 2 2 1 4 4
                     3 1 1 4 0
                       1 4 4 0
                         4 0 0
                           0 0
                            0
             (* PLANE NB 3 *)
             3 2 2 2 3 2 2 1 4
               2 2 2 2 2 2 1 4
                 3 2 2 2 1 1 4
                   2 2 2 1 4 4
                     3 1 1 4 0
                       1 4 4 0
                         4 0 0
                          0 0
             (* PLANE NB 4 *)
             5 4 4 4 5 4 4 4 4
               4 4 4 4 4 4 4 4
                 5 4 4 4 4 4 4
                   4 4 4 4 4 4
                     5 4 4 4 0
                       4 4 4 0
                         4 0 0
                          0 0
                            0
MACRO := MAC: ::
EDIT 2 NGRO 2 NMIX 5
READ INPUT
MIX 1
     DIFF 1.500E+00 4.0000E-01
    TOTAL 3.000E-02 8.0000E-02
   NUSIGF 0.000E+00 1.3500E-01
 H-FACTOR 0.000E+00 1.3500E-01
     SCAT 1 1 0.0 2 2 0.0 0.2E-01
XIM
     2
     DIFF 1.500E+00 4.0000E-01
    TOTAL 3.000E-02 8.5000E-02
   NUSIGF 0.000E+00 1.3500E-01
 H-FACTOR 0.000E+00 1.3500E-01
     SCAT 1 1 0.0 2 2 0.0 0.2E-01
XIM
     3
     DIFF 1.500E+00 4.00000E-01
    TOTAL 3.000E-02 1.30000E-01
   NUSIGF 0.000E+00 1.35000E-01
 H-FACTOR 0.000E+00 1.35000E-01
     SCAT 1 1 0.0 2 2 0.0 0.2E-01
MIX 4
```

```
DIFF 2.000E+00 3.0000E-01
    TOTAL 4.000E-02 1.0000E-02
    SCAT 1 1 0.0 2 2 0.0 0.4E-01
MIX 5
    DIFF 2.000E+00 3.0000E-01
    TOTAL 4.000E-02 5.5000E-02
    SCAT 1 1 0.0 2 2 0.0 0.4E-01
TRACK := TRIVAT: IAEA3D ::
     TITLE 'TEST IAEA 3D'
     EDIT 5 MAXR 405 DUAL 3 1;
SYSTEM := TRIVAA: MACRO TRACK ::
     EDIT 5;
FLUX := FLUD: SYSTEM ::
     EDIT 2 ;
EDIT := OUT: FLUX ::
     EDIT 2 INTG
      (* PLANE NB 1 *)
      0 0 0 0 0 0 0 0
        0 0 0 0 0 0 0 0
           0 0 0 0 0 0 0
              0 0 0 0 0
                0 0 0 0 0
                   0 0 0 0
                     0 0 0
                        0 0
                           0
      (* PLANE NB 2 *)
      1 2 3 4 5 6 7 8 0
        9 10 11 12 13 14 15 0
          16 17 18 19 20 21 0
             22 23 24 25 0 0
               26 27 28 0 0
                  29 0 0 0
                     0 0 0
                        0 0
      (* PLANE NB 3 *)
      30 31 32 33 34 35 36 37 0
        38 39 40 41 42 43 44 0
           45 46 47 48 49 50 0
             51 52 53 54 0 0
                55 56 57 0 0
                   58 0 0 0
                      0 0 0
                         0 0
      (* PLANE NB 4 *)
      0 0 0 0 0 0 0 0
        0 0 0 0 0 0 0
           0 0 0 0 0 0 0
              0 0 0 0 0
                0 0 0 0 0
                   0 0 0 0
                     0 0 0
```

```
0 0
0
;
END: ;
```

# 2.4 S30 hexagonal benchmark in 2-D

The S30 hexagonal benchmark in 2-D is defined in Ref. 15. Its geometry is represented in Fig. 14. Here, it is solved using a mesh centered finite difference method without mesh splitting of the hexagonal elements:

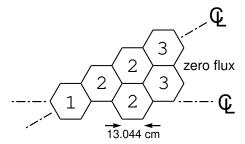


Figure 14: Description of the S30 hexagonal benchmark.

```
LINKED_LIST HEX MACRO TRACK SYSTEM FLUX EDIT ;
MODULE GEO: MAC: TRIVAT: TRIVAA: FLUD: OUT: END: ;
HEX := GEO: :: HEX
                     6
       EDIT 2
       HBC S30 ZERO
       SIDE 13.044
       SPLITH 0
       MIX
       1
       2
       2
         2
       3
         3
MACRO := MAC: ::
 EDIT 2 NGRO 2 NMIX 3
READ INPUT
 XIM
      DIFF
            1.5E+00
                    4.00E-01
     TOTAL
            3.0E-02
                     1.30E-01
    NUSIGF
            0.0E+00
                    1.35E-01
  H-FACTOR
            0.0E+00 1.35E-01
            1 1 0.0 2 2 0.0 0.2E-01
      SCAT
 MIX
         2
     DIFF
                     4.00E-01
            1.5E+00
     TOTAL
            3.0E-02
                     8.50E-02
    NUSIGF
            0.0E+00
                     1.35E-01
           0.0E+00 1.35E-01
  H-FACTOR
      SCAT
            1 1 0.0 2 2 0.0 0.2E-01
 XIM
         3
      DIFF
            2.0E+00 3.0E-01
     TOTAL 4.0E-02 1.0E-02
```

```
SCAT 1 1 0.0 2 2 0.0 0.4E-01;

TRACK := TRIVAT: HEX ::
    TITLE 'S30 HEXAGONAL BENCHMARK IN 2-D.'
    EDIT 5 MAXR 50 MCFD (* IELEM= *) 1;

SYSTEM := TRIVAA: MACRO TRACK ::
    EDIT 5;

FLUX := FLUD: SYSTEM ::
    EDIT 2;

EDIT := OUT: FLUX ::
    EDIT 2 INTG IN;

END:;
```

#### 2.5 LMW benchmark in 2-D

The LMW benchmark in 2-D is a space-time kinetics problem introduced by Greenman<sup>[21]</sup> and used by Monier<sup>[14]</sup>. Its geometry is represented in Fig. 15. Here, it is solved using a parabolic nodal collocation method with  $2 \times 2$  mesh splitting of each element. A reactivity transient is induced by the rapid withdrawal of the control rod in material mixture 6. The control rod is removed in 26.7 s, causing a negative ramp variation in total cross section.

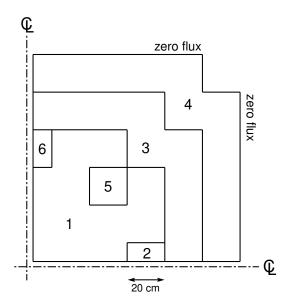


Figure 15: Description of the LMW benchmark in 2-D.

```
* TEST CASE LMW 2D

* REF: G. Greenman, "A Quasi-Static Flux Synthesis Temporal Integration

* Scheme for an Analytic Nodal Method," Nuclear Engineer's Thesis,

* Massachusetts Institute of Technology, Department of Nuclear

* Engineering (May 1980).

*

*----

* Define STRUCTURES and MODULES used

*----

LINKED_LIST LMW TRACK MACRO1 SYSTEM1 MACRO2 SYSTEM2 FLUX KINET;

MODULE GEO: MAC: TRIVAT: TRIVAA: FLUD: INIKIN: KINSOL: GREP: DELETE:
```

```
END: ;
REAL fnorm sigt1 sigt2;
REAL TIME := 0.0;
PROCEDURE assertS assertS2;
LMW := GEO: :: CAR2D 6 6
     X- REFL X+ ZERO
     Y- REFL Y+ ZERO
     MIX 1 1 1 2 3 4
         1 1 1 1 3 4
         1 1 5 1 3 4
         6 1 1 3 3 4
         3 3 3 3 4 4
         4 4 4 4 4 0
     MESHX 0.0 10. 30. 50. 70. 90. 110.
     MESHY 0.0 10. 30. 50. 70. 90. 110.
     SPLITX 2 2 2 2 2 2
     SPLITY 2 2 2 2 2 2
MACRO1 := MAC: ::
EDIT O NGRO 2 NMIX 6
READ INPUT
     1
XIM
     DIFF 1.423910E+00 3.563060E-01
    TOTAL 2.795756E-02 8.766216E-02
   NUSIGF 6.477691E-03 1.127328E-01
 H-FACTOR 2.591070E-03 4.509310E-02
     SCAT 1 1 0.0 2 2 0.0 0.175555E-01
     OVERV 0.800E-07 4.000E-06
XIM
       2
     DIFF 1.423910E+00 3.563060E-01
    TOTAL 2.850756E-02 9.146219E-02
   NUSIGF 6.477691E-03 1.127328E-01
 H-FACTOR 2.591070E-03 4.509310E-02
     SCAT 1 1 0.0 2 2 0.0 0.175555E-01
     OVERV 0.800E-07 4.000E-06
MIX
       3
     DIFF 1.425610E+00 3.505740E-01
    TOTAL 2.817031E-02 9.925634E-02
   NUSIGF 7.503282E-03 1.378004E-01
 H-FACTOR 3.001310E-03 5.512106E-02
     SCAT 1 1 0.0 2 2 0.0 0.171777E-01
     OVERV 0.800E-07 4.000E-06
MIX
      4
     DIFF 1.634220E+00 2.640020E-01
    TOTAL 3.025750E-02 4.936351E-02
     SCAT 1 1 0.0 2 2 0.0 0.275969E-01
     OVERV 0.800E-07 4.000E-06
MIX
      5
     DIFF 1.423910E+00 3.563060E-01
    TOTAL 2.795756E-02 8.766216E-02
   NUSIGF 6.477691E-03 1.127328E-01
 H-FACTOR 2.591070E-03 4.509310E-02
     SCAT 1 1 0.0 2 2 0.0 0.175555E-01
     OVERV 0.800E-07 4.000E-06
```

```
XIM
     DIFF 1.423910E+00 3.563060E-01
    TOTAL 2.850756E-02 9.146217E-02
   NUSIGF 6.477691E-03 1.127328E-01
  H-FACTOR 2.591070E-03 4.509310E-02
     SCAT 1 1 0.0 2 2 0.0 0.175555E-01
     OVERV 0.800E-07 4.000E-06
TRACK := TRIVAT: LMW ::
     TITLE 'LMW 2-D BENCHMARK'
     EDIT 1 MAXR 144 MCFD 2;
SYSTEM1 := TRIVAA: MACRO1 TRACK ::
     EDIT 1 UNIT ;
FLUX := FLUD: SYSTEM1 TRACK ::
     EDIT 1 EXTE 5.0E-7;
assertS FLUX :: 'K-EFFECTIVE' 1 1.014803;
* Crank-Nicholson space-time kinetics
EVALUATE TIME := 0.0;
KINET := INIKIN: MACRO1 TRACK SYSTEM1 FLUX :: EDIT 1
           0.000247 0.0013845 0.001222 0.0026455 0.000832 0.000169
     BETA
     LAMBDA 0.0127 0.0317 0.115 0.311 1.40 3.87
     CHID 1.0 1.0 1.0 1.0 1.0 1.0
            0.0 0.0 0.0 0.0 0.0 0.0
     NORM POWER-INI 1.0E4;
EVALUATE sigt1 := 2.850756E-02;
EVALUATE sigt2 := 9.146217E-02 ;
WHILE TIME 26.7 <= DO
  EVALUATE sigt1 := sigt1 5.5E-4 0.1 26.7 / * - ;
  EVALUATE sigt2 := sigt2 3.8E-3 0.1 26.7 / * - ;
  MACRO2 := MAC: MACRO1 ::
     EDIT 0
     READ INPUT
     MIX 6
        TOTAL <<sigt1>> <<sigt2>>
  SYSTEM2 := TRIVAA: MACRO2 TRACK ::
     EDIT 1 UNIT;
  KINET := KINSOL: KINET MACRO2 TRACK SYSTEM2 MACRO1 SYSTEM1 ::
     EDIT 5 DELTA 0.1
     SCHEME FLUX CRANK PREC CRANK EXTE 1.0E-6;
  GREP: KINET :: GETVAL 'TOTAL-TIME' 1 >>TIME<< ;</pre>
  ECHO "TIME=" TIME "S" "sigt=" sigt1 sigt2;
  IF TIME 1.0 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 1.986270E+02;
   assertS2 KINET :: 'CTRL-PREC' 1 1.095509E-01;
   assertS2 KINET :: 'E-POW'
                                1 1.008753E+04;
  ELSEIF TIME 5.0 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 2.090369E+02 ;
   assertS2 KINET :: 'CTRL-PREC' 1 1.097266E-01;
   assertS2 KINET :: 'E-POW' 1 1.063990E+04;
  ELSEIF TIME 10.0 - ABS 1.0E-3 < THEN
    assertS2 KINET :: 'CTRL-FLUX' 1 2.305455E+02;
```

```
assertS2 KINET :: 'CTRL-PREC' 1 1.104699E-01;
   assertS2 KINET :: 'E-POW' 1 1.176902E+04;
 ELSEIF TIME 15.0 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 2.641221E+02;
   assertS2 KINET :: 'CTRL-PREC' 1 1.121002E-01;
   assertS2 KINET :: 'E-POW' 1 1.352433E+04;
 ELSEIF TIME 20.0 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 3.157370E+02 ;
   assertS2 KINET :: 'CTRL-PREC' 1 1.150681E-01;
   assertS2 KINET :: 'E-POW' 1 1.621938E+04;
 ELSEIF TIME 25.0 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 3.971426E+02;
   assertS2 KINET :: 'CTRL-PREC' 1 1.200883E-01;
   assertS2 KINET :: 'E-POW' 1 2.047011E+04;
 ELSEIF TIME 26.7 - ABS 1.0E-3 < THEN
   assertS2 KINET :: 'CTRL-FLUX' 1 4.351272E+02;
   assertS2 KINET :: 'CTRL-PREC' 1 1.224600E-01;
   assertS2 KINET :: 'E-POW' 1 2.245449E+04;
 ENDIF ;
 MACRO1 SYSTEM1 := DELETE: MACRO1 SYSTEM1 ;
 MACRO1 := MACRO2 ;
 SYSTEM1 := SYSTEM2 ;
 MACRO2 SYSTEM2 := DELETE: MACRO2 SYSTEM2 ;
ENDWHILE ;
ECHO "test lmw2D completed" ;
```

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