Automatic Control Systems

NYQUIST-II

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- **Reading:**
  - **chapter 8 (**Section 8.2)
  - **chapter 9 (**Section 9.5)
- **Practice problems**
  - Study table 9.6 on pages 704-711
  - Solve problems at the end of chapter 9

The Nyquist Stability Criterion

Case 1

$\mathbf{L(S) = G(s)H(}\boldsymbol{s}\mathbf{)}$ has no **open-loop poles** in the RHP, $(\boldsymbol{P} = \mathbf{0})$

"A feedback system is stable if and only if the contour $\boldsymbol{\Gamma_L}$ in the $\mathbf{G(s)H(}\boldsymbol{s}\mathbf{)}$-plane DOES NOT encircle the $(-\mathbf{1}, \mathbf{0})$ point when the number of poles of $\boldsymbol{L(s)}$ in the RHP is zero $(\boldsymbol{P} = \mathbf{0})$"

$$\boxed{\begin{gathered} \boldsymbol{Z = N + P} \\ \therefore \boldsymbol{Z} = \mathbf{0} \\ \Rightarrow \boldsymbol{N} = \mathbf{0} \end{gathered}}$$

- For no zeros of the characteristic equation on the RHP, then there should be **no encirclement** of the **point $-\mathbf{1}$ in the $\boldsymbol{L(s)}$ plane**

The Nyquist Stability Criterion

Case 2

<u>$G(s)H(s)$ has **poles** in the RHP, $(P \neq 0)$</u>

"A feedback system is stable <u>if and only if</u>, for the contour Γ_L, the number of <u>counterclockwise</u> encirclements of the $(-1, 0)$ point equals to the number of poles of $\mathrm{G}(s)\mathrm{H}(s)$ in the RHP of the s-plane"

$$\boldsymbol{Z = N + P}$$

$$\therefore\ \boldsymbol{Z = 0}$$

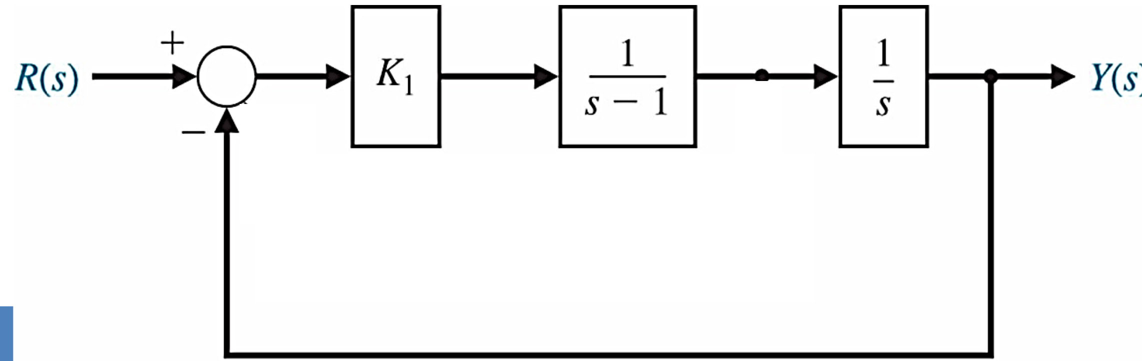
$$\Rightarrow \boldsymbol{N = -P}$$

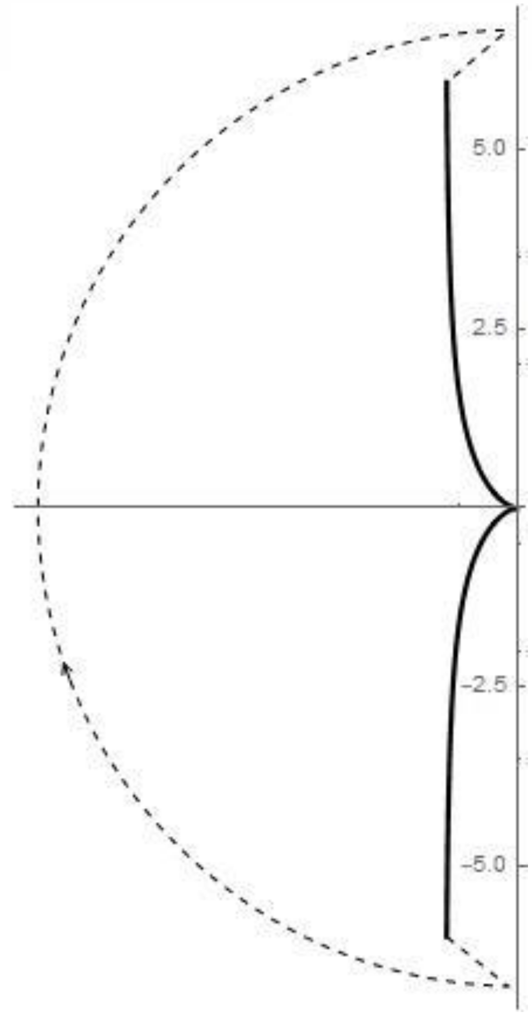
- For no zeros of the characteristic equation on the RHP, we must have P **<u>counterclockwise</u>** encirclement of the -1 point in the $\mathrm{G}(s)H(s)$ plane

- ***example***

$$\boxed{G(s) = K_1/s(s-1)}$$

ω	$GH(j\omega)$
$j\omega \to 0_+$	$\infty\angle -270^0$
$j\omega \to +\infty$	$0\angle -180^0$

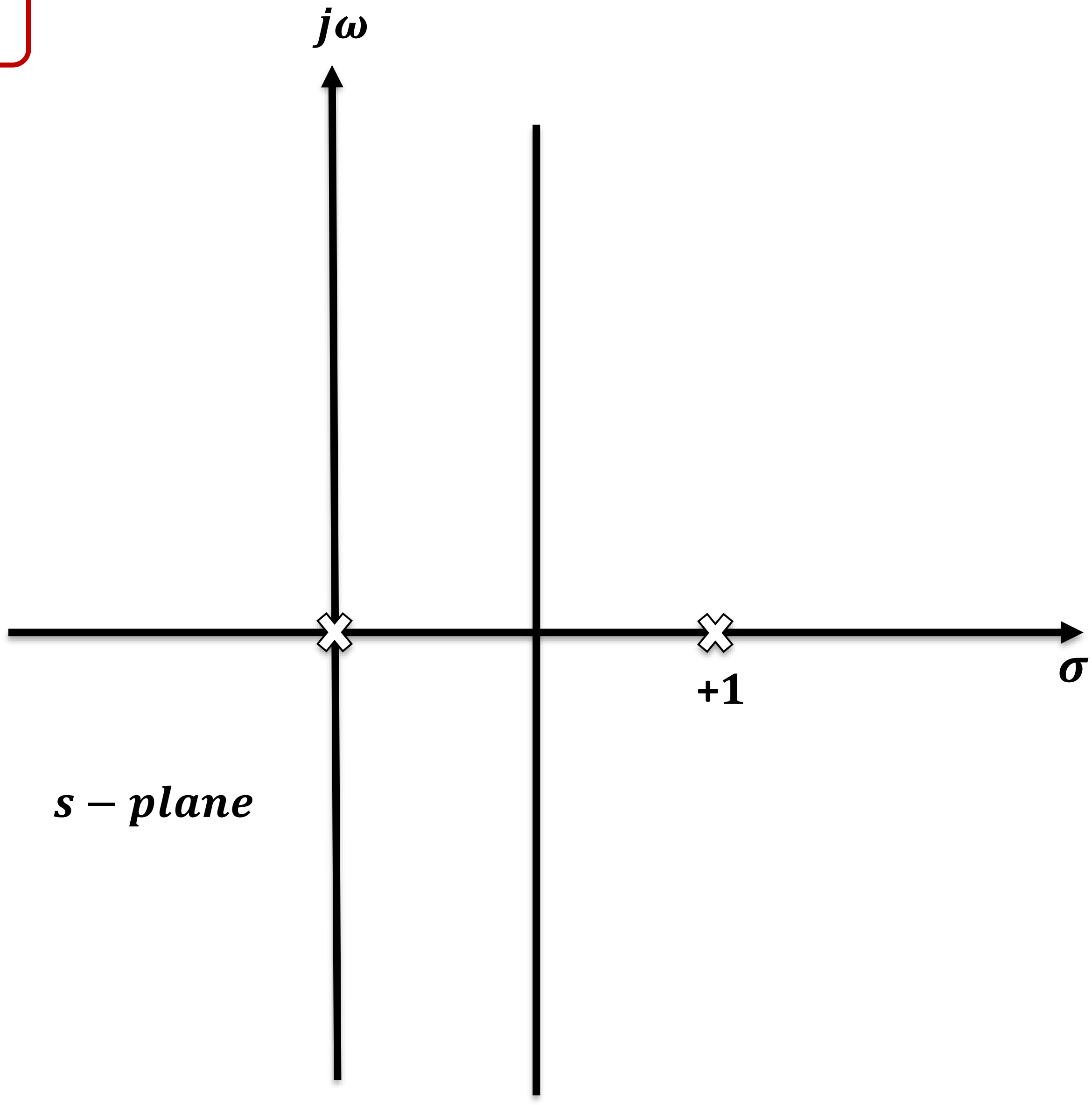




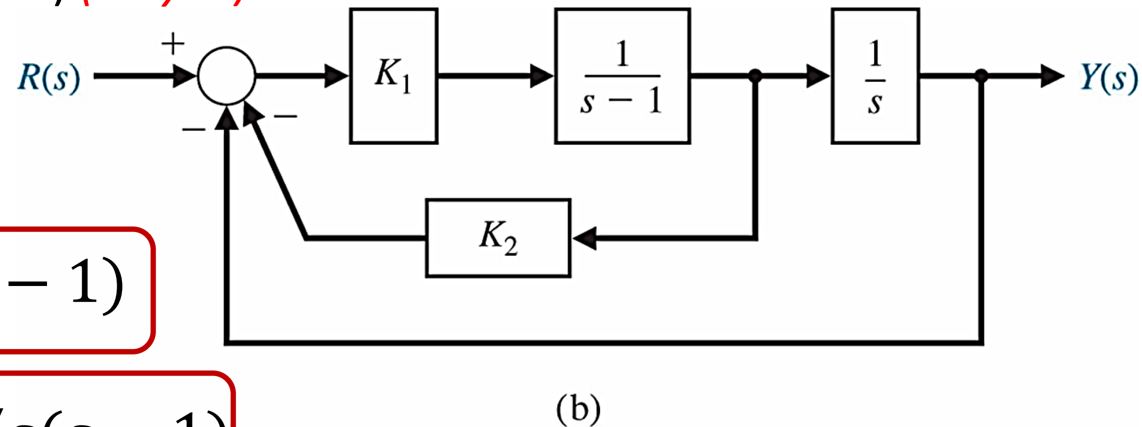
- → This is CASE-II
- $P = 1, N = +1 \Rightarrow Z = 2$

- The system is unstable because there are **two roots** in the **RHS** of the s-plane regardless of the value of K_1

$$G(s) = K_1/s(s-1)$$



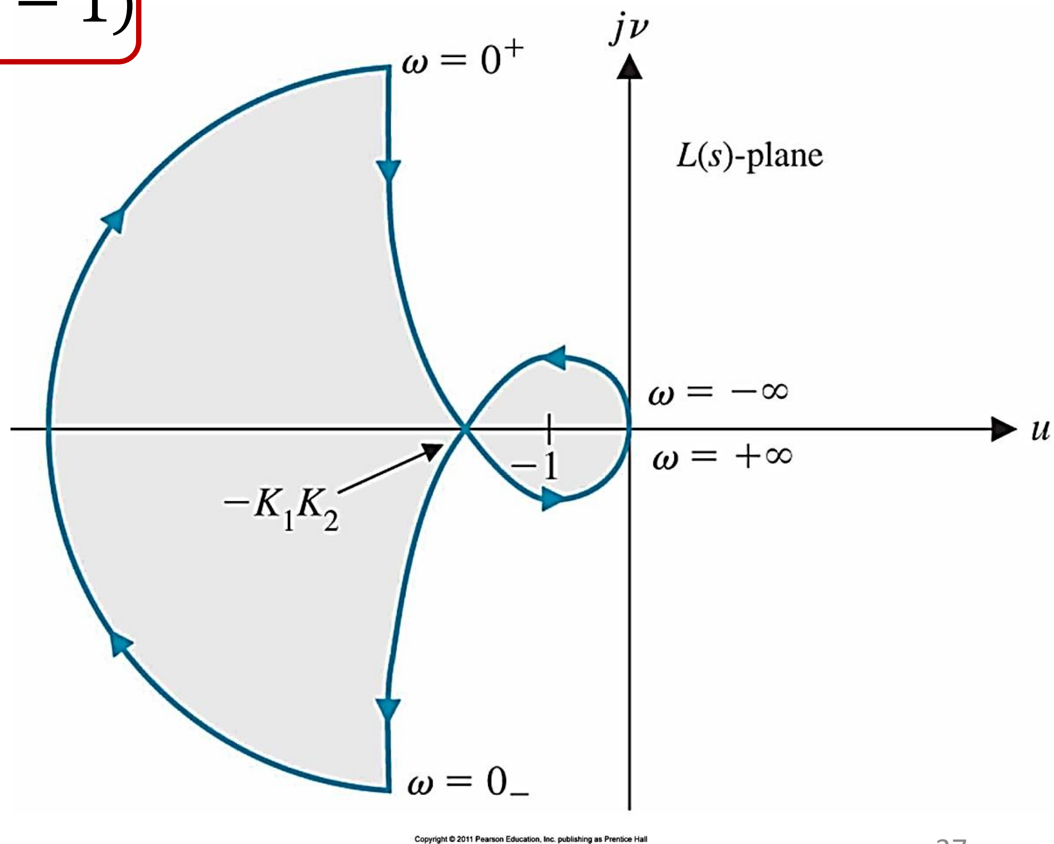
- Add a **derivative** feedback (**PD**) *(why??)*



$$G(s) = K_1(1 + K_2 s)/s(s - 1)$$

$$G(s) = K_1 K_2(s + 1/K_2)/s(s - 1)$$

ω	$GH(j\omega)$
$j\omega \to 0_+$	$\infty \angle -270^0$
$j\omega \to +\infty$	$0 \angle -90^0$



- $P = 1$ *(case II)*

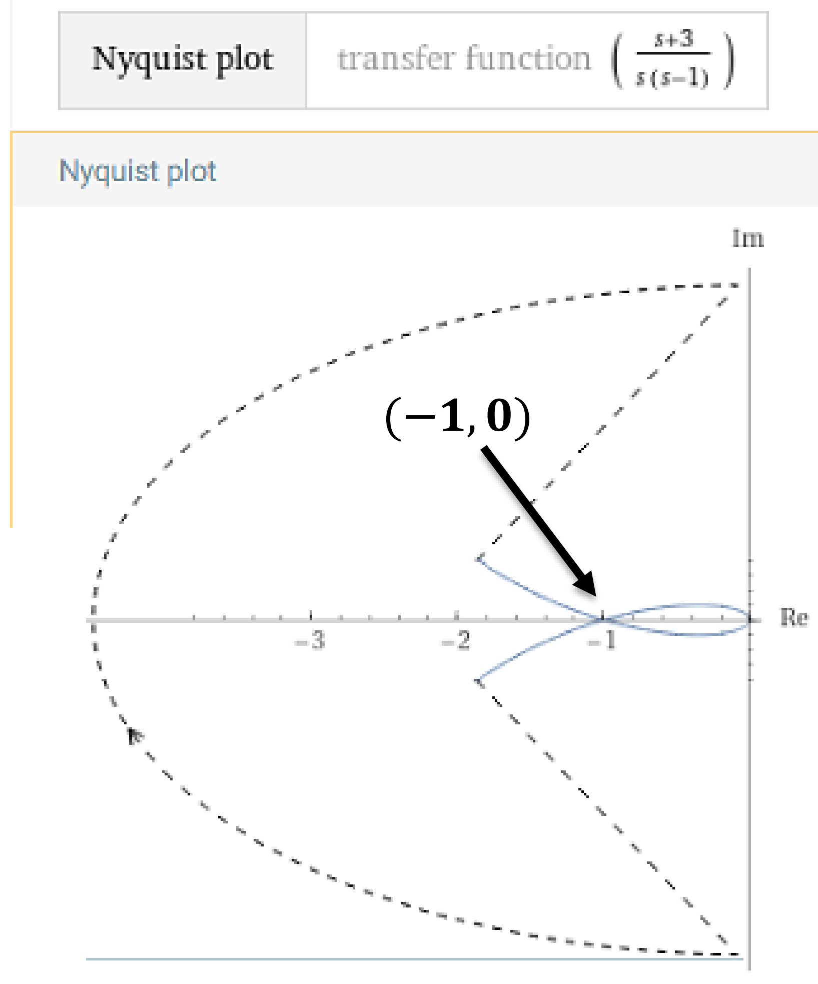
- Condition for stability:

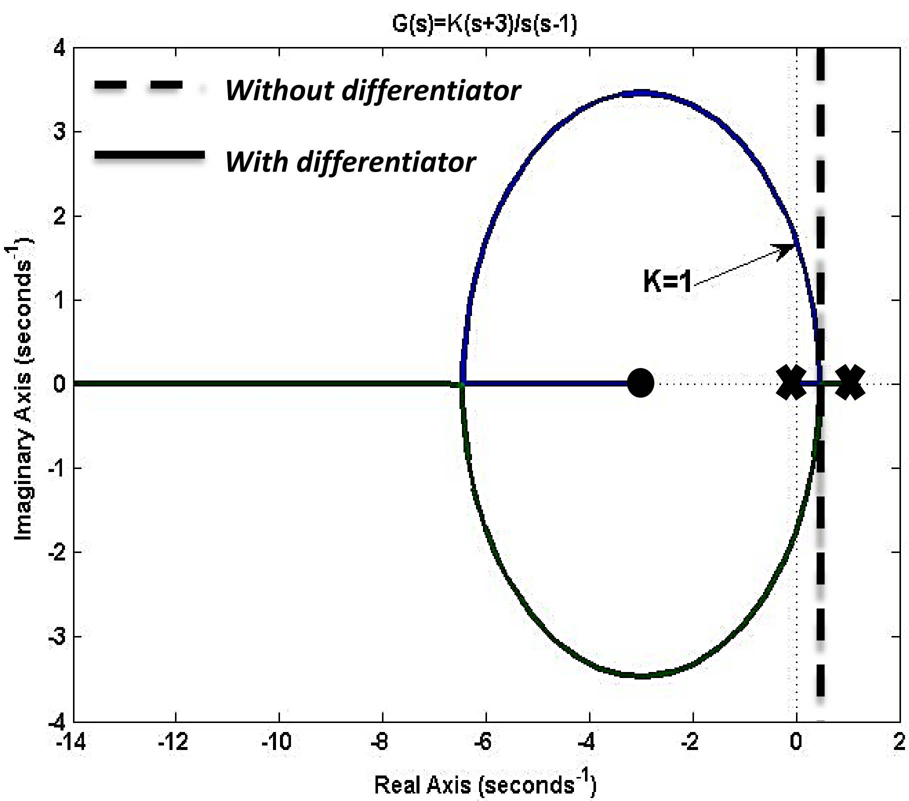
$N = -1 \Rightarrow K = K_1 K_2 > 1$

- $G(s) = K_1 K_2 (s + 1/K_2)/s(s-1)$
- Let: $1/K_2 = 3,\ K_1 K_2 = K$

$$\boxed{G(s)H(s) = \frac{K(s+3)}{s(s-1)}}$$

- verify using root locus



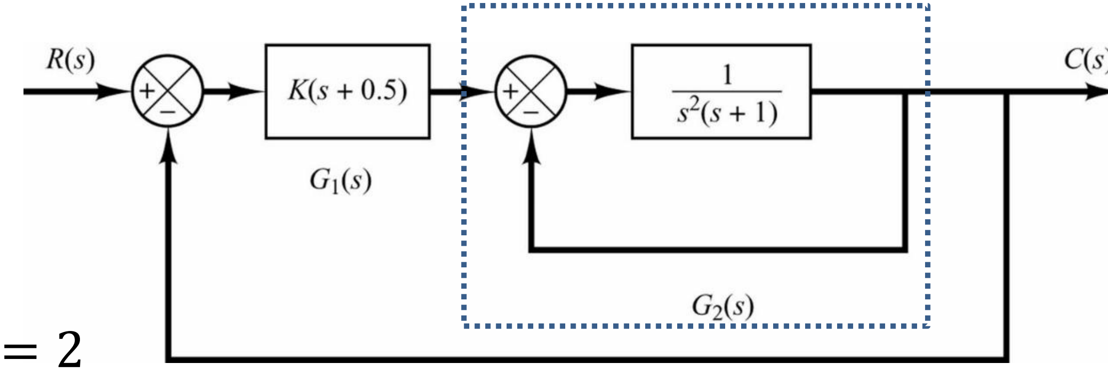



```
b=0.5*[1 3]; a=[1 -1 0]; nyquist(b,a)
```

```
b=[1 3]; a=[1 -1 0]; rlocus(b,a)
```

- **example**: determine the range of K for stability?
- *Inner-loop* stability

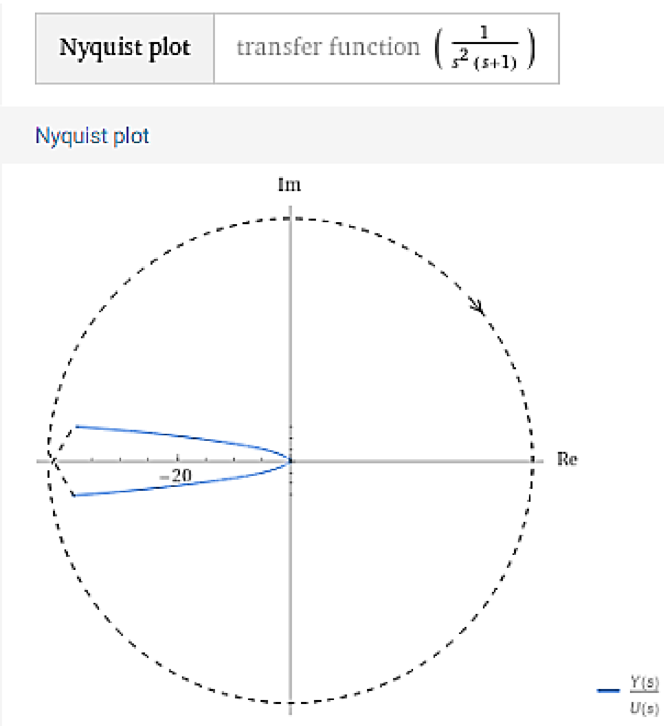
$$\boxed{G(s) = \frac{1}{s^2(s+1)}}$$

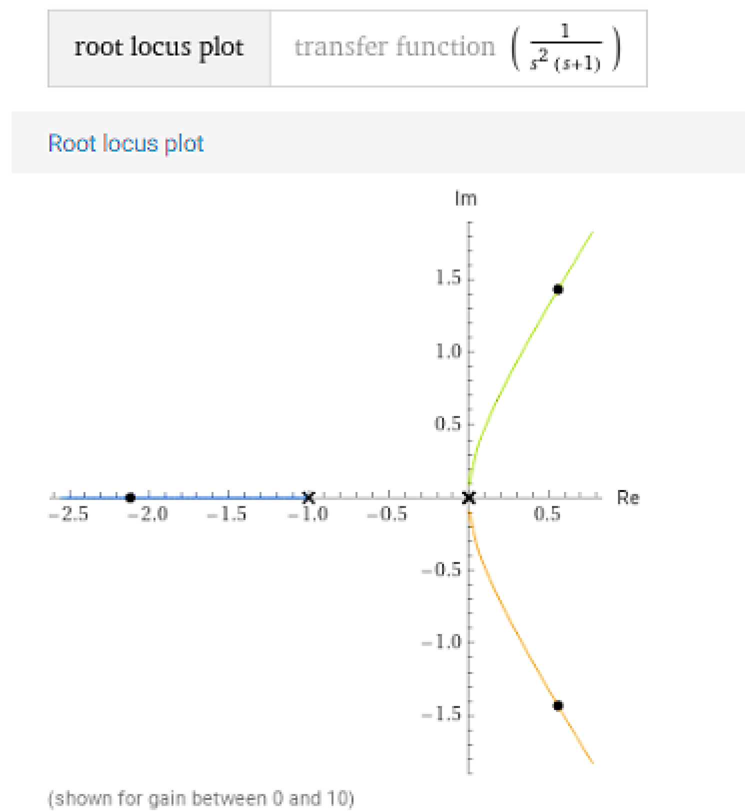


$\Rightarrow \quad \text{case} - \text{I}$

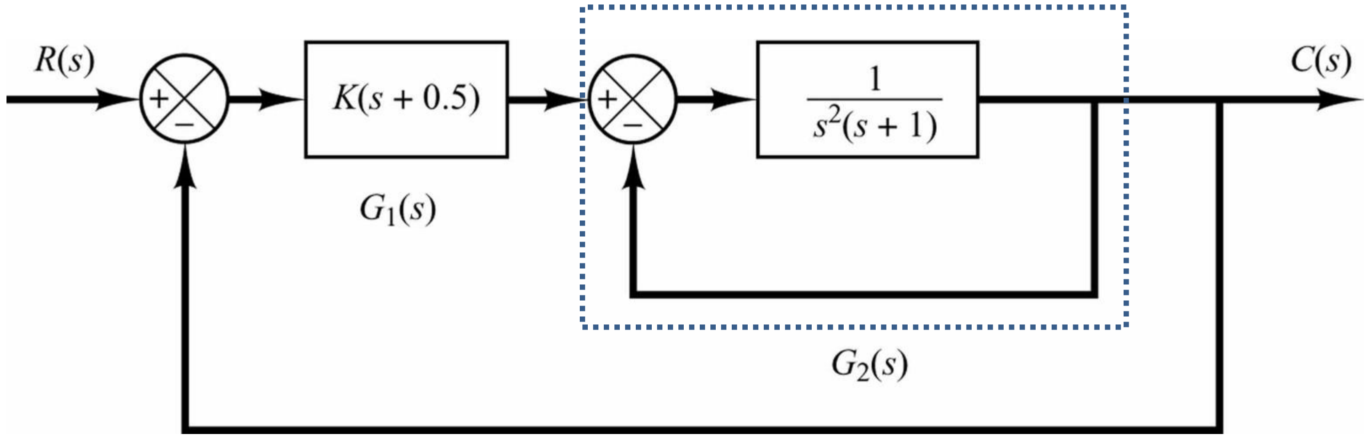
$P = 0, N = 2 \implies Z = N + P = 2$

Inner-loop is *unstable*





- After adding a differentiator



- The open-loop system with **PD** $\boxed{G(s)=G(s)_1G_2(s)=\dfrac{K(s+0.5)}{s^3+s^2+1}}$

- $\boldsymbol{P=2}$

- For stability of the closed-loop system,
- $\Rightarrow Z=0 \Longrightarrow \boxed{\boldsymbol{N=Z-P=-2}}$

(two **<u>counterclockwise</u>** encirclement of the critical point -1)

- Open-loop poles, $\boldsymbol{P = 2}$ ***(case II)***

$$G(s) = G(s)_1 G_2(s) = \frac{K(s+0.5)}{s^3+s^2+1}$$

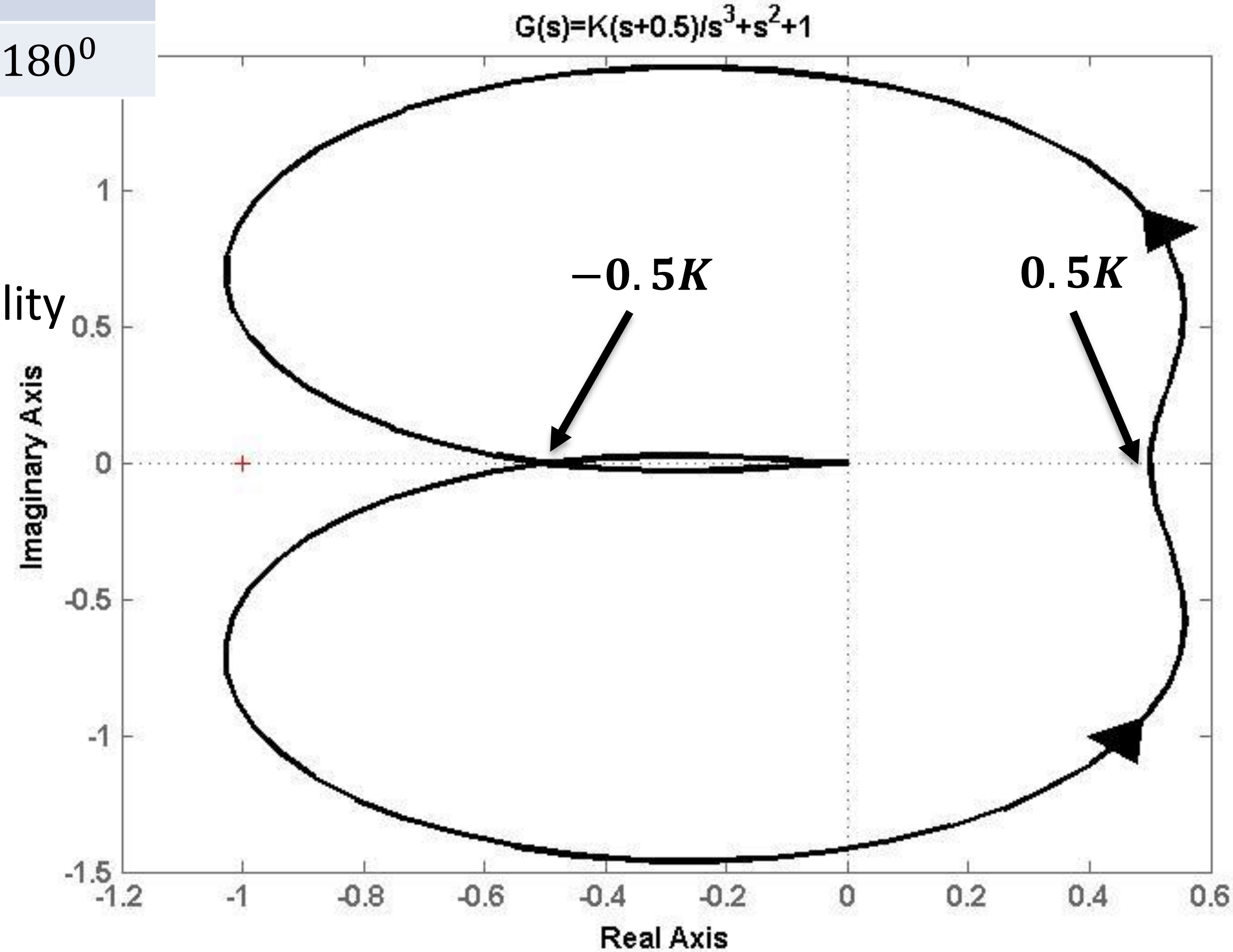
$\boldsymbol{\omega}$	$\boldsymbol{G(j\omega)}$
$j\omega \to 0_+$	$0.5K$
$j\omega \to +\infty$	$0\angle -180^0$

- Condition for stability

$$Z = 0 \Longrightarrow N = -2$$

$$\Rightarrow 0.5K > 1$$

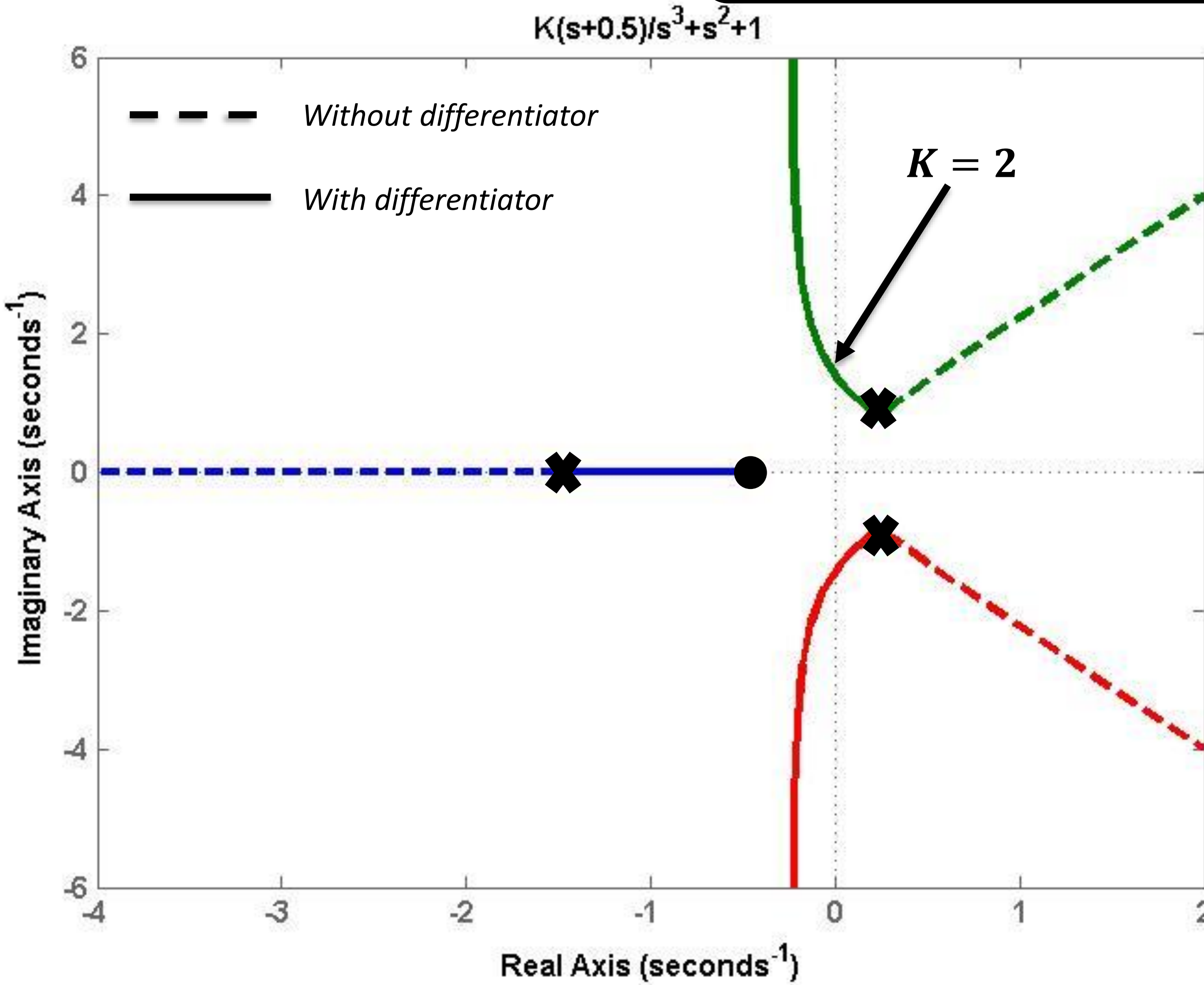
$$\Longrightarrow \boxed{K > 2}$$



b=[1 0.5]; a=[1 1 0 1]; sys=tf(b,a); nyquist(sys)

- verify using root locus

$$G(s) = G(s)_1 G_2(s) = \frac{K(s+0.5)}{s^3 + s^2 + 1}$$

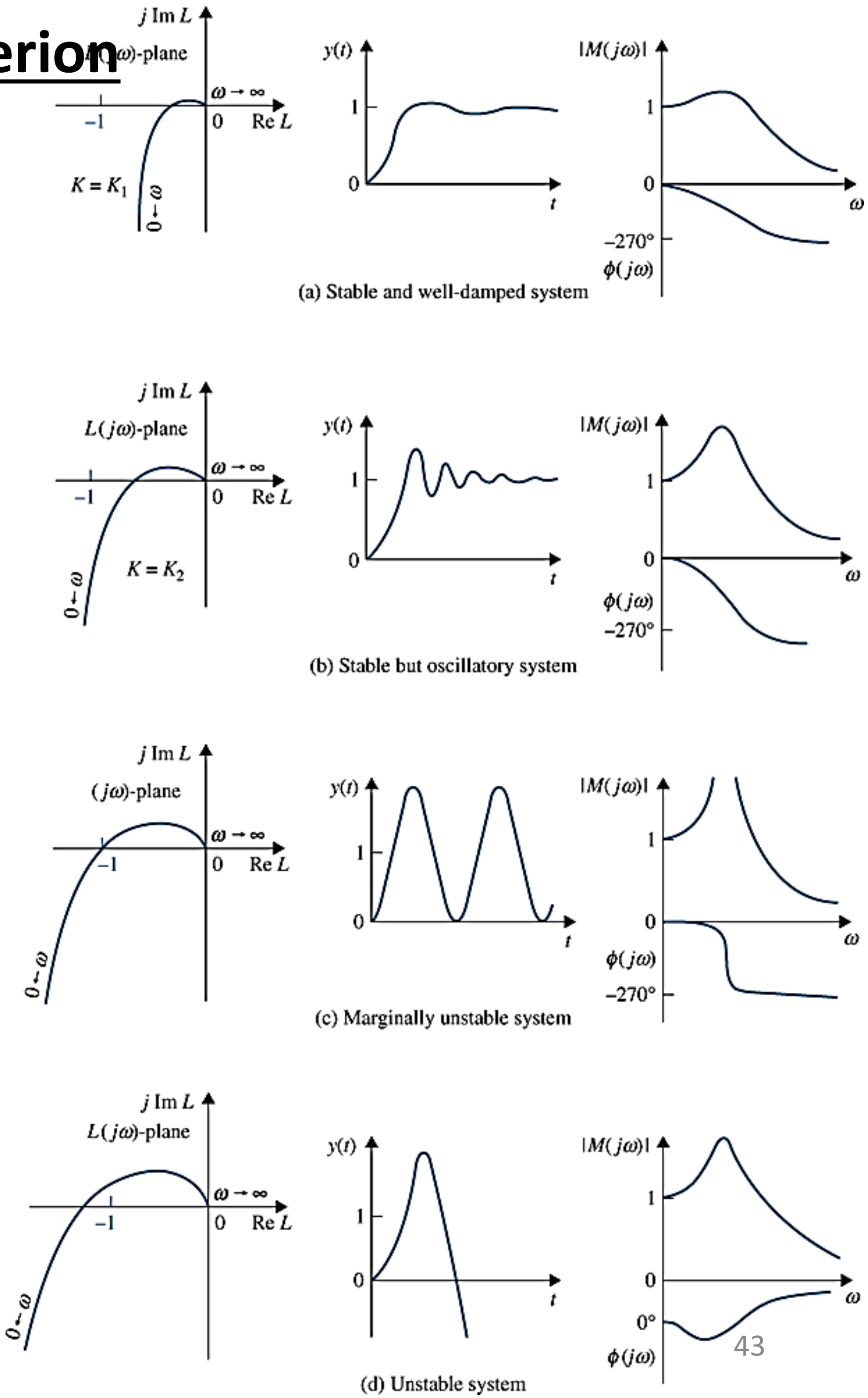



```
b=[1 0.5]; a=[1 1 0 1]; sys=tf(b,a); rlocus(sys)
```

Relative stability and the Nyquist criterion

- The proximity of the $L(j\omega)$ - locus to the $(-1, j0)$ critical point is a measure of the relative stability of the system.

- Case I
- Case II



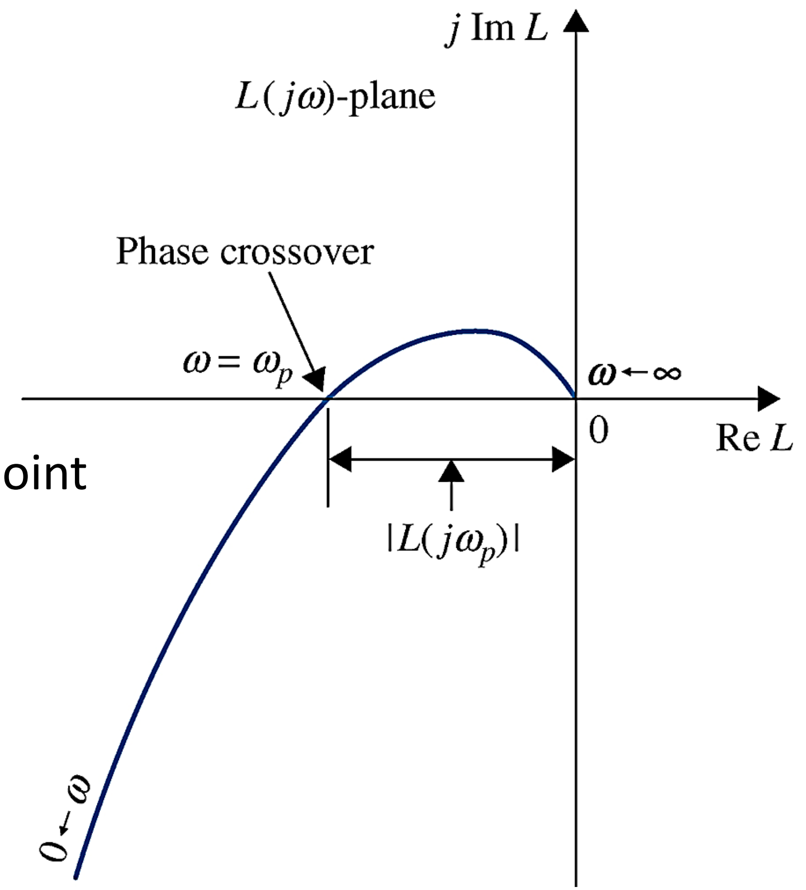
(a) Stable and well-damped system
(b) Stable but oscillatory system
(c) Marginally unstable system
(d) Unstable system

Gain Margin (GM)

Gain margin is the amount of gain in decibels (dB) that is allowed to be <u>INCREASED/DECREASED</u> In the loop before the closed-loop system reaches instability

PHASE-CROSSOVER POINT:

the point at which the Nyquist plot $L(j\omega)$ intersects the negative real axis



PHASE-CROSSOVER FREQUENCY

is the frequency ω_p at the phase-crossover point $\angle L(j\omega) = 180^o$

$$\boxed{G.M. = 20\log_{10}\frac{1}{\left|L(j\omega_p)\right|} \quad \text{dB}}$$

- A rule of thumb $\boldsymbol{G.M > 6dB}$

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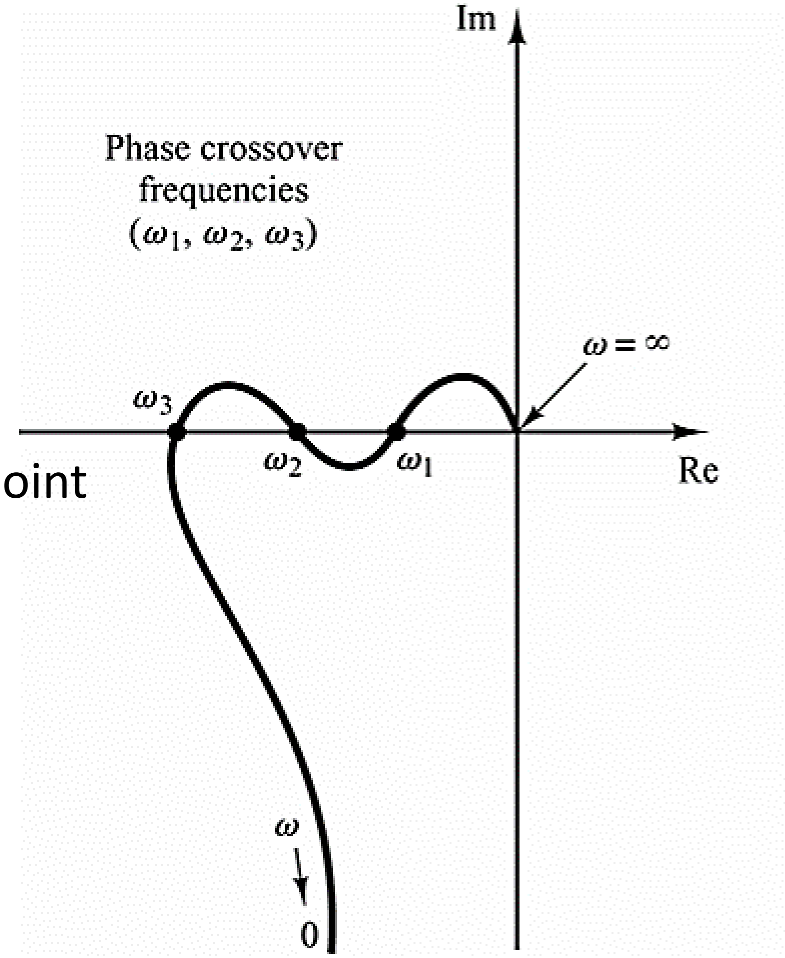


- A rule of thumb $\boldsymbol{G.M > 6dB}$

- ***example***

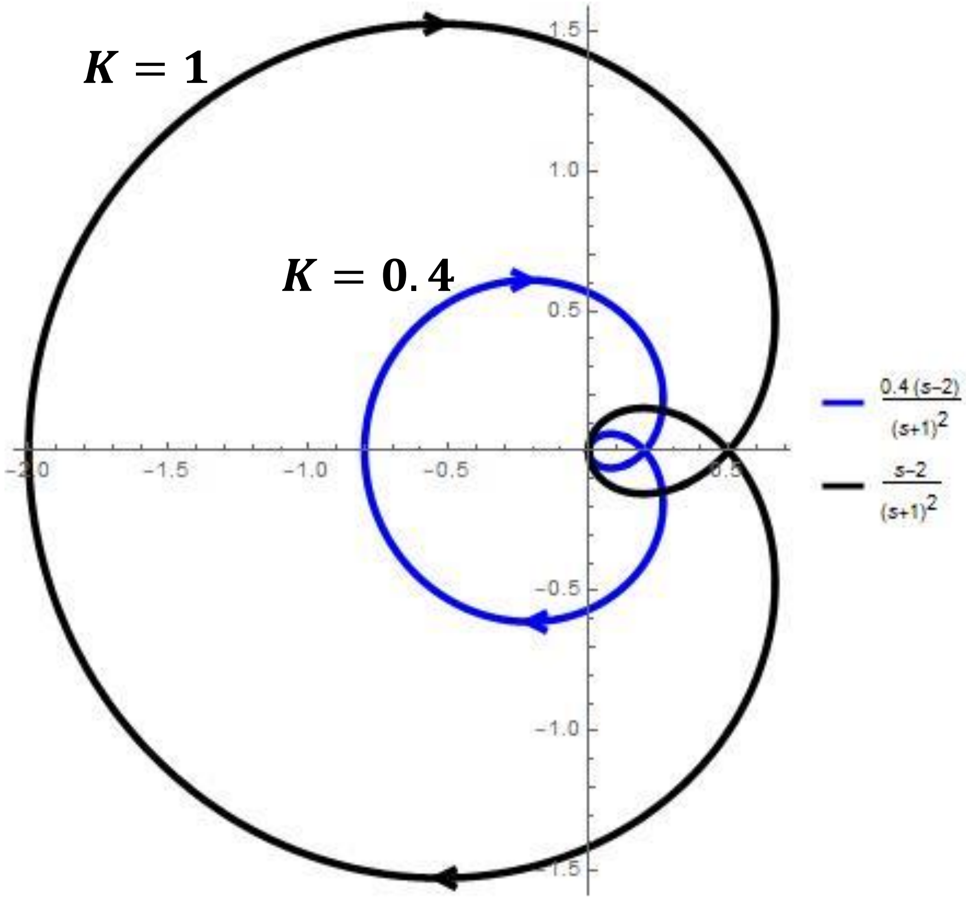
$$\boxed{GH(s) = K(s-2)/(s+1)^2}$$

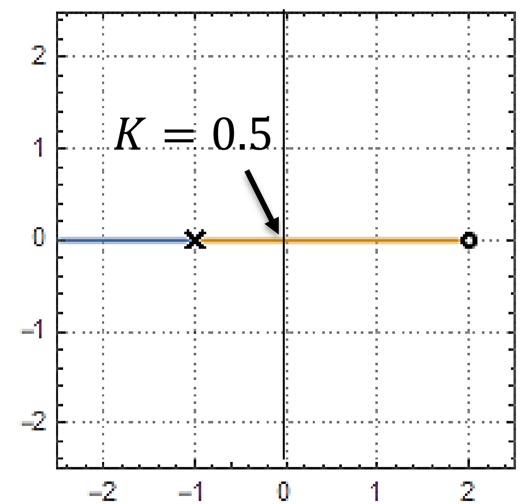
- $P = 0$ *(case-I)*

ω	$GH(j\omega)$
$j\omega \to 0_+$	$-2K$
$\omega = \sqrt{5}$	$K/2$
$j\omega \to +\infty$	$0\angle -90^0$

- $\boxed{G.M. = 20log_{10}(1/2K)}$

K	N	Z	stability
$0 < K < 1/2$	0	0	stable
$K = 1/2$	$--$	$--$	Marginally stable
$K > 1/2$	1	1	unstable





- ***Example***

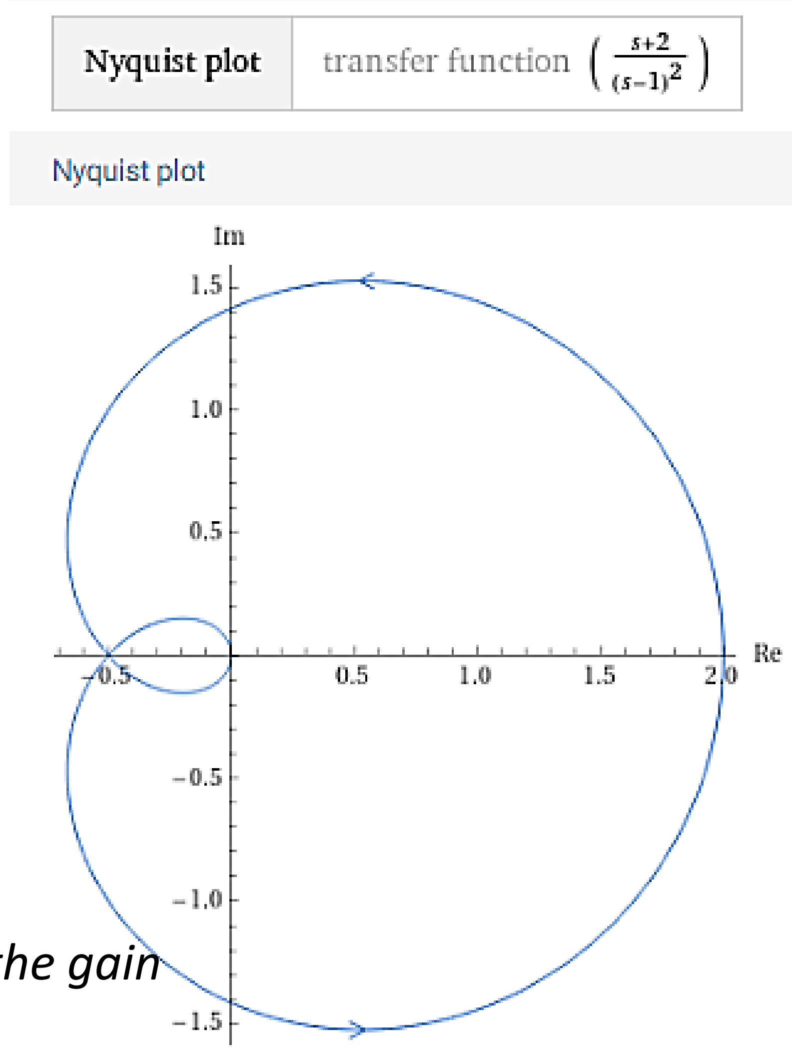
$$\boxed{GH(s) = K(s+2)/(s-1)^2}$$

- $P = 2$ *(case-II)*

$\boldsymbol{\omega}$	$\boldsymbol{GH(j\omega)}$
$j\omega \to 0_+$	$2K$
$\omega = \sqrt{5}$	$-K/2$
$j\omega \to +\infty$	$0\angle -90^0$

- for stability, $Z = 0 \implies N = Z - P = -2$

- $G.M.$ *Is defined* ***in this case*** *by how much the gain Is* ***decreased*** *to reach instability*



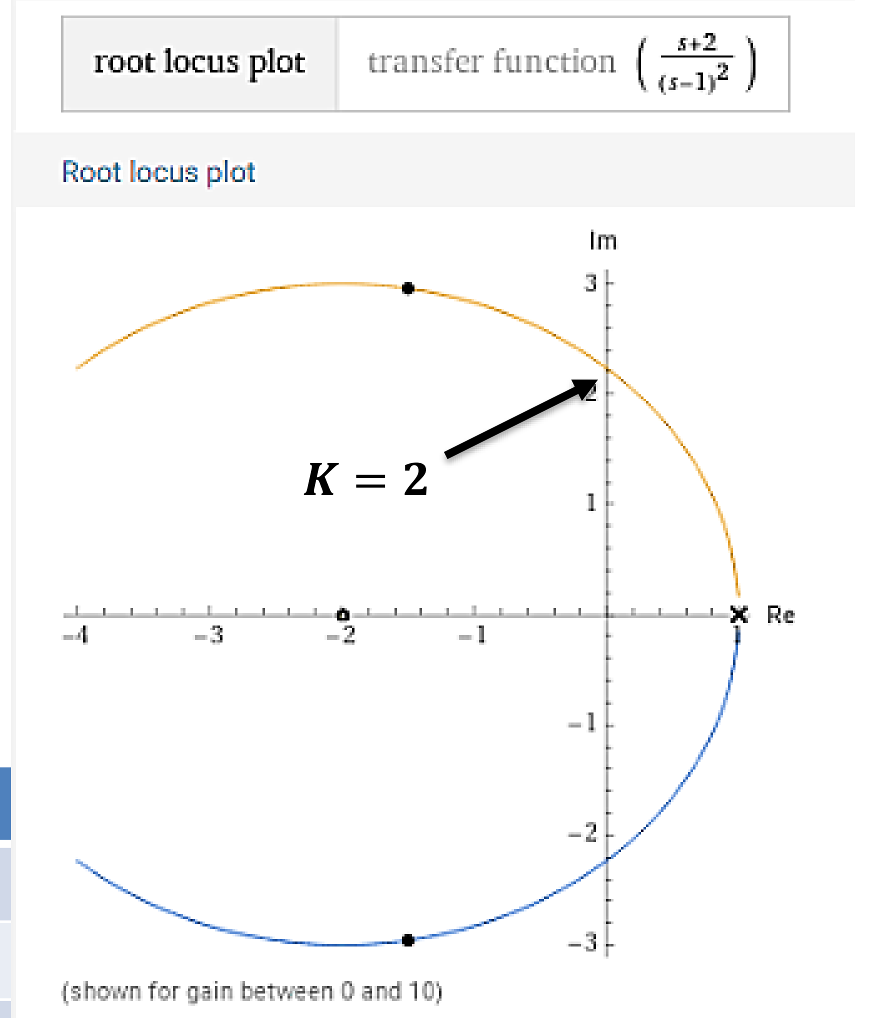
$\boldsymbol{K}$	$\boldsymbol{N}$	$\boldsymbol{Z}$	stability
$\mathbf{0 < K < 2}$	$\mathbf{0}$	$\mathbf{2}$	UNSTABLE
$\boldsymbol{K = 2}$	$- -$	$- -$	Marginally stable
$\boldsymbol{K > 2}$	$\mathbf{-2}$	$\mathbf{0}$	STABLE

- ***Example***

$$\boxed{GH(s) = K(s+2)/(s-1)^2}$$

- $P = 2$ *(case-II)*

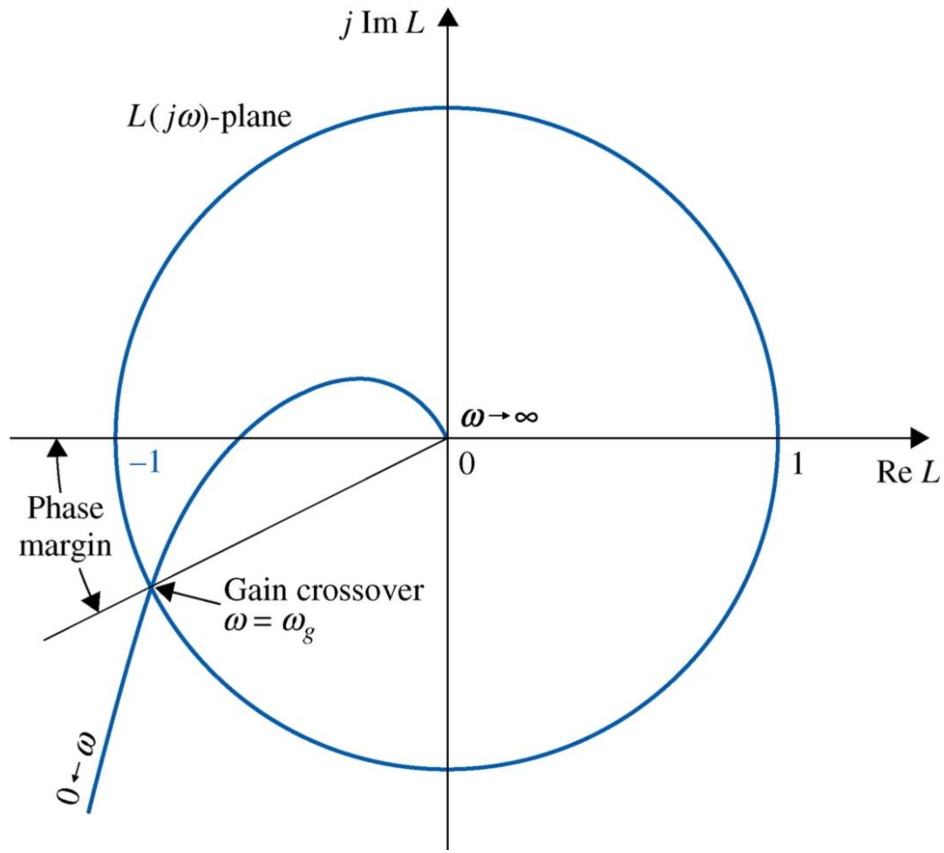
- $G.M.$ *Is defined **in this case** by how much the gain is **decreased** to reach **instability***



K	N	Z	stability
$0 < K < 2$	0	2	UNSTABLE
$K = 2$	$--$	$--$	Marginally stable
$K > 2$	-2	0	STABLE

Phase Margin (PM)

Phase margin is defined as the angle in degrees in which the $L(j\omega)$ plot must be rotated about the origin in order that the gain-crossover point on the locus passes through the $(-1, j0)$ point



GAIN-CROSSOVER POINT:

The point on the $L(j\omega)$ plot at which $|L(j\omega)| = 1$

GAIN-CROSSOVER FREQUENCY:

The gain-crossover frequency ω_g is the frequency of $L(j\omega)$ at the gain-crossover point, i.e., $|L(j\omega_g)| = 1$

$$\boxed{P.M. = \angle L(j\omega_g) - 180^o}$$

- A rue of thumb, $\boldsymbol{30^o < P.M < 60^0}$

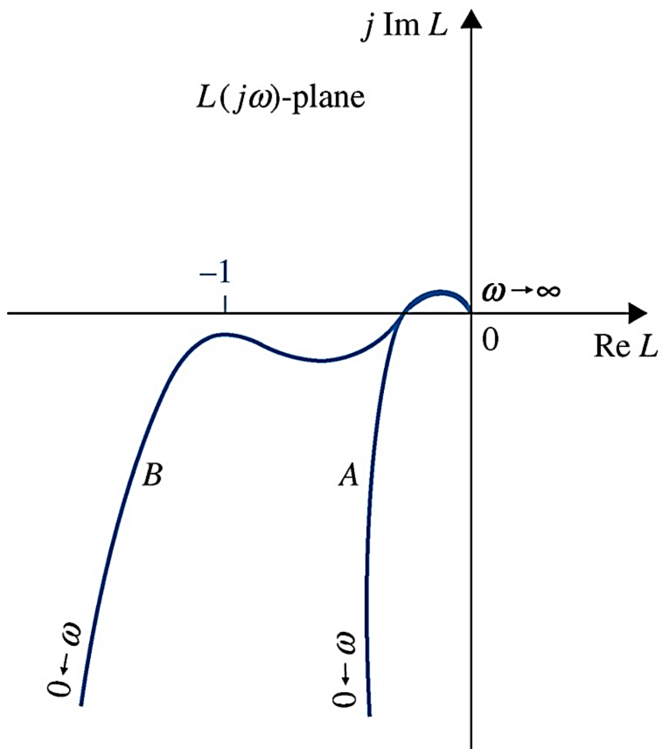
Phase Margin (PM)

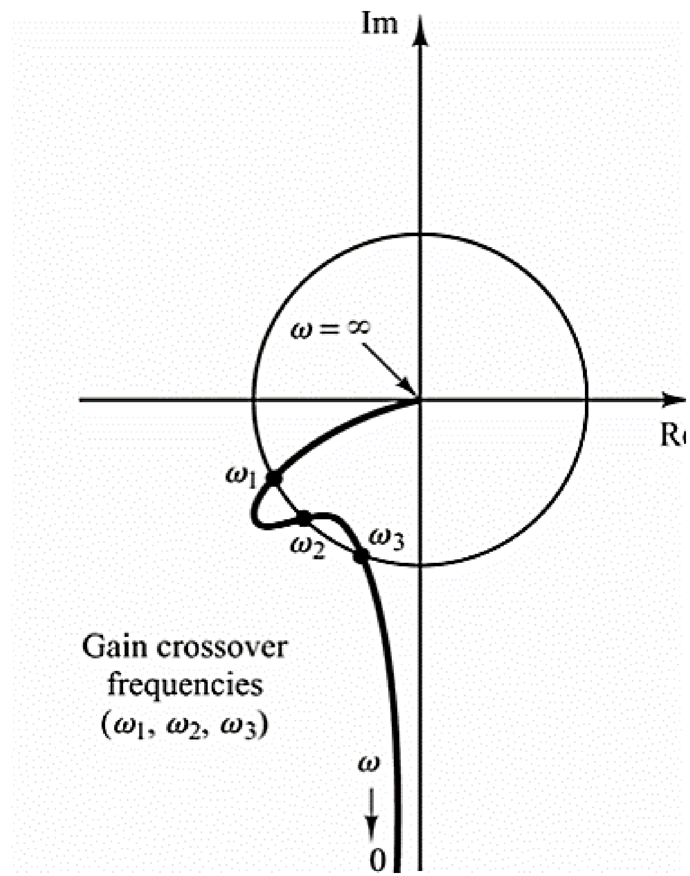
Phase margin is defined as the angle in degrees in which the $L(j\omega)$ plot must be rotated about the origin in order that the gain-crossover point on the locus passes through the $(-1, j0)$ point

locus A and locus B have **the same**

Gain margin but have different relative stability

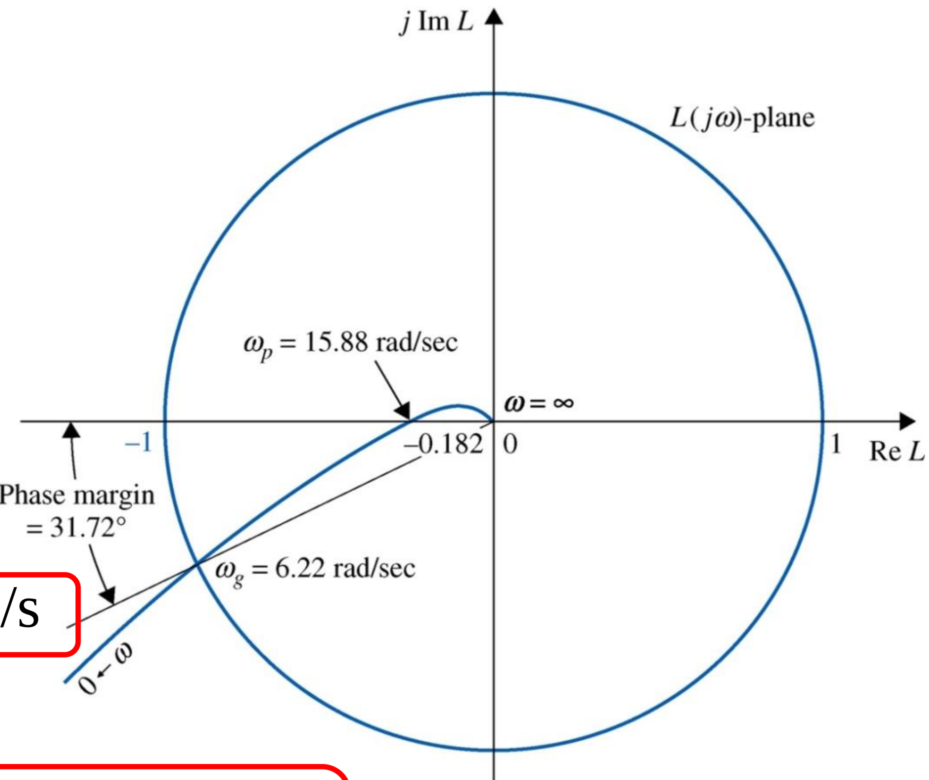
Because they have **different phase margins**





- ***example***

$$\boxed{L(s) = \frac{2500}{s(s+50)(s+5)}}$$



- **Phase-crossover:**

$$\mathrm{Im}\big(L(j\omega)\big) = 0$$

$$\Rightarrow L(j\omega_p) = -0.182, \quad \boxed{\omega_p = 15.88 \quad \text{rad/s}}$$

- **Gain margin**

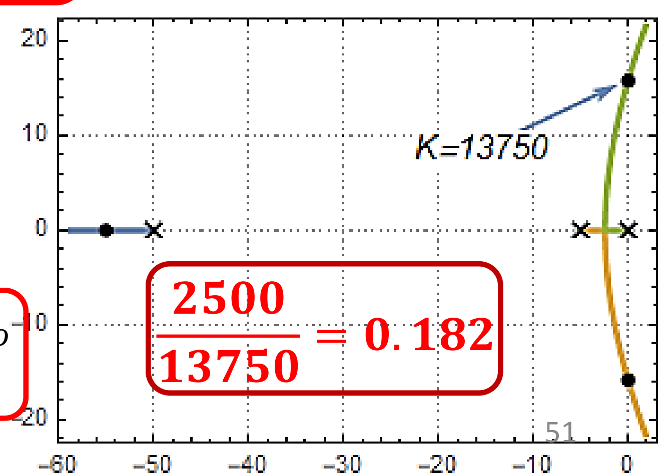
$$\boxed{G.M. = 20\log_{10}\frac{1}{|L(j\omega_p)|} = 20\log_{10}\frac{1}{0.182} = 14.82\ \text{dB}}$$

- **Gain-crossover:**

$$|L(j\omega)| = 1 \quad \Rightarrow \boxed{\omega_g = 6.22 \quad \text{rad/s}}$$

- **Phase margin**

$$\boxed{P.M. = \angle L(j\omega_g) - 180^o = 211.72^o - 180^o = 31.72^o}$$



$$\boxed{\frac{2500}{13750} = 0.182}$$

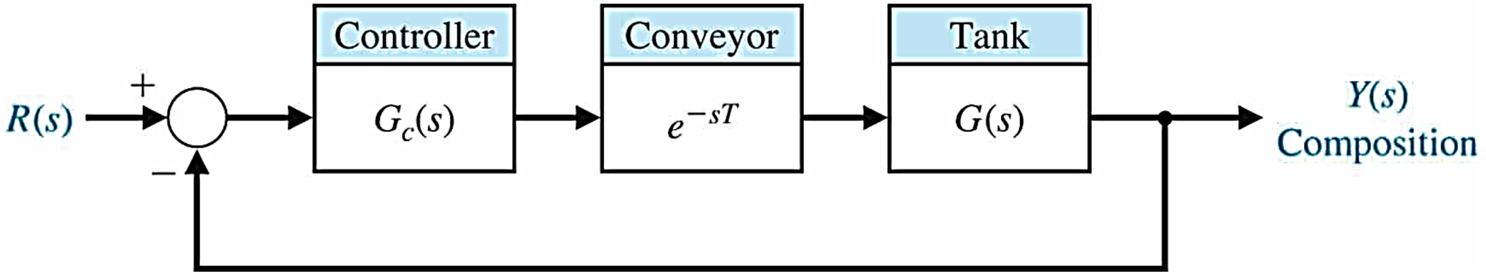
The stability of control systems with time delays

- Usually, we cannot eliminate the effect of delay in real systems

- The Routh-Hurwitz criterion is **not applicable** for systems with time delays because **the characteristic equation is no longer algebraic**

- a pure time delay, $\boldsymbol{T}$, is modeled as $\boxed{\boldsymbol{G_d(s) = e^{-sT}}}$

- the loop transfer function $L(s)$, becomes $\boxed{L(s) = G_c(s)G(s)e^{-sT}}$



The stability of control systems with time delays

- A pure time delay, T, has **<u>no effect on the gain</u>** of $L(j\omega)$

$$\boxed{|\boldsymbol{L}(\boldsymbol{j\omega})| = |\boldsymbol{G_c}(\boldsymbol{j\omega})\boldsymbol{G}(\boldsymbol{j\omega})|}$$

- A pure delay, T, **<u>reduces the phase</u> of $L(j\omega)$**

$$\boxed{L(s) = \frac{e^{-sT}}{(1+sT)}}$$

$$\boxed{\angle \boldsymbol{L}(\boldsymbol{j\omega}) = \angle \boldsymbol{G_c}(\boldsymbol{j\omega})\boldsymbol{G}(\boldsymbol{j\omega}) - \boldsymbol{\omega T}}$$

- Time delay **<u>reduces</u>** the stability of the system
- Reduces Phase Margin