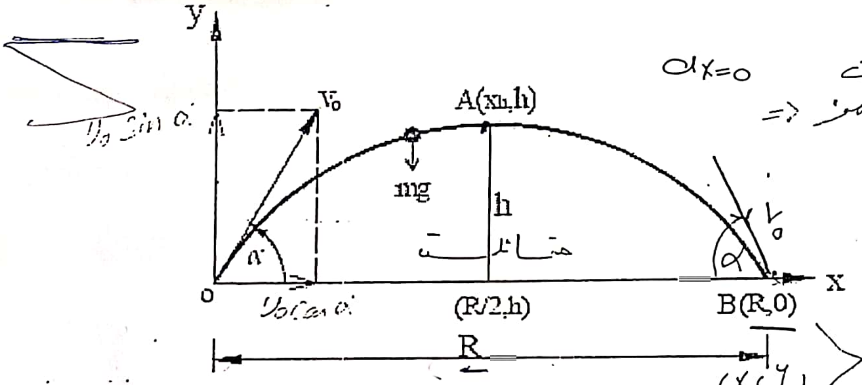
Projectiles Motion

حركة المقذوفات

The free-flight motion of a projectile is often studied in term of its rectangular components. Simply analyze if the following two assumptions are made:-

FIRST: - the free-fall acceleration $a_y = -g$ in y-direction is constant over the range of motion and its direction downwards.
SECOND:- the effect of air resistance is negligible, i.e. The horizontal acceleration $a_x = 0$



$a_x = 0$

ثابت V_x
متغير V_y

$y=0$ $x=R$

1- Velocity components

$$\boxed{v_x = v_o \cos\alpha} \qquad \boxed{v_y = v_o \sin\alpha - gt}$$

2- Position of the particle

$$\boxed{x = (v_o \cos\alpha)t} \qquad \boxed{y = (v_o \sin\alpha)t - \frac{1}{2}gt^2}$$

3- Path equation

$$\boxed{y = x\tan\alpha - \left(\frac{g x^2}{2v_0^2}\right)(1+\tan^2\alpha)}$$

4- Maximum height

V_y = zero

$$\boxed{\therefore h = \frac{v_0^2 \sin^2\alpha}{2g}}$$

at $v_y = 0$ $\quad U_0 \sin\alpha - gt = 0$

$t_h = \frac{U_0 \sin\alpha}{g}$

نعوض في

5- Range

=> at $y=0$

$$\boxed{R = \frac{v_0^2 \sin 2\alpha}{g} = 2x_h}$$

$y=0$ $\quad U_0 \sin\alpha (t) - \frac{1}{2}gt^2 = 0$

$t(U_0\sin\alpha - \frac{1}{2}gt) = 0$

$U_0 \sin\alpha - \frac{1}{2}gt = 0$

$t = \frac{2U_0 \sin\alpha}{g}$

6- Time of flight

$$\boxed{t_f = \frac{2v_0 \sin\alpha}{g} = 2t_h}$$

Max. Range at the horizontal plane occurs at $\alpha = 45^0$

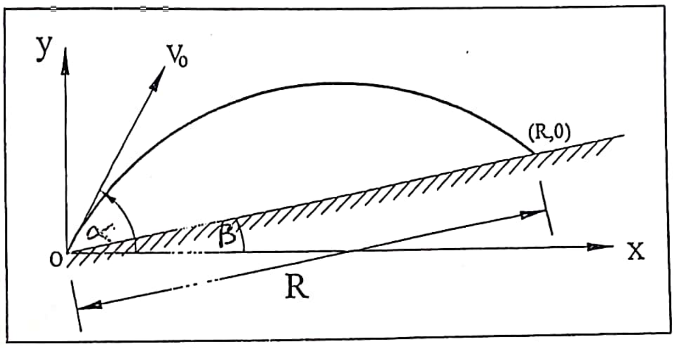
ojectile on inclined plane (upward plane)

$v_x = v_0 \cos(\alpha-\beta) - gt \sin\beta$

$v_y = v_0 \sin(\alpha-\beta) - gt \cos\beta$

$x = v_0 \cos(\alpha-\beta)\, t - \frac{1}{2} gt^2 \sin\beta$

$y = v_0 \sin(\alpha-\beta)\, t - \frac{1}{2} gt^2 \cos\beta$



$$R = \frac{2V_0^2 \cos\alpha\,(\sin(\alpha-\beta))}{g \cos^2\beta}$$

$$t = \frac{2V_0 \sin(\alpha-\beta)}{g \cos\beta}$$

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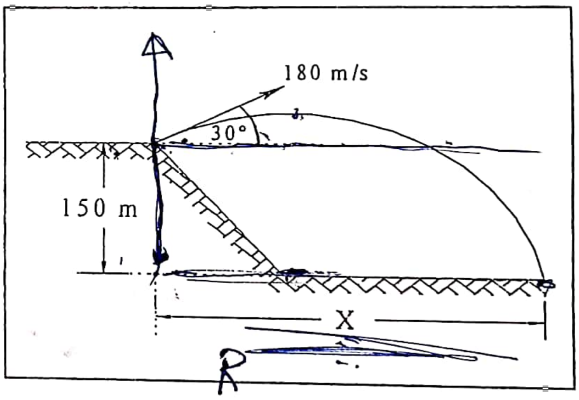
1) A projectile is fired from the edge of a **150** m cliff with an initial velocity of **180** m/s , at an angle of **30°** with the horizontal. Find

(a) The horizontal distance from the gun to the point where the projectile strikes the ground

(b) The greatest elevation above the ground.



$y = (V_0 \sin\alpha)t - \frac{1}{2} g t^2$

$-150 = (180 \sin 30°)t - \frac{1}{2}(9.8)t^2$

$\therefore t^2 - 18.36t - 30.61 = 0$

$\therefore \quad t = 20$ Sec $\qquad t = -1.5$ sec (xx)

$x = (V_0 \cos\alpha)t = (180 \cos 30°)(20) = 3100$ m #

$h = 150 + \frac{V_0^2 \sin^2\alpha}{2g} = 150 + \frac{(180)^2 \sin^2 30°}{2(9.8)} = 563.3$ m #

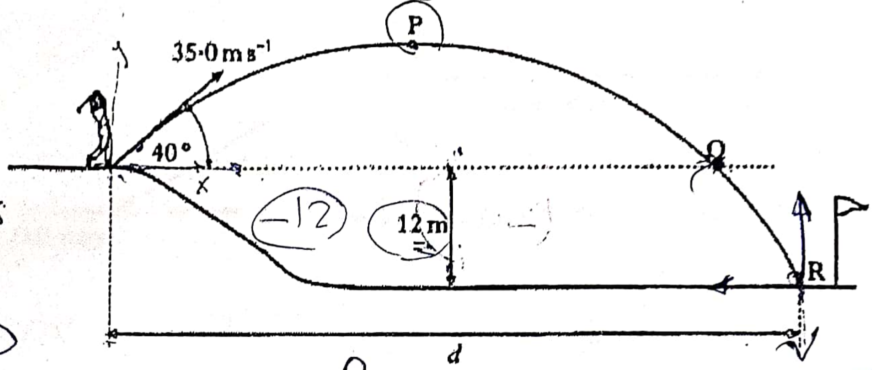
A golfer on an elevated tee hits a golf ball with an initial velocity of 35 m/s at an angle of 40° to the horizontal. The ball travels through the air and hits the ground at point R which is **12m** below the tee as shown

a) Calculate the horizontal and vertical components of the initial velocity of the ball

b) The time taken for the ball to reach its maximum height at point **P**.

c) Calculate the total horizontal distance **d** travelled by the ball.

d) Find the direction of the ball velocity just before it hit the ground at **R**



given $V_o = 35$ $\alpha = 40°$

a) $V_x = V_o \cos\alpha = 26.8$ m/s

$V_y = V_o \sin\alpha = 22.5$ m/s

b) at the max. height $V_y = 0$

$\therefore V_o \sin\alpha - gt = 0$

$$\therefore t = \frac{35 \sin 40°}{9.8} = 2.3 \text{ Sec. } \#$$

c) at Point (R) $x = d$ $y = -12$

$$y = (V_o \sin\alpha)t - \frac{1}{2} g t^2 \Rightarrow -12 = (35 \sin 40°)t - \frac{1}{2}(9.8)t^2$$

$\therefore t = 5$ sec $t = -0.48$ X X

$\therefore d = (V_o \cos\alpha)t$

$$d = (35 \cos 40°)(5) = 134 \text{ m. } \#$$

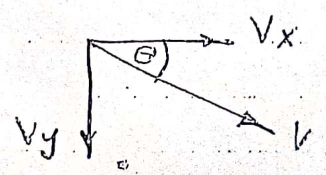
d) at Point (R) $t = 5$ Sec

$V_x = (V_o \cos\alpha) = 26.8$ m/Sec $\longrightarrow$

$V_y = V_o \sin\alpha - gt = -26.5$ m/s $\downarrow$

$$\therefore \tan\theta = \frac{+V_y}{+V_x} = 0.988 \qquad \theta = 44.6°$$

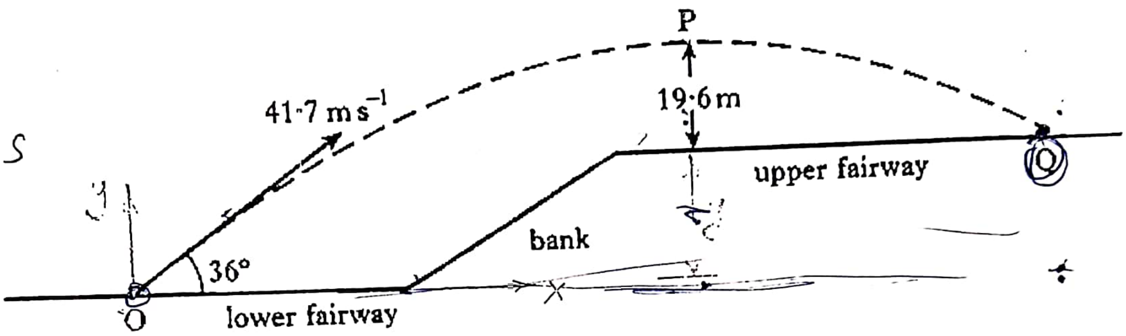
عن الزاوية بين المحصلة وال x



a golf ball at point O is given an initial velocity of **41.7** m/s at an angle **36º** to the horizontal. The ball reaches the max height at point p and then continues to hit the ground at point Q.

a) Find the time from O to Q
b) Calculate the horizontal distance travelled by the ball

$\alpha = 36^\circ$
$V_o = 41.7 \text{ m/s}$



max height $h = \dfrac{V_o^2 \sin^2\alpha}{2g} = \dfrac{(41.7)^2 \sin^2 36^\circ}{2(9.8)} = 30.65 \text{ m}$

elevation of upper Level $y = h - 19.6 = 11.05 \text{ m}$

$\therefore y = (V_o \sin\alpha)t - \frac{1}{2} g t^2 \Rightarrow 11.05 = (41.7 \sin 36)t - \frac{1}{2}(9.8)t^2$

$\therefore 4.9t^2 - 24.5t + 11.05 = 0$ $\qquad t = 4.5 \text{ sec}$, $t = 0.5 \text{ sec}$

to check the value of the time
زمن أقصى ارتفاع $t_h = \dfrac{V_o \sin\alpha}{g} = 2.5 \text{ sec}$

So take $t = 4.5 \text{ sec}$ #

$\therefore x = (V_o \cos\alpha)t = (41.7 \cos 36^\circ)(4.5) = 151.8^m$ #

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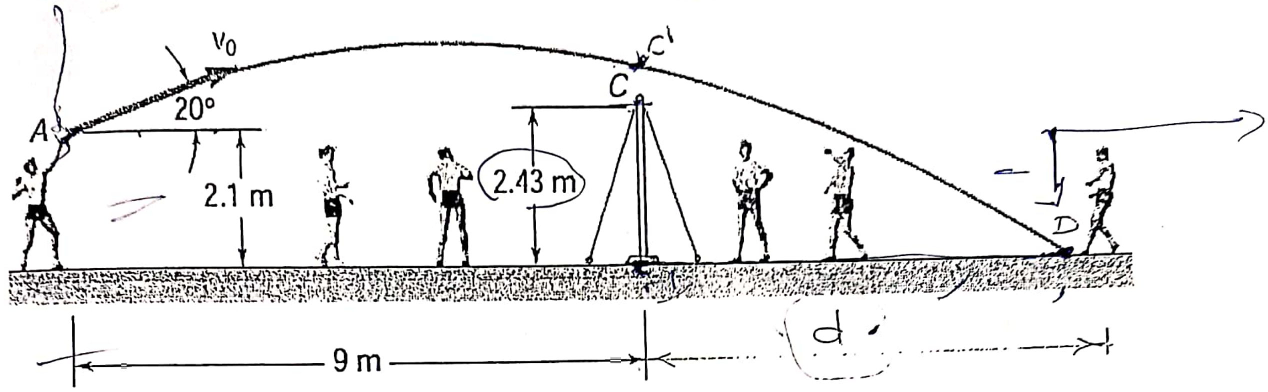
A volleyball player serves the ball with an initial velocity v_0 of magnitude **13.4**^{0} m/s at an angle of **20°** with the horizontal. Determine (**a**) if the ball will clear the top of the net, (**b**) how far from the net the ball will land.



at Point C' $\quad X = 9m \quad y = ?$

$$y = x\tan\alpha - \frac{gx^2}{2V_0^2}(1+\tan^2\alpha) = 9\tan 20^\circ - \frac{9.8\,(9)^2}{2\,(13.4)^2}(1+\tan^2 20^\circ)$$

$\therefore\ y = 0.766\ m$

$\therefore$ total height from the ground is $2.1 + 0.766 = 2.86\,m > 2.43\,m$

therefore, the ball will clear the top of the wall (net)

at Point (D) $\quad y = -2.1\,m \quad x = S$

$$y = x\tan\alpha - \frac{gx^2}{2V_0^2}(1+\tan^2\alpha)$$

$$-2.1 = S\tan 20^\circ - \frac{9.8\,S^2}{2\,(13.4)^2}(1+\tan^2 20^\circ)$$

$$S^2 - 11.77S - 67.95 = 0$$

$S = 16m \qquad S = -4.2\,m$ rejected

$\therefore\ d = S - 9m = 16 - 9 = 7m$ #

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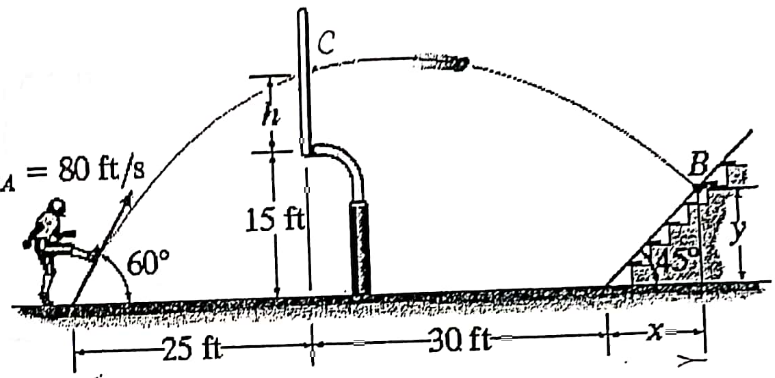
The football is to be kicked over the goalpost, which is 15 ft high. If it's initial speed is 15 ft/s,

a) Determine the point B (x, y) where it strikes the bleachers.

b) Find the direction of the ball velocity just before it hit the plane at point B .

$V_A = 80$ ft/s

$\alpha = 60°$



at Point (C) $x = 25$

$$y = x \tan\alpha - \frac{g x^2}{2V_o^2}(1+\tan^2\alpha)$$

$$y = 25 \tan 60° - \frac{32.2(25)}{2(80)^2}(1+\tan^2 60°)$$

$y = 37$ ft $\quad \therefore h = 37 - 15 = 22$ ft

at Point (B) $\quad x = y \quad$ due to angle 45° $\quad (x = y = S - 55)$

$$y = S \tan\alpha - \frac{g S^2}{2V_o^2}(1+\tan^2\alpha)$$

$$y = \sqrt{3}\, S - \frac{32.2 S^2}{2(80)^2}(1+\tan^2 60°) = \sqrt{3}\, S - 0.01 S^2$$

$\therefore \quad S - 55 = \sqrt{3}\, S - 0.01 S^2 \Rightarrow S = 119$ ft

$\therefore x = y = 119 - 55 = 64$ ft #

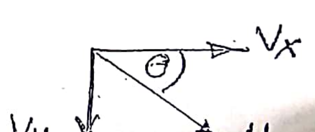
$\therefore x = (V_o \cos\alpha)t \Rightarrow 119 = (80 \cos 60)\, t$

$\therefore t = 2.97$ Sec #

$V_x = V_o \cos\alpha = 80 \cos 60° = 40$ ft/s

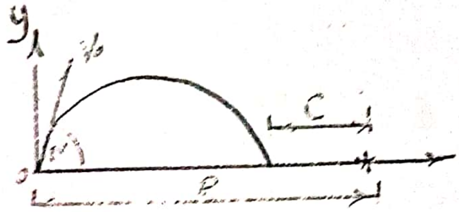
$V_y = V_o \sin\alpha - gt = 80 \sin 60° - 32.2(2.97) = -26.5$ ft/s

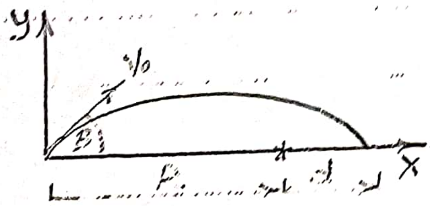
$\therefore \tan\theta = \frac{V_y}{V_x} = \frac{26.5}{40} = 0.66 \quad \theta = 33.5°$

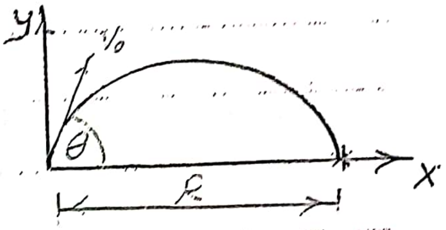


A Projectile aimed at a mark which is in a horizontal plane through the point of projection. The projectile fall at (C) meter short of the aim when the angle of projection be α and goes (d) meter too far when the angle of projection be β show that if the velocity of projection be the same in all cases the correct angle of project should be

$$\theta = \frac{1}{2}\sin^{-1}\left\{\frac{c\sin 2\beta + d\sin 2\alpha}{c+d}\right\}$$







$$R - c = \frac{V_0^2 \sin 2\alpha}{g} \qquad R + d = \frac{V_0^2 \sin 2\beta}{g} \qquad R = \frac{V_0^2 \sin 2\theta}{g} \quad (3)$$

$$Rd - cd = \frac{V_0^2}{g} d\sin 2\alpha \quad (I) \qquad Rc + cd = \frac{V_0^2}{g} c\sin 2\beta \quad (II)$$

adding I, II

$$R(c+d) = \frac{V_0^2}{g}[d\sin 2\alpha + c\sin 2\beta]$$

$$R = \frac{V_0^2}{g}\left[\frac{d\sin 2\alpha + c\sin 2\beta}{c+d}\right] \quad (4)$$

By equating (3), (4)

$$\therefore \frac{V_0^2}{g}\sin 2\theta = \frac{V_0^2}{g}\left[\frac{d\sin 2\alpha + c\sin 2\beta}{c+d}\right]$$

$$\therefore 2\theta = \sin^{-1}\frac{d\sin 2\alpha + c\sin 2\beta}{c+d}$$

$$\theta = \frac{1}{2}\sin^{-1}\left[\frac{d\sin 2\alpha + c\sin 2\beta}{c+d}\right] \quad \#$$

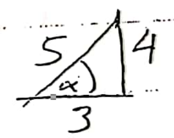
particle is projected from a certain point. It is noticed that its range on the orizontal plane which passes through the point of projection is equal to **three** times the **maximum height** above the point of projection, and the velocity after **two** seconds from the time of projection is equal to the velocity of projection Find the velocity of projection, also find the position of the projectile after **5** sec from the beginning of projection.

given $R=3h$ $\qquad V(t=2)=V_0$

$$\frac{V_0^2\sin 2\alpha}{g} = 3\,\frac{V_0^2\sin^2\alpha}{2g} \Rightarrow \sin 2\alpha = \frac{3}{2}\sin^2\alpha$$

$$2\sin\alpha\cos\alpha = \frac{3}{2}\sin^2\alpha \Rightarrow 4\cos\alpha = 3\sin\alpha$$

$$\therefore \tan\alpha = \frac{4}{3}$$



at $t=2$ $\quad V=V_0$ $\quad\therefore$ the flight time is 2 sec

$$2 = \frac{2V_0\sin\alpha}{g} \Rightarrow 2 = \frac{2V_0\left(\frac{4}{5}\right)}{32.2} \Rightarrow \boxed{V_0 = 40 \text{ ft/s}}$$

ft/sec

<u>at $t = 5$ sec</u>

$$x = (V_0\cos\alpha)t = 40\left(\frac{4}{5}\right)(5) = 120 \text{ ft}$$

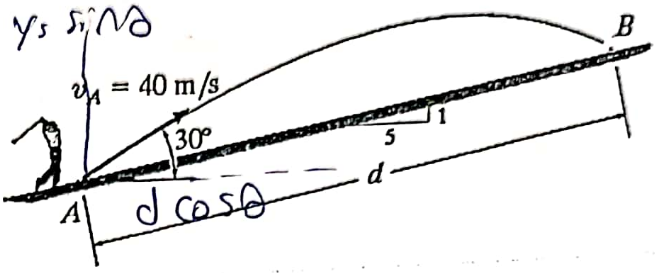
$$y = (V_0\sin\alpha)t - \frac{1}{2}gt^2 = 40\left(\frac{4}{5}\right)(5) - \frac{1}{2}(32.2)(5)^2$$

$$= -240 \text{ ft}$$

The golf ball is hit at A with a speed of and directed at an angle of with the horizontal as shown.

a) Determine the distance d where the ball strikes the slope at B.

b) Find the direction of the ball velocity just before it hit the plane at point B



at Point (B)

$$x = d\cos\theta = \frac{5}{\sqrt{26}}d$$
$$y = d\sin\theta = \frac{1}{\sqrt{26}}d$$

$$y = x\tan\alpha - \frac{gx^2}{2V_o^2}(1+\tan^2\alpha)$$

$$\frac{d}{\sqrt{26}} = \frac{5}{\sqrt{26}}d\tan 30^\circ - \frac{9.8\left(\frac{25}{26}\right)x^2}{2(40)^2}(1+\tan^2 30^\circ)$$

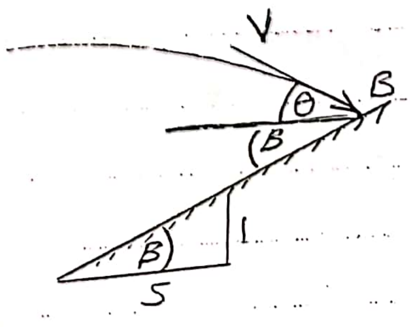
$$1 = \frac{5}{\sqrt{3}} - 0.02d \Rightarrow d = 94.4\text{ m}\ \#$$

$$x = (V_o\cos\alpha)t \quad \therefore \quad \frac{5}{\sqrt{26}} = (40\cos 30^\circ)t$$

$$\therefore t = 2.67\text{ Sec}\ \#$$

$$V_x = V_o\cos\alpha = 40\cos 30^\circ = 34.6\text{ m/s}$$
$$V_y = V_o\sin\alpha - gt = 40\sin 30^\circ - 9.8(2.67) = -6.17\text{ m/s}$$

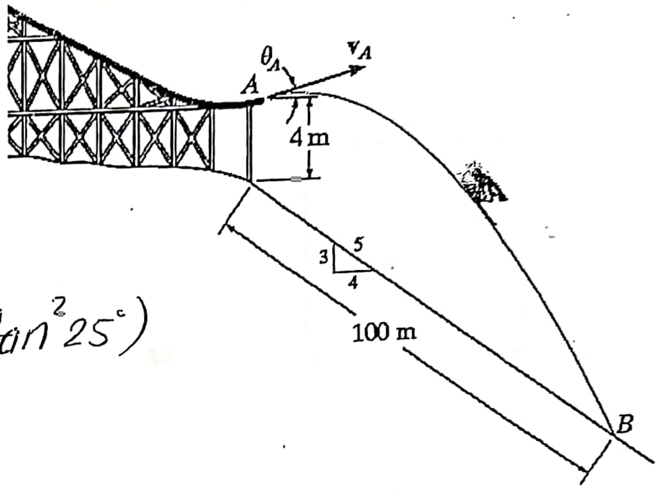
$$\tan\theta = \frac{V_y}{V_x} = 0.178 \Rightarrow \theta = 10^\circ$$



$\therefore$ angle with the inclined plane $= \theta^\circ + \beta^\circ$
$= 10 + 11.3 = 21.3^\circ$

$\beta \doteq 11.3^\circ$

t is observed that the skier leaves the ramp A at an angle 25° with the horizontal. he strikes the ground at **B**, determine his initial speed v_A and the time of flight t_{AB} nd its velocity direction at point **B**



at Point (B)

$x = 80m \quad y = -64m$

$$-64 = 80\tan 25° - \frac{9.8(80)^2}{2V_A^2}(1+\tan^2 25°)$$

$\therefore V_A = 19.4 \ m/Sec$

$\because x = (V_o \cos\alpha)t \Rightarrow 80 = (19.4 \cos 25°)t$

$\therefore t = 4.54 \ Sec$

$V_x = V_o \cos\alpha = 19.4 \cos 25° = 17.6 \ m/S$

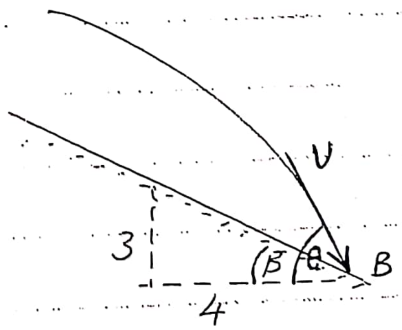
$V_y = V_o \sin\alpha - gt = 19.4 \sin 25° - 9.8(4.54) = -36.3 \ m/S$

$\tan\theta = \frac{V_y}{V_x} = 2 \Rightarrow \theta = 64°$ with X-axis

angle with the inclined Plane $= \theta - \beta$

$= 64° - 36.8°$

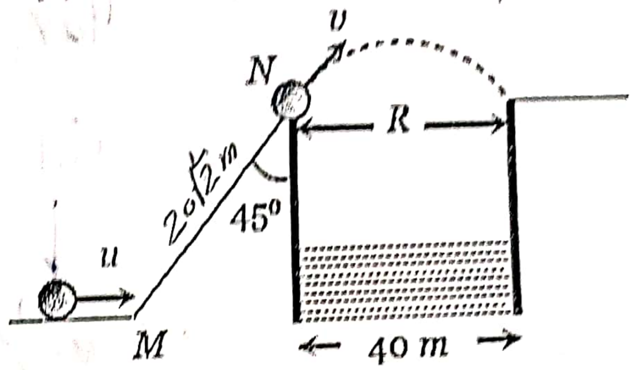
$= 27.2°$ #



$\beta = 36.8°$

A body is projected up a smooth inclined plane (length = $20\sqrt{2}$ m) with velocity u om the point M as shown in the figure. The angle of inclination is 45o and the top is onnected to a well of diameter 40 m. If the body just manages to cross the well,

a) What is the value of v
b) Calculate the velocity u of the body at M



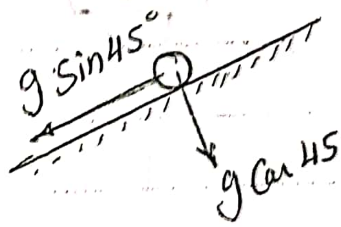
$R = 40\,m \qquad \alpha = 45°$

$\therefore R = \dfrac{V_o^2 \sin 2\alpha}{g} \qquad \therefore 40 = \dfrac{V^2 \sin 90°}{9.8} \Rightarrow V = 20\,m/Sec$ #

Motion in straight line MN

with $a = g \sin 45°$

$X = 20\sqrt{2}$



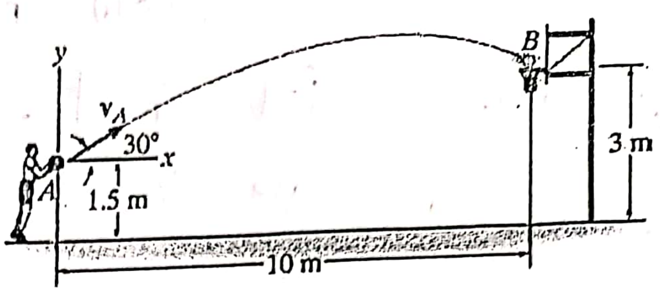
$\therefore V^2 = V_o^2 + 2aX$

$(20)^2 = u^2 + 2(g \sin 45°)(20\sqrt{2})$

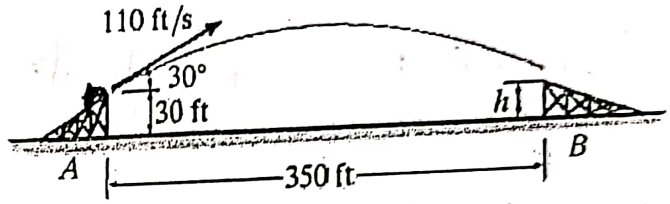
$u = 20\sqrt{2}\,m/Sec$ #

Assignment to be worked

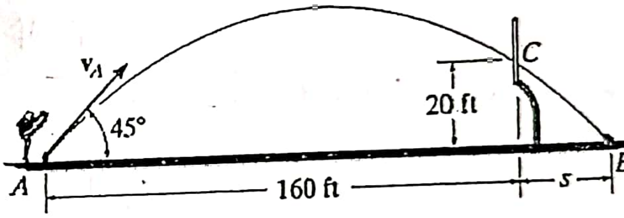
Determine the speed at which the basketball at **A** must be thrown at the angle **30º** so that it makes it to the basket at **B**.



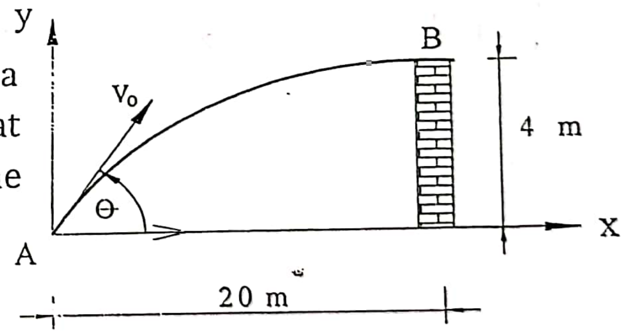
2-If the motorcycle leaves the ramp traveling at **110** ft/s, determine the height ***h*** ramp **B** must have so that the motorcycle lands safely



3-If the football is kicked at the **45°**, determine its minimum initial speed V_A so that it passes over the goal post at **C**. At what distance s from the goal post will the Football strikes the ground at **B**?



4- A ball is kicked from **A**, just to clear the top of a wall at **B** as it reaches its max. height, knowing that the distance from A to the wall is **20m** and the wall **4m** height determine the initial velocity of the ball.



5-A particle is projected with velocity **u** so that its range on the horizontal plane is twice the greatest height .prove that the range is $\mathbf{4u^2/5g}$

6- A helicopter is flying with a constant horizontal velocity of **180** km/h and is directly above point A when a loose part begins to fall. The part lands **6.5** s later at point B on an inclined surface. Determine

(a) The distance d between points A and B
(b) The initial height h.