Although simple circuits can be analyzed using KCL, KVL and Ohm's Law alone, it becomes cumbersome for large circuits with many nodes & loops and is not a viable option. As a result, two other techniques based on these laws are developed that can be used to analyze large circuits. These two techniques are Nodal analysis and Mesh analysis

## Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

To simplify matters, initially we shall assume that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed later.

In nodal analysis, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

1. ***Select a node as the reference node. Assign voltages $v_1$, $v_2$,…,$v_{n-1}$ to the remaining n − 1 nodes. The voltages are referenced with respect to the reference node.***
2. ***Apply KCL to each of the n − 1 non reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.***
3. ***Solve the resulting simultaneous equations to obtain the unknown node voltages.***

The first step in nodal analysis is selecting a node as the *reference* or *datum* node. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in Fig. 1. The type of ground in Fig. 1(b) is called a chassis ground and is used in devices where the case, enclosure, or chassis acts as a reference point for all circuits. When the potential of the earth is used as reference, we use the earth ground in Fig. 1(a) or (c). We shall always use the symbol in Fig. 1(b).

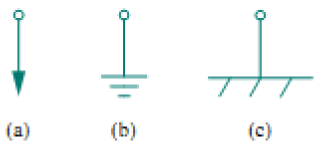


Figure 1 Common symbols for indicating reference node

Once we have selected a reference node, we assign voltage designations to non-reference nodes. Consider, for example, the circuit in Fig. 2(a). Node 0 is the reference node (v = 0), while nodes 1 and 2 are assigned voltages $v_1$ and $v_2$, respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in Fig. 2(a), each node voltage is the voltage rise from the reference node to the corresponding non-reference node or simply the voltage of that node with respect to the reference node.

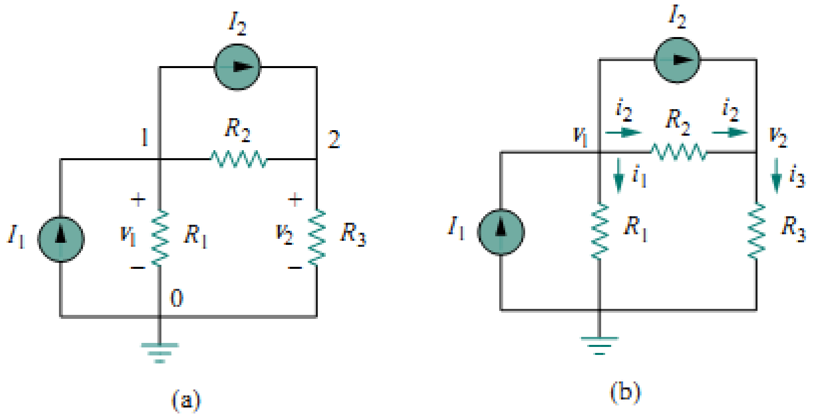


Figure 2 Typical circuit for nodal analysis



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

As the second step, we apply KCL to each non-reference node in the circuit. To avoid putting too much information on the same circuit, the circuit in Fig. 2(a) is redrawn in Fig. 2(b), where we now add $i_1$, $i_2$, and $i_3$ as the currents through resistors $R_1$, $R_2$, and $R_3$, respectively. At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2$$

At node 2

$$I_2 + i_2 = i_3 \qquad (3.1)$$

We now apply Ohm's law to express the unknown currents $i_1$, $i_2$, and $i_3$ in terms of node voltages. The key idea to bear in mind is that, since resistance is a passive element, by the passive sign convention, ***current must always flow from a higher potential to a lower potential.***

We can express this principle as

$$i = \frac{v_{higher} - v_{lower}}{R} \qquad (3.2)$$

For the circuit in Fig. 2(b) we can obtain

$$\left.\begin{aligned} i_1 &= \frac{v_1 - 0}{R_1} \\ i_2 &= \frac{v_1 - v_2}{R_2} \\ i_3 &= \frac{v_2 - 0}{R_3} \end{aligned}\right\} \qquad (3.3)$$

Substituting equations 3.3 in equations 3.1 we get

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$or, \quad I_1 - I_2 = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$or, \quad I_1 - I_2 = \frac{v_1}{R_1} + \frac{v_1}{R_2} - \frac{v_2}{R_2} \qquad (3.4)$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \qquad (3.5)$$

We can write equations 3.4 and 3.5 in matrix form

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \qquad (3.6)$$

This matrix can be solved to find the node voltages $v_1, v_2$.

## Nodal analysis (Format approach)

With the observation of equation 3.6 we can develop another approach for generating the equation. This approach is called format approach. The steps are as follows

1. *Choose a reference node and assign a subscripted voltage label to the (N-1) remaining nodes of the network.*
2. *The number of equations required for a complete solution is equal to the number of subscripted voltages (N-1). Column 1 of each equation is formed by summing the conductance tied to the node of interest and multiplying the result by that subscripted nodal voltage.*
3. *We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal*



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

*voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.*

4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.*
5. *Solve the resulting simultaneous equations for the desired voltages.*

Consider the example

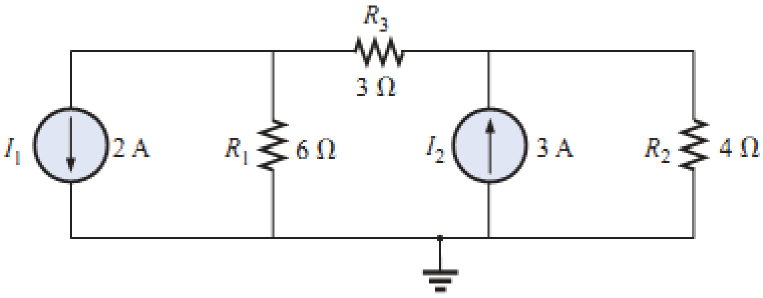


**Figure 3 Figure for nodal analysis (format approach)**

Step 1: The figure is redrawn with assigned subscripted voltages in Fig. 4

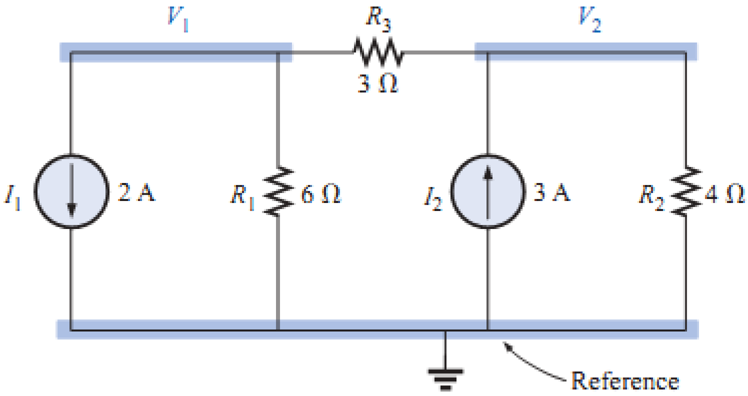


**Figure 4 Defining the nodes for the network of figure 3**

Steps 2 to 4

$$V_1:\ \underbrace{\left(\frac{1}{6\ \Omega}+\frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = \overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\ \text{A}}$$

$$V_2:\ \underbrace{\left(\frac{1}{4\ \Omega}+\frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = \overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\ \text{A}}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

## Nodal analysis with voltage source

We now consider how voltage sources affect nodal analysis. We use the circuit in Fig. 5 for illustration. Consider the following two possibilities.

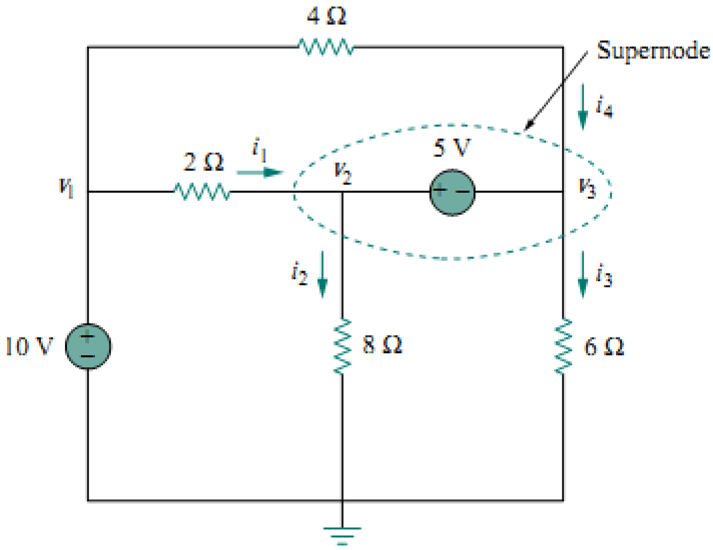


Figure 5 A circuit with voltage source and supernode

**Case 1:** If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source. In Fig. 5, for example,

$$v_1 = 10 \qquad (3.7)$$

So our analysis is simplified by the knowledge of this node.

**Case 1:** If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages.

***A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.***

In Fig. 5 nodes 2 and 3 form a supernode.

We analyze a circuit with supernodes using the same three steps mentioned in the previously except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, KCL must be satisfied at a supernode like any other node. Hence, at the supernode in Fig. 5,

$$i_1 + i_4 = i_2 + i_3 \qquad (3.8a)$$

Or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \qquad (3.8b)$$

To apply Kirchhoff's voltage law to the supernode in Fig. 5, we redraw the circuit as shown in Fig. 6. Going around the loop in the clockwise direction gives

$$-v_2 + 5 + v_3 = 0 \quad so, \quad v_2 - v_3 = 0 \qquad (3.9)$$

From Eqs. (3.7), (3.8b), and (3.9), we obtain the node voltages.

Note the following properties of Supernode

1. ***The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.***

2. ***A supernode has no voltage of its own.***
3. ***A supernode requires the application of both KCL and KVL.***

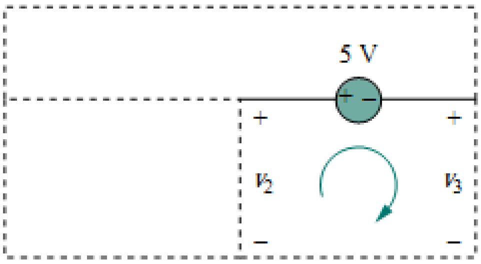


**Figure 6 Applying KVL to a supernode**

**EXAMPLE 3.1**

Calculate the node voltages in the circuit shown in Fig. 3.3(a).

**Solution:**

Consider Fig. 3.3(b), where the circuit in Fig. 3.3(a) has been prepared for nodal analysis. Notice how the currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that $i_2$ enters the 4 Ω resistor from the left-hand side, $i_2$ must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages $v_1$ and $v_2$ are now to be determined.

At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Longrightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

or

$$3v_1 - v_2 = 20 \qquad (3.1.1)$$

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Longrightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \qquad (3.1.2)$$

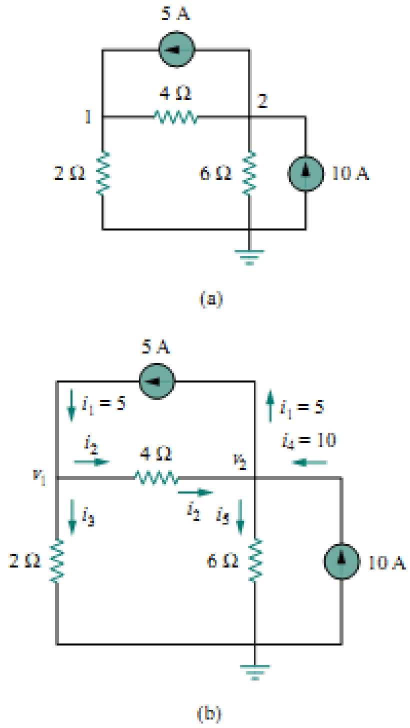


**Figure 3.3** For Example 3.1: (a) original circuit, (b) circuit for analysis.

Now we have two simultaneous Eqs. (3.1.1) and (3.1.2). We can solve the equations using any method and obtain the values of $v_1$ and $v_2$.

**METHOD 1** Using the elimination technique, we add Eqs. (3.1.1) and (3.1.2).

$$4v_2 = 80 \quad \Longrightarrow \quad v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \quad \Longrightarrow \quad v_1 = \frac{40}{3} = 13.33 \text{ V}$$

**METHOD 2** To use Cramer's rule, we need to put Eqs. (3.1.1) and (3.1.2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \qquad (3.1.3)$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain $v_1$ and $v_2$ as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.33 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}$$

giving us the same result as did the elimination method.

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5 \text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6667 \text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666$$

$$i_4 = 10 \text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that $i_2$ is negative shows that the current flows in the direction opposite to the one assumed.

## EXAMPLE 3.2

Determine the voltages at the nodes in Fig. 3.5(a).

**Solution:**

The circuit in this example has three nonreference nodes, unlike the previous example which has two nonreference nodes. We assign voltages to the three nodes as shown in Fig. 3.5(b) and label the currents.

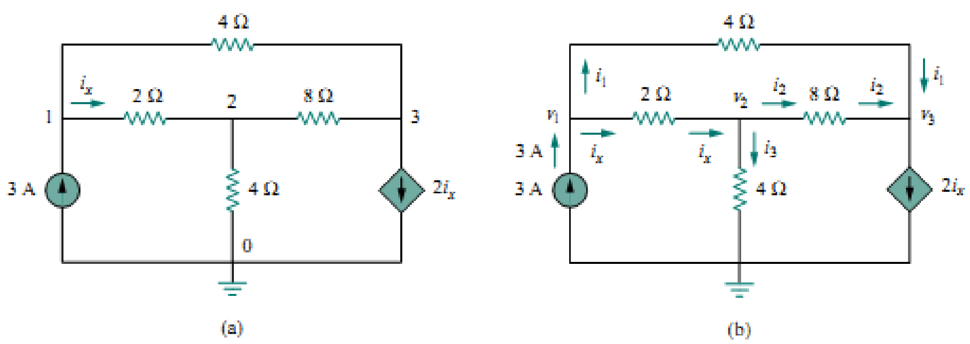


Figure 3.5 For Example 3.2: (a) original circuit, (b) circuit for analysis.

At node 1,

$$3 = i_1 + i_x \qquad \Longrightarrow \qquad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \tag{3.2.1}$$

At node 2,

$$i_x = i_2 + i_3 \qquad \Longrightarrow \qquad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \tag{3.2.2}$$

At node 3,

$$i_1 + i_2 = 2i_x \qquad \Longrightarrow \qquad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3.2.3)$$

We have three simultaneous equations to solve to get the node voltages $v_1$, $v_2$, and $v_3$. We shall solve the equations in two ways.

**METHOD 1** Using the elimination technique, we add Eqs. (3.2.1) and (3.2.3).

$$5v_1 - 5v_2 = 12$$

or

$$v_1 - v_2 = \frac{12}{5} = 2.4 \qquad (3.2.4)$$

Adding Eqs. (3.2.2) and (3.2.3) gives

$$-2v_1 + 4v_2 = 0 \qquad \Longrightarrow \qquad v_1 = 2v_2 \qquad (3.2.5)$$

Substituting Eq. (3.2.5) into Eq. (3.2.4) yields

$$2v_2 - v_2 = 2.4 \qquad \Longrightarrow \qquad v_2 = 2.4, \qquad v_1 = 2v_2 = 4.8\text{ V}$$

From Eq. (3.2.3), we get

$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4\text{ V}$$

Thus,

$$v_1 = 4.8\text{ V}, \qquad v_2 = 2.4\text{ V}, \qquad v_3 = -2.4\text{ V}$$

**METHOD 2** To use Cramer's rule, we put Eqs. (3.2.1) to (3.2.3) in matrix form.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

From this, we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \qquad v_2 = \frac{\Delta_2}{\Delta}, \qquad v_3 = \frac{\Delta_3}{\Delta}$$

where $\Delta$, $\Delta_1$, $\Delta_2$, and $\Delta_3$ are the determinants to be calculated as follows. As explained in Appendix A, to calculate the determinant of a 3 by 3 matrix, we repeat the first two rows and cross multiply.

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{c|ccc|c} & 3 & -2 & -1 & \\ & -4 & 7 & -1 & \\ & 2 & -3 & 1 & \\ - & 3 & -2 & -1 & + \\ - & -4 & 7 & -1 & + \\ - & & & & + \end{array}$$

$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

Similarly, we obtain

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \\ 12 & -2 & -1 \\ 0 & 7 & -1 \end{vmatrix} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \\ 3 & 12 & -1 \\ -4 & 0 & -1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & 12 \\ -4 & 7 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$

as we obtained with Method 1.

## EXAMPLE 3.3

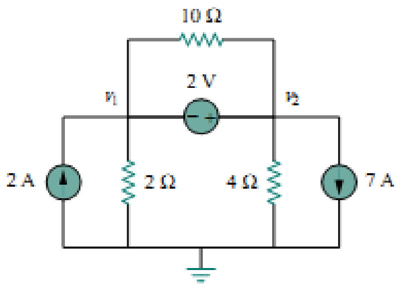


Figure 3.9 For Example 3.3.

For the circuit shown in Fig. 3.9, find the node voltages.

**Solution:**

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

$$2 = i_1 + i_2 + 7$$

Expressing $i_1$ and $i_2$ in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Longrightarrow \quad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \tag{3.3.1}$$

To get the relationship between $v_1$ and $v_2$, we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \quad \Longrightarrow \quad v_2 = v_1 + 2 \tag{3.3.2}$$

From Eqs. (3.3.1) and (3.3.2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Longrightarrow \quad v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

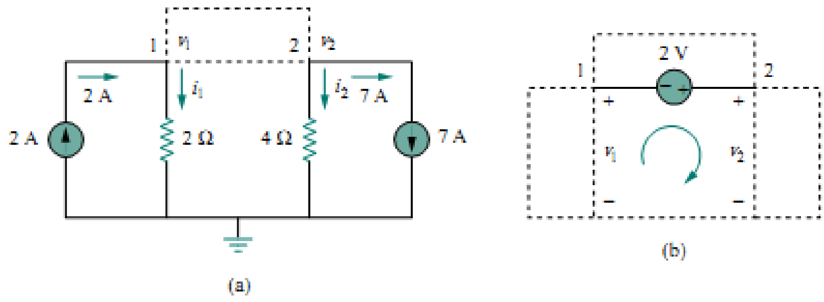


Figure 3.10 Applying: (a) KCL to the supernode, (b) KVL to the loop.

1. Alexander Example 3.4
2. Alexander Practice Problem 3.1-3.4



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

## Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it.

Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is non-planar. A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches. For example, the circuit in Fig. 7(a) has two crossing branches, but it can be redrawn as in Fig. 7(b). Hence, the circuit in Fig. 7(a) is planar. However, the circuit in Fig. 8 is non-planar, because there is no way to redraw it and avoid the branches crossing. Non-planar circuits can be handled using nodal analysis, but they will not be considered in this course.

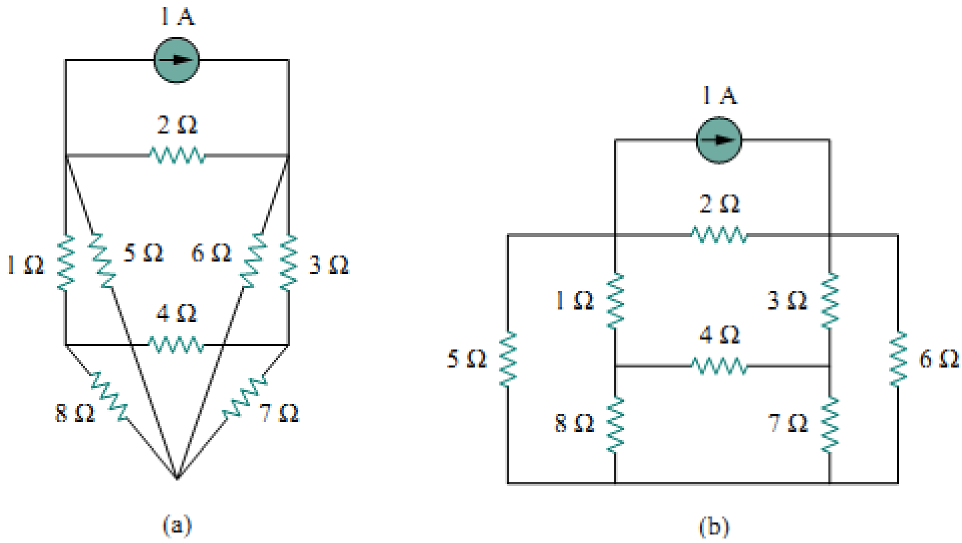


Figure 7 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

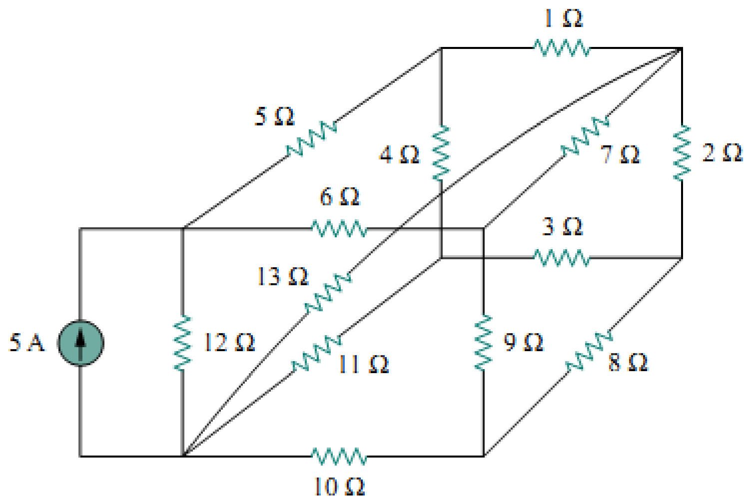


Figure 8 A non-planar circuit

To understand mesh analysis, we should first explain more about what we mean by a mesh.

***A mesh is a loop which does not contain any other loops within it.***

In Fig. 9, for example, paths abefa and bcdeb are meshes, but path abcdefa is not a mesh. The current through a mesh is known as mesh current. In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit.

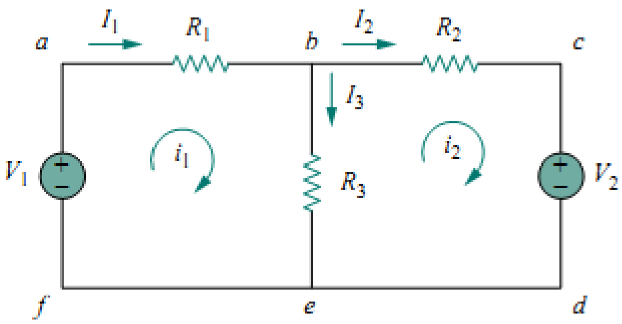


Figure 9 A circuit with two meshes

In the mesh analysis of a circuit with n meshes, we take the following three steps.

1. ***Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.***
2. ***Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.***
3. ***Solve the resulting n simultaneous equations to get the mesh currents.***

To illustrate the steps, consider the circuit in Fig. 9. The first step requires that mesh currents $i_1$ and $i_2$ are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain

$$-V_1 + R_1 i_i + R_3(i_1 - i_2) = 0$$

Or

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1 \qquad (3.10)$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

Or

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \qquad (3.11)$$

Note in Eq. (3.10) that the coefficient of $i_1$ is the sum of the resistances in the first mesh, while the coefficient of $i_2$ is the negative of the resistance common to meshes 1 and 2. Now observe that the same is true in Eq. (3.11). This can serve as a shortcut way of writing the mesh equations. We will exploit this idea later.

The third step is to solve for the mesh currents. Putting Eqs. (3.10) and (3.11) in matrix form yields

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (3.12)$$

which can be solved to obtain the mesh currents $i_1$ and $i_2$. Any appropriate method can be used to solve the simultaneous equations.

Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use $i$ for a mesh current and $I$ for a branch current. The current elements $I_1, I_2, and\ I_3$ are algebraic sums of the mesh currents. It is evident from Fig. 9 that

$$I_1 = i_1, \qquad I_2 = i_2, \qquad I_3 = i_1 - i_2 \qquad (3.13)$$

**Mesh analysis (Format Approach)**

Now that the basis for the mesh-analysis approach has been established, we will now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, a circuit is drawn in Fig. 10 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

Which can be written as

$$7I_1 - 6I_2 = -5$$
$$8I_2 - 6I_1 = 10$$

And expanded as

| Col. 1 | Col. 2 | Col. 3 |
|---|---|---|
| $(1+6)I_1 -$ | $6I_2$ | $= (5-10)$ |
| $(2+6)I_2 -$ | $6I_1$ | $= 10$ |

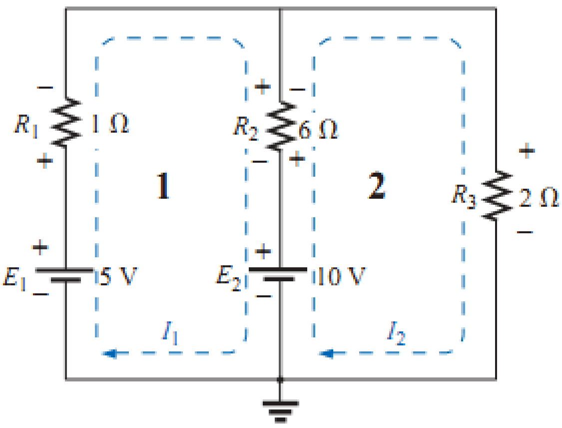


Figure 10 Circuit for mesh analysis

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the algebraic sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following format approach to mesh analysis:

1. *Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.*
2. *The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.*
3. *We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A mutual term is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.*
4. *The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.*
5. *Solve the resulting simultaneous equations for the desired loop currents.*

### Mesh analysis with current source

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

**Case 1**: When a current source exists only in one mesh: Consider the circuit in Fig. 11, for example. We set $i_2 = -5\ A$ and write a mesh equation for the other mesh in the usual way, that is,

$$-10 + 4i + 6(i_1 - i_2) = 0 \qquad \therefore i_1 = -2A \qquad (3.14)$$

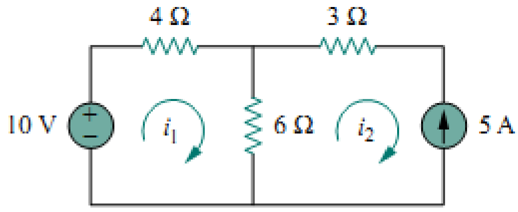


Figure 11 A circuit with current source

**Case 2**: When a current source exists between two meshes: Consider the circuit in Fig. 12(a), for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. 3.23(b). Thus,

***A supermesh results when two meshes have a (dependent or independent) current source in common.***

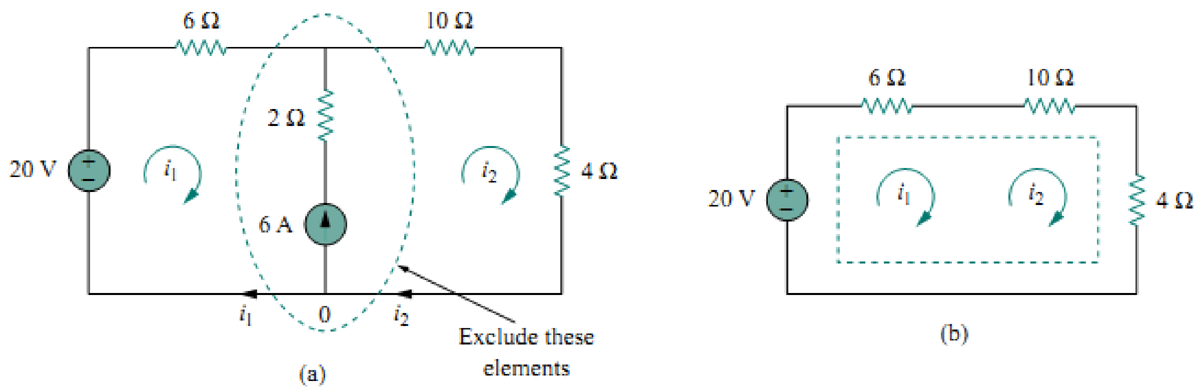


Figure 12 (a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

As shown in Fig. 12(b), we create a supermesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.) Why treat the supermesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh in Fig. 12(b) gives

$$-20 + 6i + 10i_2 + 4i_2 = 0$$

Or

$$6i_1 + 14i_2 = 20 \qquad (3.15)$$

We apply KCL to a node in the branch where the two meshes intersect. Applying KCL to node 0 in Fig. 12(a) gives

$$i_2 = i_1 + 6 \qquad (3.16)$$

Solving equations 3.15 and 3.16, we get

$$i_1 = -3.2A, \qquad i_2 = 2.8A$$

Note the following properties of a supermesh:

1. ***The current source in the supermesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.***
2. ***A supermesh has no current of its own.***
3. ***A supermesh requires the application of both KVL and KCL.***

*Check Alexander section3.6 for Format approach of nodal analysis and mesh analysis, which is slightly different than discussed in this lecture. If you have trouble grasping the format approach discussed here, you should definitely check this.*

**EXAMPLE 3.5**

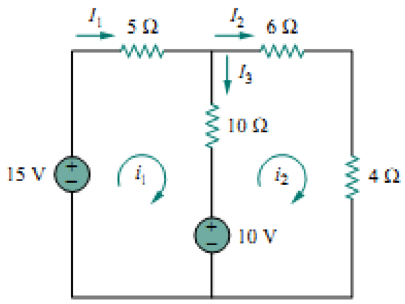


Figure 3.18 For Example 3.5.

For the circuit in Fig. 3.18, find the branch currents $I_1$, $I_2$, and $I_3$ using mesh analysis.

**Solution:**

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

**METHOD 1** Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Longrightarrow \quad i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

**METHOD 2** To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.



PREPARED BY
SHAHADAT HUSSAIN PARVEZ

**Example 3.6**

Use mesh analysis to find the current $i_o$ in the circuit in Fig. 3.20.

**Solution:**

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (3.6.1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (3.6.2)$$

For mesh 3,

$$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $i_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3.6.3)$$

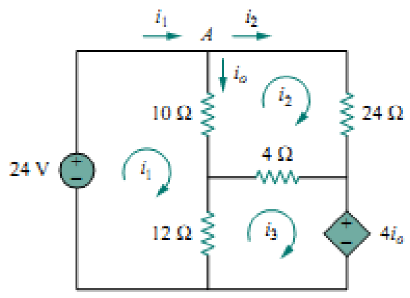


Figure 3.20 For Example 3.6.

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtained the determinant as

$$\Delta = \left| \begin{array}{ccc} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{array} \right|$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \left| \begin{array}{ccc} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{array} \right| = 456 - 24 = 432$$

$$\Delta_2 = \left| \begin{array}{ccc} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{array} \right| = 24 + 120 = 144$$

$$\Delta_3 = \left| \begin{array}{ccc} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{array} \right| = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $i_o = i_1 - i_2 = 1.5$ A.

## EXAMPLE 3.7

For the circuit in Fig. 3.24, find $i_1$ to $i_4$ using mesh analysis.

**Solution:**

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh

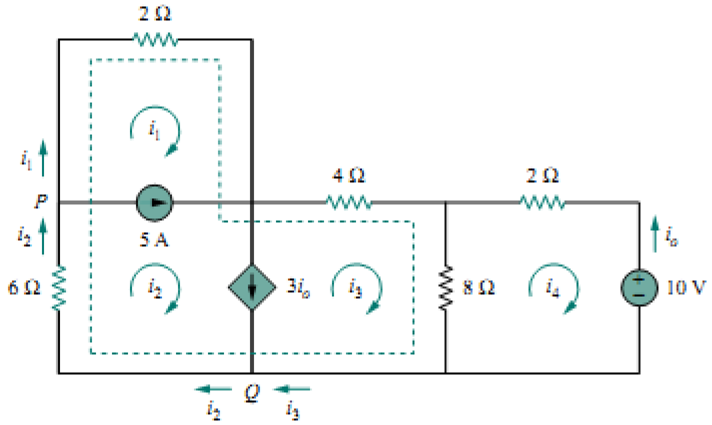


Figure 3.24 For Example 3.7.

because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \tag{3.7.1}$$

For the independent current source, we apply KCL to node $P$:

$$i_2 = i_1 + 5 \tag{3.7.2}$$

For the dependent current source, we apply KCL to node $Q$:

$$i_2 = i_3 + 3i_o$$

But $i_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \tag{3.7.3}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \tag{3.7.4}$$

From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

3. Alexander Example 3.8-3.9
4. Alexander Practice Problem 3.5-3.9