Assignment #

Quiz #01

Name: Ahmed Siddique

Reg. Id: 200903

Subject: Multi variable calculus

Submitted to:

Date:

(9)

$$29x^2 - 24xy + 36y^2 + 118x - 24y - 55 = 0$$

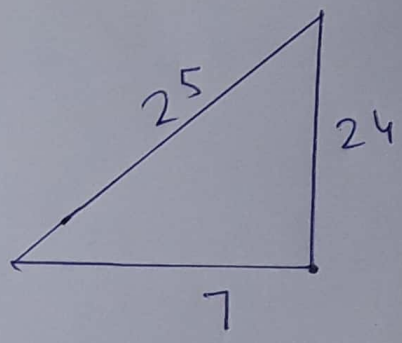
$$A = 29 \quad , \quad B = -24 \quad , \quad C = 36$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{29-36}{-24} = \frac{-7}{-24} = \frac{7}{24}$$

$$\tan 2\theta = \frac{24}{7}$$

$$\cos 2\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

$$= \sqrt{\frac{1+\frac{7}{25}}{2}}$$



$$\boxed{\cos 2\theta = \frac{4}{5}}$$

$$\sin 2\theta = \sqrt{\frac{1-\frac{7}{25}}{2}} = \boxed{\frac{3}{5}}$$

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$x = \frac{4}{5}x' - \frac{3}{5}y'$$

$$= \frac{1}{5}(4x' - 3y')$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

$$= \frac{1}{5}(3x' + 4y')$$

Put the x and y these values in eq

$$\frac{29}{25}(4x'-3y')^2 + \frac{24}{25}(4x'-3y')(3x'+4y') + \frac{36}{21}(3x'+4y)^2$$

$$+ \frac{118}{5}(4x'-3y') - \frac{24}{5}(3x'+4y') - 55 = 0$$

$$= \frac{464x'^2 + 261y'^2 - 696x'y' - 288x'^2 - 168x'y' + 288y'^2}{25}$$

$$= \frac{324x'^2 + 576y'^2 + 864x'y}{25} + \frac{1}{5}(472x' - 354y'$$

$$\frac{-72x' - 96y'}{5}) - 55 = 0$$

$$= \frac{500x'^2 + 1125y'^2}{25} + \frac{(400x' - 450y'')}{5} - 55 = 0$$

$$20x'^2 + 45y'^2 + 80x' - 90y' - 55 = 0$$

$$20(x'^2 + 4x') + 45(y'^2 - 2y) - 55 = 0$$

$$20(x'^2 + 4x + (2)^2 - (2)^2) + 45(y'^2 - 2y + (1)^2 - (1)^2) - 55 = 0$$

$$20(x+2)^2 - 80 + 45(y-1)^2 - 45 - 55 = 0$$

$$20(x+2)^2 + 45(y-1)^2 = 180$$

divide by 180

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$$

It makes it Ellipse

(b)

$$16x^2 - y^2 - 32x - 6y = 57$$

$$16x^2 - 32x - y^2 - 6y = 57$$

$$16(x^2 - 2x) - (y^2 + 6y) = 57$$

$$16(x^2 - 2x + (1)^2 - (1)^2) - (y^2 + 6y + (3)^2 - (3)^2) = 57$$

$$16(x-1)^2 - (y+3)^2 - 16 + 9 = 57$$

$$16(x-1)^2 - (y+3)^2 = 57 + 7$$

$$16(x-1)^2 - (y+3)^2 = 64$$

Divide by 64

$$\frac{(x-1)^2}{4} - \frac{(y+3)^2}{64} = 1$$

Compare it with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

So $a^2 = 4 \qquad b^2 = 64$

$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 64$$

$$c = \sqrt{68}$$

foci foci $(\pm c, 0)$

axis are translated so

$X = x - 1$ and $Y = y + 3$

$x - 1 = \pm\sqrt{68}$ $\quad y = -3$

$x = -1 + \sqrt{68}, -1 - \sqrt{68}$

so foci are $(+1+\sqrt{68}, -3)$ and $(+1-\sqrt{68}, -3)$

Vertices

$V(\pm a, 0)$

$x - 1 = \pm 4$ $\quad y + 3 = 0$

$x = 5, \quad x = -3$ $\quad y = -3$

so vertices are $(-3, -3)$, $(5, -3)$

Sketching