TD Algebre serie 1°

Ex2:

$A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix}$

$$A + B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 2 & 9 \end{pmatrix}$$

$$2A - B = 2 \times \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ 4 & 10 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 4 & 6 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 2 + 3 \times 0 & 1 \times 2 + 3 \times 4 \\ 2 \times 2 + 5 \times 0 & 2 \times 2 + 5 \times 4 \end{pmatrix} = \begin{pmatrix} 2 & 14 \\ 4 & 24 \end{pmatrix}$$

$$B \times A = \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 8 & 20 \end{pmatrix}$$

$${}^t(A.B) = {}^t\begin{pmatrix} 2 & 14 \\ 4 & 24 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 14 & 24 \end{pmatrix}$$

$${}^tB \times {}^tA = \begin{pmatrix} 2 & 0 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 14 & 24 \end{pmatrix}$$

$$\boxed{{}^t(AB) = {}^tA \times {}^tB}$$

c) soit $X \in M_2(\mathbb{R})$ tq :

$A - 3X = 2B \iff A - 2B = 3X$

$\iff X = \frac{1}{3}(A - 2B) \Rightarrow$

$\Rightarrow X = \frac{1}{3}\left[\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} - 2\begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix}\right]$

$= \frac{1}{3}\begin{pmatrix} -3 & -1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{3} \\ \frac{2}{3} & -1 \end{pmatrix}$

$S_{M_2(\mathbb{R})} = \left\{\begin{pmatrix} -1 & -\frac{1}{3} \\ \frac{2}{3} & -1 \end{pmatrix}\right\}$

Ex3:

$L = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix} = M_{13}$

$C = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \in M_{31}$

$LC \in M_{11}$, $CL \in M_{33}$

$$LC = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = -1$$

$$CL = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 6 \\ 2 & 0 & -4 \\ -1 & 0 & 2 \end{pmatrix}$$

Ex5

$A = (a_{ij})_{1 \leq i,j \leq n} \in M_n(\mathbb{R})$

$$S(A) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}$$

$J = (1)_{1 \leq i,j \leq n} = (b_{ij})_{1 \leq i,j \leq n}$, $b_{ij} = 1$ $\forall\ 1 \leq i,j \leq n$

$JA = (c_{ij})_{1 \leq i,j \leq n}$, $c_{ij} = \sum_{k=1}^{n} b_{ik} a_{kj} \Rightarrow$

$= \sum_{k=1}^{n} a_{kj}$

$JAJ = (d_{ij})_{1 \leq i,j \leq n}$

$$d_{ij} = \sum_{m=1}^{n} c_{im} b_{mj} = \sum_{m=1}^{n} c_{im}$$

or $c_{im} = \sum_{k=1}^{n} a_{km} \Rightarrow d_{ij} = \sum_{m=1}^{n} \sum_{k=1}^{n} a_{km}$

$= S(A)$

$$JAJ = \begin{pmatrix} S(A) & S(A) & \cdots & S(A) \\ S(A) & S(A) & \cdots & S(A) \\ \vdots & & & \vdots \\ S(A) & & \cdots & S(A) \end{pmatrix}$$

$$= S(A) \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

$$= S(A) \cdot J$$

Ex6:

$M = (a_{lk})_{1 \leq k, l \leq n}$

$E_{ij} M = (C_{lk})_{1 \leq k l \leq n}$

$$C_{lk} = \sum_{m=1}^{m} C_{lm} \, a_{mk}$$

si $l \neq i$

$\Rightarrow C_{lm} = 0 \quad \forall \, 1 \leq m \leq n$

$\Rightarrow C_{lk} = 0 \quad \forall \, 1 \leq k \leq n_{\, l \neq i}$

si $l = i$

$$C_{ik} = \sum_{m=1}^{n} C_{in} \, a_{mk}$$

$$C_{ik} = \sum_{m=1}^{k} \underbrace{l_{im}}_{0} a_{mk} + \underbrace{l_{ij}}_{1} a_{jk} + \sum_{m=j+1}^{n} \underbrace{l_{in}}_{0} a_{nk}$$

$$= a_{jk}$$

Ex7:

soit $M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in M_3(\mathbb{R})$ tq:

$MA = AM$

$$\Leftrightarrow \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -2a & 3b & 5c \\ -2d & 3e & 5f \\ -2g & 3h & 5i \end{pmatrix} = \begin{pmatrix} -2a & -2b & -2c \\ 3d & 3e & 3f \\ 5g & 5h & 5i \end{pmatrix}$$

$$\begin{cases} -2a = -2a \\ 3b = -2b \\ 5c = -2c \\ -2d = 3d \\ 3e = 3e \\ 5f = 3f \\ -2g = 5g \\ 3h = 5h \\ 5i = 5i \end{cases} \qquad \begin{cases} a = \mathbb{R} \\ b = 0 \\ c = 0 \\ d = 0 \\ e = \mathbb{R} \\ f = 0 \\ g = 0 \\ h = 0 \\ i = \mathbb{R} \end{cases}$$

$$\Leftrightarrow M = \begin{pmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{pmatrix}$$

Ex9:

Soit $B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in M_3(\mathbb{R})$ tq:

$AB = 0$

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} a+2d & b+2e & c+2f \\ 2a+d & 2b+e & 2c+f \\ d & e & f \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} d = 0 \\ e = 0 \\ f = 0 \\ a + 2d = 0 \\ b + 2e = 0 \\ c + 2f = 0 \\ 2a + d = 0 \\ 2b + e = 0 \\ 2c + f = 0 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ b = 0 \\ c = 0 \\ d = 0 \\ e = 0 \\ f = 0 \end{cases} \Leftrightarrow B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & h & i \end{pmatrix}$$

$$\Leftrightarrow S_{M_3(\mathbb{R})} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & h & i \end{pmatrix} \right\}$$

$g, h, i \in \mathbb{R}$

Ex 18 :

a) $A^2 = A \cdot A = \begin{pmatrix}0&1&0\\-1&2&0\\1&0&-1\end{pmatrix}\begin{pmatrix}0&1&0\\-1&2&0\\1&0&-1\end{pmatrix} = \begin{pmatrix}-1&2&0\\-2&3&0\\-1&1&1\end{pmatrix}$

$A^3 = A^2 \cdot A = \begin{pmatrix}-1&2&0\\-2&3&0\\-1&1&1\end{pmatrix}\begin{pmatrix}0&1&0\\-1&2&0\\1&0&-1\end{pmatrix} = \begin{pmatrix}-2&3&0\\-3&4&0\\0&1&-1\end{pmatrix}$

$A^3 - A^2 - A + I =$

$\begin{pmatrix}-2&3&0\\-3&4&0\\0&1&-1\end{pmatrix} - \begin{pmatrix}-1&2&0\\-2&3&0\\-1&1&1\end{pmatrix} - \begin{pmatrix}0&1&0\\-1&2&0\\1&0&1\end{pmatrix} + \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$= \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \Leftrightarrow A^3 - A^2 - A + I_3 = 0$

b) $A^3 - A^2 - A + I_3 = 0$

$\Leftrightarrow -A^3 + A^2 + A = I \Leftrightarrow \begin{cases} A(-A^2 + A + I_3) = I_3 \\ (-A^2 + A + I_3)A = I_3 \end{cases}$

(=> Alors A est inversible

et $A^{-1} = -A^2 + A + I$

c) $B^3 - 3B^2 + 2B = 0$

d) soit $A \neq 0$ tq

$\exists B \neq 0$ et $A \cdot B = 0$

$\exists C$

$CA = I$

$\Leftrightarrow CA \cdot B = B \Rightarrow 0 \neq B = 0$

impossible

$B(B^2 - 3B + 2I) = 0$

on a $B^2 - 3B + 2I \neq 0$

on suppose que B inversible

$\Rightarrow \exists A$ tq :

$AB = I_3 \Leftrightarrow AB(B^2 - 3B + 2I)$
$= B^2 - 3B + 2I$

$\Leftrightarrow A \cdot 0 = B^2 - 3B + 2I$

$\Leftrightarrow 0 = B^2 - 3B + 2I$

B n'est pas inversible

Ex 23 :

a)
$A = \begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}$

$B = A - I_3 = \begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix} - \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = \begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}$

$B^2 = B \times B = \begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$

$B^3 = B \cdot B^2 = \begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$

si $n \geq 3$

$B^n = B^{n-3} B^3 = B^{n-3} \times 0 = 0$

$\boxed{\text{si } AB = BA \quad (A+B)^n = \sum_{k=0}^{n} C_n^k A^k B^{n-k}}$

$A = B + I$

$A^n = (B + I)^n$

on a $BI = IB = B \Rightarrow A^n = (B+I)^n = \sum_{k=0}^{n} C_n^k B^k I^{n-k}$

b) soit $C = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in M_3(\mathbb{R})$ tq :

$AC = CA = 0$

$\iff AC = 0$ et $CA = 0$

$\iff C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & h & i \end{pmatrix}$ et

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & h & i \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\iff \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g+2h & 2g+h+i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\iff \begin{cases} g + 2h = 0 \\ 2g + h + i = 0 \end{cases} \iff \begin{cases} g = -2h \\ 3h + i = 0 \end{cases}$

$\iff \begin{cases} g = -2h \\ i = 3h \end{cases}$

$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2h & h & 3h \end{pmatrix}$

Ex 1

triangulaire superieures (a) (f)

" inferieures (c) (e)

Diagonale (g) (h)

inversible :

a) $A = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix}$

soit $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ tq : et $AB = I_2$, $BA = I_2$

on a $AB = I_2 \iff \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\iff \begin{pmatrix} a+3c & b+3d \\ -c & -d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$a + 3c = 1$
$b + 3d = 0$
$-c = 0$
$-d = 1$

$\iff \begin{cases} a = 1 \\ b = 3 \\ c = 0 \\ d = 0 \end{cases}$

$\iff B = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} = A$

Ex16 :

$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^2 = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

$2I_3 - A = 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

$= A^2$

$\implies A^2 = 2I_3 - A$

b) on a :

$A^2 = 2I - A \iff \dfrac{A^2 - A}{2} = I_3$

$\iff \dfrac{A.A + AI_3}{2} = I_3$

et $\dfrac{A.A + I_3 A}{2} = I_3$

$\implies$

$\implies A\left(\dfrac{A+I_3}{2}\right) = I_3$ $\implies$ A inversible

$\left(\dfrac{A+I_3}{2}\right) A = I_3$ $\quad A^{-1} = \dfrac{A+I_3}{2}$

TD 2 : Determinants

Exercice 1.2 :

$\det\begin{pmatrix}1&2\\3&4\end{pmatrix} = 1\times4 - 3\times2 = -2$

$\begin{vmatrix}1&2\\3&4\end{vmatrix} = L_2 \leftarrow L_2 - 3L_1 \begin{vmatrix}1&2\\0&-2\end{vmatrix} = 1\times(-2) = -2$

b) $\det\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix} = 1\times2\times3 = 6$

c) $\det\begin{pmatrix}1&0&0\\2&-1&0\\3&4&3\end{pmatrix} = 1\times-1\times3 = -3$

d) $\det\begin{pmatrix}1&1&2\\0&2&-1\\0&0&1\end{pmatrix} = 1\times2\times1 = 2$

E) $\det\begin{pmatrix}1&0&0\\0&2&4\\0&3&3\end{pmatrix} = 1\begin{vmatrix}2&4\\3&3\end{vmatrix} = 2\times3 - 4\times3 = -6$

H) $\begin{vmatrix}3&1&1&1\\1&3&1&1\\1&1&3&1\\1&1&1&3\end{vmatrix} = 3\begin{vmatrix}3&1&1\\1&3&1\\1&1&3\end{vmatrix} - 1\begin{vmatrix}1&1&1\\1&3&1\\1&1&3\end{vmatrix}$

$+\begin{vmatrix}1&1&1\\3&1&1\\1&1&3\end{vmatrix} - 1\begin{vmatrix}1&1&1\\3&1&1\\1&3&1\end{vmatrix}$

ou bien

$L_2 \leftarrow L_2 - L_3 = \begin{vmatrix}3&1&1&1\\0&2&2&0\\1&1&3&1\\1&1&1&3\end{vmatrix}$

$=$

$L_3 \leftarrow L_3 - L_4 = \begin{vmatrix}3&1&1&1\\0&2&-2&0\\0&0&2&2\\1&1&1&3\end{vmatrix}$

$L_4 \leftarrow L_4 - \frac{1}{3}L_1 = \begin{vmatrix}3&1&1&1\\0&2&-2&0\\0&0&2&-2\\0&\frac{2}{3}&\frac{2}{3}&\frac{8}{3}\end{vmatrix}$

$= \frac{1}{3}\begin{vmatrix}3&1&1&1\\0&2&-2&0\\0&0&2&-2\\0&2&2&8\end{vmatrix} = L_4 \leftarrow L_4 - L_2$

$= \frac{1}{3}\begin{vmatrix}3&1&1&1\\0&2&-2&0\\0&0&2&-2\\0&0&4&8\end{vmatrix}$

$L_4 \leftarrow L_4 - 2L_3$

$= \frac{1}{3}\begin{vmatrix}3&1&1&1\\0&2&-2&0\\0&0&2&-2\\0&0&0&12\end{vmatrix} = \frac{1}{3}(3\times2\times2\times12) = 48$

Exercice 1.3 :

$\begin{vmatrix}2a&2b&2c\\d&e&f\\g&h&i\end{vmatrix} = 2\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix} = 2\times3 = 6$

$\begin{vmatrix}d&e&f\\a&b&c\\g&h&i\end{vmatrix} = -\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix} = -3$

$\begin{vmatrix}2c&b&a\\2f&e&d\\2i&h&g\end{vmatrix} = -2\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix} = -2\times3 = -6$

$\begin{vmatrix}a&b&c\\2d-3g&2e-3h&2f-3i\\g&h&i\end{vmatrix} = L_2 \leftarrow L_2 + 3L_3$

$= \begin{vmatrix}a&b&c\\2d&2e&2f\\g&h&i\end{vmatrix} = 2\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix} = 6$

si $n \geqslant 3$

$$A^n = \sum_{k=0}^{2} C_n^k B^k + \sum_{k=3}^{n} C_n^k B^k$$

on $B^k = 0 \quad \forall\, k \geqslant 3$

$$\Rightarrow A^n = \sum_{k=0}^{2} C_n^k B^k = C_n^0 B^0 + C_n^1 B^1 + C_n^2 B^2$$

$$C_n^0 = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1$$

$$C_n^1 = \frac{n!}{1!(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$$

$$C_n^2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)(n-2)!}{2(n-2)!}$$

$$C_n^2 = \frac{n(n-1)}{2}$$

$$\Rightarrow A^n = 1B^0 + nB^1 + \frac{n(n-1)}{2}B^2$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + n\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \frac{n(n-1)}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & n & \frac{n(n-1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

Exercice 3.5 :

$$B_m = \begin{pmatrix} 1 & -1 & 2 \\ m & 1-m & 2m-2 \\ 2 & m & -3m-1 \end{pmatrix}$$

1) Calculer $\det B_m$

2) Determiner $rg(B_m)$

3) chercher la matrice Echelonnée en ligne équivalente à B_m

1) $\det B_m =$

$$\begin{matrix} L_2 \leftarrow L_2 - mL_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{matrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & m+2 & -3m+5 \end{pmatrix}$$

$$= \begin{vmatrix} 1 & -2 \\ m+2 & -3m+5 \end{vmatrix} = -3m - 5 + 2m + 4 = -(m+1)$$

si $m \neq -1 \rightarrow m+1 \neq 0 \Rightarrow \det B_m \neq 0$

B_m inversible $\Rightarrow rg(B_m) = ord(B_m) = 3$

si $m = -1 \Rightarrow \det B_{-1} = 0$ $\quad B_{-1}$ n'est pas inversible

$rg(B_{-1}) \neq 3$

$\rightarrow rg(B_{-1}) \in \{1, 2\}$ $\qquad B_{-1} \begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & -4 \\ 2 & -1 & 2 \end{pmatrix}$

$$\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 1 \neq 0 \rightarrow rg(B_{-1}) = 2$$

si $m \neq -1 \Rightarrow B_m \sim I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

une matrice echelonné en ligne

si $m = -1$, $B_{-1} = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & -4 \\ 2 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{pmatrix}$

$L_2 \leftarrow L_2 + L_1$

$L_3 \leftarrow L_3 - 2L_1$

$$\sim \begin{matrix} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 + L_2 \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

Exercice:

soit $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Par deux methodes donnée l'expression de A^n pour tout entrer $n \geqslant 0$

1) on prend

$$X = I_2 \ , \ Y = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$I_2^n = I_2$

$Y^2 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$Y^m = \begin{cases} I_2, \text{si } m = 0 \\ Y, \text{si } m = 1 \\ O_2, \text{si } m \geqslant 2 \end{cases}$$

$$A^n = (X+Y)^n = \sum_{k=0}^{n} C_n^k \, Y^{n-k}$$

$$= C_n^n Y^0 + C_n^{n-1} Y$$

$$= I_2 + nY = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$$

Pour $n = 0$, $x_0 = 0$

$$A^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Pour $n = 1$, $x_1 = 2$

$$A^1 = \begin{pmatrix} 1 & x_1 \\ 0 & 1 \end{pmatrix}$$

onsuppose que les propriéte est vrai ~~Ruiques~~ ~~yiaa~~ jusqu'à l'ordre n

Mq vrais à l'ordre $n+1$

$$A^{n+1} = A^n A = \begin{pmatrix} 1 & x_n \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2+x_n \\ 0 & 1 \end{pmatrix}$$

$x_{n+1} = x_n + 2$

$x_0 = 0$

suite arith de raison 2 premier terme 0

$$x_n = 2n$$
$$= x_0 + 2(n-0)$$