A major advantage of analyzing circuits using Kirchhoff's laws as we did in earlier is that we can analyze a circuit without tampering with its original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved.

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to linear circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer.

Linearity Property

Linearity is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the homogeneity (scaling) property and the additivity property.

The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input i to the output v,

$$v = iR \qquad (4.1)$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$kiR = kv \qquad (4.2)$$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$v_1 = i_1R \qquad (4.3a)$$

and

$$v_2 = i_2R \qquad (4.3b)$$

then applying $(i_1 + i_2)$ gives

$$v = (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2 \qquad (4.4)$$

We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Throughout this course we consider only linear circuits. Note that since $p = i^2R = V^2/R$ (making it a quadratic function rather than linear), the relationship between power and voltage (or current) is nonlinear. Therefore, the theorems covered in this lecture are not applicable to power.

To understand the linearity principle, consider the linear circuit shown in Fig. 1. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s, which serves as the input. The circuit is terminated by a load R. We may take the current i through R as the output. Suppose $v_s = 10V$ gives $i = 2A$. According to the linearity principle, $v_s = 1\ V$ will give $i = 0.2\ A$.

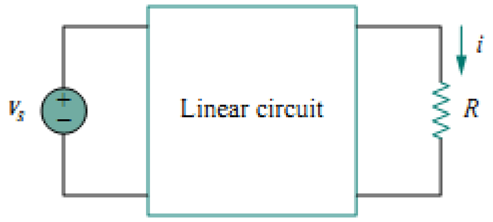

Figure 1 Linear circuit with input v_s and output *I*

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Example 4.1

For the circuit in Fig. 4.2, find i_o when $v_s = 12$ V and $v_s = 24$ V.

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0 \qquad (4.1.1)$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad (4.1.2)$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \qquad (4.1.3)$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Longrightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Longrightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$i_o = i_2 = \frac{12}{76} \text{ A}$$

When $v_s = 24$ V,

$$i_o = i_2 = \frac{24}{76} \text{ A}$$

showing that when the source value is doubled, i_o doubles.

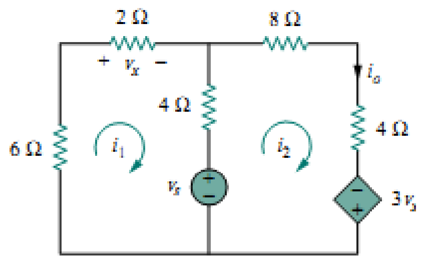

Figure 4.2 For Example 4.1.

Example 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit in Fig. 4.4.

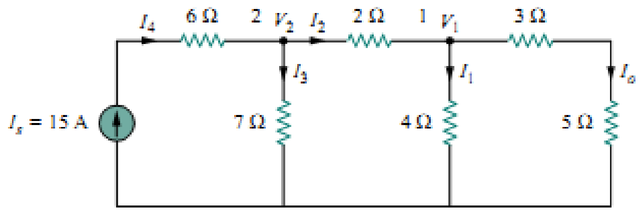

Figure 4.4 For Example 4.2.

Solution:

If $I_o = 1$ A, then $V_1 = (3+5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A; the actual source current of 15 A will give $I_o = 3$ A as the actual value.

1. Alexander, practice problem 4.1,4.2

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Superposition Theorem

If a circuit has two or more independent sources, one way to determine the value of a specific variable is to determine the contribution of each independent source to the variable and then add them up. The approach is known as the ***superposition***.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This approach is shown in figure 2
2. Dependent sources are left intact because they are controlled by circuit variables.

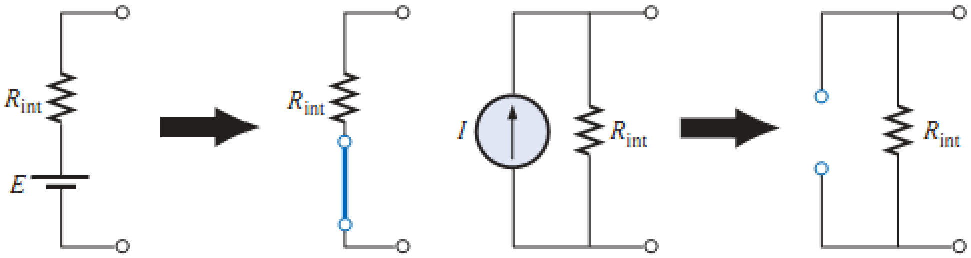

Figure 2 Removing effect of practical sources

The superposition theorem can be summarized in the following three steps

1. ***Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.***
2. ***Repeat step 1 for each of the other independent sources.***
3. ***Find the total contribution by adding algebraically all the contributions due to the independent sources.***

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

Since superposition is based on linearity, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Example 4.3

Use the superposition theorem to find v in the circuit in Fig. 4.6.

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and

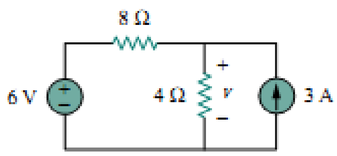

Figure 4.6 For Example 4.3.

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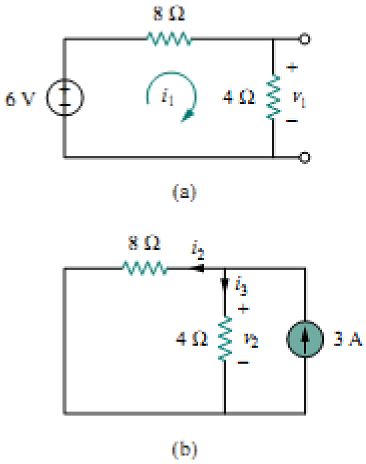

Figure 4.7 For Example 4.3: (a) calculating v_1, (b) calculating v_2.

the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Longrightarrow \quad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Example 4.4

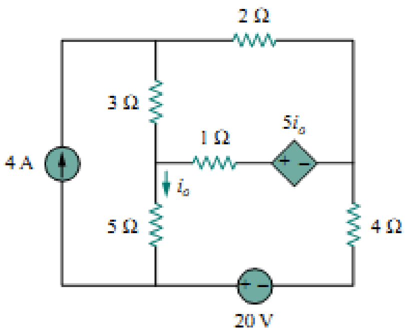

Figure 4.9 For Example 4.4.

Find i_o in the circuit in Fig. 4.9 using superposition.

Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i'_o + i''_o \tag{4.4.1}$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o, we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o. For loop 1,

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

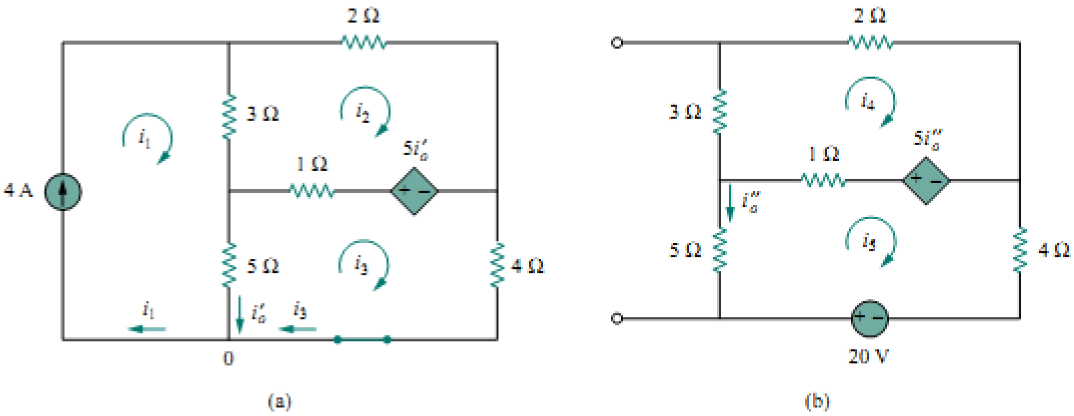

Figure 4.10 For Example 4.4: Applying superposition to (a) obtain i'_0, (b) obtain i''_0.

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i_o' = 4 - i_o' \qquad (4.4.5)$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i_o' = 8 \qquad (4.4.6)$$

$$i_2 + 5i_o' = 20 \qquad (4.4.7)$$

which can be solved to get

$$i_o' = \frac{52}{17}\text{ A} \qquad (4.4.8)$$

To obtain i_o'', we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i_o'' = 0 \qquad (4.4.9)$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i_o'' = 0 \qquad (4.4.10)$$

But $i_5 = -i_o''$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i_o'' = 0 \qquad (4.4.11)$$

$$i_4 + 5i_o'' = -20 \qquad (4.4.12)$$

which we solve to get

$$i_o'' = -\frac{60}{17}\text{ A} \qquad (4.4.13)$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

2. Alexander Example 4.5
3. Alexander Practice Problem 4.3,4.4,4.5

Source Transformation

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. Source transformation is another tool for simplifying circuits. Basic to these tools is the concept of equivalence. An equivalent circuit is one whose v-i characteristics are identical with the original circuit.

Previously, we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources. It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in Fig. 3. Either substitution is known as a ***source transformation***.

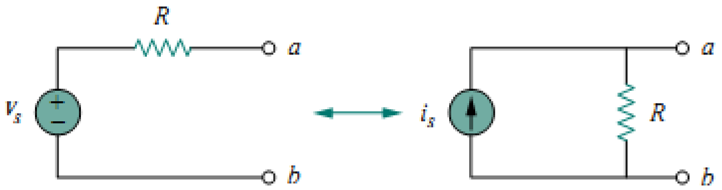

Figure 3 Transformation of independent Sources

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

The voltage current relationship can be found using the formula

$$v_s = i_s R \quad or \quad i_s = \frac{v_s}{R} \qquad (4.5)$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 4, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

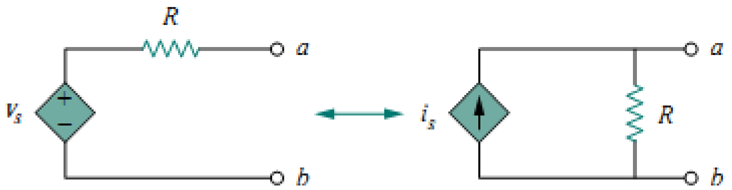

Figure 4 Transformation of Dependent sources

Like the wye-delta transformation studied earlier, a source transformation does not affect the remaining part of the circuit. When applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis. However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 3 (or Fig. 4) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from Eq. (4.5) that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, non-ideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source.

Example 4.6

Use source transformation to find v_o in the circuit in Fig. 4.17.

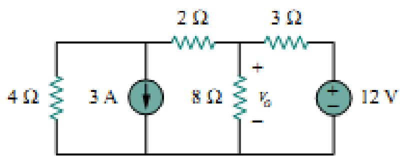

Figure 4.17 For Example 4.6.

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

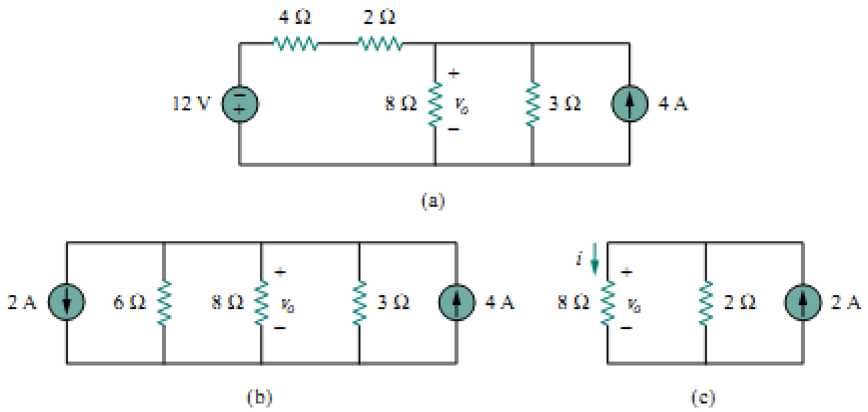

Figure 4.18 For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2+8}(2) = 0.4$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Example 4.7

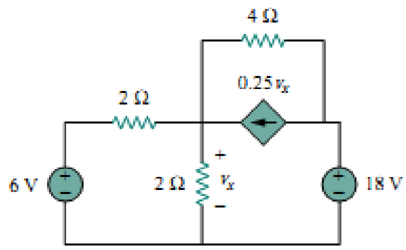

Figure 4.20 For Example 4.7.

Find v_x in Fig. 4.20 using source transformation.

Solution:

The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \tag{4.7.1}$$

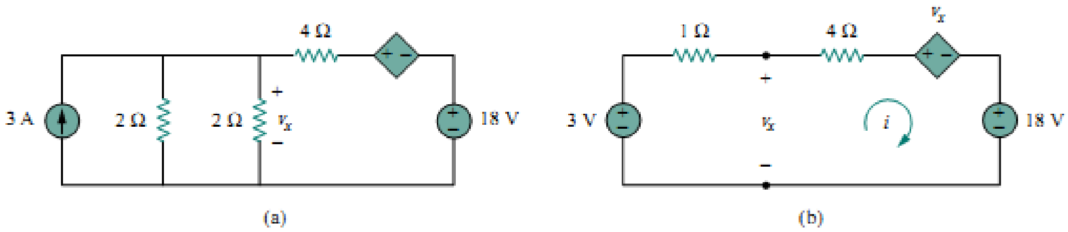

Figure 4.21 For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and v_x yields

$$-3 + 1i + v_x = 0 \quad \Longrightarrow \quad v_x = 3 - i \tag{4.7.2}$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Alternatively, we may apply KVL to the loop containing v_x, the 4-Ω resistor, the voltage-controlled dependent voltage source, and the 18-V voltage source in Fig. 4.21(b). We obtain

$$-v_x + 4i + v_x + 18 = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5$ V.

4. Alexander Practice Problem 4.6,4.7

Thevenin's Theorem

It often occurs in practice that a particular element in a circuit is variable (usually called the load) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, ***Thevenin's theorem*** provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in Fig. 5(a) can be replaced by that in Fig. 5(b). (The load in Fig. 5 may be a single resistor or another circuit.) The circuit to the left of the terminals a-b in Fig. 5(b) is known as the *Thevenin equivalent circuit*;

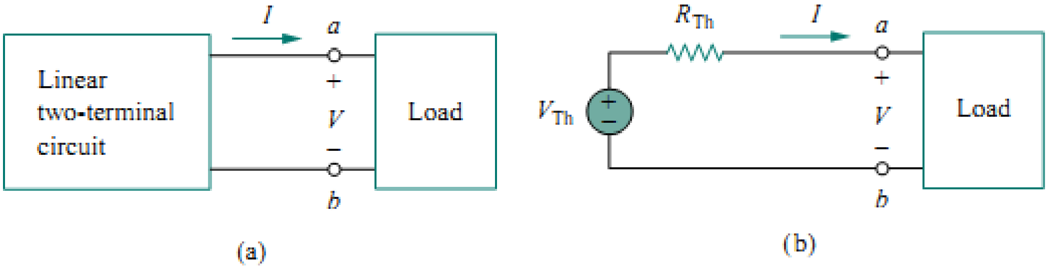

Figure 5 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

For the purpose of analyzing the circuit we have to find Thevenin equivalent voltage V_{Th} and resistance R_{Th}.

To do so, suppose the two circuits in Fig. 5 are equivalent. Two circuits are said to be equivalent if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. 5 equivalent. If the terminals a-b are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals a-b in Fig. 5(a) must be equal to the voltage source V_{Th} in Fig. 5(b), since the two circuits are equivalent. Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 6(a); that is,

$$V_{Th} = v_{oc} \qquad (4.6)$$

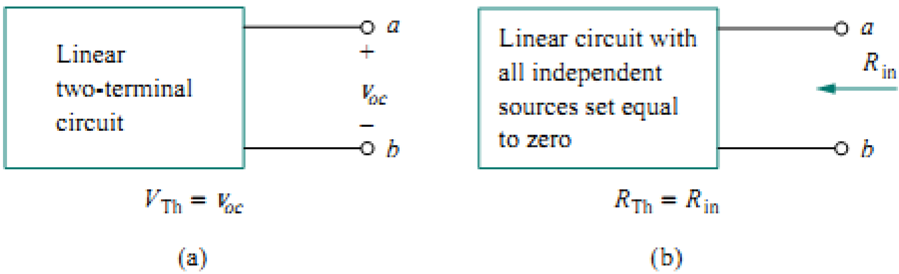

Figure 6 Finding V_{Th} and R_{Th}

Again, with the load disconnected and terminals a-b open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals a-b in Fig. 5(a) must be equal to R_{Th} in Fig. 5(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. 6(b); i.e.

$$R_{Th} = R_{in} \qquad (4.7)$$

To apply this idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.

CASE 1: If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b, as shown in Fig. 6(b).

CASE 2: If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals a and b and determine the resulting current i_o. Then $R_{Th} = v_o/i_o$, as shown in Fig. 7(a). Alternatively, we may insert a current source i_o at terminals a-b as shown in Fig. 7(b) and find the terminal voltage v_o. Again $R_{Th} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o. For example, we may use $v_o = 1V \ or \ \ i_o = 1A$, or even use unspecified values of $v_o \ or \ i_o$.

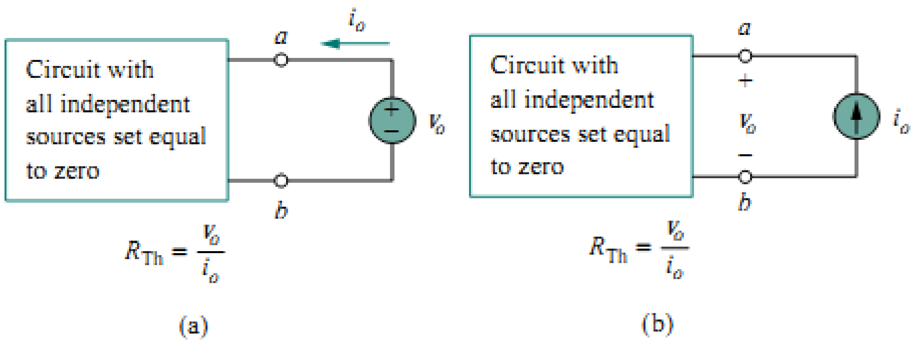

Figure 7 Finding R_{Th} when circuit has dependent sources

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v = -iR$) implies that the circuit is supplying power.

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load RL, as shown in Fig. 8(a). The current IL through the load and the voltage VL across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. 8(b). From Fig. 8(b), we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \qquad (4.8a)$$

$$V_L = R_L I_L = \frac{R_L}{R_{TH} + R_L} V_{Th} \qquad (4.8b)$$

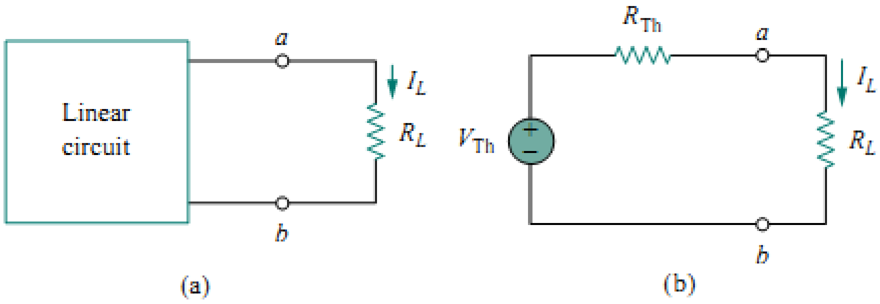

Figure 8 A circuit with a load; (a) Original circuit (b)Thevenin Equivalent ciricuit

It can be noted from Fig. 8(b), that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

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Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 Ω.

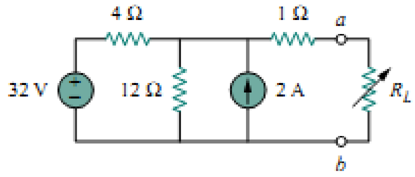

Figure 4.27 For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

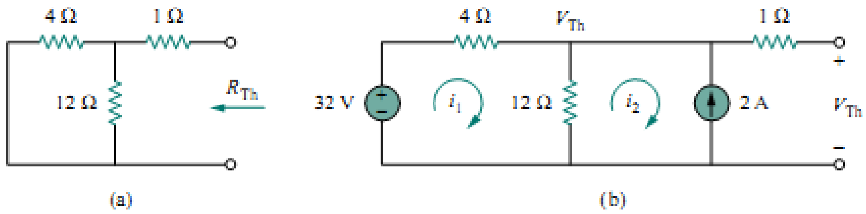

Figure 4.28 For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{\text{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\ \text{V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{\text{Th}}}{4} + 2 = \frac{V_{\text{Th}}}{12}$$

or

$$96 - 3V_{\text{Th}} + 24 = V_{\text{Th}} \quad \Longrightarrow \quad V_{\text{Th}} = 30\ \text{V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

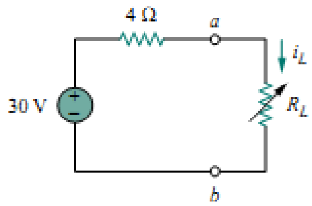

Figure 4.29 The Thevenin equivalent circuit for Example 4.8.

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3\ \text{A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5\ \text{A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75\ \text{A}$$

Example 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31.

Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1$ V to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o, and obtain $R_{Th} = v_o/1$.)

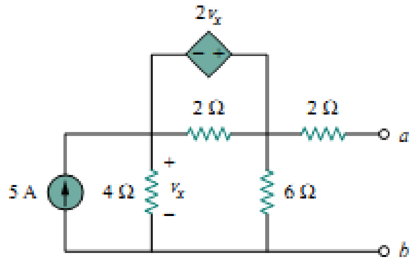

Figure 4.31 For Example 4.9.

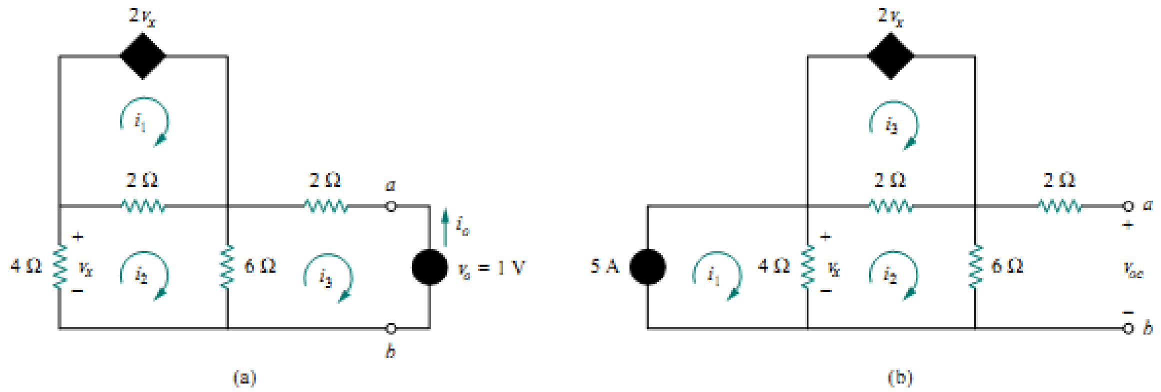

Figure 4.32 Finding R_{Th} and V_{Th} for Example 4.9.

Applying mesh analysis to loop 1 in the circuit in Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \tag{4.9.1}$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \tag{4.9.2}$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \tag{4.9.3}$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \tag{4.9.4}$$

$$-2v_x + 2(i_3 - i_2) = 0 \implies v_x = i_3 - i_2 \tag{4.9.5}$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

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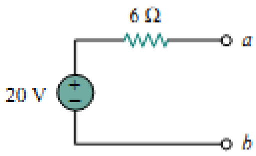

Figure 4.33 The Thevenin equivalent of the circuit in Fig. 4.31.

or

$$12i_2 - 4i_1 - 2i_3 = 0 \tag{4.9.6}$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{Th} = v_{oc} = 6i_2 = 20 \text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

Example 4.10

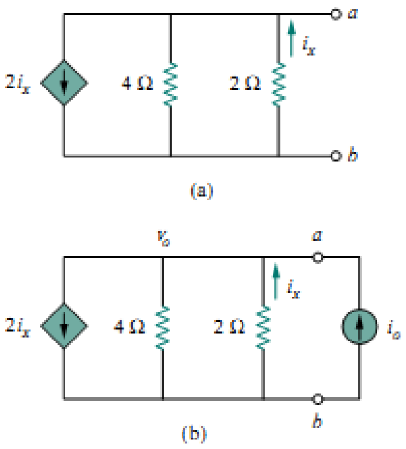

Figure 4.35 For Example 4.10.

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a).

Solution:

Since the circuit in Fig. 4.35(a) has no independent sources, $V_{Th} = 0$ V. To find R_{Th}, it is best to apply a current source i_o at the terminals as shown in Fig. 4.35(b). Applying nodal analysis gives

$$i_o + i_x = 2i_x + \frac{v_o}{4} \tag{4.10.1}$$

But

$$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2} \tag{4.10.2}$$

Substituting Eq. (4.10.2) into Eq. (4.10.1) yields

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \quad \text{or} \quad v_o = -4i_o$$

Thus,

$$R_{Th} = \frac{v_o}{i_o} = -4 \ \Omega$$

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in Fig. 4.35(a) is supplying power. Of course, the resistors in Fig. 4.35(a) cannot supply power (they absorb power); it is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

5. Alexander Practice Problem 4.8,4.9, 4.10

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Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 9(a) can be replaced by the one in Fig. 9(b).

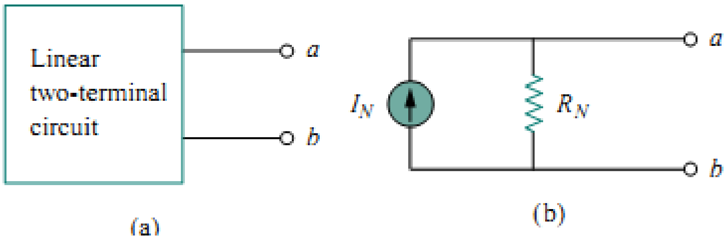

Figure 9 (a)Original Circuit (b) Norton equivalent circuit

For the Norton equivalent circuit finding $R_N and\ I_N$ is necessary. We find R_N in the same way we find R_{Th}. In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th} \qquad (4.9)$$

To find the Norton current IN, we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 9. It is evident that the short-circuit current in Fig. 9(b) is I_N. This must be the same short-circuit current from terminal a-b in Fig. 9 (a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \qquad (4.10)$$

Dependent and independent sources are treated the same way as in Thevenin's theorem.

There is a close relationship between Thevenin's and Norton's theorem: $R_N = R_{Th}$ and

$$I_N = \frac{V_{Th}}{R_{Th}} \qquad (4.11)$$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since VTh, IN, and RTh are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage voc across terminals a and b.
- The short-circuit current isc at terminals a and b.
- The equivalent or input resistance Rin at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Since

$$V_{Th} = v_{oc} \qquad (4.12a)$$
$$I_N = i_{sc} \qquad (4.12b)$$
$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N \qquad (4.12c)$$

The open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent.

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Example 4.11

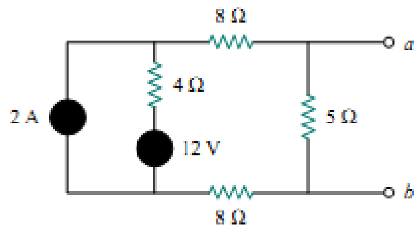

Figure 4.39 For Example 4.11.

Find the Norton equivalent circuit of the circuit in Fig. 4.39.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \parallel (8+4+8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2\text{ A}$$
$$25i_4 - 4i_3 - 12 = 0 \quad \Longrightarrow \quad i_4 = 0.8\text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4\text{ V}$$

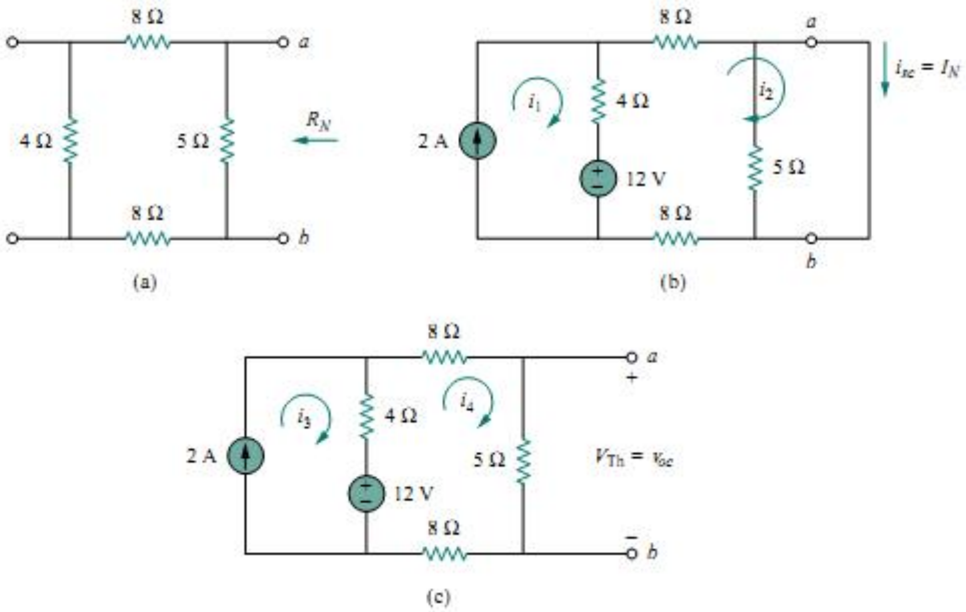

Figure 4.40 For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.7) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

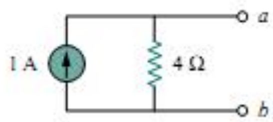

Figure 4.41 Norton equivalent of the circuit in Fig. 4.39.

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Example 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals a-b.

Solution:

To find R_N, we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = v_o/5 = 1/5 = 0.2$. At node a, $-i_o = i_x + 2i_x = 3i_x = 0.6$, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{-0.6} = -1.67\ \Omega$$

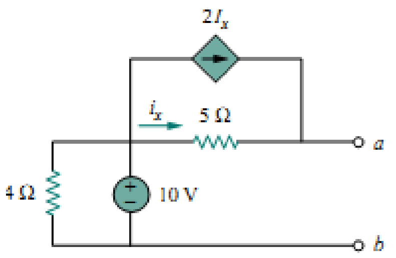

Figure 4.43 For Example 4.12.

To find I_N, we short-circuit terminals a and b and find the current i_{sc}, as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10 - 0}{5} = 2\ \text{A}$$

At node a, KCL gives

$$i_{sc} = i_x + 2i_x = 2 + 4 = 6\ \text{A}$$

Thus,

$$I_N = 6\ \text{A}$$

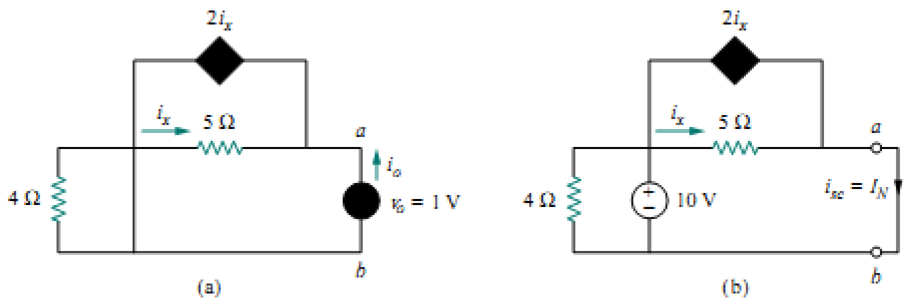

Figure 4.44 For Example 4.12: (a) finding R_N, (b) finding I_N.

6. Alexander Practice Problem 4.11,4.12

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Maximum Power Transfer theorem

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance RL. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 10, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \qquad (4.13)$$

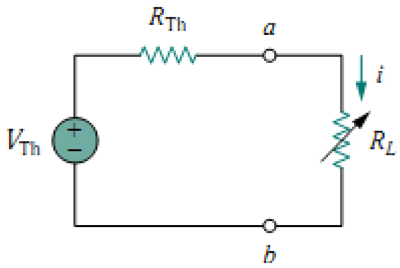

Figure 10 The circuit used for maximum power transfer theorem

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig. 11. We notice from Fig. 11 that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the ***maximum power theorem***.

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.13) with respect to R_L and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$
$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$$0 = (R_{Th} + R_L - 2R_L) = (R_{Th} - R_L) \qquad (4.14)$$

Which yields

$$\boldsymbol{R_L = R_{Th}} \qquad (4.15)$$

The maximum power transferred is obtained by the formula

$$\boldsymbol{P_{max} = \frac{V_{Th}^2}{4T_{Th}}} \qquad (4.16)$$

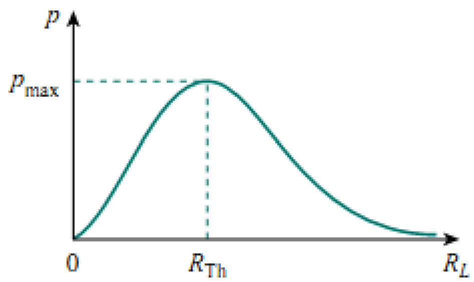

Figure 11 Power Delivered to the load as a function of R_L

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Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

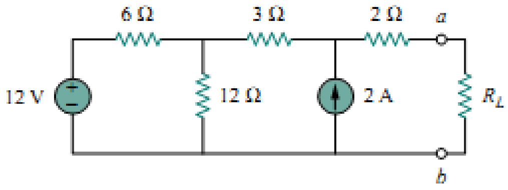

Figure 4.50 For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

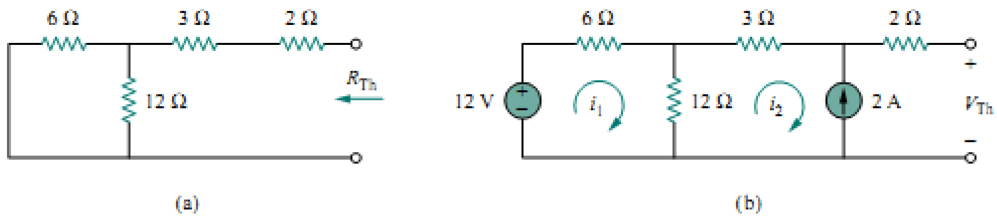

Figure 4.51 For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \qquad \Longrightarrow \qquad V_{Th} = 22\ \text{V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \text{W}$$

7. Alexander Practice Problem 4.13

Reciprocity Theorem

The ***reciprocity theorem*** is applicable only to single-source networks. It is, therefore, not a theorem employed in the analysis of multisource networks. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

In the representative network of Fig. 12(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 12(b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network of Fig. 13(a), in which values for the elements of Fig. 12(a) have been assigned.

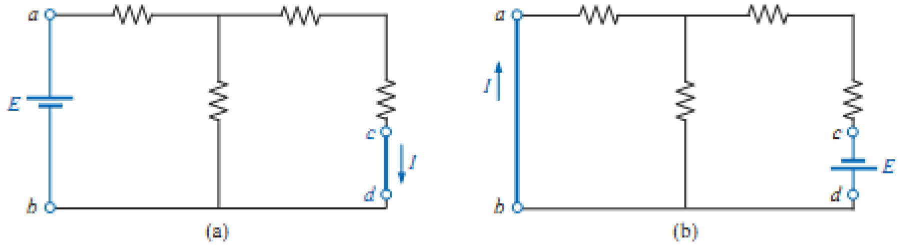

Figure 12 Demonstrating the impact of reciprocity theorem

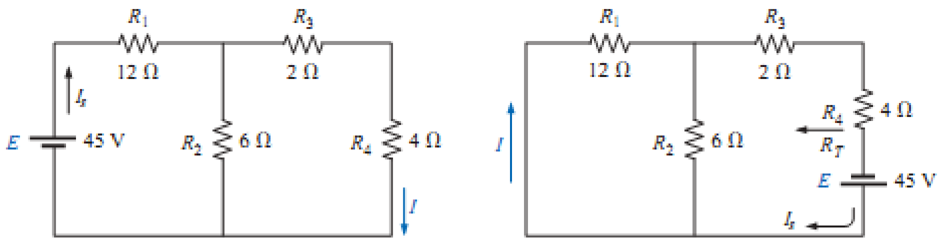

Figure 13 Finding current I due to a source E

The total resistance is

$$R_T = R_1 + R_2||(R_3 + R_4) = 12\Omega + 6\Omega||(2\Omega + 4\Omega)$$
$$= 12\Omega + 6\Omega||6\Omega = 15\Omega$$

And

$$I_s = \frac{E}{R_T} = \frac{45}{15} = 3A$$

With

$$I = \frac{3}{2} = 1.5A$$

For the network of Fig. 13(b), which corresponds to that of Fig. 12(b), we find

$$R_T = R_4 + R_3 + R_1 \| R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \| 6\ \Omega = 10\ \Omega$$

And

$$I_s = \frac{E}{R_T} = \frac{45}{10} = 4.5A$$

So that

$$I = \frac{6\Omega}{12\Omega + 6\Omega} 4.5A = 3A$$

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