- Combinatorics

* $^nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$ * $^nP_r = \dfrac{n!}{(n-r)!}$

* $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$

* $\binom{n}{0} + \binom{n}{2} + \cdots + \binom{n}{x} = \binom{n}{1} + \binom{n}{3} + \cdots + \binom{n}{y}$

x = largest even no. $\le n$ y = largest odd no. $\le n$

* $\binom{n}{r} = \binom{n}{n-r}$

** Pascal identities:

* $\binom{n}{r} = \binom{n-1}{r} + \binom{n-2}{r-1} + \binom{n-3}{r-2} + \cdots + \binom{n-k-1}{0}$

* $\binom{n}{r} = \binom{r-1}{r-1} + \binom{r}{r-1} + \binom{r+1}{r-1} + \cdots + \binom{n-1}{r-1}$

** Some identities & formula :

* $\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2$

* $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n2^{(n-1)}$

* Fibonacci numbers : $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$

* $F_{n+1} = \binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \cdots$

* Catalan numbers : $C_0 = C_1 = 1$

for $n \geq 2$: $C_n = \sum_{k=0}^{n-1} C_k \cdot C_{n-1-k} = C_0C_{n-1} + C_1C_{n-2} + \cdots + C_{n-1}C_0$

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

* $1+2+3+\cdots+n = \dfrac{n(n+1)}{2}$

* $1^2+2^2+3^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

* $1^3+2^3+3^3+\cdots+n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

* $2+4+6+\cdots+2n = n(n+1)$

* $1+3+5+\cdots+2n-1 = n^2$

* triangular numbers are in form of $\dfrac{n(n+1)}{2}$

  1^{st} few triangular numbers : $1, 3, 6, 10, 15, \ldots$

Sum of 1^{st} n triangular numbers :

$$\frac{1*(1+1)}{2} + \frac{2*(2+1)}{2} + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$