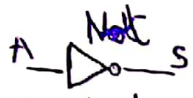
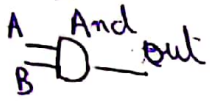
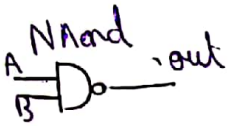
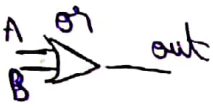
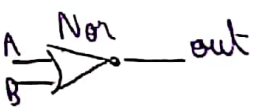
Logique Booléenne

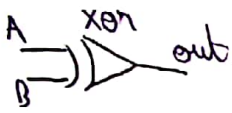


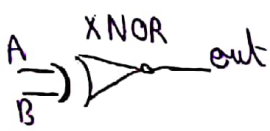












Not

A	S = $\bar{A}$
0	1
1	0

And

A	B	S = A . B
0	0	0
0	1	0
1	0	0
1	1	1

NAnd

A	B	S = $\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

or

A	B	S = A + B
0	0	0
0	1	1
1	0	1
1	1	1

Nor

A	B	S = $\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Xor

A	B	S = A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

XNor

A	B	S ($\overline{A \oplus B}$)
0	0	1
0	1	0
1	0	0
1	1	1

Associativité :

$(A+B)+C = A+(B+C)$

$A.(B.C) = (A.B).C$

Distributivité :

$A(B+C) = AB + AC$

Theorème de Boole :

$A+1=1$	$A+A=A$	$\bar{\bar{A}} = A$
$A.0=0$	$A.A=A$	
$A.1=A$	$A+\bar{A}=1$	
$A+0=A$	$A.\bar{A}=0$	

Loi de Morgan

* $\overline{A+B} = \bar{A}.\bar{B}$
* $\overline{A.B} = \bar{A}+\bar{B}$
* $\overline{A \oplus B} = \overline{\bar{A}B + A\bar{B}} = \overline{\bar{A}B}.\overline{A\bar{B}}$

$= (\bar{\bar{A}}+\bar{B}).(\bar{A}+\bar{\bar{B}})$

$= (A+\bar{B}).(\bar{A}+B) = \underbrace{A\bar{A}}_{0} + AB + \bar{A}\bar{B} + \underbrace{B\bar{B}}_{0}$

$= AB + \bar{A}\bar{B}$

A	B	C	S
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$S = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + AB\bar{C}$

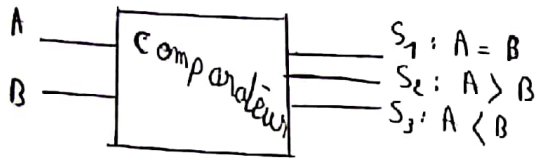
$= \bar{A}C(\bar{B}+B) + A(\underbrace{\bar{B}C + B\bar{C}}_{1})$

$= \bar{A}C + A(B \oplus C)$

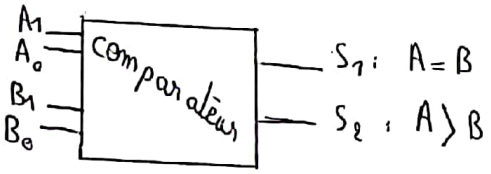
AB / C	00	01	11	10
0	0	0	1	0
1	1	1	0	1

$S = AB\bar{C} + \bar{A}C + \bar{B}C$

Exercice 1 :



1)



2)

A		B			
A_1	A_0	B_1	B_0	$S_1 : A=B$	$S_2 : A>B$
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	0	0
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	0	1
1	1	1	1	1	0

S_1 : $A = B$

$A_1 = B_1$ et $A_0 = B_0$

S_2 : $A > B$

$A_1 > B_1$ ou $(A_1 = B_1$ et $A_0 > B_0)$

Inférieur :

$A_1 < B_1$ ou $(A_1 = B_1$ et $A_0 < B_0)$

3) $S_1 = \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0 + A_1 A_0 B_1 B_0$

$S_2 = \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 B_0 + A_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 B_1 \bar{B}_0$

4)

$S_1 = \bar{A}_1 \bar{B}_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0)$

$= \bar{A}_1 \bar{B}_1 \overline{(A_0 \oplus B_0)} + A_1 B_1 \overline{(A_0 \oplus B_0)}$

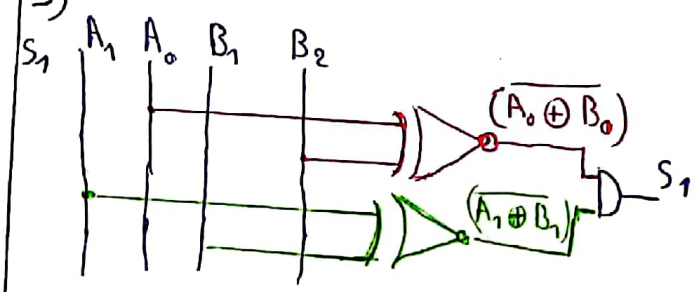
$= \overline{(A_0 \oplus B_0)} (\bar{A}_1 \bar{B}_1 + A_1 B_1)$

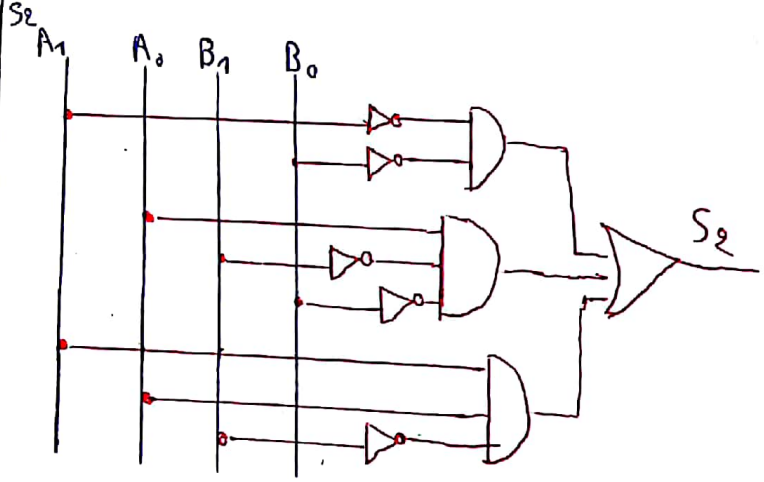
$S_1 = \overline{(A_0 \oplus B_0)} \overline{(A_1 \oplus B_1)}$

A_1A_0 / B_0B_1	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

$S_2 = \bar{A}_1 \bar{B}_0 + A_0 \bar{B}_0 \bar{B}_1 + A_1 A_0 B_1$

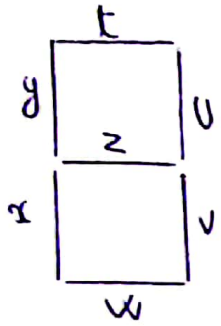
5)





Exercice:

	A	B	C	D	u	v	w	x	y	z	t
0	0	0	0	0	1	1	1	1	1	0	1
1	0	0	0	1	1	1	0	0	0	0	0
2	0	0	1	0	1	0	1	1	0	1	1
3	0	0	1	1	1	1	1	0	0	1	1
4	0	1	0	0	1	1	0	0	1	1	0
5	0	1	0	1	0	1	0	0	1	1	1
6	0	1	1	0	0	1	1	1	1	1	1
7	0	1	1	1	1	1	0	0	0	0	1
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	0	1	1	1



2)

u

CD \ AB	00	01	11	10
00	1	1	X	1
01	1	0	X	1
11	1	1	X	X
10	1	0	X	X

$u = \bar{C}\bar{D} + CD + \bar{B}$

$= (\overline{C \oplus D}) + \bar{B}$

v

CD \ AB	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	X
10	0	1	X	X

$V = \bar{C} + D + B$

w

CD \ AB	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	0	X	X
10	1	1	X	X

$W = A + B\bar{C}D + C\bar{D} + C\bar{B} + \bar{B}\bar{D}$

x

CD \ AB	00	01	11	10
00	1	0	X	1
01	0	0	X	0
11	0	0	X	X
10	1	1	X	X

$x = C\bar{D} + \bar{B}\bar{D}$

y

CD \ AB	00	01	11	10
00	1	1	X	1
01	0	1	X	1
11	0	0	X	X
10	0	1	X	X

$y = \bar{C}\bar{D} + B\bar{C} + B\bar{D} + A$

z

CD \ AB	00	01	11	10
00	0	1	X	1
01	0	1	X	1
11	1	0	X	X
10	1	1	X	X

$z = B\bar{C} + C\bar{D} + \bar{B}C + A$

$= \bar{B} \oplus C + C\bar{D} + A$

t

CD \ AB	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	1	X	X
10	0	1	X	X

$t = A + C + DB + \bar{B}\bar{D}$

$= A + C + \overline{(B \oplus D)}$

$u = \overline{C \oplus D} + \bar{B}$

$V = \bar{C} + D + B$

$W = A + B\bar{C}D + C\bar{D} + C\bar{B} + \bar{B}\bar{D}$

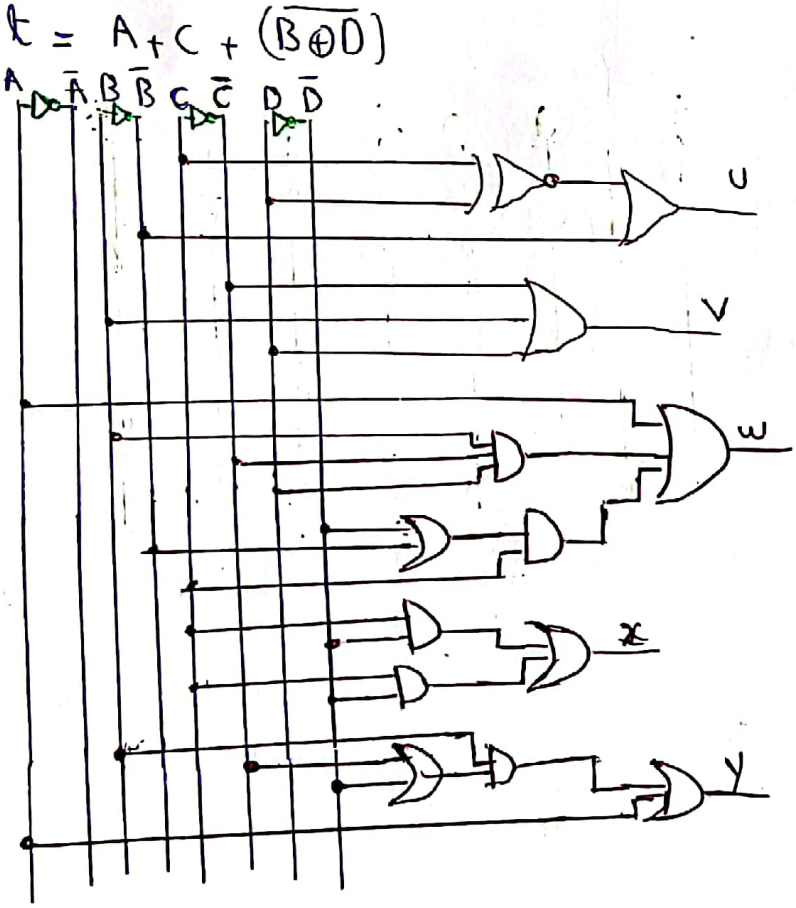
$= A + BCD + C\bar{D} + C\bar{B} = A + B\bar{C}D + C(\bar{D} + \bar{B})$

$x = C\bar{D} + \bar{B}\bar{D}$

$y = C\bar{D} + B\bar{C} + B\bar{D} + A = B(\bar{C} + \bar{D}) + A$

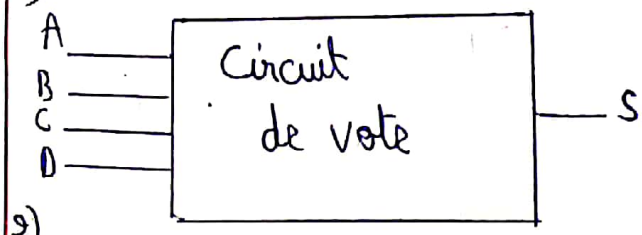
$z = \overline{(B \oplus C)} + C\bar{D} + A$

$t = A + C + \overline{(B \oplus D)}$



Exercice 3 :

1)



2)

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

⇒ a partir nombre de vot

A = 1 ≡ 2 votes

B ou C ou D = 1 = 1 voi

S ⩽ 2 voix = 0

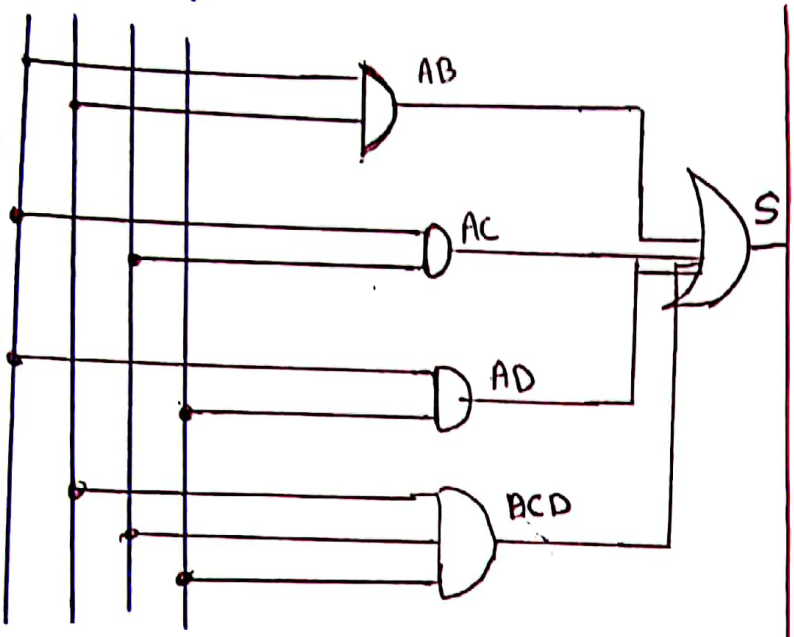
S ⩾ 2 voix = 1

3)

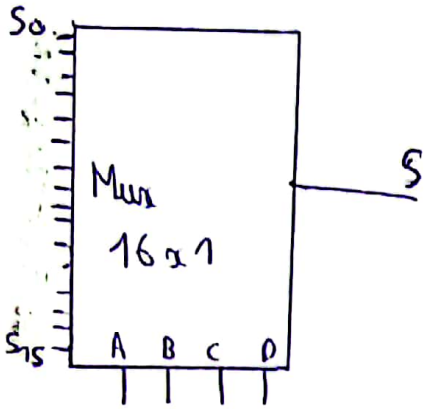
CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	0	1	1	1
10	0	0	1	1

$S = AB + AC + AD + BCD$

4)



5)



bits de selection

a) Multiplexeur 16 x 1

b) A : ligne selection

B, C, D : ligne d'activation

$S_1, .. S_{15}$: ligne d'entrée

S : sortie

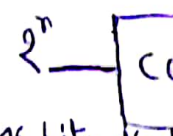
$S = \Sigma \{ S_7, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15} \}$

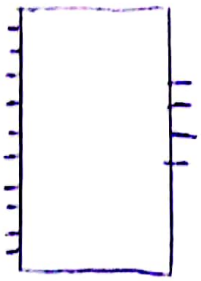
d)

	S'_0	S'_1	S'_2	S'_3	S'_4	S'_5	S'_6	S'_7
$\bar{A}$	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
A	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}

Exercice 4 :

1)





2) 16 bits d'entrée et 4 bits sortie

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	A	B	C	D
1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	0	0	0	1	1	0
0	0	0	0	0	0	0	1	0	0	0	1	1	1
0	0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	1	0	0	1

$A = e_9 + e_{10}$

$B = e_5 + e_6 + e_7 + e_8$

$C = e_3 + e_4 + e_7 + e_8$

$D = e_2 + e_4 + e_6 + e_8 + e_{10}$