# Homework - 1
# CSE 431: Digital Signal Processing
# Fall 2020

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Assigned Date: 24th November, 2020
Due Date for Written Task: 4th December, 2020; 11.58PM
Date for Viva: Will be announced later

## Basic Instructions:

This homework, accompanied by a viva-voce will replace your mid semester examination. In this section, you will find a general overview about the homework, along with its grading policy.

- **Exam Format** The homework has two components:
    1. A single PDF file outlining handwritten answers to all questions asked in this file
    2. A viva-voce based on everything taught in the class before the viva date

  **Marks Distribution** Questions asked in this file is graded on points, instead of on marks. Total points that you obtain will later be converted to 15 marks based on your answers and a verification viva-voce. Rest of the 15 marks of your mid semester examination will come entirely from a general viva-voce based on everything taught in the class, along with reading materials from the textbook. The syllabus for this viva will be everything taught before the viva date. This viva will be comprehensive and will be your ticket to good grades.

  **Format of your submitted PDF** You will solve all questions by hand in a notebook. Then you will capture images of your solutions, combine them in a PDF and submit this PDF. Remember that, you are required to submit a SINGLE PDF, not multiple image files or multiple PDF files. Also remember that, you cannot type your solutions in any word processing application (eg: MS Word or Latex).

  **Submission Deadline** Please look at the due date mentioned above.

- All submissions must follow the honor code mentioned in the class, which in summary is: **Do not copy anything from any sources including online resources, your classmates, friends, seniors and so on. You are NOT PERMITTED to do group study for this homework**. Of course, you can use the class lecture or the textbook or any other textbooks available to you **(except any solution manual)** for solving your homework problems. If you are using any book, mention the name of the book in the acknowledgement section of your PDF file. Violating the honor code will incur the following penalties:
    1. The homework will not be graded. You will also lose marks of the general viva-voce. This in summary means, you will receive 0 marks against this mid-semester examination.
    2. In addition to the above penalty, you will lose another 15 marks from the total marks you obtain at the end of the semester. You will also be reported for further disciplinary actions to the head of the department.

- Understand that, even 1% copy will be regarded as violation of the honor code. **All parties involved in the process will be penalized**. Also remember that, any suspected work will only be penalized after a thorough discussion with you. This discussion will start by giving you a chance

to accept or reject my suspicion. If you actually were involved in this violation in any way and you accept it, then the second 15 marks reduction penalty will not apply to your case. If you reject my suspicion, then further investigation will be conducted and you will also face a viva related to the **TOPICS** of the assignment. I kindly request you to obey the honor code so that we both can be saved from this unwanted hassle.

# Homework Questions

1. {20 points} Draw the following signals:
   (a) $u[n] + \delta[n]; \qquad for\ -\infty \leq n \leq \infty$
   (b) $u_r[n-2] + u[n+1]; \qquad for\ -10 \leq n \leq 5$

2. {10 points} Define:
   (a) Energy and Power Signal
   (b) Even and Odd Signal

3. {15 points} Consider you are given an input signal $x[n]$ as:

$$x[n] = \{..., -2, \mathbf{1}, 2, ...\}$$

   Consider you are also given an impulse response $h[n]$ as:

$$h[n] = \{..., 2, 1, \mathbf{0}, -2, -3...\}$$

   In both equations, the enlarged bold digit represents the sequence value at $n = 0^{th}$ sample. Now your task is to find **graphically** the response of the linear time invariant system.

4. {20 points} Consider you are given an input signal $x[n]$ which is periodic with $period = N$ (that is, $x[n] = x[n+N]$). Also consider that the linear time invariant response of an impulse is given as $h[n]$. Prove that, the response of the linear time invariant system $y[n]$ under the given condition will also be periodic. Note that, you proof must be sound and formal, with no additional assumptions.

5. {$15 + 20 = 35$ points} Let me remind you about two general formula of mathematics and then ask the actual question.

   **Formula 1:** The first formula is about the Geometric Series. You know the equation of geometric series is:

$$1 + a^1 + a^2 + a^3 + ... = \sum_{k=0}^{\infty} a^k = \frac{1}{1-|a|}; \qquad whenever\ |a| < 1$$

   Have you asked yourself what would happen if the sum starts from some other number, say 2, instead of 0? Let us look closer to this case:

$$a^2 + a^3 + a^4 + a^5 + ... = a^2(1 + a^1 + a^2 + a^3 + ...) = a^2 \sum_{k=0}^{\infty} a^k = \frac{a^2}{1-|a|}; \qquad whenever\ |a| < 1$$

**Formula 2:** Now let us look at the second formula. Say we have a series that looks like this: $\sum_{k=0}^{\infty} a^k$. What happens if we want to go to the opposite direction? That is, we want to swap the bounds of the summation, so that the summation becomes: $\sum_{k=-\infty}^{0} a^k$. The trick to solve this problem is to make a change of variable. We will change the variable $n$ to $-t$ and see what follows:

$$\begin{aligned} a^k \quad & \textit{becomes} \quad a^{-t} \\ k = 0 \quad & \textit{becomes} \quad t = 0 \\ k = \infty \quad & \textit{becomes} \quad t = -\infty \end{aligned}$$

Thus the summation becomes: $\sum_{k=0}^{-\infty} a^{-t}$. Basically this summation is running backwards. We can then easily run it to forward directions as:

$$\sum_{k=0}^{-\infty} a^{-t} = \sum_{k=-\infty}^{0} a^{-t}$$

**Actual Question Now:** Consider you are given the response of a linear time invariant system as:

$$h[n] = \left(\frac{1}{4}\right)^n \{u[n] - u[n-1]\} + 2^{2n} u[-5-n]$$

Prove whether the above system is:

(a) Causal

(b) Stable

Your answer needs to have clear, **complete** and **detailed** explanation and justification. There should not be any assumptions or any fill it yourself portion left for me. When I read your solution, I should not have any question like, "How did he derived this?" or "Why this is true?". Understand that, if your answer is not well explained or justified or if you answer is in Yes/No, then you will not receive any (partial) points for this problem.

# Special Note:

- Your handwriting must be neat, with enough spaces in between lines. If your handwriting is unclear, you will be penalized.
- Before submitting your solutions, check the quality of your captured photographs. Your PDF file should be clear (without any blur) and readable. If your submitted PDF is hard to read, then I will not grade it (you get zero).