



Faculty of Engineering & Technology
Electrical & Computer Engineering Department

Machine Learning and Data Science - ENCS5341

Assignment #2

Prepared by:

Ahmed Zubaidia 1200105

Instructor: Dr. Yazan Abu Farha

Section: 2

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Solution:

Part1

Model Selection and Hyper-parameters Tunning

The `data_reg.csv` file contains a set of 200 examples. Each row represents one example which has two attributes x_1 and x_2 , and a continuous target label y .

Using python, implement the solution of the following tasks:

- 1- Read the data from the csv file and split it into training set (the first 120 examples), validation set (the next 40 examples), and testing set (the last 40 examples). Plot the examples from the three sets in a scatter plot (each set encoded with a different color). Note that the plot here will be 3D plot where the x and y axes represent the x_1 and x_2 features, whereas the z-axis is the target label y .

Figure 1: part 1 text

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression, Ridge, LogisticRegression
from sklearn.metrics import mean_squared_error, accuracy_score
```

Figure 2: libraries that I used.

At first we will read the data, and slice it to three sets, train data with 120 sample , validation with 40 samples , and test set with 40 samples also.

```
25 # read the data
26
27 data = pd.read_csv('data_reg.csv') data: ['x1', 'x2', 'y'] [0 0.548814 0.311796 0.547818] [1 0.715189 0.696343 0.576032] [2 0.602763 0.377752 0.113475] [3
28
29 # split the data into train and test and validation
30
31 train_data = data.iloc[:120] # train data from 0 to 120 train_data: ['x1', 'x2', 'y'] [0 0.548814 0.311796 0.547818] [1 0.715189 0.696343 0.576032] [2 0.602763 0.377752 0.113475] [3
32
33 validation_data = data.iloc[120:160] # validation data from 120 to 160 validation_data: ['x1', 'x2', 'y'] [120 0.725254 0.278328 0.486485] [121 0.501324 0.131483 0.8
34
35 test_data = data.iloc[160:] # test data from 160 to the end test_data: ['x1', 'x2', 'y'] [160 0.697429 0.187131 -0.265093] [161 0.453543 0.903984 0.963340] [162 0.72
36
37
38
```

Figure 3: code which I used to make train , validation, test sets.

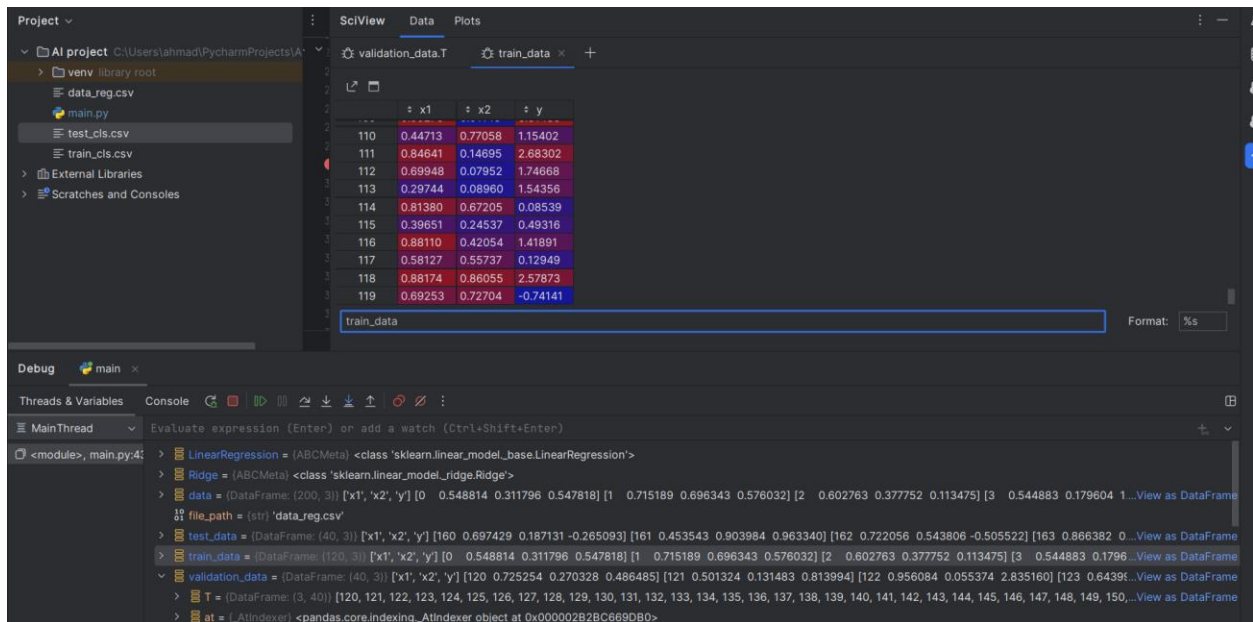


Figure 4: train data 120 sample

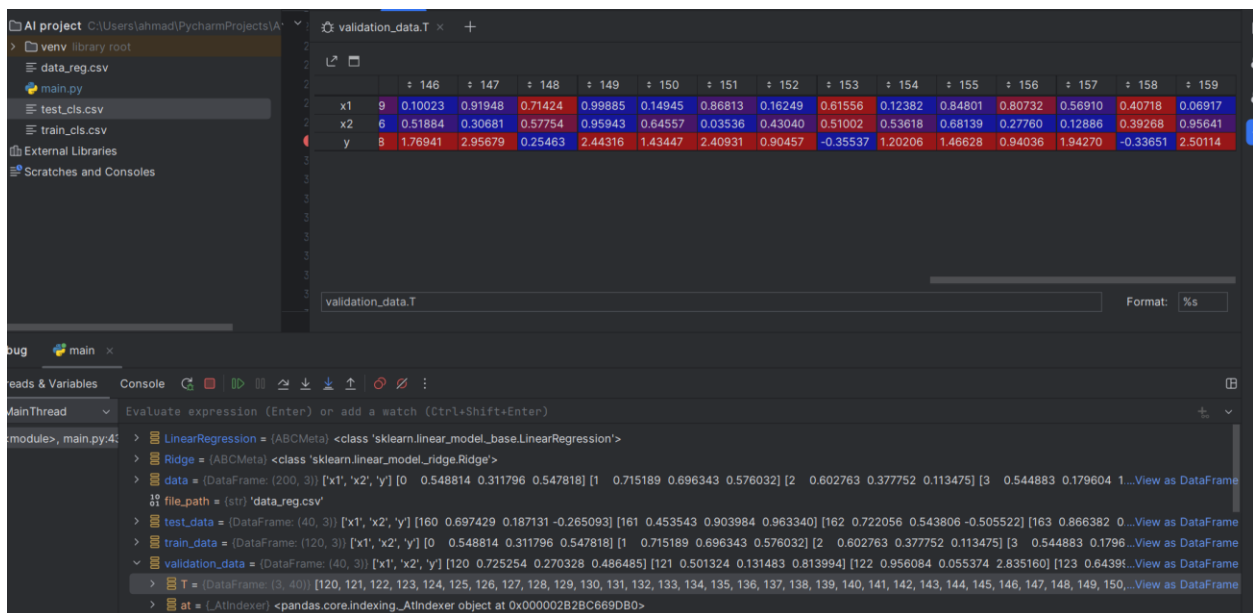


Figure 5: validation set 40 samples.

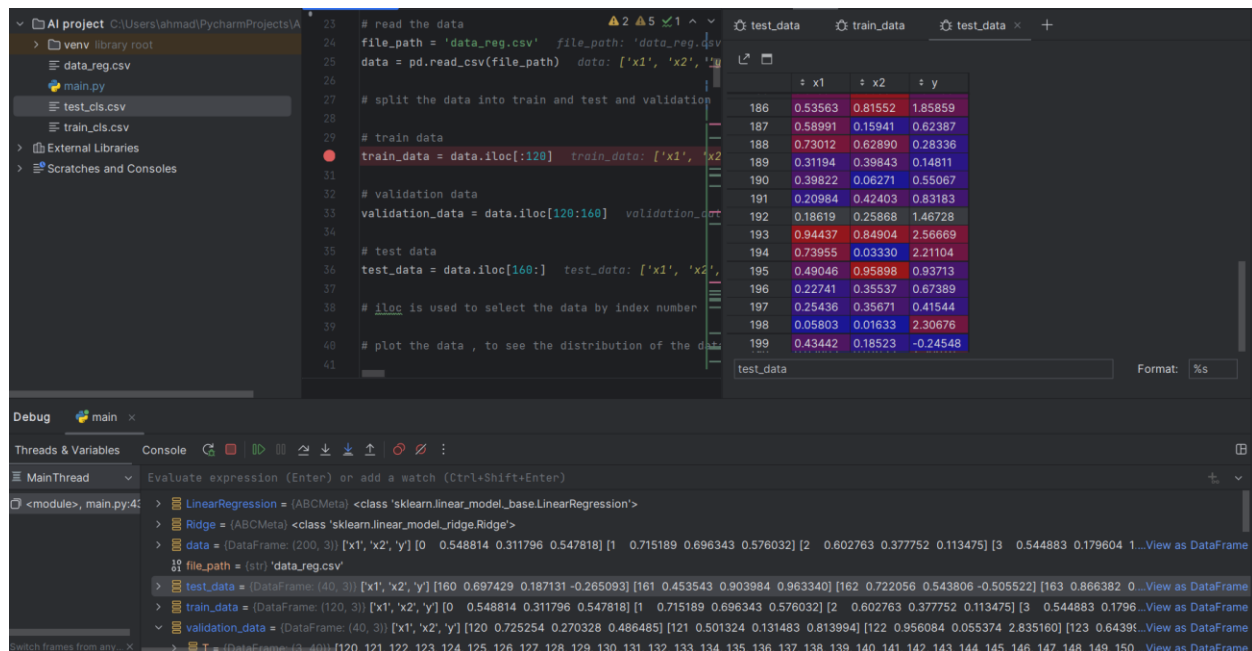


Figure 6:test set 40 samples.

Now we will plot these three sets with different colors, in 3D plane as required:

First, I create an object with (10,8) width and height, to make the plots on it, then telling the object to make sub plot which takes all the space of the object and make it in 3d plane.

Red for training data, green for validation, and blue for testing data.

```

39 # plotting the data in 3D
40
41 fig = plt.figure(figsize=(10, 8)) # Create a figure object with size 10x8 inches.
42 ax = fig.add_subplot(111, projection='3d')
43
44 # Plot each dataset with a different color
45 ax.scatter(train_data['x1'], train_data['x2'], train_data['y'], color='r', label='Training Set') # * Plot the training data red
46
47 ax.scatter(validation_data['x1'], validation_data['x2'], validation_data['y'], color='g', label='Validation Set') # * Plot the validation data green
48
49 ax.scatter(test_data['x1'], test_data['x2'], test_data['y'], color='b', label='Testing Set') # * Plot the testing data blue
50
51 # Labeling
52 ax.set_xlabel('X1')
53 ax.set_ylabel('X2')
54 ax.set_zlabel('Y')
55 ax.set_title('3D Scatter Plot of the Data')
56 ax.legend() # Show the legend in the plot (the labels of the datasets)
57
58 # Show the plot
59 plt.show()

```

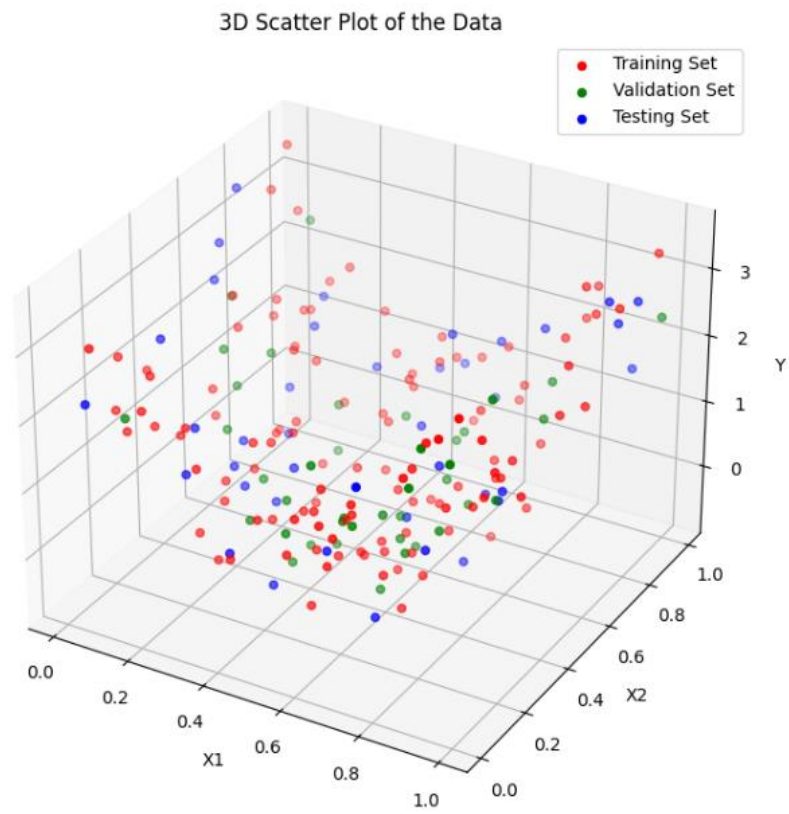


Figure 7: plot of data sets in 3D

Part 2

- 2- Apply polynomial regression on the training set with degrees in the range of 1 to 10. Which polynomial degree is the best? Justify your answer by plotting the validation error vs polynomial degree curve. For each model plot the surface of the learned function alongside with the training examples on the same plot. (hint: you can use `PolynomialFeatures` and `LinearRegression` from `scikit-learn` library)

Non-Linear Regression

Example of non-linear basis functions:

- Radial basis functions

$$f(x) = e^{\frac{-(x-\alpha)^2}{\lambda}}$$

- Arctan Functions

- Monomials

$$\begin{aligned}x &\rightarrow x, x^2, \dots, x^m \\(x_1, x_2) &\rightarrow x_1, x_2, x_1 x_2, x_1^2, x_2^2\end{aligned}$$

Figure 8: universal basis function

In this section , I used in my code the monomials to generate features according to each degree from 1 to 10 , by using the function **PolynomialFeatures()**,for example if the degree is 2 and the original features are **X**=(x1,x2) the polynomial function will generate **Xpoly** = [1, x1 ,x2 ,(x1)^2 ,(x2)^2, x1 * x2] features. Now in my code also when the **Xpoly** generate the features functions , I used it to substitute the values of that features and get the final result . for example here is snapshot of my code :

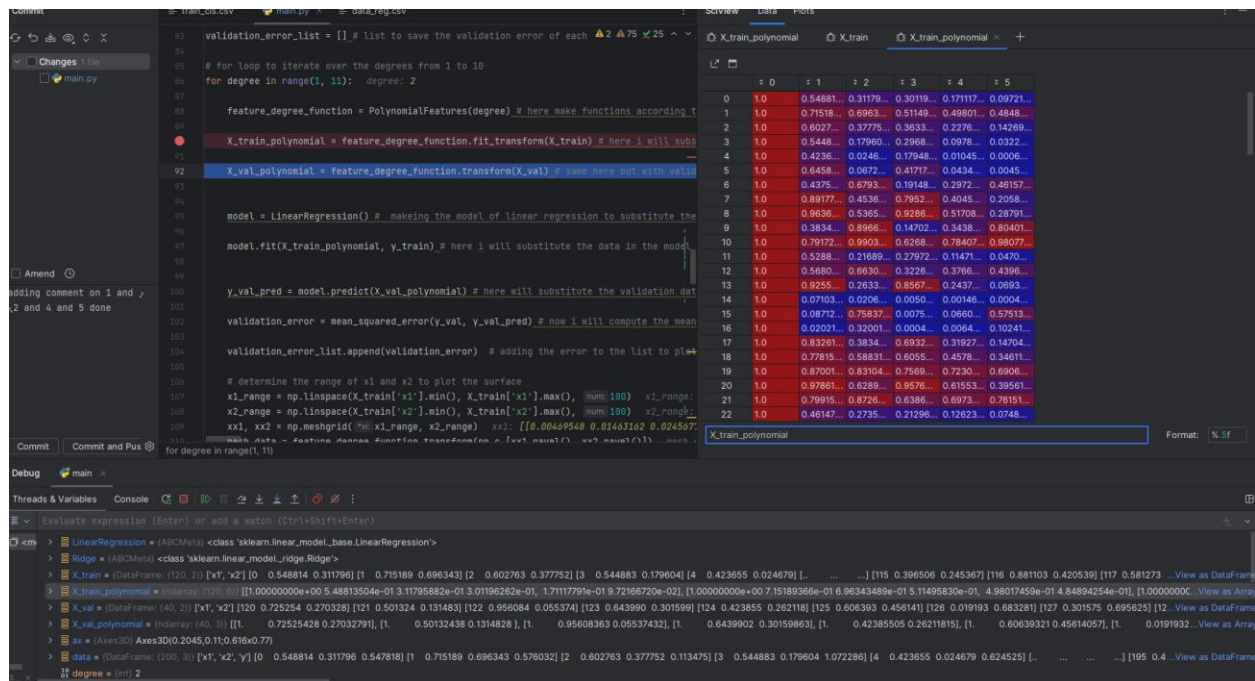


Figure 9: part 2 features degree 2 displayed

As we see here the array on the right shows the features of train data after substitute them, by poly function. and finally, we just substitute validation set using transform function.

We used `fit` transform in training data because we want it at first understand the structure of data and the degree of poly , then we don't need that on validation because it's already done above so we used transform alone.

Then by using `linearRegression ()` and model fit we found the module according to the degree assigned to it , then we make the prediction values by `model.predict(value_of_features_according_to_the_degree)`

Then we found the mean square error between the real values and predictions by mean function , then store that value in `val_errors` list to use it later to find the best degree for predictions , and plot figure showing that.

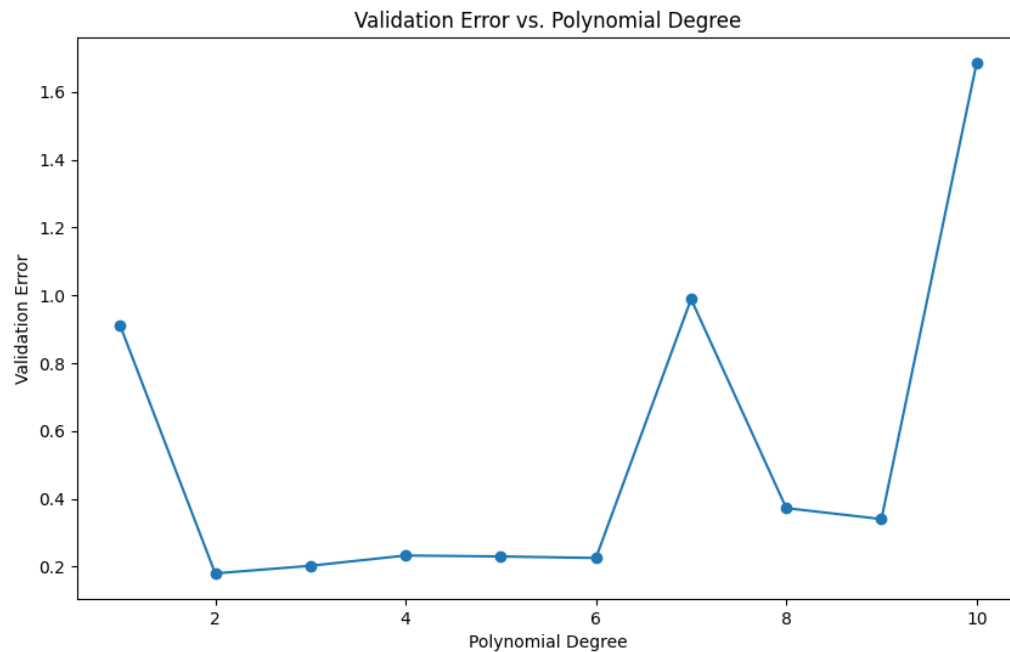


Figure 10: Validation Error vs Polynomial Degree

```
y_val_pred = model.predict(X_val_polynomial) # here will substitute the validation data in the model to predict the target feature.

validation_error = mean_squared_error(y_val, y_val_pred) # now i will compute the mean squared error of the validation data.

validation_error_list.append(validation_error) # adding the error to the list to plot it later.
```

Figure 11: how we get the validation error

```
validation_error_list = [0.911824583849479, 0.17990349576391818, 0.2024332725287566, 0.23268739290547966, 0.2299800431885386, 0.22539808216993928, 0.9890112652181108, 0.3725126622710879, 0.34018777376608433, 1.6855103035270538]

00 * (float64, ()) 0.911824583849479
01 * (float64, ()) 0.17990349576391818
02 * (float64, ()) 0.2024332725287566
03 * (float64, ()) 0.23268739290547966
04 * (float64, ()) 0.2299800431885386
05 * (float64, ()) 0.22539808216993928
06 * (float64, ()) 0.9890112652181108
07 * (float64, ()) 0.3725126622710879
08 * (float64, ()) 0.34018777376608433
09 * (float64, ()) 1.6855103035270538
__len__ * (int) 10
```

Figure 12: validation error for each degree

As we see, when we found the validation error by using mean square error between the predicted values and validation true values of y , we found that the least mean square error when the degree of the regression is 2.

And here is the surface plot for each degree from 1 to 10.

Polynomial Regression (Degree 1) with Training Data

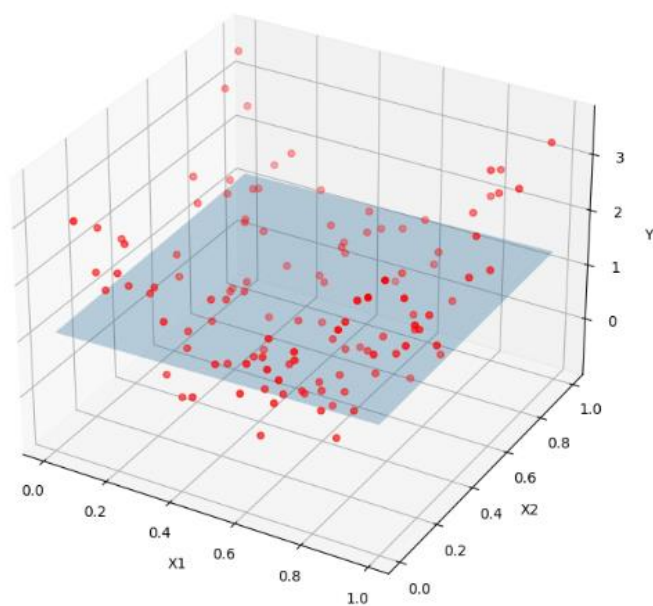


Figure 13: degree 1 plot regression

Polynomial Regression (Degree 2) with Training Data

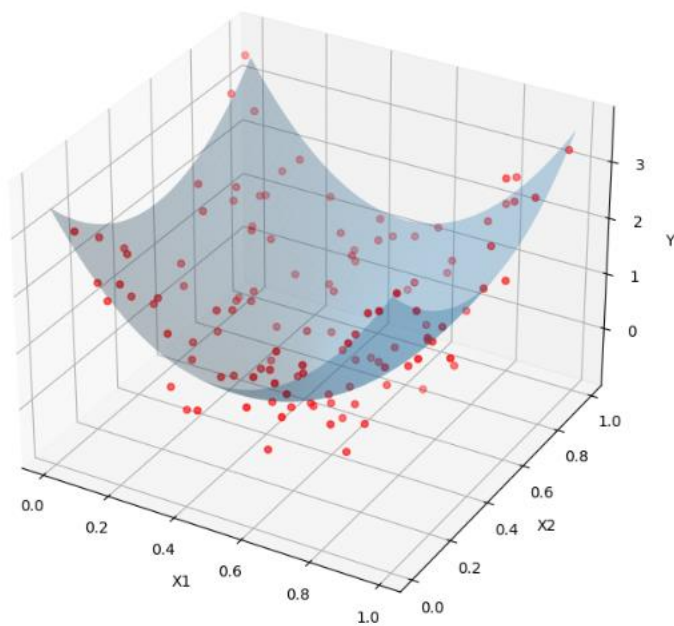


Figure 14: degree 2

Polynomial Regression (Degree 3) with Training Data

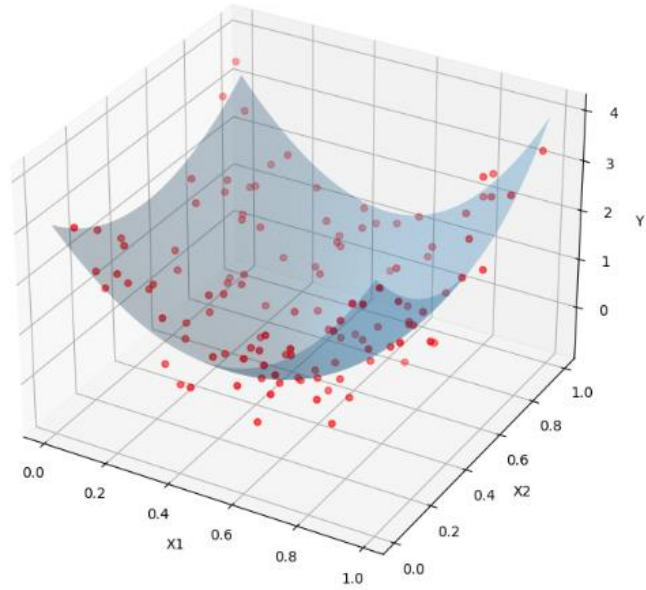


Figure 15: degree 3

Polynomial Regression (Degree 4) with Training Data

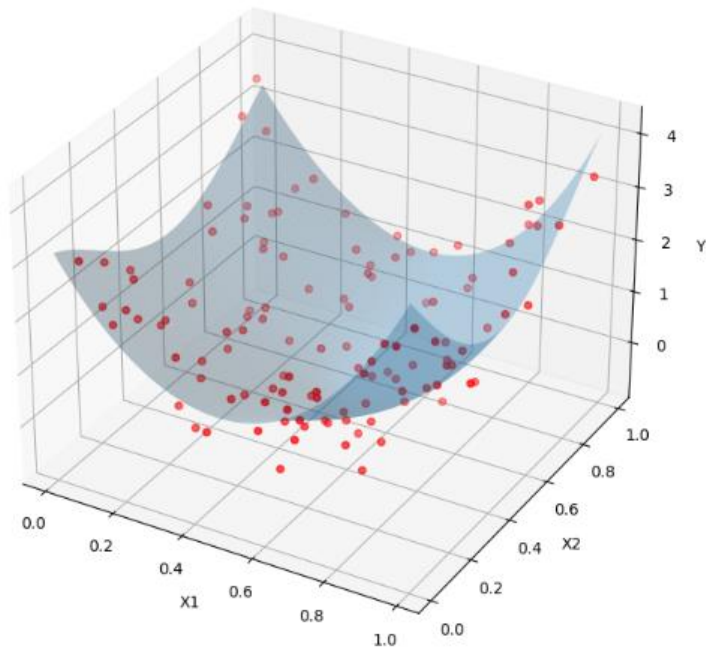


Figure 16: degree 4

Polynomial Regression (Degree 5) with Training Data

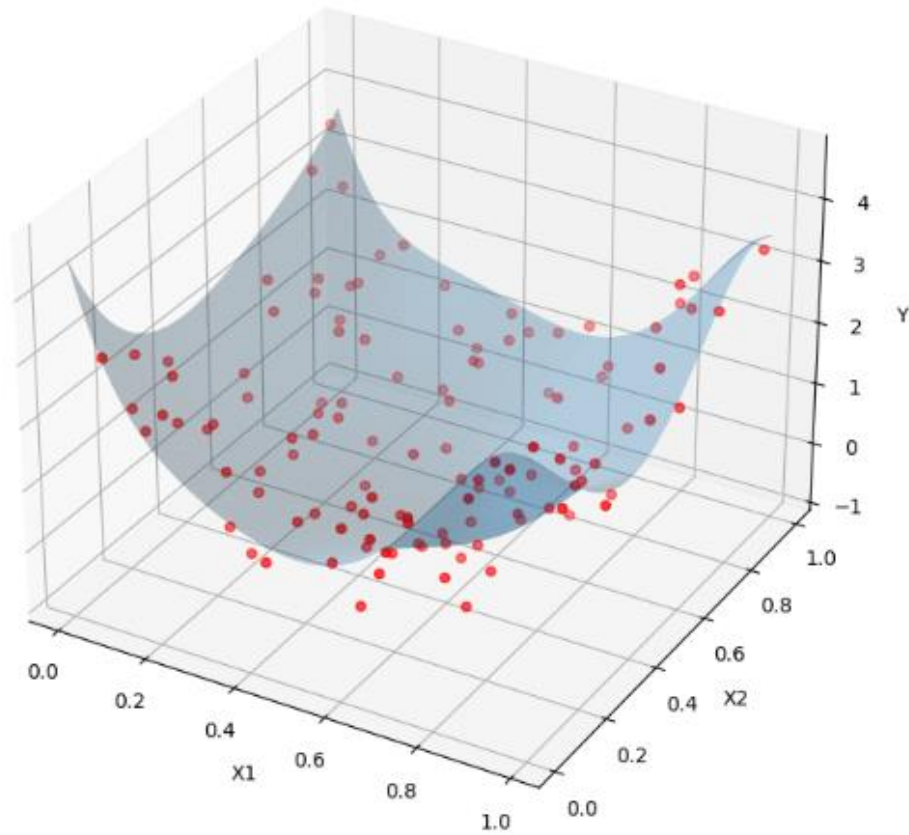


Figure 17: degree 5

Polynomial Regression (Degree 6) with Training Data

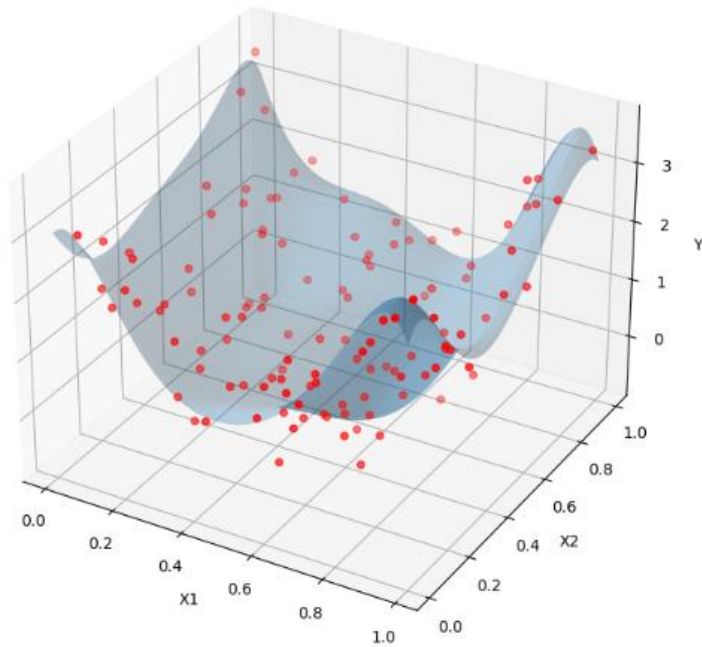


Figure 18: degree 6

Polynomial Regression (Degree 7) with Training Data

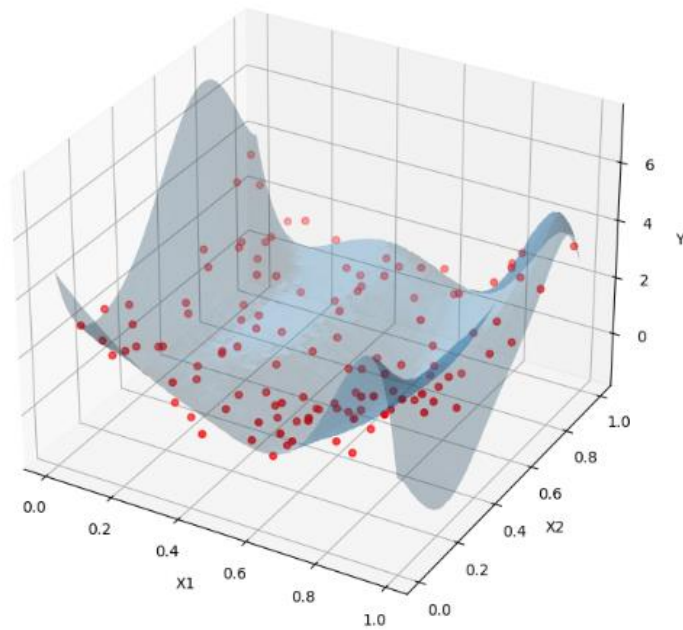


Figure 19: degree 7

Polynomial Regression (Degree 8) with Training Data

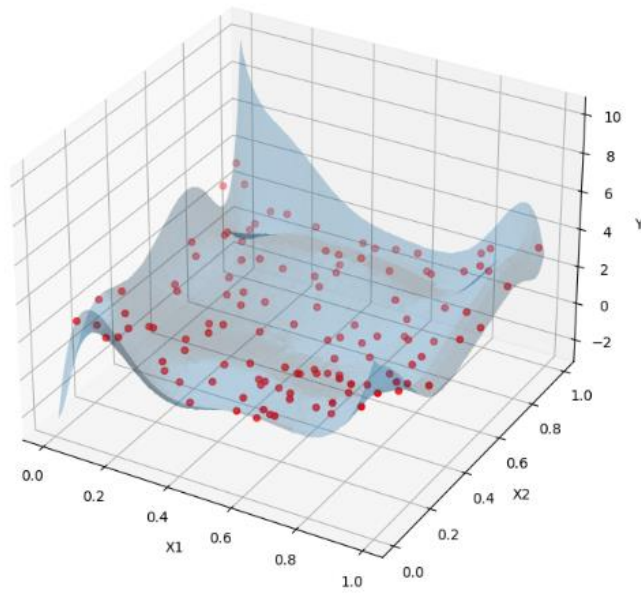


Figure 20: degree 8

Polynomial Regression (Degree 9) with Training Data

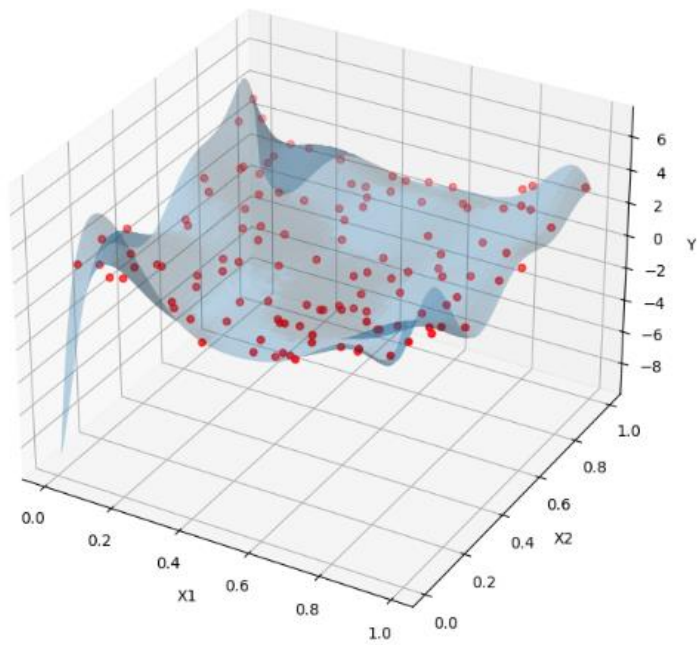


Figure 21: degree 9

Polynomial Regression (Degree 10) with Training Data

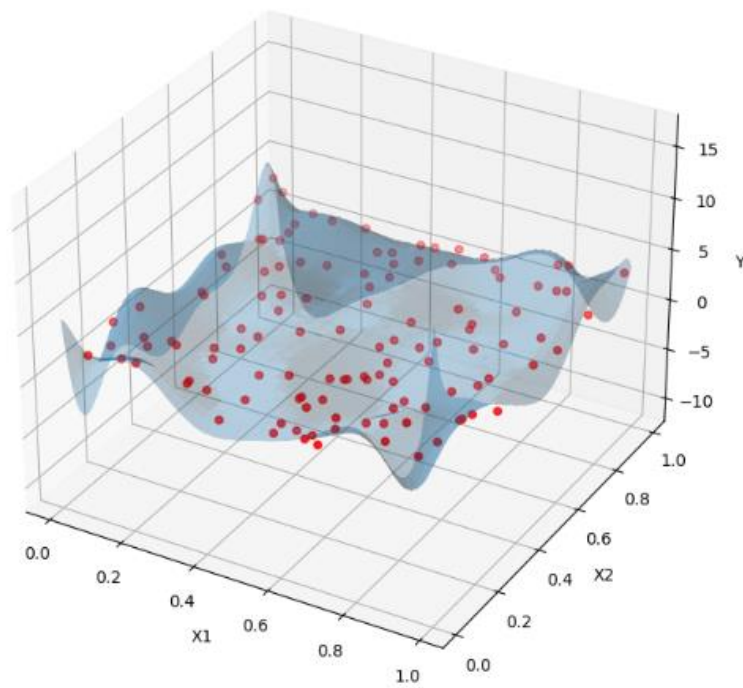


Figure 22: degree 10

Part 3:

- 3- Apply ridge regression on the training set to fit a polynomial of degree 8. For the regularization parameter, choose the best value among the following options: {0.001, 0.005, 0.01, 0.1, 10}. Plot the MSE on the validation vs the regularization parameter.
(hint: you can use `Ridge` regression implementation from `scikit-learn`)

Figure 23: part 3 text

Ridge Regression

- Alternatively, we can choose a regularization term that penalizes the squares of the parameter magnitudes. Then, our regularized loss function is:

$$L_{Ridge}(\mathbf{w}) = \frac{1}{n} \sum_{i=1}^n (y_i - \mathbf{w}^T \mathbf{x}_i)^2 + \lambda \sum_{j=1}^d w_j^2$$

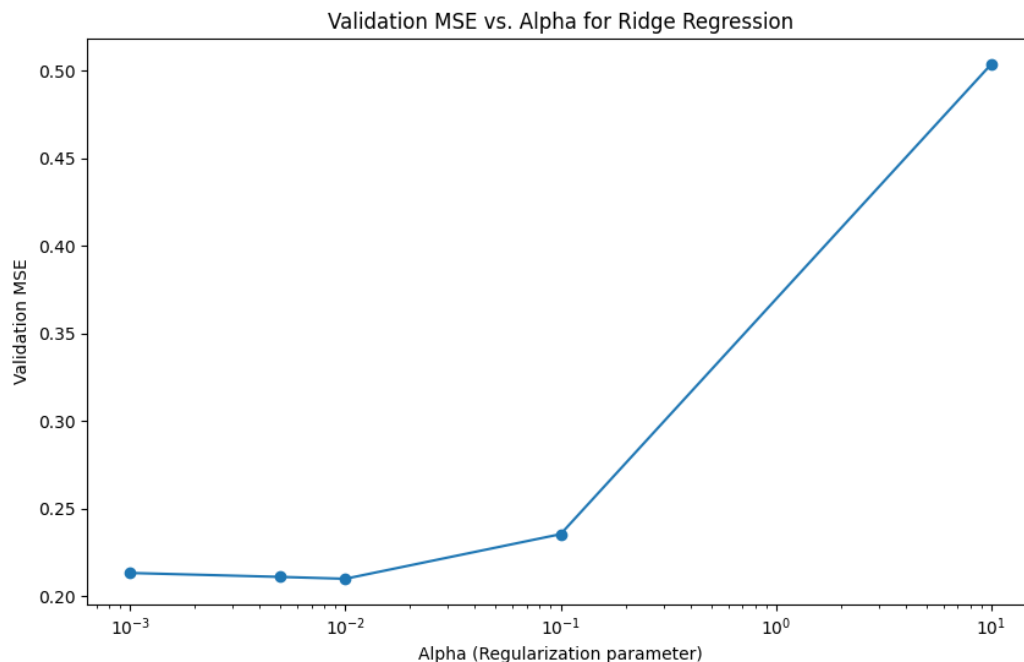


Figure 24: validation error vs alpha values with Ridge regression

```

01 alpha = (int) 10
✓ alphs = {list: 5} [0.001, 0.005, 0.01, 0.1, 10]
  10 0 = {float} 0.001
  10 1 = {float} 0.005
  10 2 = {float} 0.01
  10 3 = {float} 0.1
  10 4 = {int} 10
  10 __len__ = {int} 5

```

Figure 25: the values of alphs

```

✓ val_errors = {list: 5} [0.21328335916735383, 0.21103328202646282, 0.20996554038850385, 0.23545304328552435, 0.5038254258404573]
  10 0 = {float64: ()} 0.21328335916735383
  10 1 = {float64: ()} 0.21103328202646282
  10 2 = {float64: ()} 0.20996554038850385
  10 3 = {float64: ()} 0.23545304328552435
  10 4 = {float64: ()} 0.5038254258404573
  10 __len__ = {int} 5

```

Figure 26: the values of validation error according to each alpha using mean error with Ridge module.

As we see here the best alpha that gave us the minimum error is alpha = 0.01 with error 0.2099 according to mean square error between the predicted values on the validation set on the ridge module and real values of output on the validation set y.

```

67 for alpha in alphs:  alpha: 10
68
69     model = Ridge(alpha=alpha) # creating the model of ridge regression with the alpha value.
70
71     model.fit(X_train_poly, y_train) # here i will substitute the data in the model to train it.
72
73
74     y_val_pred = model.predict(X_val_poly) # getting the predicted values of the validation data.
75
76     val_error = mean_squared_error(y_val, y_val_pred) # computing the mean squared error of the validation data. val_error: 0.5038254258404573
77
78     val_errors.append(val_error) # adding the error to the list to plot it later.
79

```

Figure 27: how we get the validation error for each alpha

First, we make the module using Ridge function given it the alpha in one iteration, then substitute the train set inside the function go get the final module when it trained by data. Then finding the error for each alpha by finding the mean error between the predicted validation set values and real values.

Part 4

Logistic Regression

The `train_cls.csv` file contains a set of training examples for a binary classification problem, and the testing examples are provided in the `test_cls.csv` file. The following figures show these examples.

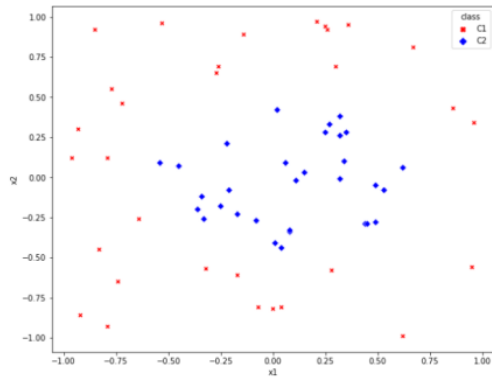


Figure 1 Training Set

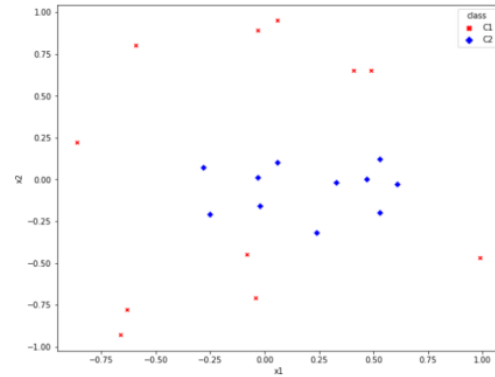


Figure 2 Testing Set

1. using the logistic regression implementation of `scikit-learn` library, Learn a logistic regression model with a linear decision boundary. Draw the decision boundary of the learned model on a scatterplot of the training set (similar to Figure 1). Compute the training and testing accuracy of the learned model.

Figure 28: part 4 text

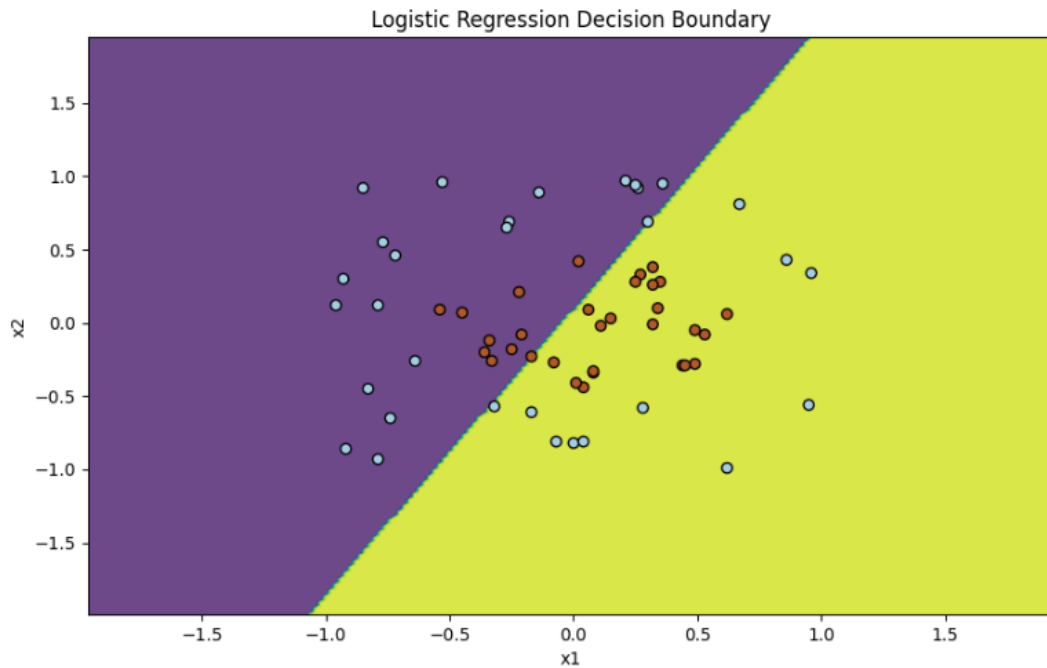


Figure 29: Logistic regression part 4 Decision Boundary

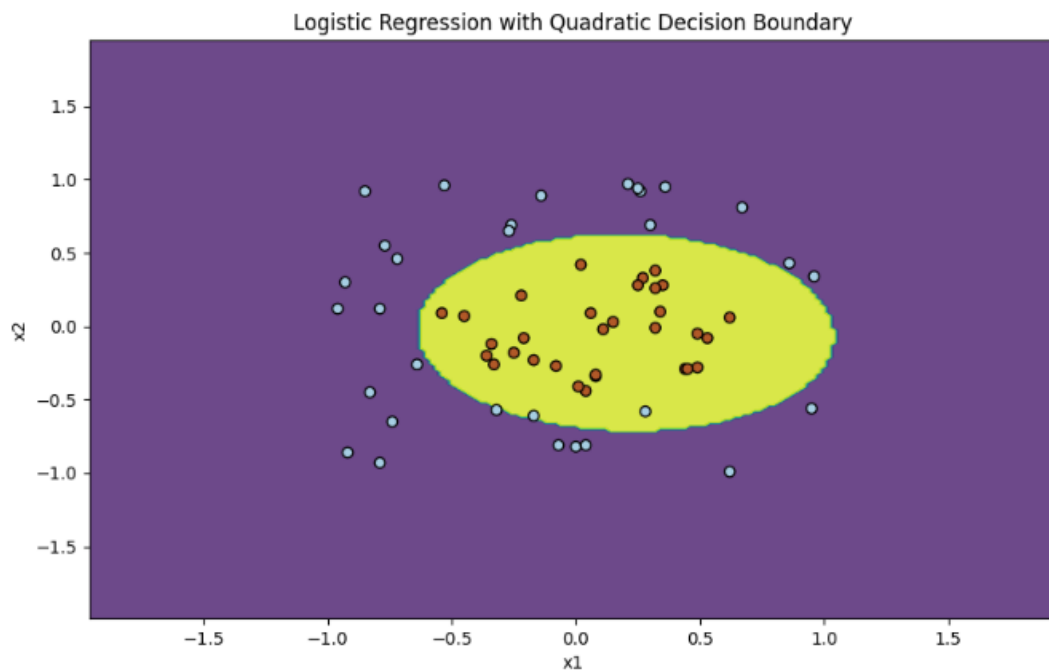
```
Training Accuracy: 0.6612903225806451
Testing Accuracy: 0.6818181818181818
```

Figure 30: accuracies of the module

According to training accuracy and testing accuracy , this module appears **underfitting** in both of them , in order to low values that we got. I think the problem that the module that we create is too simple to get higher accuracy, and we need to make it more complex than liner , to better degree which gives us better results.

Part 5

2. Repeat part 1 but now to learn a logistic regression model with quadratic decision boundary.



```
training accuracy of quadratic: 0.967741935483871
testing accuracy of quadratic: 0.9545454545454546
```

Now according to the both of training accuracy and testing, we see a big difference here in the results, this model gave us a higher accuracy in both training and testing, compared to linear one above, and in terms of overfitting this model not showing overfitting **due to the testing accuracy** which is high , and it more stable than the above one with linear model.

Part 6

3. Comment on the learned models in 1 and 2 in terms of overfitting/underfitting.

Figure 31: part 6 text

Answered above.