

$$F(x) = \text{TrailingZeros}(n)$$

$$\text{Let } f(x) = n$$

$$\therefore n = \frac{x}{5} + \frac{x}{25} + \frac{x}{125} + \dots$$

$$n = x \sum_{i=1}^{\infty} \frac{1}{5^i} = x \cdot \sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i$$

$$\therefore \sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$$

$$\therefore \sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i = \frac{(\frac{1}{5})}{1-(\frac{1}{5})} = \frac{1}{4}$$

$$\therefore \sum_{i=1}^m \left(\frac{1}{5}\right)^i \leq \frac{1}{4}$$

Because we loop until $\frac{x}{5^i} < 1$

$$\therefore n \leq x \left(\frac{1}{4}\right)$$

$$\therefore \boxed{x \geq 4n}$$

$$\therefore \text{Low} = 4 * n$$

$$\text{high} = 5 * n \quad (\text{why?!})$$

$$\downarrow \text{ This means } x \leq 5n$$

Proof:

$$\text{assume } x > 5n$$

\Rightarrow TrailingZeros(x) is monotonous function
which means
for all $x \leq y$
we have $f(x) \leq y$

$$\text{so if } x > 5n \\ \therefore f(x) > f(5n)$$

$$\therefore f(x) = n$$

$$\therefore n > f(5n)$$

$$\therefore n > (5n) \cdot \sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i$$

$$n > 5n \left(\frac{1}{4}\right)$$

$$n > \frac{5}{4} n \quad \# \text{ contradiction}$$

$$\therefore \boxed{x \leq 5n}$$