Till now we studied only resistive circuits. But there are other passive elements like Capacitors and Inductors, which stores energy. Capacitors and inductors are widely used in practical circuits. So we need to study their properties

Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.
A capacitor is typically constructed as depicted in Fig. 1a.

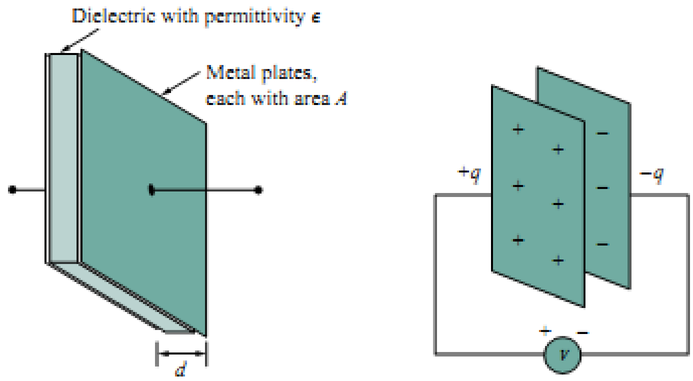

Figure 1 (a) A typical capacitor (b) Capacitor connected with a voltage source

A capacitor consists of two conducting plates separated by an insulator (or dielectric).
In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.
When a voltage source v is connected to the capacitor, as in Fig. 1b, the source deposits a positive charge q on one plate and a negative charge − q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$q = Cv \qquad (5.1)$$

where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F). 1 F is equal to 1 C/V (Coulomb per volt)

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

$$C = \frac{q}{v} \qquad (5.2)$$

The unit of capacitance is the farad F (or more usually μF = 10^{-6} F or pF = 10−12 F), which is defined as the capacitance when a p.d. of one volt appears across the plates when charged with one coulomb.
Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. 1, the capacitance is given by

$$C = \frac{\epsilon A}{d} \qquad (5.3)$$

The equation above is for parallel plate capacitors.

Here
A = Surface area of plates
ϵ = permittivity of dielectric
d = Plate separation

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In general, three factors determine the value of the capacitance:

1. *The surface area of the plates—the larger the area, the greater the capacitance.*
2. *The spacing between the plates—the smaller the spacing, the greater the capacitance.*
3. *The permittivity of the material—the higher the permittivity, the greater the capacitance.*

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range. They are described by the dielectric material they are made of and by whether they are of fixed or variable type. Figure 2 shows the circuit symbols for fixed and variable capacitors. Note that according to the passive sign convention, current is considered to flow into the positive terminal of the capacitor when the capacitor is being charged, and out of the positive terminal when the capacitor is discharging.

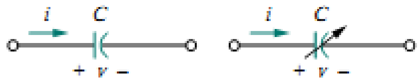

Figure 2 Circuit symbol for capacitor (a) fixed (b) variable

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (5.1). Since

$$i = \frac{dq}{dt} \qquad (5.4)$$

Differentiating both sided of equation 5.1 gives

$$\boldsymbol{i = C\frac{dv}{dt}} \qquad (5.5)$$

Equation 5.4 shows the current-voltage relationship of a capacitor, assuming the pos-itive sign convention. The relationship is illustrated in Fig. 3 for a capacitor whose capacitance is independent of voltage. Capacitors that satisfy Eq. (5.5) are said to be linear. For a nonlinear capacitor, the plot of the current-voltage relationship is not a straight line. Although some capacitors are nonlinear, most are linear. We will assume linear capacitors in this course.

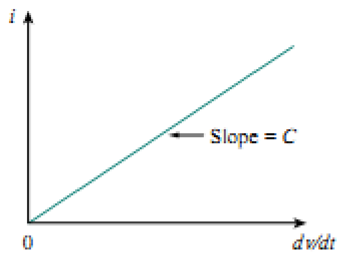

Figure 3 Current-voltage relationship of a capacitor

The voltage-current relation of the capacitor can be obtained by integrating both sides of the equation 5.5. We get

$$\boldsymbol{v = \frac{1}{C}\int_{-\infty}^{t} i\,dt} \qquad (5.6)$$

Or

$$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0) \qquad (5.7)$$

Where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at timet_0. Equation (5.7) shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt} \qquad (5.8)$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p\,dt = C\int_{-\infty}^{t} v\frac{dv}{dt}dt = C\int_{-\infty}^{t} v\,dv = \frac{1}{2}Cv^2\Big|_{t=-\infty}^{t} \qquad (5.9)$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$\boldsymbol{w = \frac{1}{2}Cv^2} \qquad (5.10)$$

Using equation 5.1 we can rewrite the equation 5.10 as

$$\boldsymbol{w = \frac{q^2}{2C}} \qquad (5.11)$$

We should note the following important properties of a capacitor:

1. When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus, ***A capacitor is an open circuit to dc.*** However, if a battery (dc voltage) is connected across acapacitor, the capacitor charges.
2. The voltage on the capacitor must be continuous. Thus ***the voltage on a capacitor cannot change abruptly***. The capacitor resists an abrupt change in the voltage across it. According to Eq. (5.5), a discontinuous change in voltage requires an infinite current, which is physically impossible.
   For example, the voltage across a capacitor may take the form shown in Fig. 4(a), whereas it is not physically possible for the capacitor voltage to take the form shown in Fig. 4(b) because of the abrupt change. Conversely, the current through a capacitor can change instantaneously.
3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.
4. A real, non-ideal capacitor has a parallel-model leakage resistance, as shown in Fig. 5. The leakage resistance may be as high as 100 MΩ and can be neglected for most practical applications. For this reason, we will assume ideal capacitors in this course.

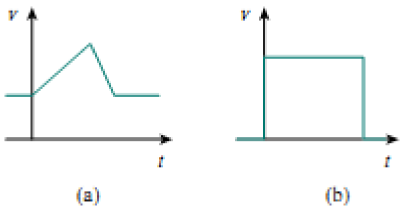

Figure 4 Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

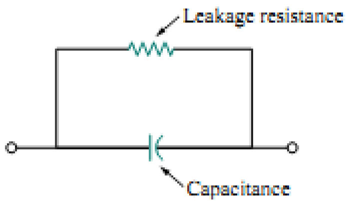

Figure 5 Circuit model of a nonideal capacitor.

Series and Parallel capacitors

We know from resistive circuits that series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor C_{eq}.

Proof of how the equations on series and parallel combination of capacitance is not necessary for this course but for anyone interested it can be found on the book: Fundamental of electric circuits by Alexander (section 6.3, pg-208)

For parallel combination the equivalent capacitance (As depicted in figure 6) can be found using the formula:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N \qquad (5.12)$$

The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

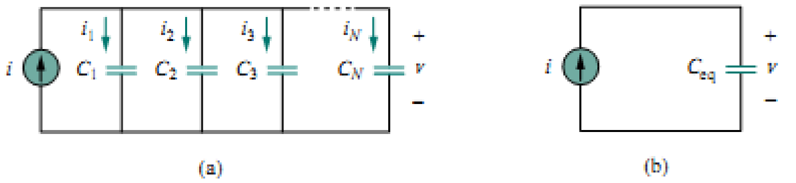

Figure 6 (a) Parallel-connected N capacitors, (b) equivalent circuit for the parallel capacitors.

For Series combination the equivalent capacitance (As depicted in figure 7) can be found using the formula:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N} \qquad (5.13)$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}} \qquad (5.14)$$

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

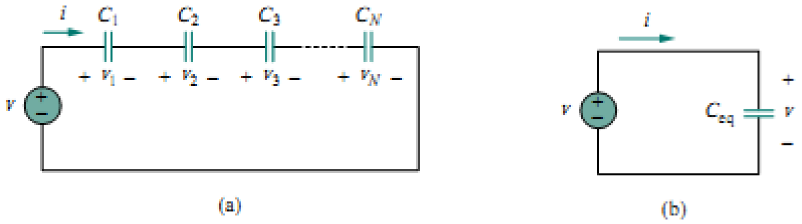

Figure 7 (a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.

For two capacitors in series the equation 5.13 and 5.14 can be simply expressed as

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad (5.14)$$

Note that the formula for finding equivalent capacitance is opposite of that of the formulae for equivalent resistance.

Example 6.1

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across i

(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Example 6.2

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10\cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10\sin 6000t = -0.3\sin 6000t \text{ A}$$

Example 6.3

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \frac{1}{C}\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$
$$= \frac{3 \times 10^3}{-3000}e^{-3000t}\Big|_0^t = (1 - e^{-3000t}) \text{ V}$$

Example 6.4

Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 6.9.

Solution:

The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\,dv/dt$ and $C = 200\ \mu$F, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Fig. 6.10.

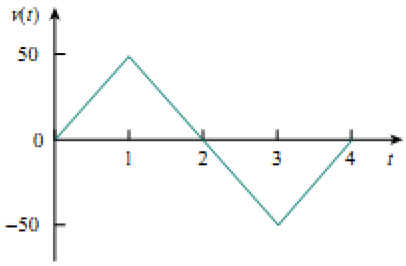

Figure 6.9 For Example 6.4.

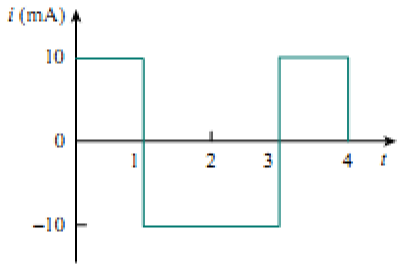

Figure 6.10 For Example 6.4.

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.

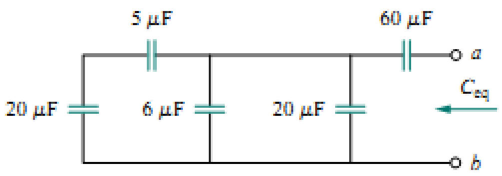

Figure 6.16 For Example 6.6.

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Solution:

The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu\text{F}$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

Example 6.7

For the circuit in Fig. 6.18, find the voltage across each capacitor.

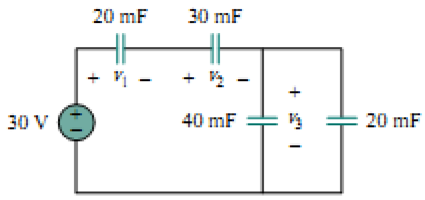


Figure 6.18 For Example 6.7.

Solution:

We first find the equivalent capacitance C_{eq}, shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get $40 + 20 = 60$ mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{\text{eq}} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}}\ \text{mF} = 10\ \text{mF}$$

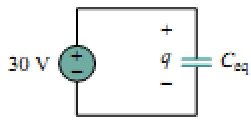


Figure 6.19 Equivalent circuit for Fig. 6.18.

The total charge is

$$q = C_{\text{eq}} v = 10 \times 10^{-3} \times 30 = 0.3\ \text{C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\ \text{V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10\ \text{V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5\ \text{V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60\ \text{mF}} = \frac{0.3}{60 \times 10^{-3}} = 5\ \text{V}$$

1. Alexander practice problem 6.1-6.4
2. Alexander practice problem 6.6-6.7

Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig. 8.

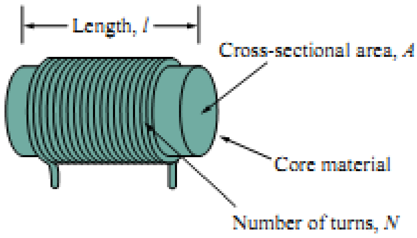


Figure 8 Typical form of inductor

An inductor consists of a coil of conducting wire

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$v = L\frac{di}{dt} \qquad (5.15)$$

where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the Henry (H). It is clear from Eq. (5.15) that 1 Henry equals 1 volt-second per ampere.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension and construction. Formulas for different shape is different and is not in the course.

There are different symbol that can be used to represent an inductor. Figure 9 below show the common inductor symbol.

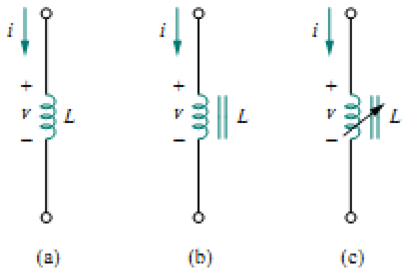


Figure 9 Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

The current-voltage relationship is obtained from Eq. (5.15) as

$$di = \frac{1}{L} v \, dt$$

Integrating gives

$$i = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt \qquad (5.16)$$

Or

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0) \qquad (5.17)$$

where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty) = 0$. The idea of making $i(-\infty) = 0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eqs. (5.15) and (5.17). The power delivered to the inductor is

$$\boldsymbol{p = vi = \left(L\frac{di}{dt}\right)i} \qquad (5.18)$$

The energy stored is

$$w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t}\left(L\frac{di}{dt}\right)i\,dt$$
$$= L\int_{-\infty}^{t} i\,di = \frac{1}{2}L\,i^2(t) - \frac{1}{2}L\,i^2(-\infty) \qquad (5.19)$$

Since $i(-\infty) = 0$

$$\boldsymbol{w = \frac{1}{2}Li^2} \qquad (5.20)$$

We should note the following important properties of an inductor.

1. Voltage across an inductor is zero when the current is constant. Thus, ***An inductor acts like a short circuit to dc.***
2. An important property of the inductor is its opposition to the change in current flowing through it. So ***the current through an inductor cannot change instantaneously.***
3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.
4. A practical, non-ideal inductor has a significant resistive component, as shown in Fig. 11. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the winding resistance R_w, and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The non-ideal inductor also has a winding capacitance C_w due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies. We will assume ideal inductors in this course.

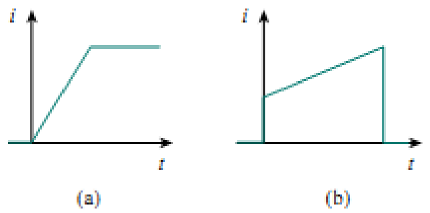


Figure 10 Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

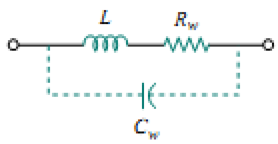


Figure 11 Circuit model of a practical inductor

Series and parallel inductors

Now that the inductor has been added to our list of passive elements, it is necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

Proof of how the equations on series and parallel combination of inductance is not necessary for this course but for anyone interested it can be found on the book: Fundamental of electric circuits by Alexander (section 6.5, pg-216)

For series combination the equivalent inductors (As depicted in figure 12) can be found using the formula:

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N \qquad (5.21)$$

The equivalent inductance of N series-connected inductors is the sum of the individual inductances.

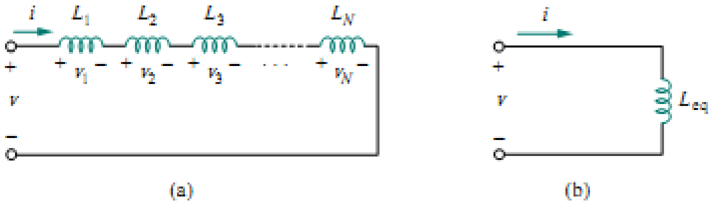


Figure 12 (a) A series connection of N inductors, (b) equivalent circuit for the series inductors

For parallel combination the equivalent inductance (As depicted in figure 13) can be found using the formula:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \qquad (5.22)$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}} \qquad (5.23)$$

The equivalent inductance of parallel-connected inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

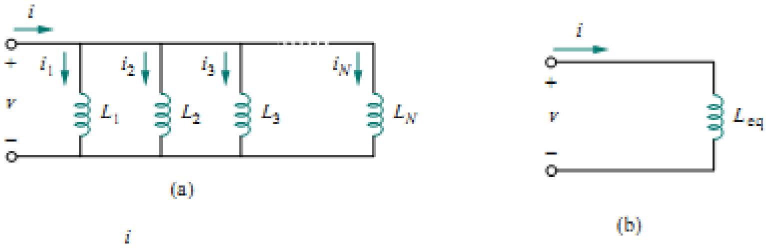


Figure 13 (a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors

For two inductors in parallel the equation 5.22 and 5.23 can be simply expressed as

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \qquad (5.24)$$

Note that the formula for finding equivalent inductance is similar of that of the formulae for equivalent resistance.

Table 1 Important characteristics of basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} i\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Example 6.8

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Example 6.9

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also find the energy stored within $0 < t < 5$ s.

Solution:

Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.13), by writing

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

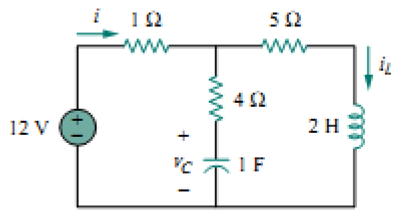
$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

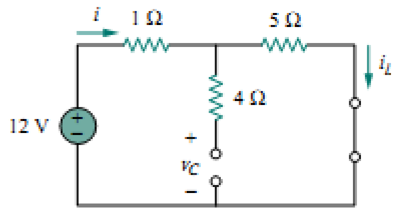
$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$



(a)



(b)

Figure 6.27 For Example 6.10.

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.

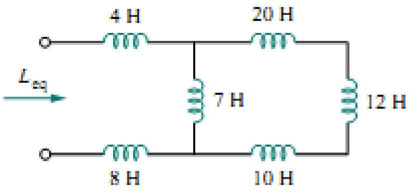


Figure 6.31 For Example 6.11.

Solution:

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

Example 6.12

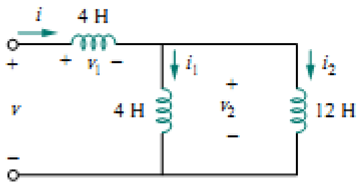


Figure 6.33 For Example 6.12.

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.

Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \parallel 12 = 2 + 3 = 5 \text{ H}$$

Thus,

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

and

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$\begin{aligned} i_1(t) &= \frac{1}{4}\int_0^t v_2\, dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\, dt + 5 \text{ mA} \\ &= -3e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA} \end{aligned}$$

Similarly,

$$\begin{aligned} i_2(t) &= \frac{1}{12}\int_0^t v_2\, dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\, dt - 1 \text{ mA} \\ &= -e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA} \end{aligned}$$

Note that $i_1(t) + i_2(t) = i(t)$.

3. Alexander practice problem 6.8-6.12

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