ESKF模型

状态量： $x=\begin{bmatrix} q \\ w_b \end{bmatrix}$

Ture-state:

$$\begin{cases} \dot{q_t}=\frac{1}{2}q_t\otimes(w_m-w_{bt}-w_n) \\ \dot{w_{bt}}=w_{bn} \end{cases} \quad (140)$$

nominal-state:

$$\begin{cases} \dot{q}\ =\frac{1}{2}q\otimes(w_m-w_b) \\ \dot{w_b}\ =0 \end{cases} \quad (142)$$

取： $q_t=q\otimes\delta q \qquad \delta q\approx\begin{bmatrix} 1 \\ \delta\,\theta/2 \end{bmatrix} \qquad w_{bt}=w_b+\delta w_b \qquad (Table2)$

得到误差状态error-state的运动模型微分形式：

$$\begin{cases} \delta\dot{\theta}=-[w_m-w_b]_\times\cdot\delta\theta-\delta w_b-w_n \\ \delta\dot{w_b}=w_{bn} \end{cases} \quad (143)$$

得到误差状态的运动模型递推表达式：

$$\begin{cases} \delta\theta\leftarrow R\{(w_m-w_b)\Delta t\}^T\cdot\delta\theta-\delta w_b\cdot\Delta t-\theta_i \\ \delta w_b\leftarrow\delta w_b+w_i \end{cases} \quad (166)$$

其中：

$$\begin{cases} (w_m-w_b)\Delta t\triangleq u\Delta\theta \\ R\{(w_m-w_b)\}^T=R\{-u\Delta\theta\}=I-[u]_\times\sin\Delta\theta+[u]_\times^2(1-\cos\Delta\theta) \end{cases}$$

其中 θ_i w_i 为高斯随机脉冲噪声，均值为0，协方差为 w_n w_{bn} 在 Δt 时间内的积分值

$$\begin{cases} \Theta_i=\sigma_{w_n}^2\Delta t^2 I \\ \Omega_i=\sigma_{w_{bn}}^2\Delta t^2 I \end{cases}$$

ESKF预测方程

$$\begin{cases} \hat{\delta x} \leftarrow F_x \cdot \hat{\delta x} \\ P \leftarrow F_x P F_x^T + F_i Q_i F_x^T \end{cases}$$

对状态变量求偏导：

$$F_x = \begin{bmatrix} R\{(w_m - w_b) \cdot \Delta t\}^T & -I \cdot \Delta t \\ 0 & I \end{bmatrix}$$

对噪声变量求偏导：

$$F_i = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \qquad Q_i = \begin{bmatrix} \Theta_i & 0 \\ 0 & \Omega_i \end{bmatrix}$$

ESKF观测方程

$$y = h(x_t) + v \qquad v \sim N\{0, V\}$$

$$\begin{cases} K = PH^T(HPH^T + V)^{-1} \\ \hat{\delta x} \leftarrow K\left(y - h\left(\hat{x}_t\right)\right) \\ P \leftarrow P - K(HPH^T + V)K^T \end{cases}$$

$$R = \begin{bmatrix} q_w^2 + q_x^2 - q_y^2 - q_z^2 & 2(q_x q_y - q_w q_z) & 2(q_x q_z + q_w q_y) \\ 2(q_x q_y + q_w q_z) & q_w^2 - q_x^2 + q_y^2 - q_z^2 & 2(q_y q_z - q_w q_x) \\ 2(q_x q_z - q_w q_y) & 2(q_y q_z + q_w q_x) & q_w^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix}$$

将加速度计的三个测量作为观测变量：

$$\Rightarrow \quad a_m = R^T \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} 2(q_x q_z - q_w q_y) \\ 2(q_y q_z + q_w q_x) \\ q_w^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix} + v$$

ESKF观测方程

$$H \triangleq \frac{\partial h}{\partial \delta x}|_x = \frac{\partial h}{\partial x_t}|_x \frac{\partial x_t}{\partial \delta x}|_x = H_x X_{\delta x}$$

其中：

$$H_x = \begin{bmatrix} -2q_y & 2q_z & -2q_w & 2q_x & 0 & 0 & 0 \\ 2q_x & 2q_w & 2q_z & 2q_y & 0 & 0 & 0 \\ 2q_w & -2q_x & -2q_y & 2q_z & 0 & 0 & 0 \end{bmatrix}$$

$$X_{\delta x} \triangleq \frac{\partial x_t}{\partial \delta x}|_x = \begin{bmatrix} Q_{\delta\theta} & 0 \\ 0 & I_3 \end{bmatrix}$$

其中：

$$Q_{\delta\theta} \triangleq \frac{\partial (q \otimes \delta q)}{\partial \delta\theta}|_q = \frac{\partial (q \otimes \delta q)}{\partial \delta q}|_q \frac{\partial \delta q}{\partial \delta\theta}|_{\hat{\delta\theta}=0} = \frac{\partial ([q]_L \delta q)}{\partial \delta q}|_q \frac{\partial \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta \end{bmatrix}}{\partial \delta\theta}|_{\hat{\delta\theta}=0} = [q]_L \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_{\delta\theta} = \frac{1}{2} \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & -q_z & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \end{bmatrix}$$

- 添加error -state状态到nominal-state

$$x \leftarrow x \oplus \hat{\delta x} \quad \Rightarrow \quad \begin{cases} q \leftarrow q \otimes q\{\hat{\delta\theta}\} \\ w_b \leftarrow w_b + \delta\hat{w}_b \end{cases}$$

- 复位error -state

$$\delta x \leftarrow g(\delta x) = \delta x \ominus \hat{\delta x}$$

$$\begin{cases} \hat{\delta x} \leftarrow 0 \\ P \leftarrow G P G^T \end{cases}$$

$$G \triangleq \frac{\partial g}{\partial \delta x}\Big|_{\hat{\delta x}} = \begin{bmatrix} I - \left[\frac{1}{2}\hat{\delta\theta}\right] & 0 \\ 0 & I_3 \end{bmatrix}$$