Automatic Control Systems

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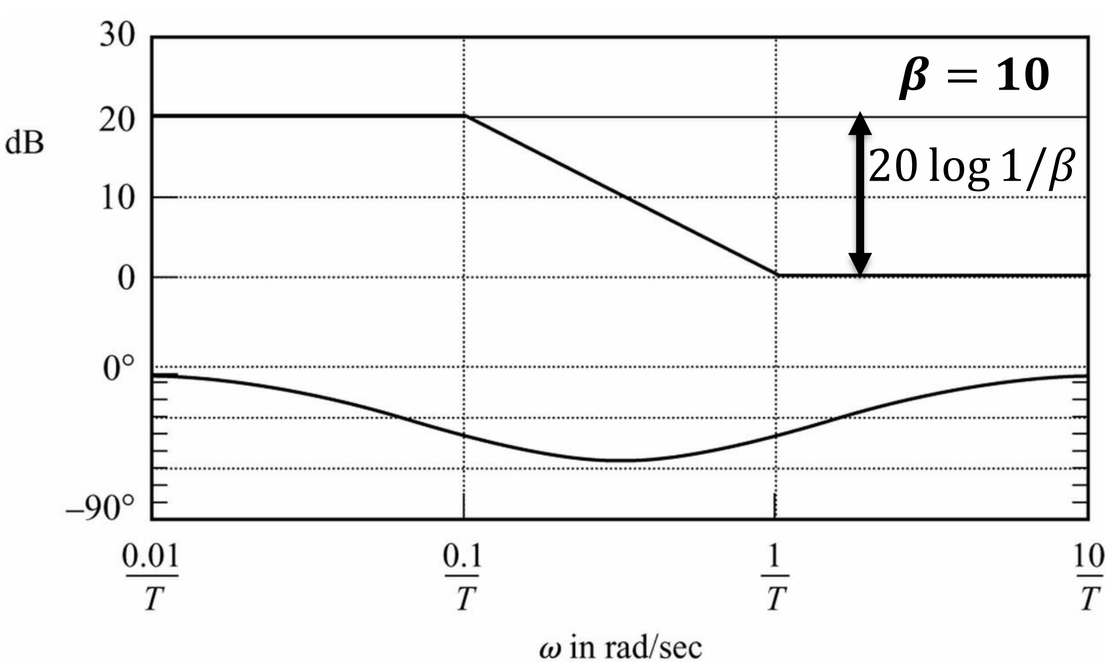
Phase-lag compensation

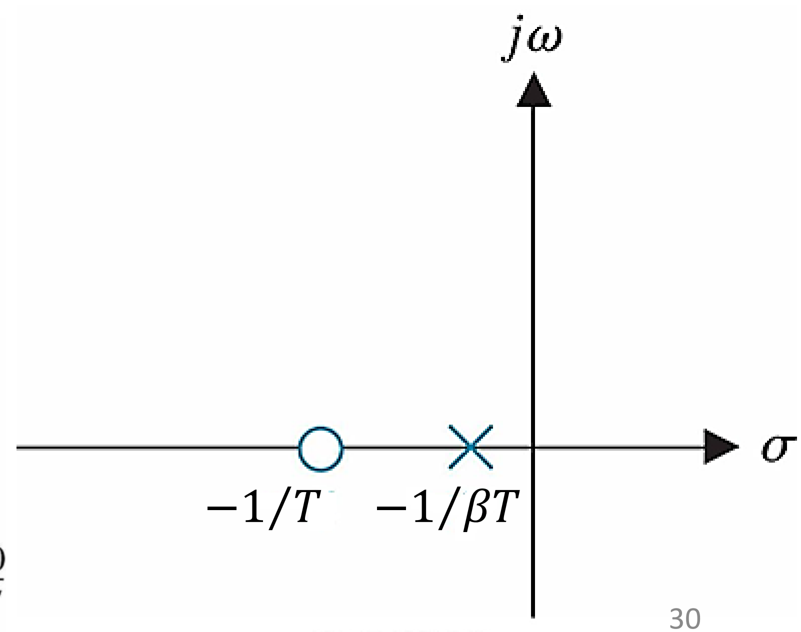
- **low-pass filter** (PI)
- suppress high-frequency noise

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K\frac{Ts+1}{\beta Ts+1}$$

$$G_c(s) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (\beta > 1)$$

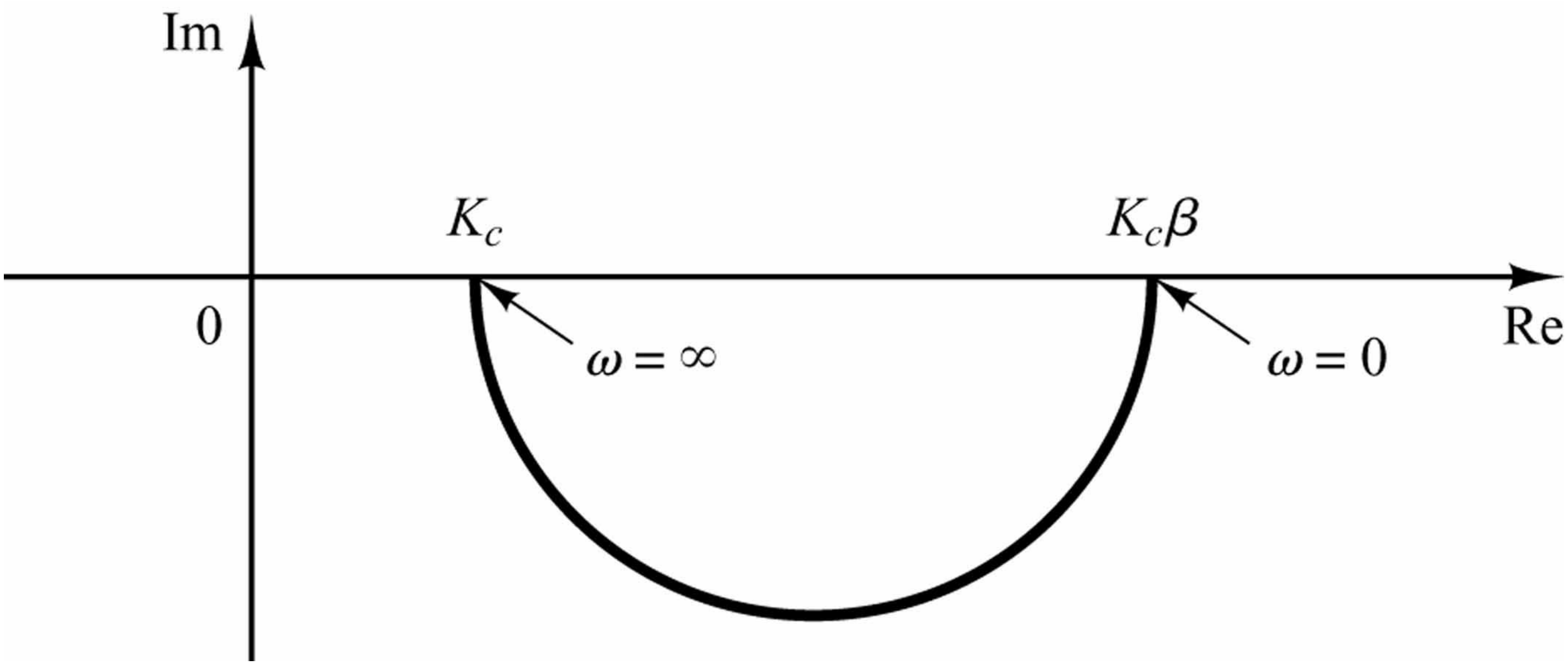
- The **pole is closer to the origin** than the zero ***(strong pole)***





- Polar plot of a lag compensator

$$\boldsymbol{G_c(s) = \frac{K_c\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)}}, \qquad \boldsymbol{\beta > 1}$$



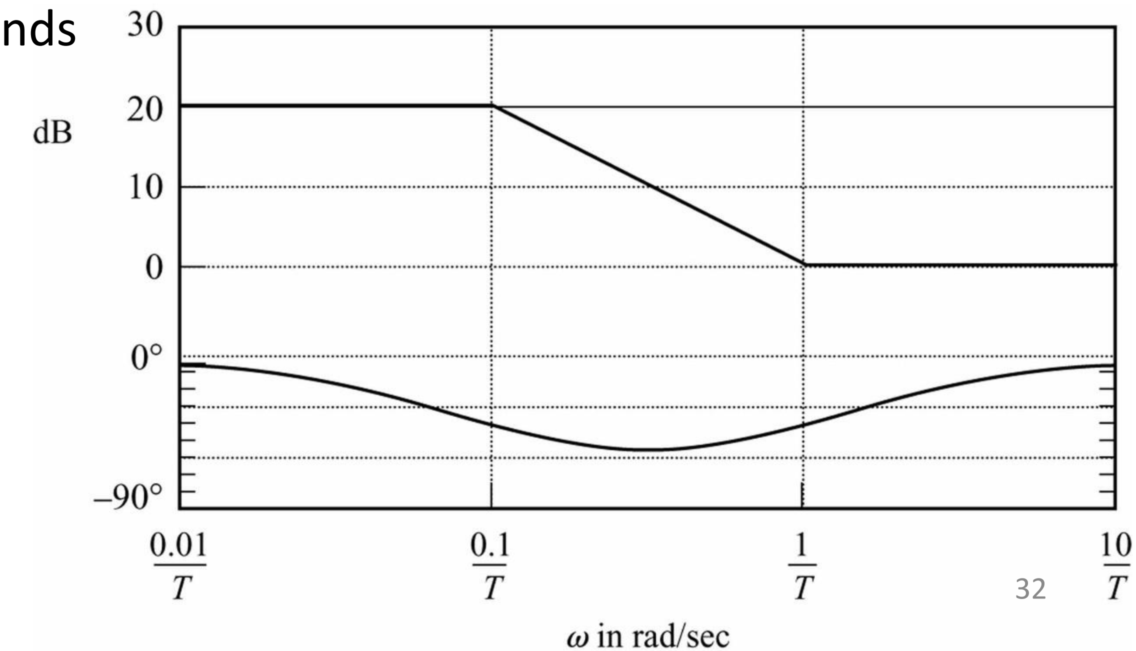
- We utilize the **attenuation characteristic** of the lag-compensator at **high-frequencies** rather than its phase characteristic.

- **The phase response has no use i**n the lag compensator

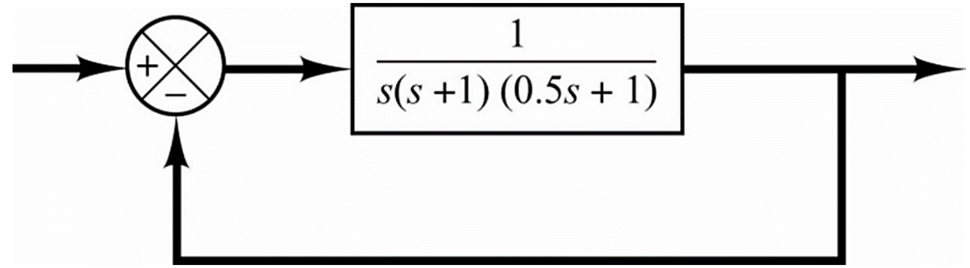
- The lag compensator **reduces** the **bandwidth** and hence **slows down the transient response**

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} \qquad (\beta > 1)$$

- A lag-compensated system tends to be **less stable (PI)**. *Hence T should be chosen >> the largest time constant of the system*



- **example**



- **Design specs:**

1. *The static velocity error constant* $K_v = 5\ sec^{-1}$
2. *The phase margin* $P.M. > 40^o$
3. *The gain margin* $G.M. > 10\ dB$

Use lag compensator

$$G_c(s) = K_c\beta\left(\frac{Ts+1}{\beta Ts+1}\right) \quad (\beta > 1)$$

$$let \quad K_c\beta = K$$

1. *The velocity error constant*

$$G_c(s)G(s) = \left(\frac{Ts+1}{\beta Ts+1}\right)\frac{K}{s(s+1)(0.5s+1)}$$

$$K_v = \lim_{s\to 0} s\, G_c(s)G(s) = K = 5$$

$$\boxed{\Rightarrow KG(s) = \frac{5}{s(s+1)(0.5s+1)}}$$

- The gain-modified system is **unstable:**

$$KG(s)=\frac{5}{s(s+1)(0.5s+1)}$$

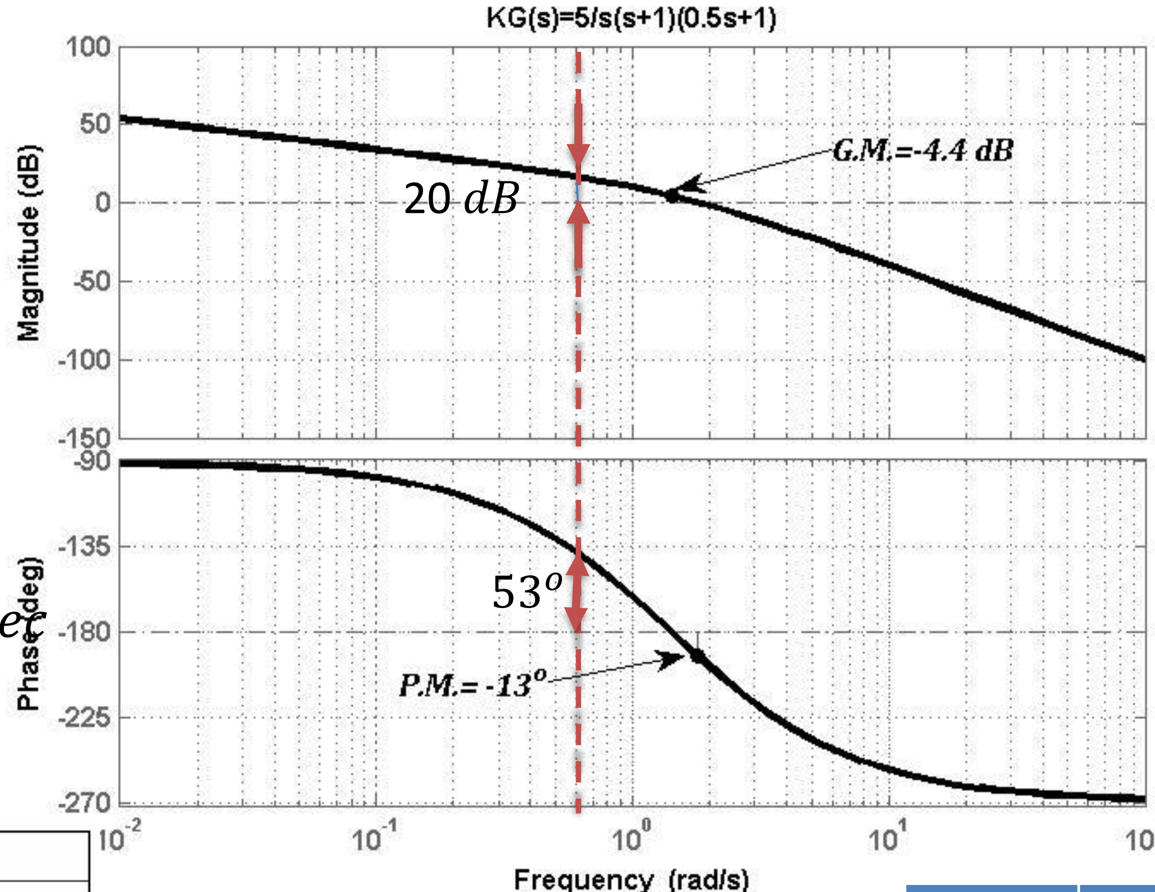
$\boldsymbol{G.M.=-4.4\ dB}$ ***"-ve G.M."***

$\boldsymbol{P.M.=-13^o}$ ***"-ve P.M."***

- $\boldsymbol{@\ \omega=0.5\frac{rad}{sec}}$
- **Gain =** $\boldsymbol{20\ dB}$
- **Phase =** $\boldsymbol{-127^o}$

$\boldsymbol{\Longrightarrow new\ P.M.=53^o}$

- Let $1/T \ll= 0.1\ rad/sec$
- Let $1/\beta T = 0.01 \frac{\text{rad}}{\text{sec}}$



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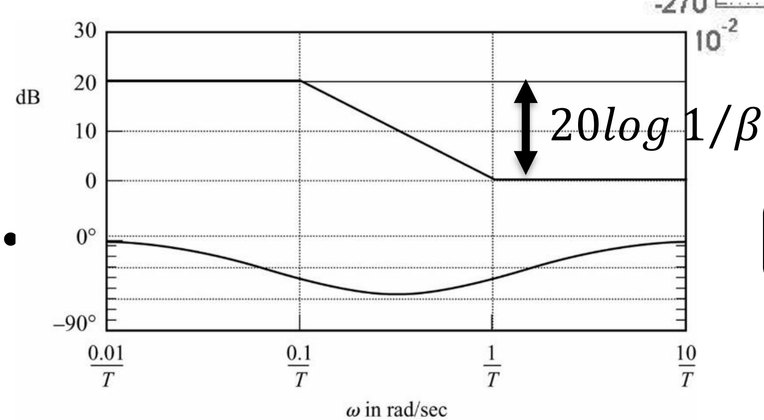
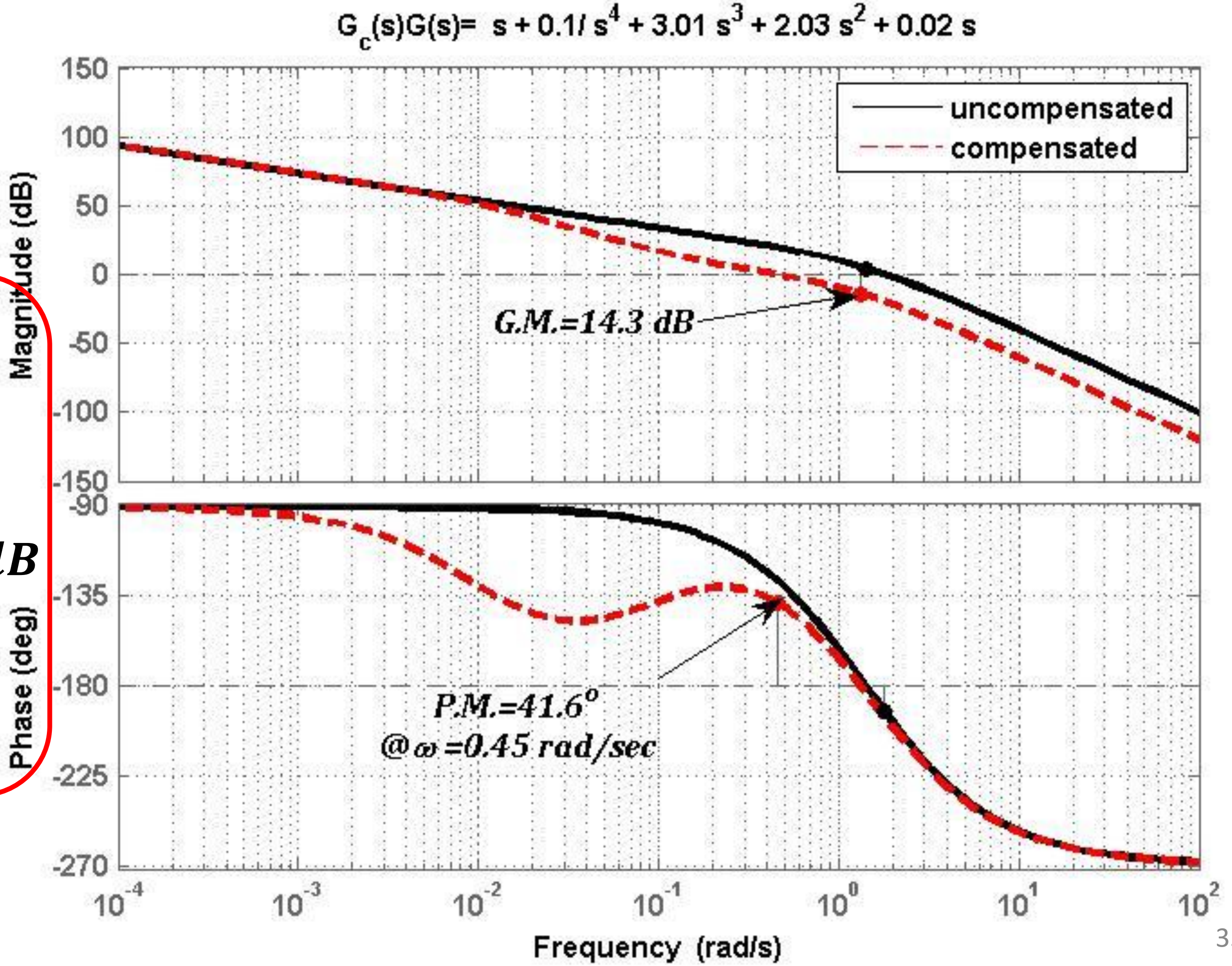


$$\boxed{20\log 1/\beta=-20 \Longrightarrow \beta=10}$$

$\boldsymbol{\beta}$	**10**
$\boldsymbol{1/T}$	**0.1**
$\boldsymbol{1/\beta T}$	**0.01**

- the lag compensator $$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = \frac{5(10s+1)}{(100s+1)}$$
- The compensated system $$\boxed{G_c(s)G(s) = \frac{5(10s+1)}{s(100s+1)(s+1)(0.5s+1)}}$$



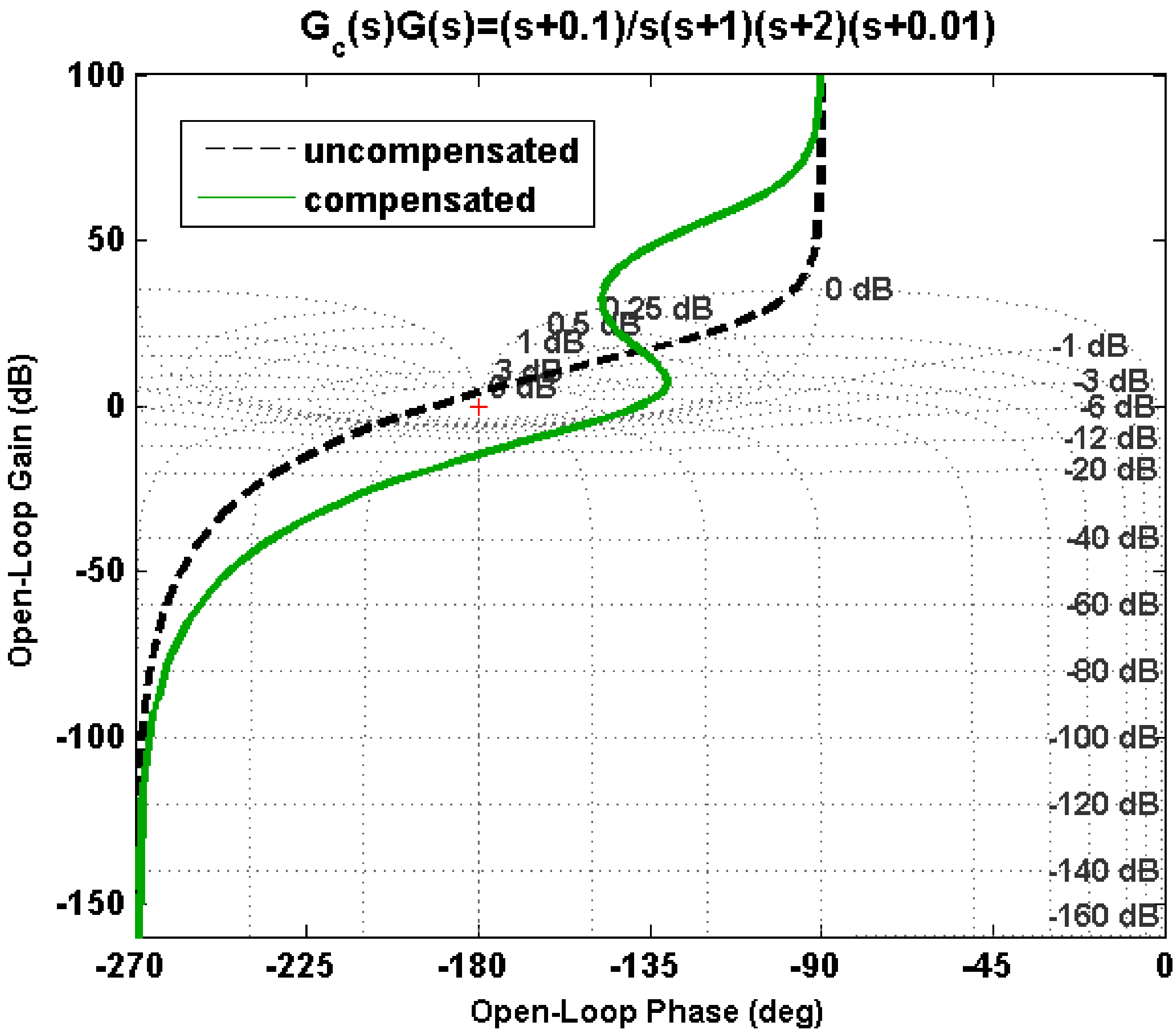
$P.M. =$
$41.6^o > 40^o$

$G.M. =$
$14.3\ dB > 10\ dB$

$B.W =$
$0.59\ rad/sec$

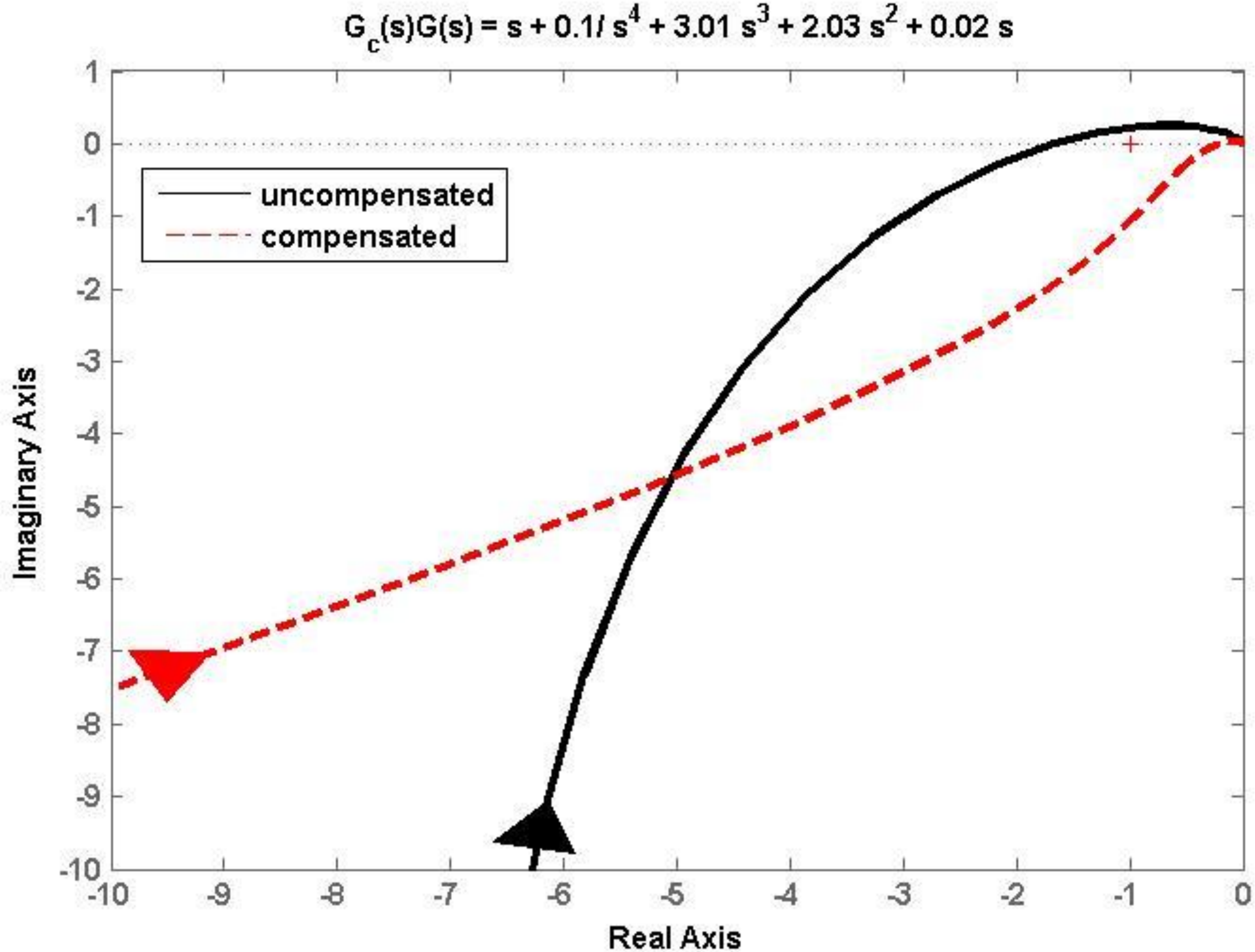
- ***Nichols chart***

ξ	≈ 0.42
$\phi.M.$	42^o
B.W.	0.592 rad/sec



- **Nyquist**

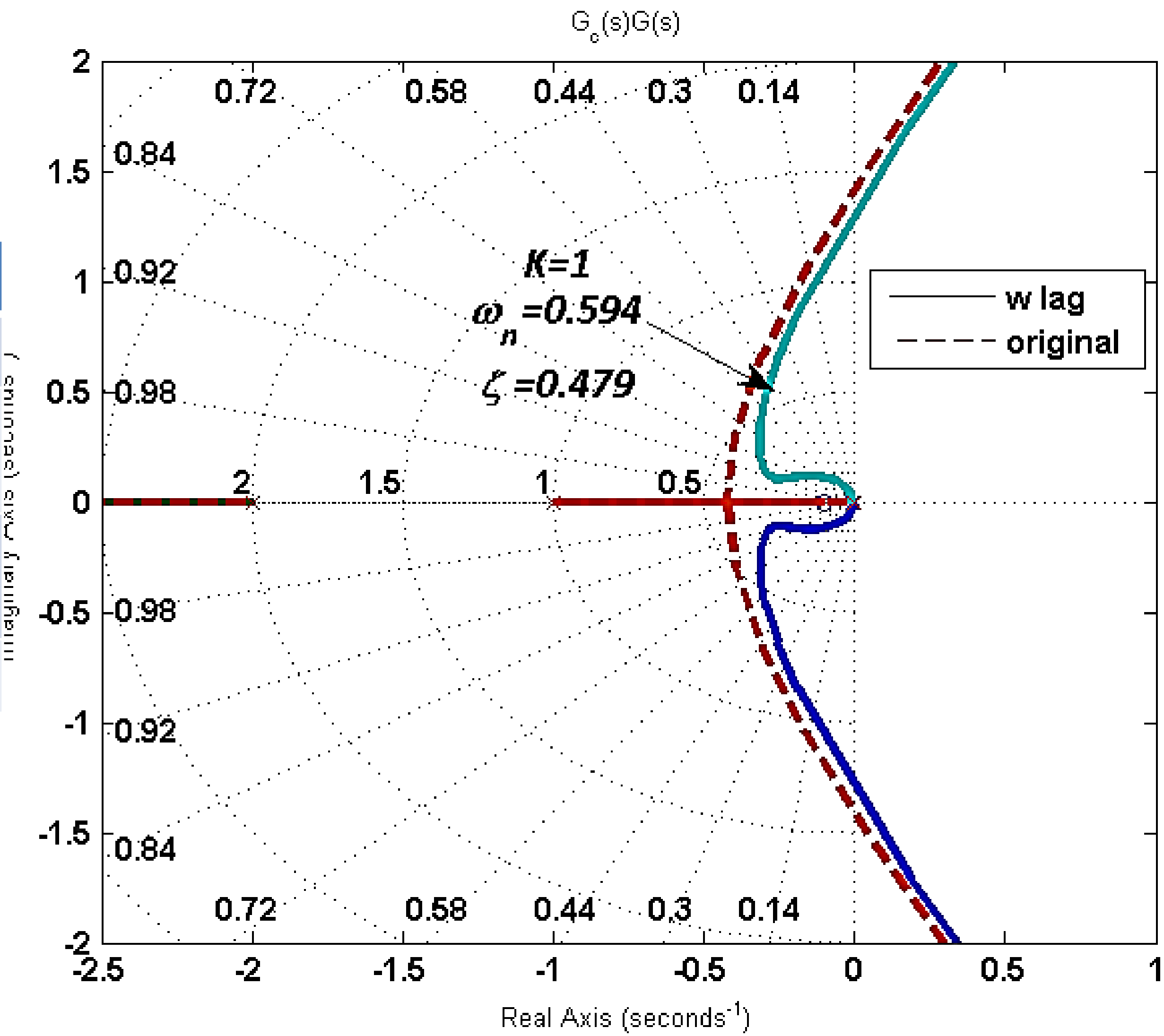
$$G_c(s)G(s) = \frac{(s+0.1)}{s(s+1)(s+2)(s+0.01)}$$



- ***Root-locus***

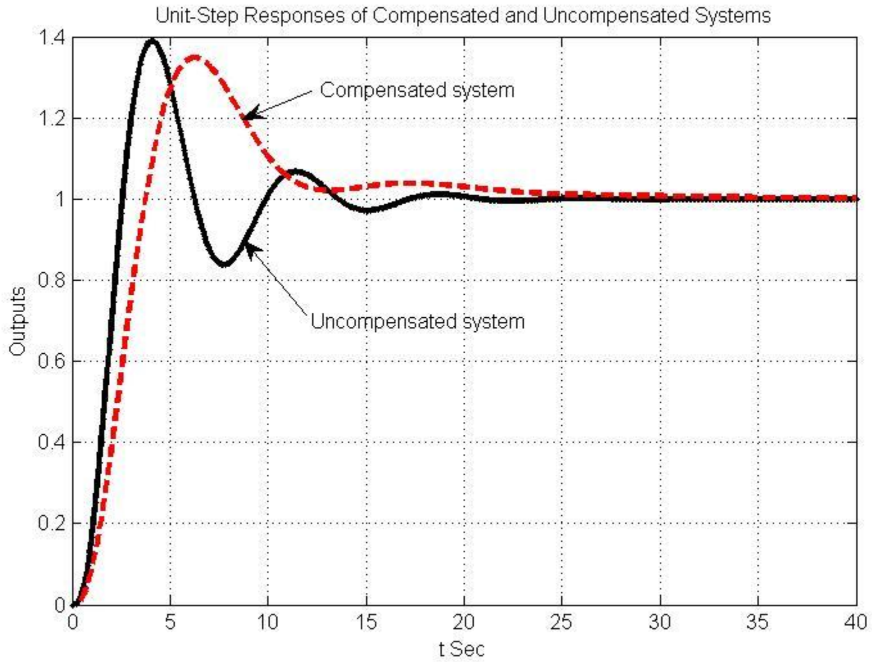
$$G_c(s)G(s) = \frac{(s+0.1)}{s(s+1)(s+2)(s+0.01)}$$

ξ	$0.479 < 0.7$
$\phi.M.$	$\sim 100\xi = 47.9^o$
B.W.	$\sim 0.59\ rad/sec$



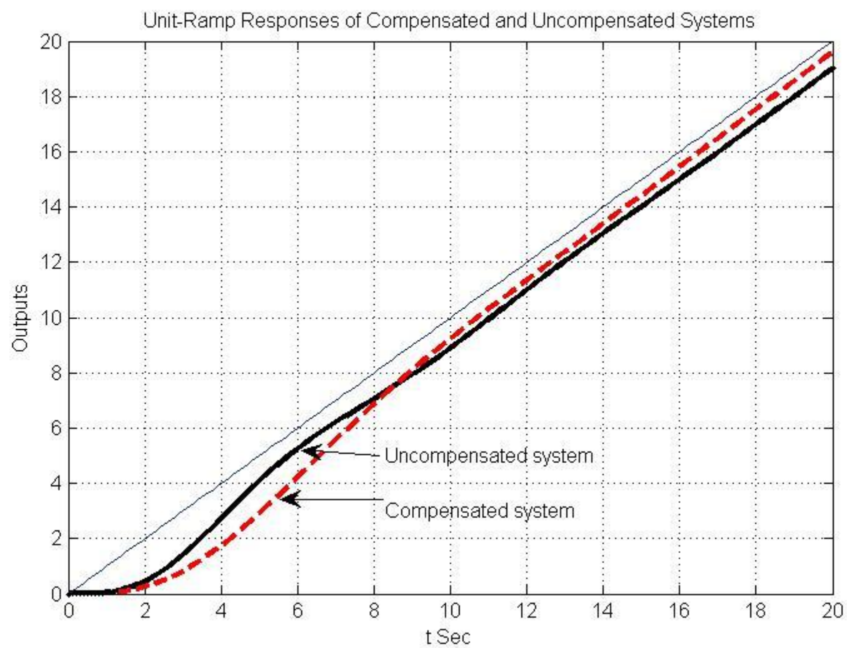
- **Step response**

- Bandwidth is reduced
- Slower transient response
- Step steady-state error is zero



- **Ramp response**

- Ramp steady-state error is reduced



Lead-compensator	Lag-compensator
Used for improving stability margins	Used for improving the steady-state performance
Uses it’s phase-lead characteristics	Uses the attenuation characteristic at high-frequencies
Increases system bandwidth, higher gain-cross over frequency	Reduces system bandwidth, smaller gain-crossover frequency
Faster transient response	Slower transient response
Amplifies high-frequency noise	Suppresses high-frequency noise
Requires larger gain in order to offset the attenuation inherent in the lead network which implies more cost	Requires less gain than the leads network, which implies less-cost

Root Velocity

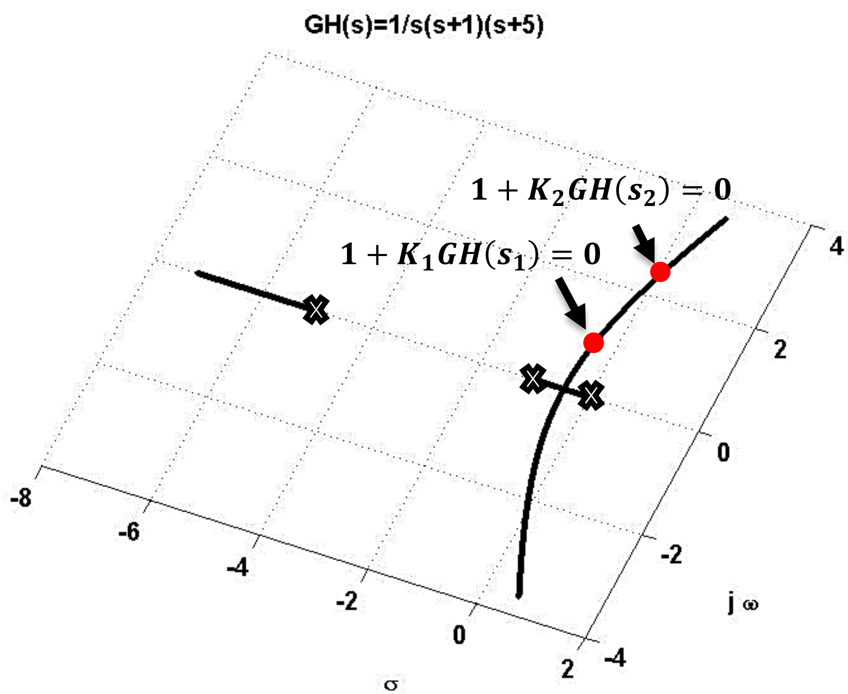
- The differential distance, D, on the s-plane can be defined as

$$\boxed{\Delta D = \Delta s}$$

- The velocity is defined as:

the rate of change in the position

$$\boxed{velocity = \frac{\partial D}{\partial t} = \frac{\partial s}{\partial t}}$$

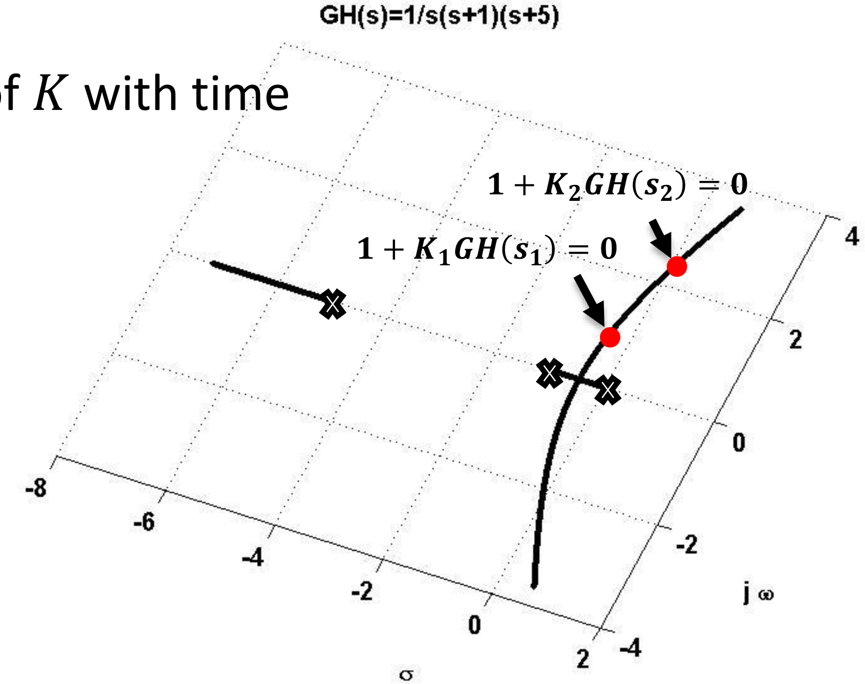


- Since each point on the root locus has a certain gain, K, value at a certain location s, then, K is a function of s.
- using chain rule:

$$\boxed{\vartheta(s,t) = \frac{\partial D}{\partial t} = \frac{\partial s}{\partial t} = \frac{\partial s}{\partial K}\frac{\partial K}{\partial t}}$$

- Where, $\dfrac{\partial K}{\partial t}$ is the rate of change of K with time

$(e.g., K(t) = t)$



- The rate of change of s with K can be obtained from the root-locus equation.

- Define the open loop transfer function as

$$\boxed{GH(s) = \frac{P(s)}{Q(s)}}$$

- Then, the characteristic equation becomes

$$\boxed{1 + KGH(s) = Q(s) + KP(s) = 0}$$

- This could be rearranged as

$$\boxed{K(s) = \frac{-Q(s)}{P(s)}}$$

- Differentiating w.r.t. (s), we have:

$$\frac{\partial K(s)}{\partial s} = \frac{Q(s)\dfrac{\partial P(s)}{\partial s} - P(s)\dfrac{\partial Q(s)}{\partial s}}{P^2(s)}$$

- The root velocity becomes: $\vartheta(s,t) = \dfrac{\partial s}{\partial K}\dfrac{\partial K}{\partial t}$

$$\vartheta(s,t) = \left(\frac{\partial K}{\partial s}\right)^{-1}\frac{\partial K}{\partial t}$$

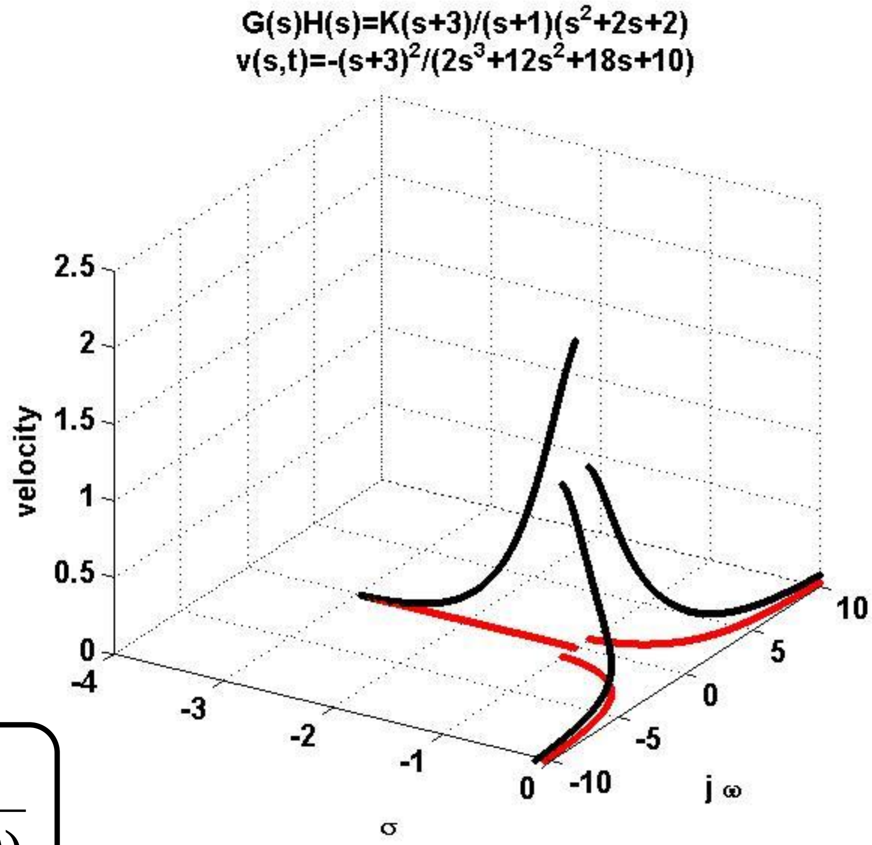
- Hence
- $$\boxed{\vartheta(s,t) = \frac{P^2}{Q\partial P - P\partial Q}\left(\frac{\partial K}{\partial t}\right)}$$

- the root velocity at **the zero** is always **zero**
- The root velocity at **the break away** point is **infinity**

- ***example 1:***

$$GH(s) = \frac{(s+3)}{(s+1)(s^2+2s+2)}$$

$$K(t) = t$$



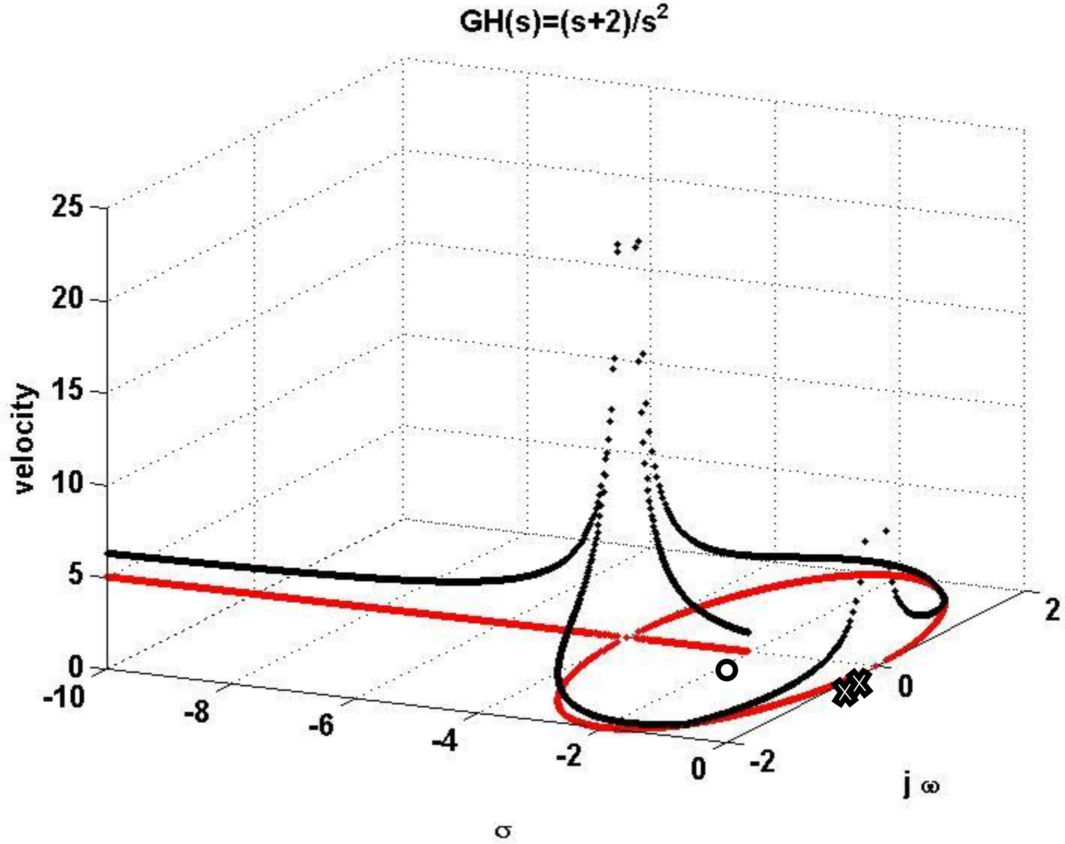
$$\vartheta(s,t) = \frac{-(s+3)^2}{(2s^3+12s^2+18s+10)}$$

- ***example 2:***

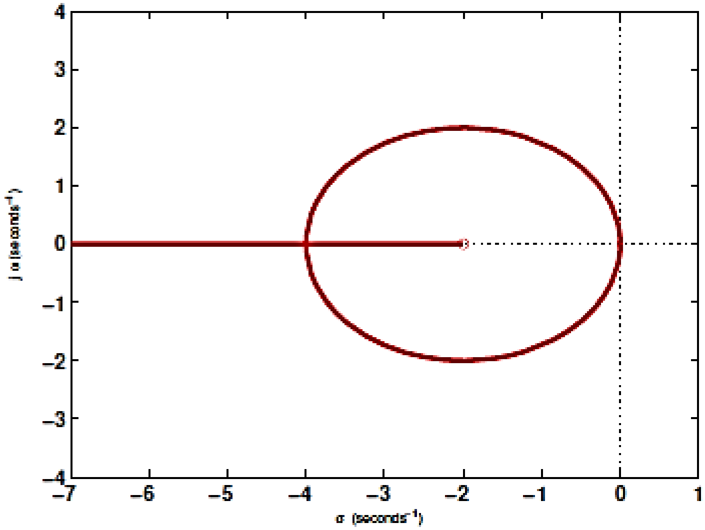
$$GH(s) = \frac{(s+2)}{s^2}$$

$$K(t) = t$$

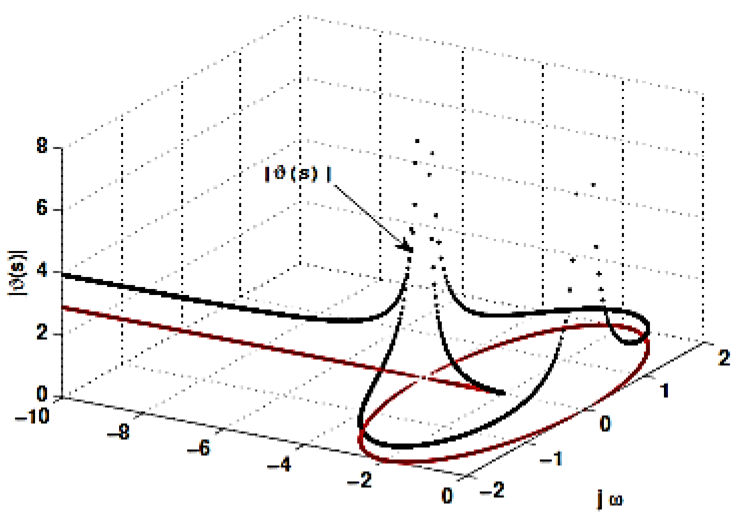
$$\vartheta(s,t) = \frac{-(s^2+2s+4)}{s(s+4)}$$



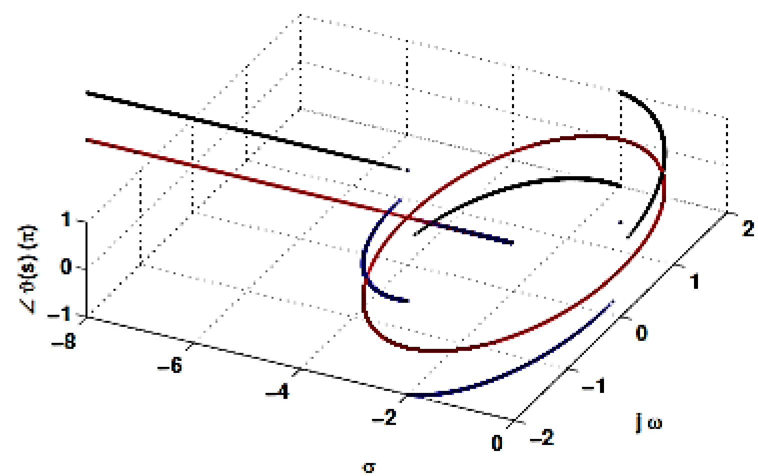
Poles direction



(a) root-locus



(b) absolute value of root velocity



(a) phase of root velocity