1) i.
$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 4 & 0 & -2 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 4 & -2 \end{vmatrix} + 3\begin{vmatrix} 3 & -1 \\ 4 & 0 \end{vmatrix} = 2 + 28 + 12 = \boxed{42}$$

ii.
$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 7 & 4 \\ 2 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} 7 & 4 \\ 3 & -2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 7 & 4 \end{vmatrix} = -26 + 39 - 26 = \boxed{-13}$$

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2) $\begin{vmatrix} 2 & -3 & 4 \\ -5 & 6 & -7 \\ 8 & -9 & x \end{vmatrix} \xrightarrow[\substack{R_2+2.5R_1 \\ R_3-4R_1}]{} \begin{vmatrix} 2 & -3 & 4 \\ 0 & -1.5 & 3 \\ 0 & 3 & x-16 \end{vmatrix} \xrightarrow[R_3+2R_2]{} \begin{vmatrix} 2 & -3 & 4 \\ 0 & -1.5 & 3 \\ 0 & 0 & x-16 \end{vmatrix}$

$$\Rightarrow 2 \times -1.5 \times (x-10) = 0 \rightarrow -3(x-10) = 0 \rightarrow \boxed{x=10}$$

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3) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow[\substack{C_2-C_1 \\ C_3-C_1}]{} \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} = 1\begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix} = (b-a)(c^2-a^2) - (c-a)(b^2-a^2)$

$$\Rightarrow (b-a)(c-a)(c+a) - (c-a)(b-a)(b+a) = (b-a)(c-a)(c+a-b-a)$$

$$\Rightarrow \boxed{(b-a)(c-a)(c-b)}$$

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4) $\begin{pmatrix} 2 & 1 & 6 \\ 1 & -1 & 4 \\ 3 & 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

$\rightarrow A^{-1} = \dfrac{1}{32}\begin{pmatrix} -6 & 14 & 10 \\ 14 & -22 & -2 \\ 5 & -1 & -3 \end{pmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} -\frac{6}{32} & \frac{14}{32} & \frac{10}{32} \\ \frac{14}{32} & -\frac{22}{32} & -\frac{2}{32} \\ \frac{5}{32} & -\frac{1}{32} & -\frac{3}{32} \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

$$\Rightarrow x = \frac{1}{2} \quad y = \frac{1}{2} \quad z = \frac{1}{4}$$

**5)** $\begin{pmatrix} 4 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -4 \end{pmatrix}$ ∴ $A^{-1} = \dfrac{1}{-5}\begin{pmatrix} 1 & -3 \\ -3 & 4 \end{pmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} -\frac{1}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}\begin{pmatrix} 13 \\ -4 \end{pmatrix} \longrightarrow \boxed{\begin{array}{l} x = -5 \\ y = 11 \end{array}}$

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**6) a.** $\left(\begin{array}{ccc|c} 4 & 3 & -17 & 0 \\ 5 & 4 & 22 & 0 \\ 4 & 2 & 19 & 0 \end{array}\right) \xrightarrow[\substack{R_2-5R_1 \\ R_3-4R_1}]{R_1/4} \left(\begin{array}{ccc|c} 1 & \frac{3}{4} & -\frac{17}{4} & 0 \\ 0 & 0.25 & 43.25 & 0 \\ 0 & -1 & 36 & 0 \end{array}\right) \xrightarrow[\substack{R_1-\frac{3}{4}R_2 \\ R_3+R_2}]{R_2\cdot 4} \left(\begin{array}{ccc|c} 1 & 0 & -134 & 0 \\ 0 & 1 & 173 & 0 \\ 0 & 0 & 209 & 0 \end{array}\right)$

$\xrightarrow[\substack{R_1+134R_3 \\ R_2-173R_3}]{R_3/209} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \longrightarrow \boxed{\begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}}$

**b.** $\left(\begin{array}{ccc|c} 5 & 5 & -1 & 0 \\ 10 & 5 & 2 & 0 \\ 5 & 15 & -9 & 0 \end{array}\right) \xrightarrow[\substack{R_2-10R_1 \\ R_3-5R_1}]{R_1/5} \left(\begin{array}{ccc|c} 1 & 1 & -\frac{1}{5} & 0 \\ 0 & -5 & 4 & 0 \\ 0 & 10 & -8 & 0 \end{array}\right) \xrightarrow[\substack{R_1-R_2 \\ R_3-10R_2}]{R_2/-5} \left(\begin{array}{ccc|c} 1 & 0 & \frac{3}{5} & 0 \\ 0 & 1 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \text{infinite sol.}$

$\to$ let $z = t \to x + \frac{3}{5}z = 0$, $y - \frac{4}{5}z = 0 \to \boxed{\begin{pmatrix} -\frac{3}{5}t \\ \frac{4}{5}t \\ t \end{pmatrix}}$

$\qquad x = -\frac{3}{5}t \qquad y = \frac{4}{5}t$

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**7)** $\left(\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 1 & 0 & 1 & b \\ 2 & 1 & 3 & c \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & -1 & -1 & b-a \\ 0 & -1 & -1 & c-2a \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & 1 & 1 & a-b \\ 0 & -1 & -1 & c-2a \end{array}\right)$

$\to \left(\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & 1 & 1 & a-b \\ 0 & 0 & 0 & c-b-a \end{array}\right) \to c-b-a = 0 \quad \boxed{c = a+b}$

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**8)** $\begin{vmatrix} 5g & 5h & 5i \\ 2d+a & 2e+b & 2F+c \\ a & b & c \end{vmatrix} \to \begin{vmatrix} 5g & 5h & 5i \\ 2d & 2e & 2F \\ a & b & c \end{vmatrix} + \begin{vmatrix} 5g & 5h & 5i \\ a & b & c \\ a & b & c \end{vmatrix} \to 5\times 2\begin{vmatrix} g & h & i \\ d & e & F \\ a & b & c \end{vmatrix} = -10\begin{vmatrix} a & b & c \\ d & e & F \\ g & h & i \end{vmatrix}$

$\Rightarrow -10 \times 7 = \boxed{-70}$

9) $Ax = B \rightarrow A^{-1}Ax = A^{-1}B \rightarrow x = A^{-1}B$

$\rightarrow \begin{pmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \\ x_5 & x_6 \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \\ x_5 & x_6 \end{pmatrix} = \begin{pmatrix} 10 & -1 \\ 9 & 10 \\ -5 & .3 \end{pmatrix}$

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10) B. $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 1 & 5 & 5 \\ 0 & 2 & 7 & 3 \end{vmatrix} = 1 \begin{vmatrix} 1 & 5 & 4 \\ 1 & 5 & 5 \\ 2 & 7 & 3 \end{vmatrix} = 1 \begin{vmatrix} 1 & 5 & 4 \\ 0 & 0 & 1 \\ 0 & -3 & -5 \end{vmatrix}$

$\Rightarrow 1 \cdot 1 \begin{vmatrix} 0 & 1 \\ -3 & -5 \end{vmatrix} = \boxed{3}$

C. $\begin{pmatrix} 0 & 5 & 1 & | & 1 & 0 & 0 \\ 1 & 4 & 0 & | & 0 & 1 & 0 \\ 3 & 6 & 2 & | & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4 & 0 & | & 0 & 1 & 0 \\ 0 & 5 & 1 & | & 1 & 0 & 0 \\ 3 & 6 & 2 & | & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4 & 0 & | & 0 & 1 & 0 \\ 0 & 5 & 1 & | & 1 & 0 & 0 \\ 0 & -6 & 2 & | & 0 & -3 & 1 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 0 & -\tfrac{4}{5} & | & -\tfrac{4}{5} & 1 & 0 \\ 0 & 1 & \tfrac{1}{5} & | & \tfrac{1}{5} & 0 & 0 \\ 0 & 0 & 3.2 & | & \tfrac{6}{5} & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & | & -\tfrac{1}{2} & \tfrac{1}{4} & \tfrac{1}{4} \\ 0 & 1 & 0 & | & \tfrac{1}{8} & \tfrac{3}{16} & -\tfrac{1}{16} \\ 0 & 0 & 1 & | & \tfrac{3}{8} & -\tfrac{15}{16} & \tfrac{5}{16} \end{pmatrix}$

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11) $\begin{pmatrix} 1 & -2 & -3 & 4 & | & 1 \\ 4 & -1 & -5 & 6 & | & 2 \\ 2 & 3 & 1 & -2 & | & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -3 & 4 & | & 1 \\ 0 & 7 & 7 & -10 & | & -2 \\ 0 & 7 & 7 & -10 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -3 & 4 & | & 1 \\ 0 & 7 & 7 & -10 & | & -2 \\ 0 & 0 & 0 & 0 & | & 2 \end{pmatrix}$

$\rightarrow$ no solution

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12) $\begin{pmatrix} 1 & 1 & 2 & 3 & | & 13 \\ 1 & -2 & 1 & 1 & | & 8 \\ 3 & 1 & 1 & -1 & | & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 2 & 3 & | & 13 \\ 0 & -3 & -1 & -2 & | & -5 \\ 0 & -2 & -5 & -16 & | & -38 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 2 & 3 & | & 13 \\ 0 & 1 & \tfrac{1}{3} & \tfrac{2}{3} & | & \tfrac{5}{3} \\ 0 & 0 & -\tfrac{13}{3} & -\tfrac{26}{3} & | & -\tfrac{104}{3} \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 1 & 2 & 3 & | & 13 \\ 0 & 1 & \tfrac{1}{3} & \tfrac{2}{3} & | & \tfrac{5}{3} \\ 0 & 0 & 1 & 2 & | & 8 \end{pmatrix} \rightarrow \boxed{x_3 + 2x_4 = 8}$

$x_2 + \tfrac{1}{3}x_3 + x_4 = \tfrac{5}{3}$

$\boxed{x_2 = -1}$

$x_1 - 1 + 2x_3 + 4x_4 - x^4 = 13$

$\boxed{x_1 - x_4 = -2}$

3)
$$\begin{pmatrix} 1 & 1 & -1 & | & 1 \\ 2 & 3 & a & | & 3 \\ 1 & a & 3 & | & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -1 & | & 1 \\ 0 & 1 & a+2 & | & 1 \\ 0 & a-1 & 4 & | & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -1 & | & 1 \\ 0 & 1 & a+2 & | & 1 \\ 0 & 0 & b-a^2-a & | & 2-a \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -1 & | & 1 \\ 0 & 1 & a-2 & | & 1 \\ 0 & 0 & (a-2)(a+3) & | & 2-a \end{pmatrix} \rightarrow$$

inf. sol. at $\boxed{a=2}$

no sol. at $(a=-3)$

unique sol. at $\boxed{a \in \mathbb{R} - \{2, -3\}}$

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4) 21. a) False, bec. reduced echelon form is unique

b) False, bec any matrix can be reduced

c) true

d) true

e) False, bec $[0\ 0\ 0\ 5\ 0]$ means $5x_4 = 0$

$x_4 = 0$

22. a) true

b) False, right most column must not be a pivot column

c) False, leading entries always in the same positions

d) true

e) False, can't justify system is consistent

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5) $a_0 + a_1 + a_2 = 6$
$a_0 + 2a_1 + 4a_2 = 15$
$a_0 + 3a_1 + 9a_2 = 28$

$\rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 1 & 2 & 4 & | & 15 \\ 1 & 3 & 9 & | & 28 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 9 \\ 0 & 2 & 8 & | & 22 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 9 \\ 0 & 0 & 2 & | & 4 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 9 \\ 0 & 0 & 1 & | & 2 \end{pmatrix} \rightarrow \boxed{a_2 = 2}$

$a_1 + 3a_2 = 9$

$\boxed{a_1 = 3}$

$a_0 + a_1 + a_2 = 6$

$\boxed{a_0 = 1}$