Automatic Control Systems

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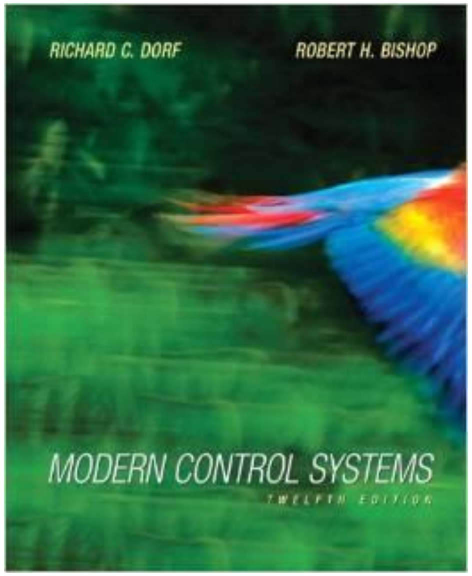
Textbook



- **Modern Control Systems, 12/E**

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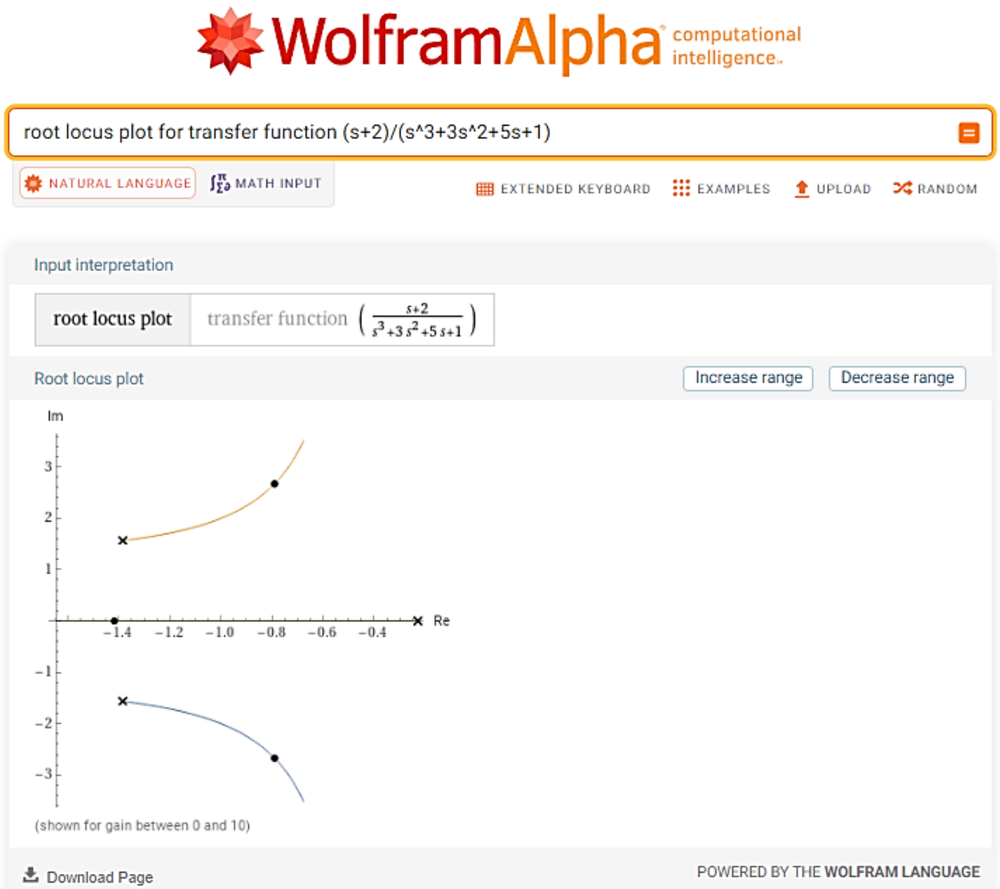
ISBN-10: 0136024580

ISBN-13: 9780136024583

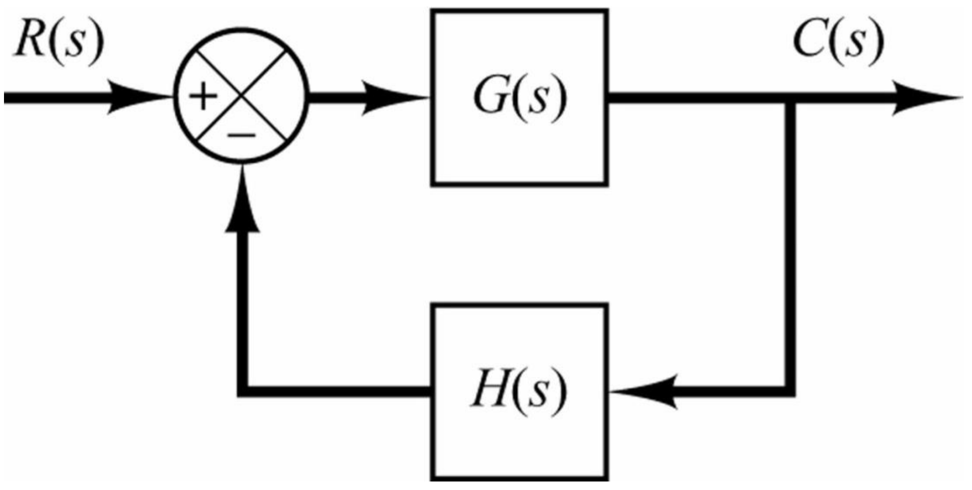
http://www.pearsonhighered.com/product?ISBN=0136024580#sthash.wTb78eoW.dpuf

Online useful tool



Root Locus Techniques

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



- Characteristic equation

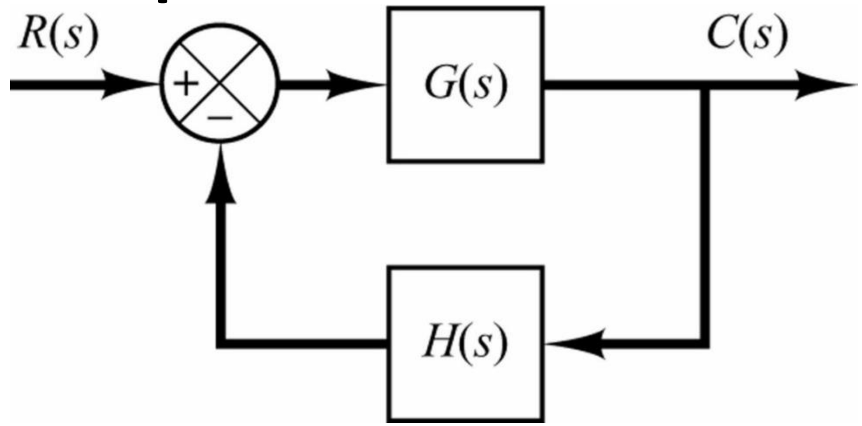
$$\boxed{1+G(s)H(s)=0}$$

- *The root locus technique uses the* ***open-loop*** *system to determine the* ***stability of the closed loop*** *system*

- *The* ***root locus*** *is a graph showing how the* ***roots*** *of the* ***characteristic equation*** *move in the* ***s-plane*** *as the loop-gain varies from* $\mathbf{0}$ ***to*** ∞

Root Locus Techniques

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



- Characteristic equation

$$1+G(s)H(s)=0$$

- The characteristic equation can be written as

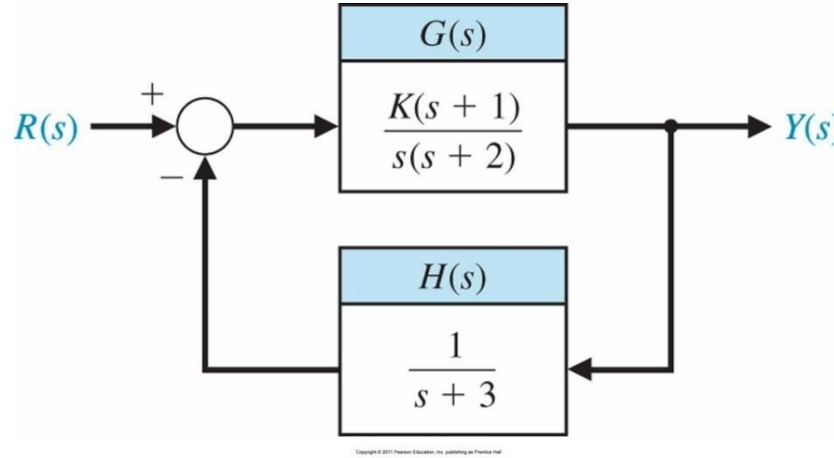
$$\boxed{G(s)H(s) = \frac{KQ(s)}{P(s)}}$$

$$\boxed{P(s)+KQ(s)=0}$$

- **<u>Rearrange the characteristic equation with $0 \le K < \infty$ as a parameter</u>**

$$\Rightarrow \boxed{1+K\frac{Q(s)}{P(s)}=0,} \qquad 0 \le K < \infty$$

- ***example***



$$1+G(s)H(s)=0$$

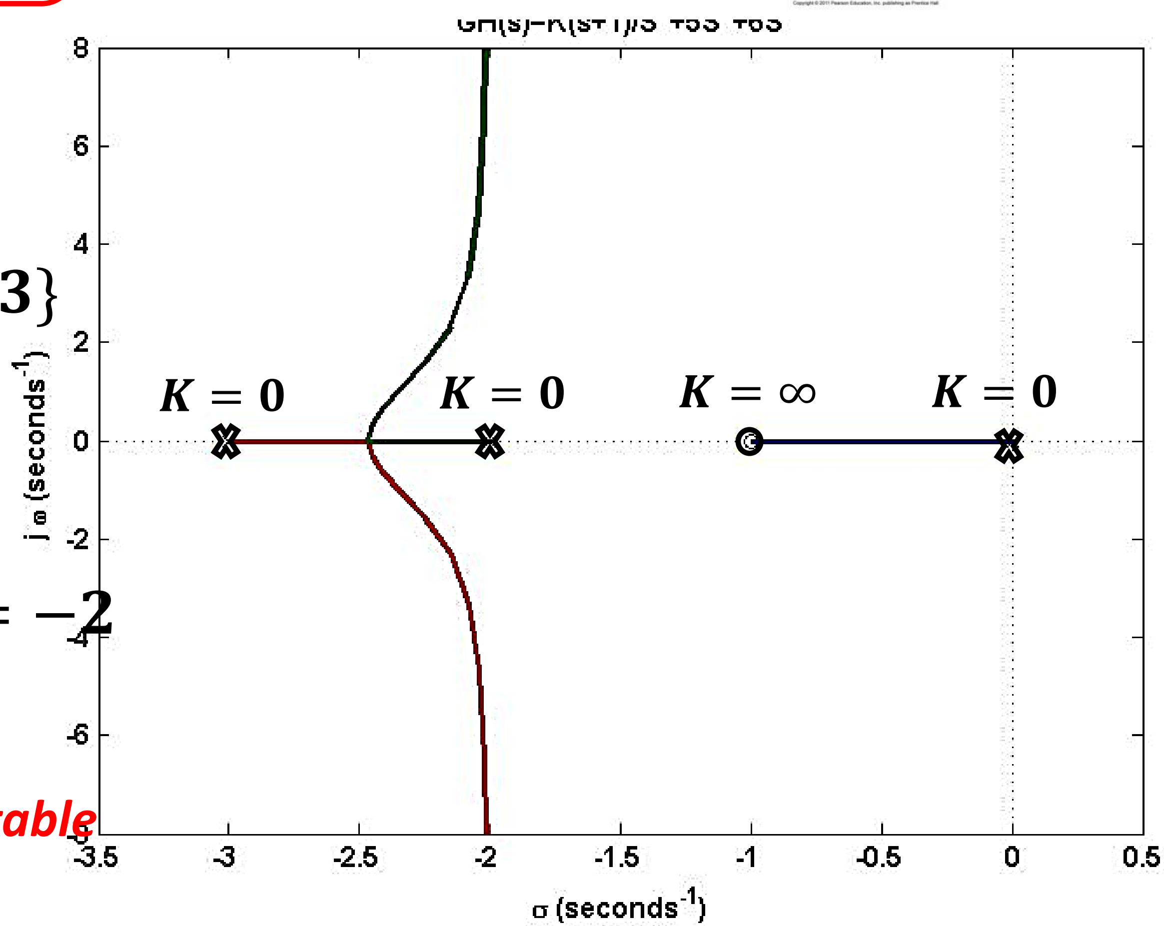
$$\boxed{1+\frac{K(s+1)}{s(s+2)(s+3)}=0}$$

$$1+K\frac{Q(s)}{P(S)}=0$$

- **Poles:** $s=\{0,-2,-3\}$
- **Zeros:** $s=-1$

- $\sigma_A=\dfrac{(-3-2)-(-1)}{2}=-2$

- ***The system is always stable***



- ***example***:

- Consider the characteristic equation

$$\boxed{\begin{aligned}&1+G(s)H(s)=0\\&1+\frac{s^2+(3+2K)s+5}{s(s+1)(s+2)}=0\end{aligned}}$$

- **<u>Rearrange</u>** the characteristic equation as $\boxed{P(s)+KQ(s)=0}$

$$s(s+1)(s+2)+s^2+(3+2K)s+5=0$$

- Rearranging

$$\boxed{1+\frac{2Ks}{s(s+1)(s+2)+s^2+3s+5}=1+K\frac{Q(s)}{P(s)}=0}$$

$$Q(s)=2s$$

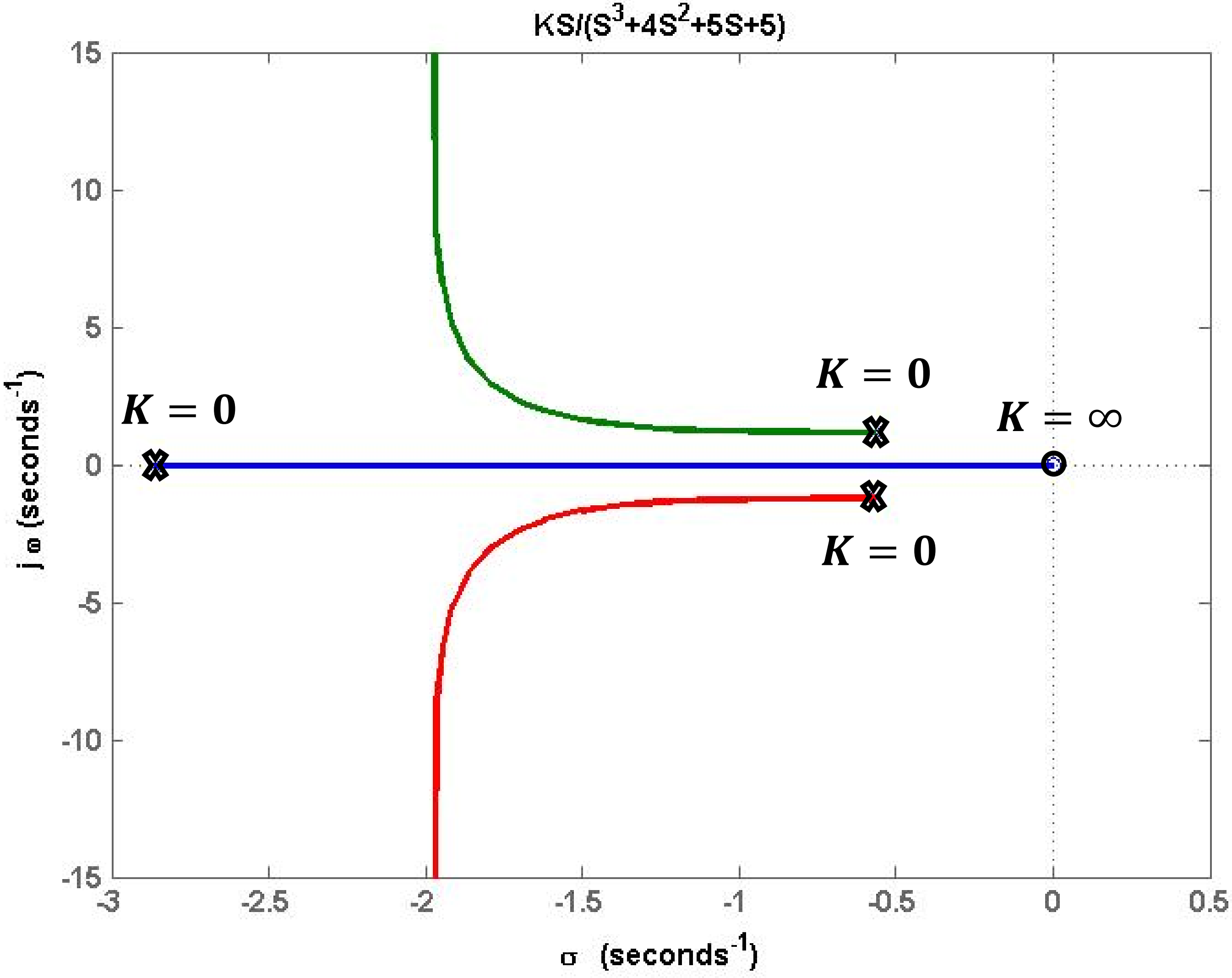
$$\Rightarrow \boxed{zeros: 0}$$

$$P(s)=s^3+4s^2+5s+5$$

$$\Rightarrow \boxed{poles: -2.8637 \qquad -0.5681 \pm 1.1930i}$$

- $$\boxed{K\frac{Q(s)}{P(s)} = \frac{2Ks}{s^3+4s^2+5s+5}}$$

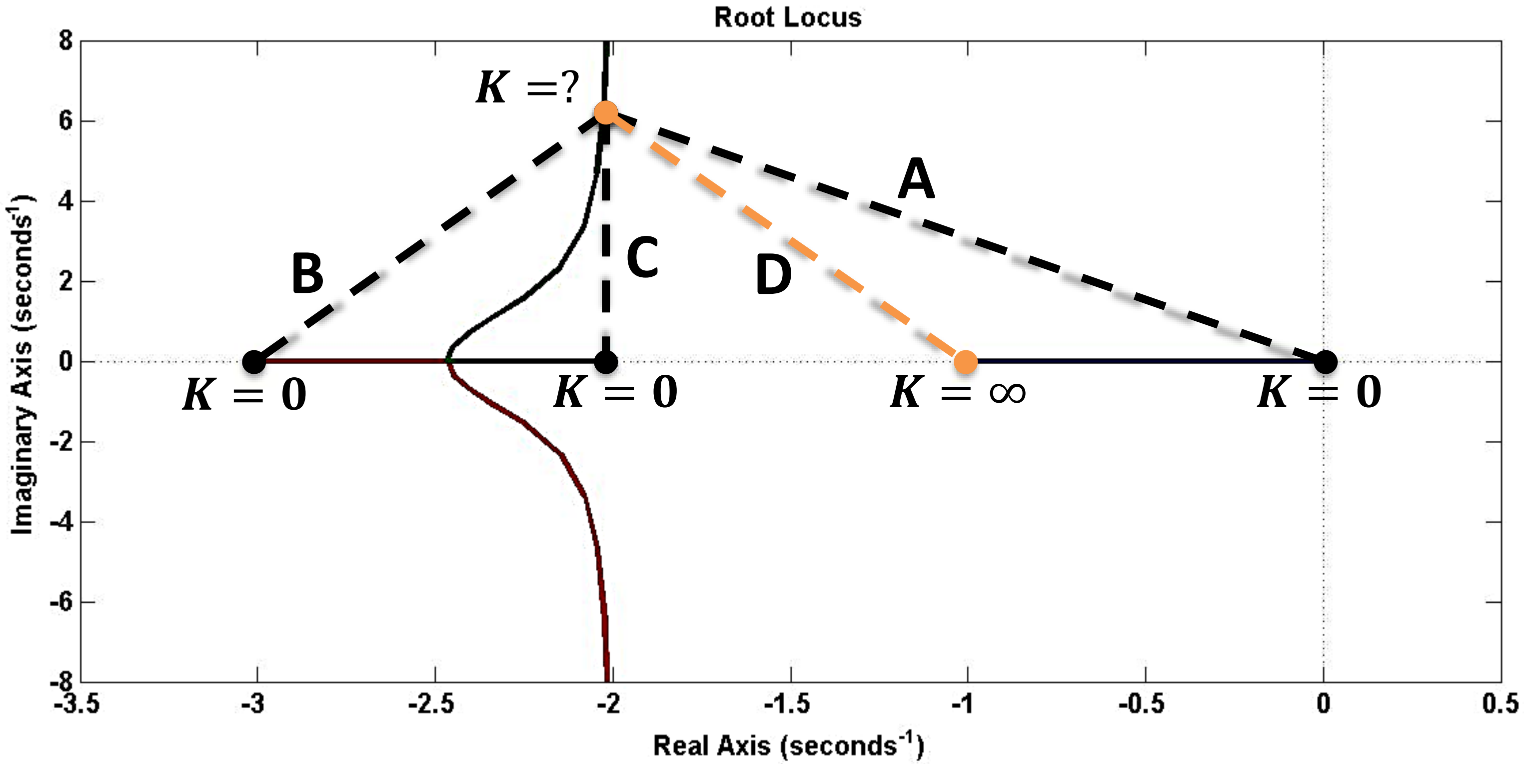
$$poles: -2.8637 \qquad -0.5681 \pm 1.1930i \qquad zeros: 0$$



- **Rule:** ***The gain value, K, at any point on the root locus can be evaluated as the ratio of the product of the lengths of the point from the poles to the lengths from the finite zeros,*** ***divided by the drawing scale***

$$K = \frac{ABC}{D}$$

- **if you use the ruler, don't forget to divide by the (drawing scale)$^{n-m}$**



- ***example:***

$$G(s)H(s) = \frac{K}{(s+1)(s+2)(s+3)}, \qquad K \geq 0$$

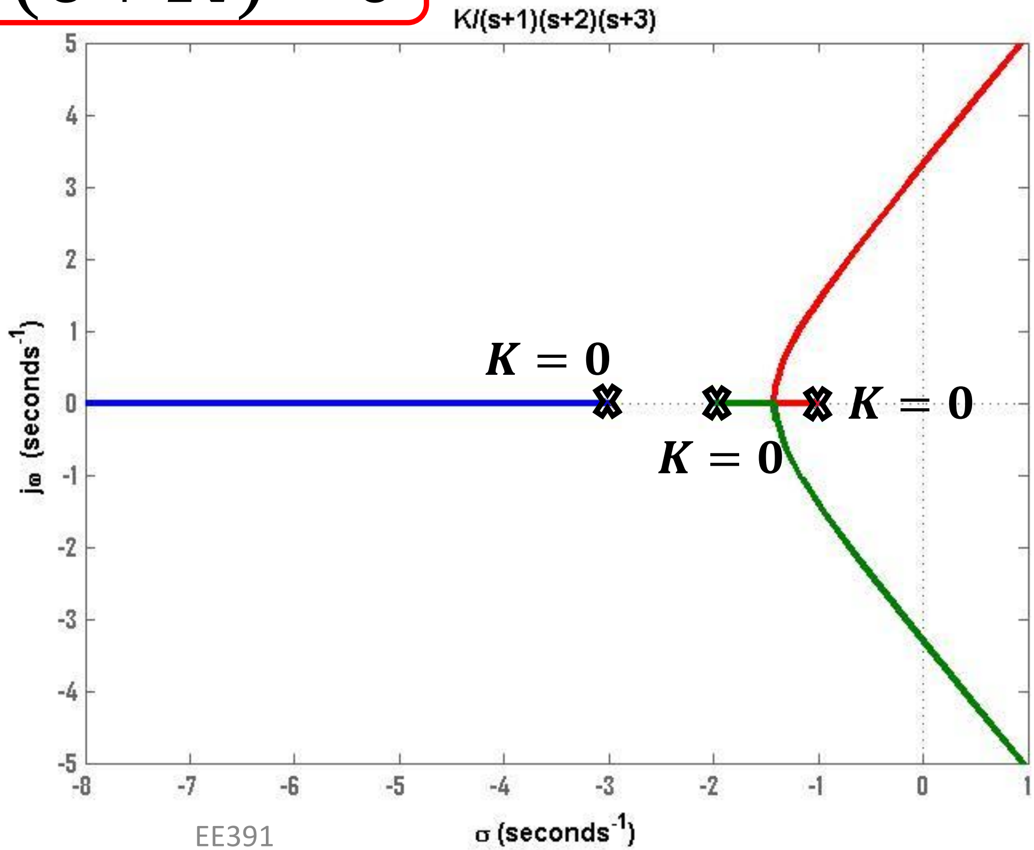
$$1 + G(s)H(s) = 0$$

$$\Rightarrow \boxed{s^3 + 6s^2 + 11s + (6+K) = 0}$$

$$K = 0$$

$$\Rightarrow \text{poles}$$

$$s = -1, -2, -3$$



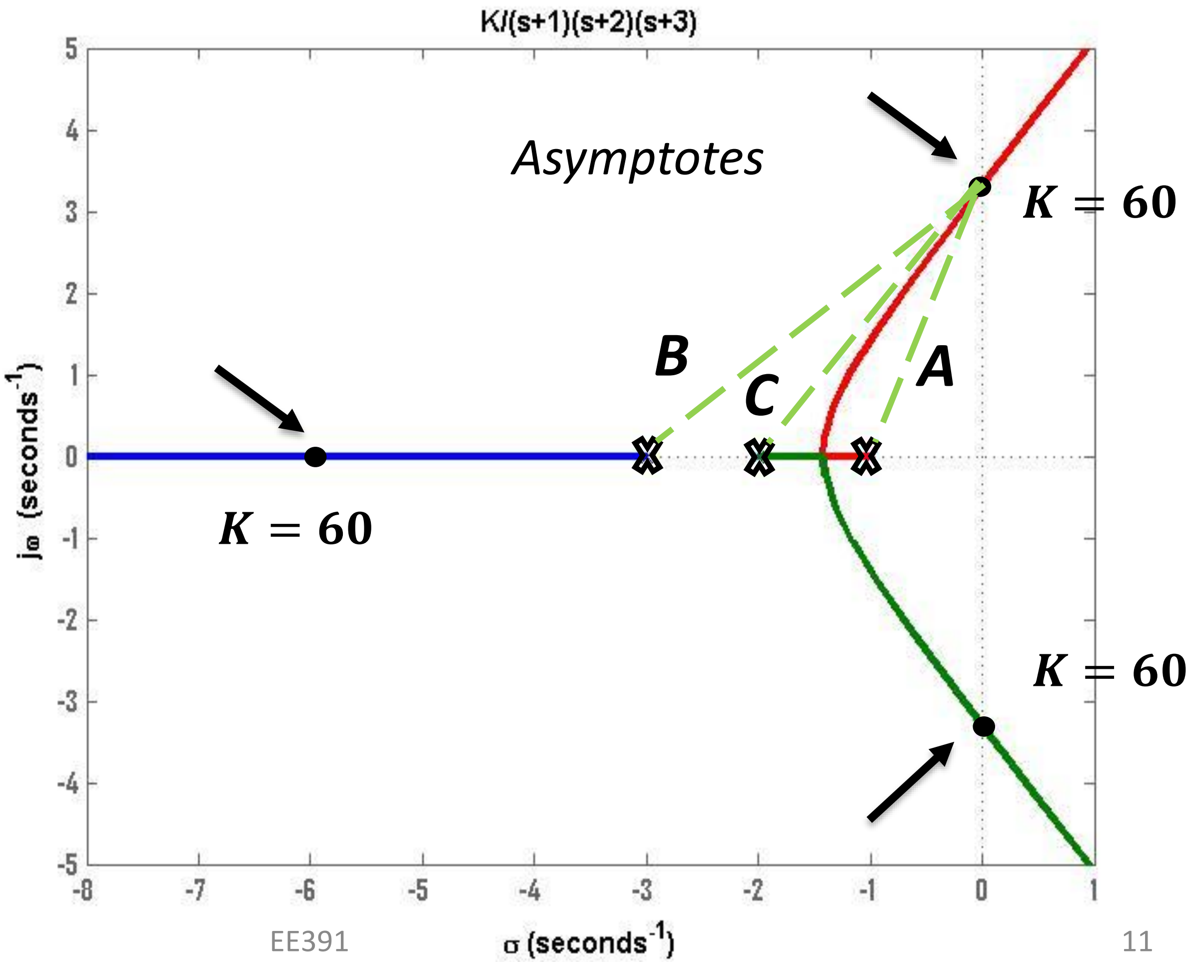
- three finite poles ($K = 0$), and three zeros at infinity ($K = \infty$)
- three branches and three asymptotes
- system is **<u>stable for $K < 60$,</u>** **(why??)**

- **Routh-Hurwitz**

OR

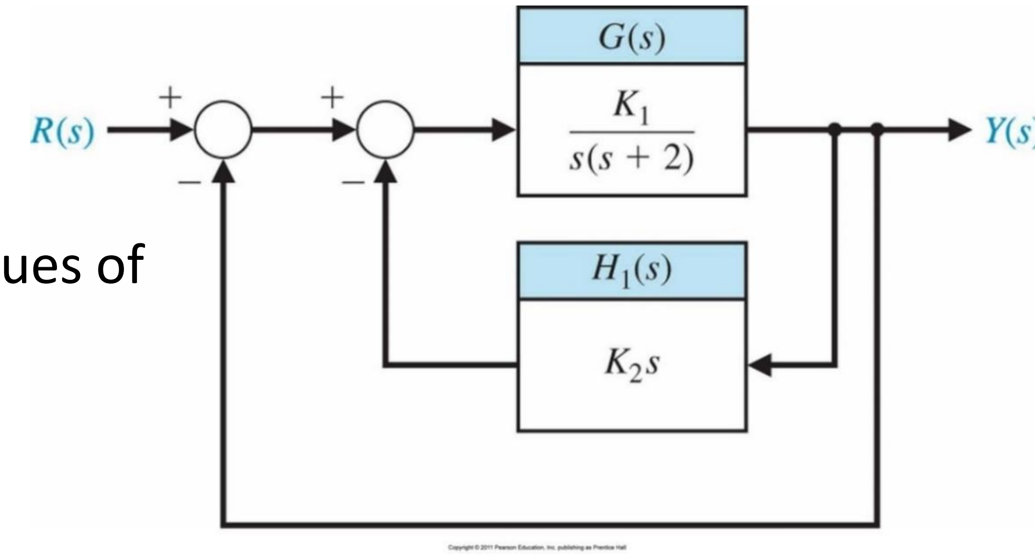
- **Graphical method**

- $\boxed{K = ABC}$



Root Contour

Multiple-Parameter Variations



- We need to determine the values of $\boldsymbol{K_1}$& $\boldsymbol{K_2}$ that satisfies certain design specs (A&B)

- Use the principle of **superposition**

- Set $\boldsymbol{K_2 = 0}$, draw the root locus with **parameter** $\boldsymbol{K_1}$. Determine the value of $\boldsymbol{K_1 = K_1^*}$ that satisfies condition A.

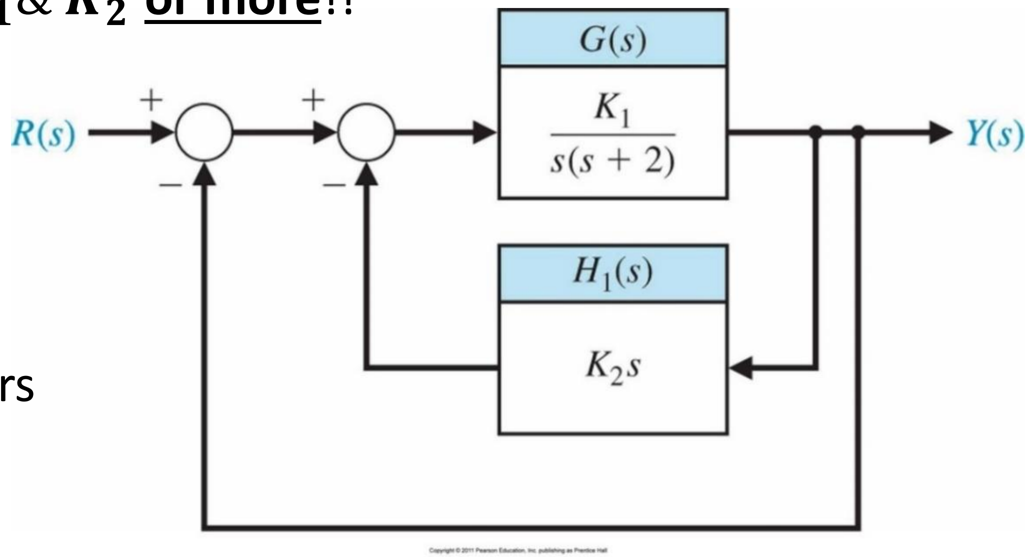
- Set $\boldsymbol{K_1 = K_1^*}$, draw the root locus with **parameter** $\boldsymbol{K_2}$. Determine the value of $\boldsymbol{K_2 = K_2^*}$ that satisfies condition B.

Root Contour

Multiple-Parameter Variations

(Reading : section 7.4)

- We need to study the locus of the roots of the characteristic equation as a function of **two** parameters $\boldsymbol{K_1}$ & $\boldsymbol{K_2}$ **or more**!!



- The characteristic equation

of the system with two parameters $\boldsymbol{K_1}$ & $\boldsymbol{K_2}$ can be written as

$$\boxed{P(s) + K_1 Q_1(s) + K_2 Q_2(s) = 0}$$

$$0 \le K_1 \le \infty,\ 0 \le K_2 \le \infty$$

- The characteristic equation with K_1& K_2 variables

$$\boxed{P(s)+K_1Q_1(s)+K_2Q_2(s)=0}$$

- since the system is **linear**, we can use **superposition,** set $K_2 = 0$

$$\boxed{0 \le K_1 \le \infty,\ \ K_2 = 0}$$

- the characteristic equation due to $\boldsymbol{K_1}$ <u>only</u> defines an ordinary root locus $\boldsymbol{RL_1}$ for $\boldsymbol{K_1}$

$$\boxed{P(s)+K_1Q_1(s)=0}$$

- rearranging

$$1+K_1\frac{Q_1(s)}{P(s)}=0$$

$$G_1(s)H_1(s)=K_1\frac{Q_1(s)}{P(s)}$$

- Consider the **effect** of $\boldsymbol{K_2}$

$$\boxed{\text{fix } K_1 = K_1^*, 0 \le K_2 \le \infty,}$$

- The characteristic equation with K_2 as a variable

$$\boxed{P(s) + K_1^* Q_1(s) + K_2 Q_2(s) = 0}$$

- rearranging in root locus format $\boldsymbol{RL_2}$

$$1 + \frac{K_2 Q_2(s)}{P(s) + K_1^* Q_1(s)} = 0$$

$\boldsymbol{RL_1}$ **at** $\boldsymbol{K_1 = K_1^*}$

$$G_2(s)H_2(s) = \frac{K_2 Q_2(s)}{P(s) + K_1^* Q_1(s)}$$

- ***the open-loop poles of G₂(s)H₂(s) are located on the root locus of G₁(s)H₁(s) at*** $\boldsymbol{K_1 = K_1^*}$

- ***example***: consider the characteristic equation

$$\boxed{s(1+K_2 s)(s^2+2s+2)+K_1=0}$$

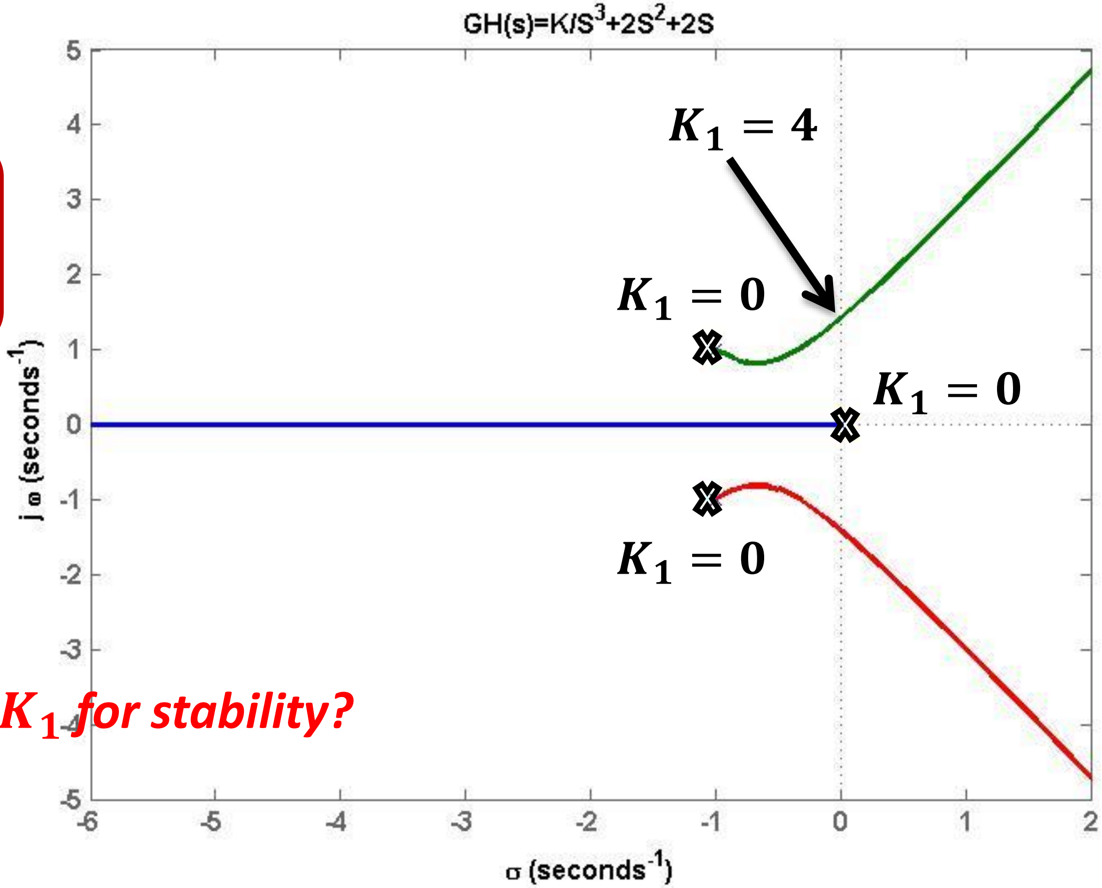
- Set $\boldsymbol{K_2=0}$, the characteristic equation becomes $s(s^2+2s+2)+K_1=0$
- Rearranging with $\boldsymbol{K_1}$ as a parameter, the characteristic equation becomes

$$1+K_1\frac{Q_1(s)}{P_1(s)}=0$$

$$\boxed{1+\frac{K_1}{s(s^2+2s+2)}=0}$$

Open-loop poles:

$\boldsymbol{s_1=0,\ s_{2,3}=-1\pm j}$

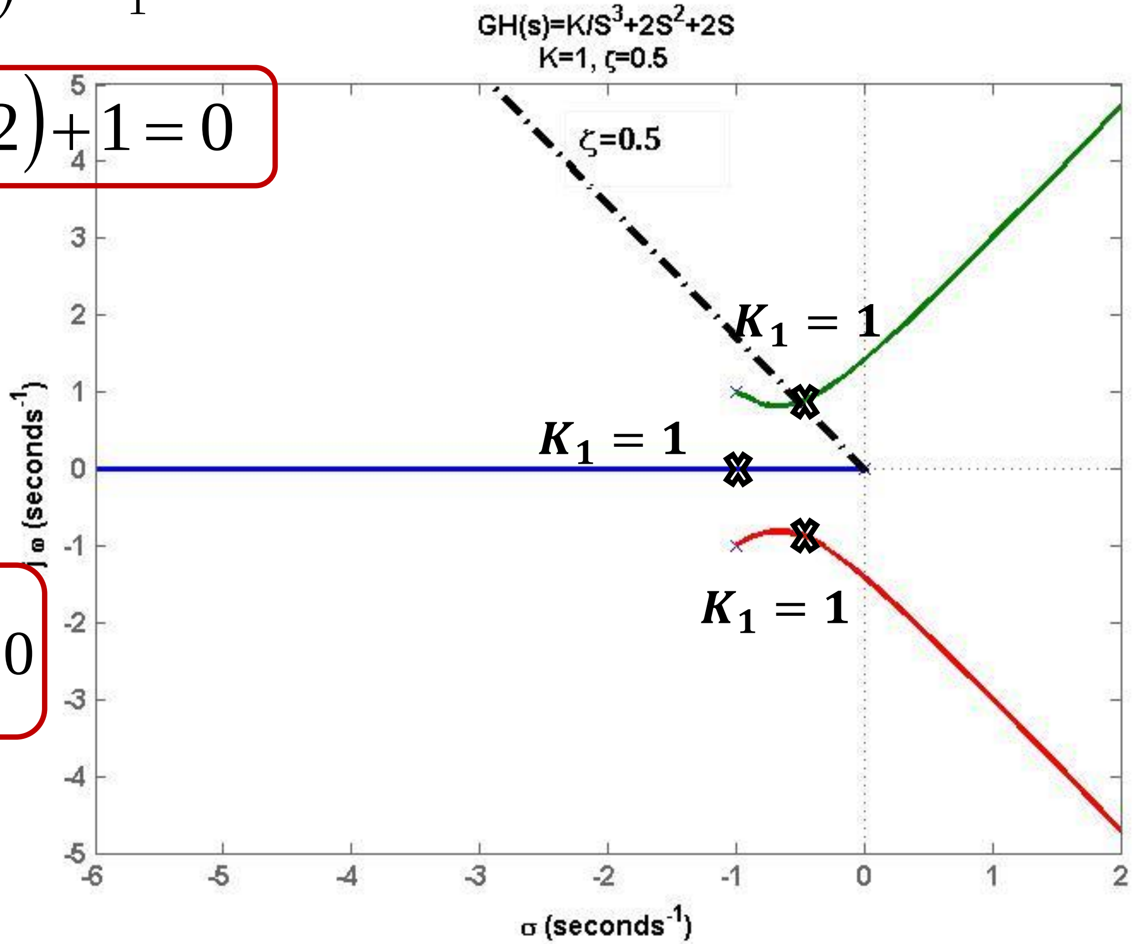


What is the critical value of K_1 for stability?

- Fix $\boldsymbol{K_1}$ that satisfies a certain design criterion. (e.g., steady state error or transient response)
- For example, let the damping ratio $\xi = 0.5 \Longrightarrow K_1 = K_1^* = 1$
- The characteristic equation with $\boldsymbol{K_1 = 1}$ becomes

$$s(1+K_2 s)(s^2+2s+2)+K_1=0$$

$$\boxed{s(1+K_2 s)(s^2+2s+2)+1=0}$$

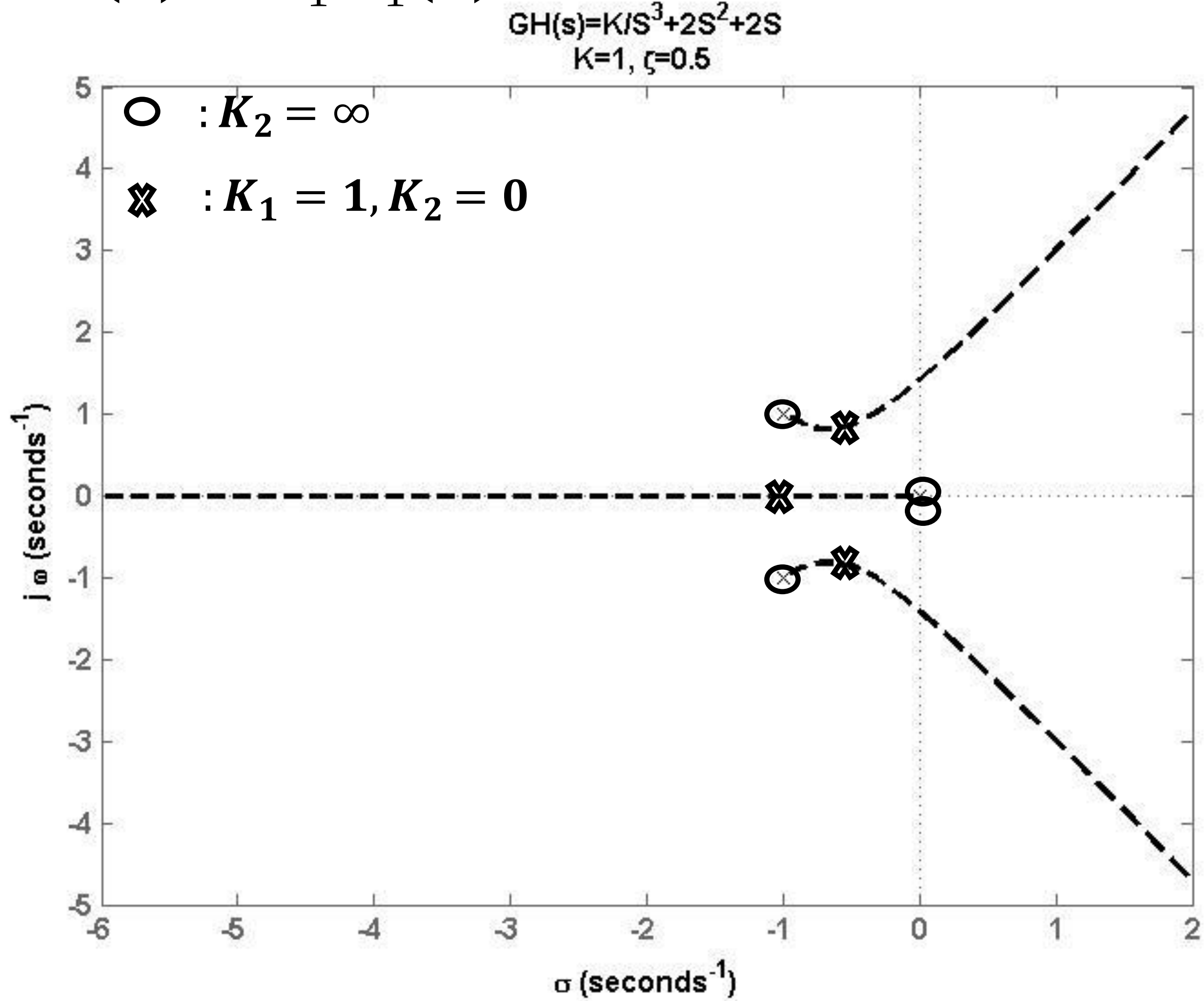


- Rearrange for $\boldsymbol{K_2}$

$$\boxed{1+\frac{K_2(s^4+2s^3+2s^2)}{s^3+2s^2+2s+1}=0}$$

- ***the Poles of $G_2(s)H_2(s)$ are located on the root locus of $G_1(s)H_1(s)$ at $K_1 = K_1^*$***
- The zeros of $G_2(s)H_2(s)$ *are the* ***zeros of*** $\boldsymbol{Q_2(s) = \{0, 0, -1 \mp j\}}$

$$G_2(s)H_2(s) = \frac{K_2 Q_2(s)}{P(s) + K_1^* Q_1(s)} = \frac{K_2\left(s^4 + 2s^3 + 2s^2\right)}{s^3 + 2s^2 + 2s + 1}$$

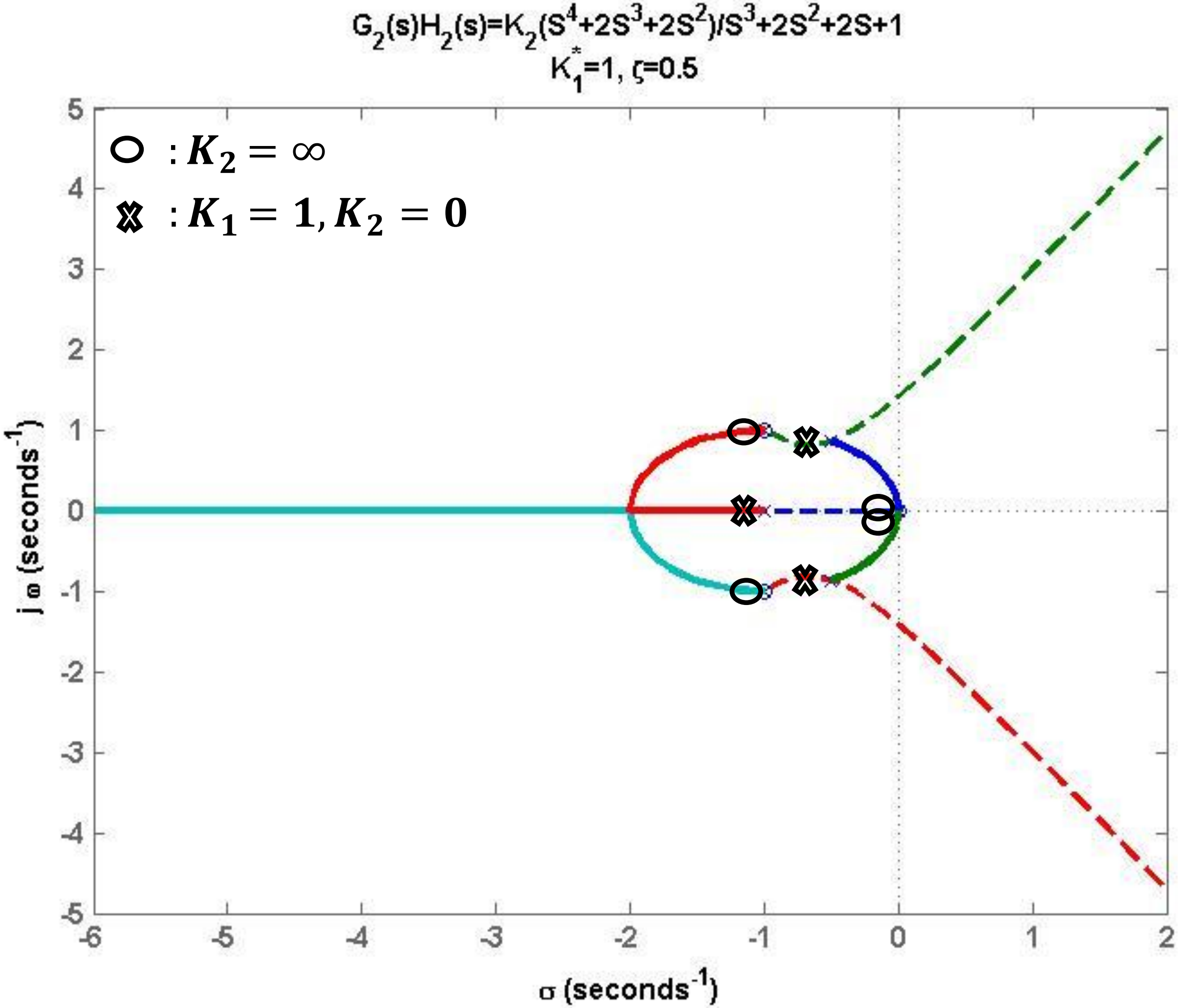


- Draw the root locus that corresponds to K_2

$$G_2(s)H_2(s) = \frac{K_2\left(s^4+2s^3+2s^2\right)}{s^3+2s^2+2s+1}$$

- The system is stable for all values of K_2

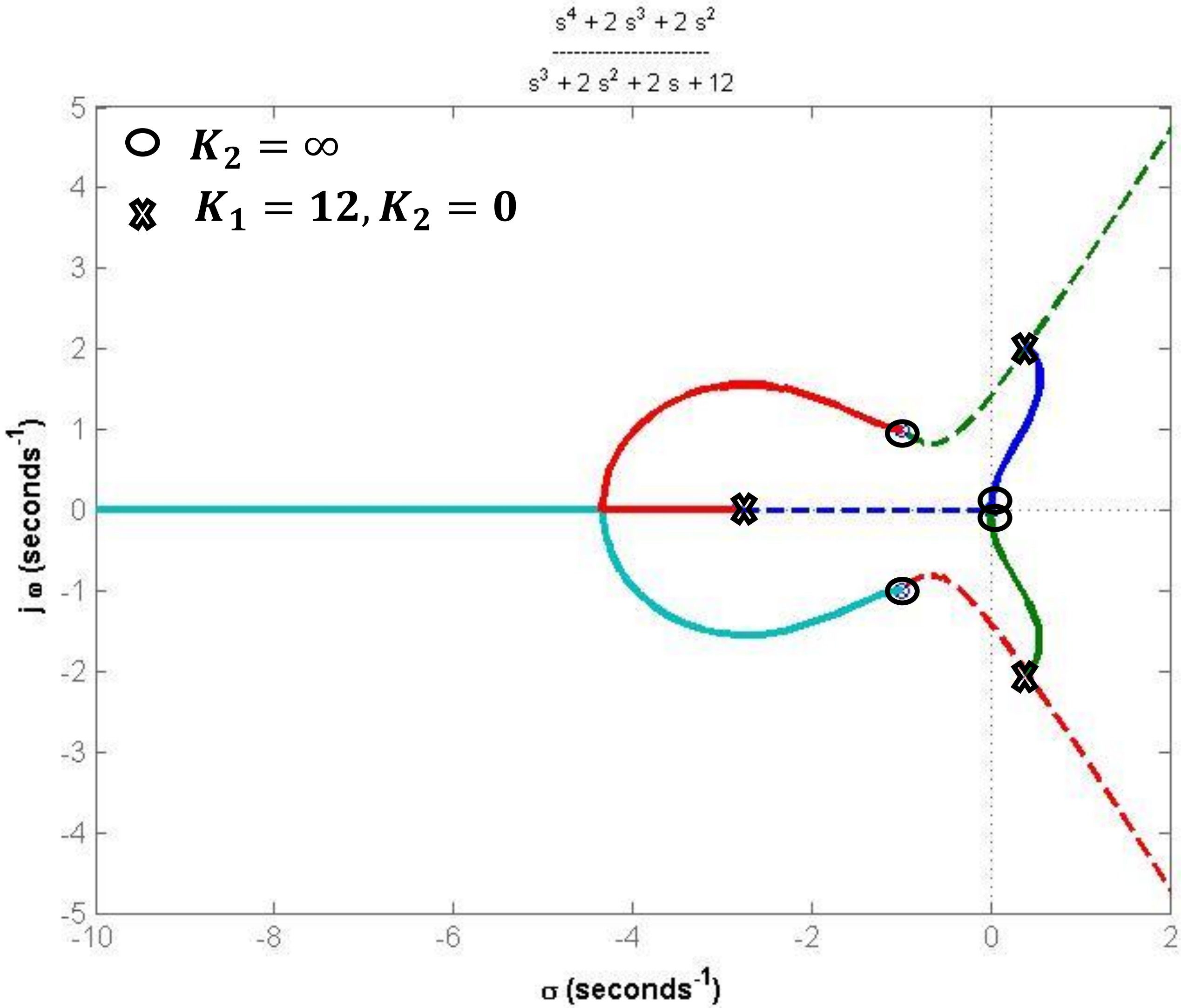
- Choose K_2 to satisfy a certain design criterion



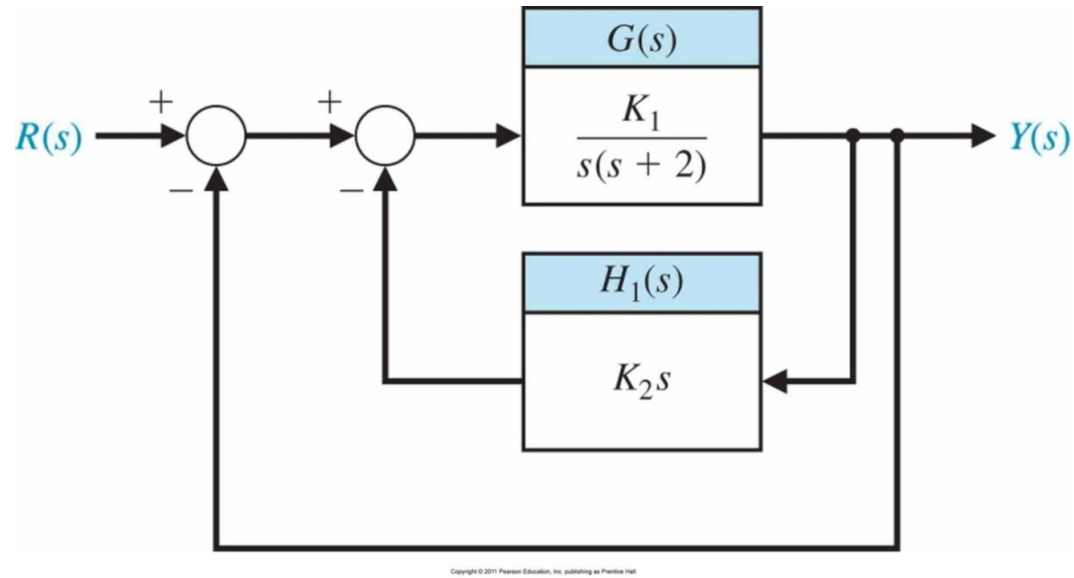
- **What if we chose $K_1 = 12 > 4$?**

$$G_2(s)H_2(s) = \frac{K_2\left(s^4 + 2s^3 + 2s^2\right)}{s^3 + 2s^2 + 2s + 12}$$

- *The system is **unstable** for all values of* K_2



- **example**



- determine the values of the amplifier gain $\boldsymbol{K_1}$ and the **derivative** feedback gain $\boldsymbol{K_2}$ such that:

1. *The natural frequency* $\omega_n = \sqrt{20}$
2. *Damping ratio of dominant roots* $\xi \geq 0.707$
3. 2% *settling time* $\leq 3\ sec$.

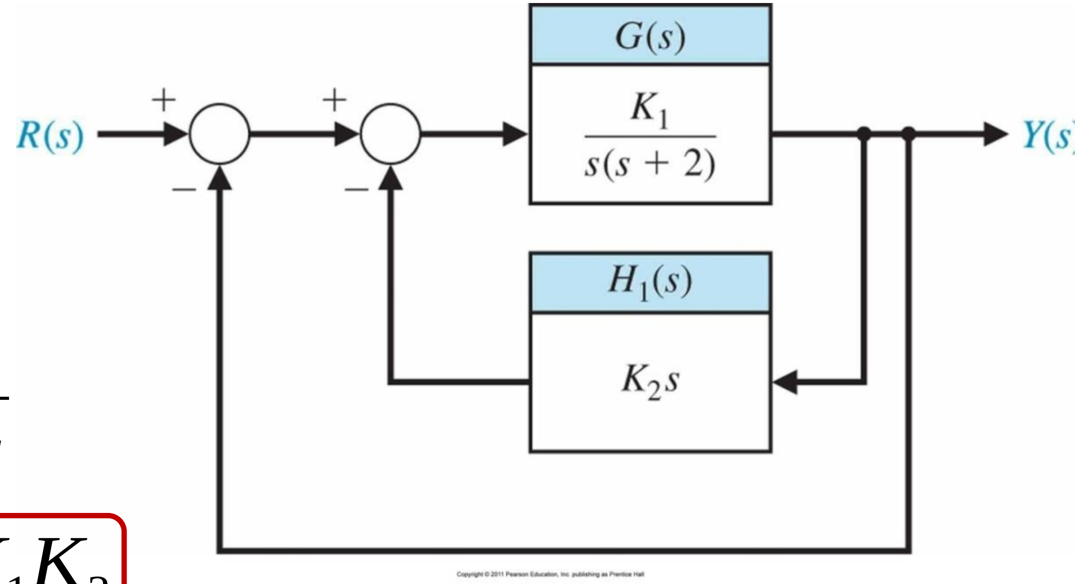
- The inner loop

$$\boxed{G_2(s) = \frac{G(s)}{1+G(s)H_1(s)}}$$

$$G_2(s) = \frac{K_1}{s(s+2)+K_1K_2s}$$

let $\boxed{\alpha = K_1 \quad \& \quad \beta = K_1K_2}$

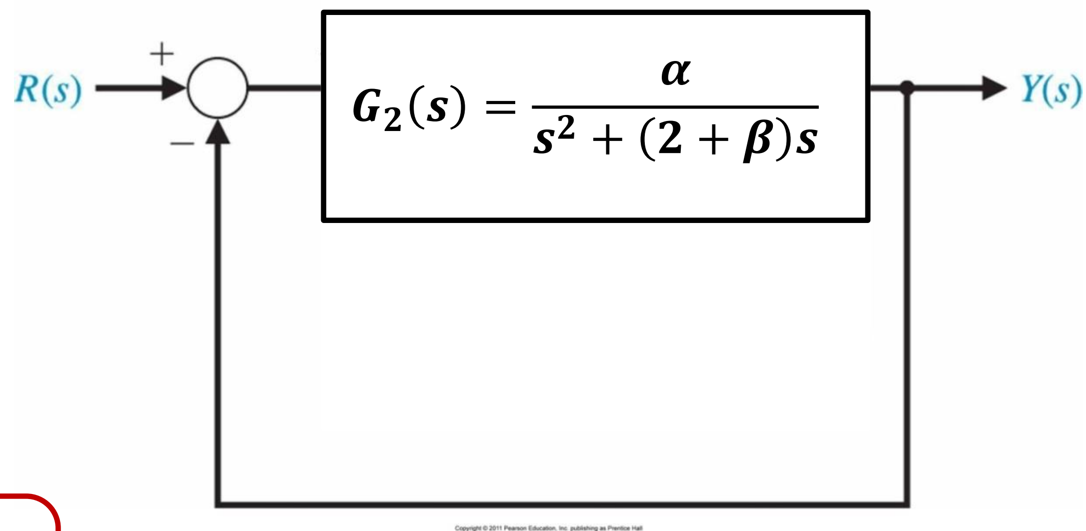
$$\boxed{G_2(s) = \frac{\alpha}{s^2+(2+\beta)s}}$$



- The transfer function

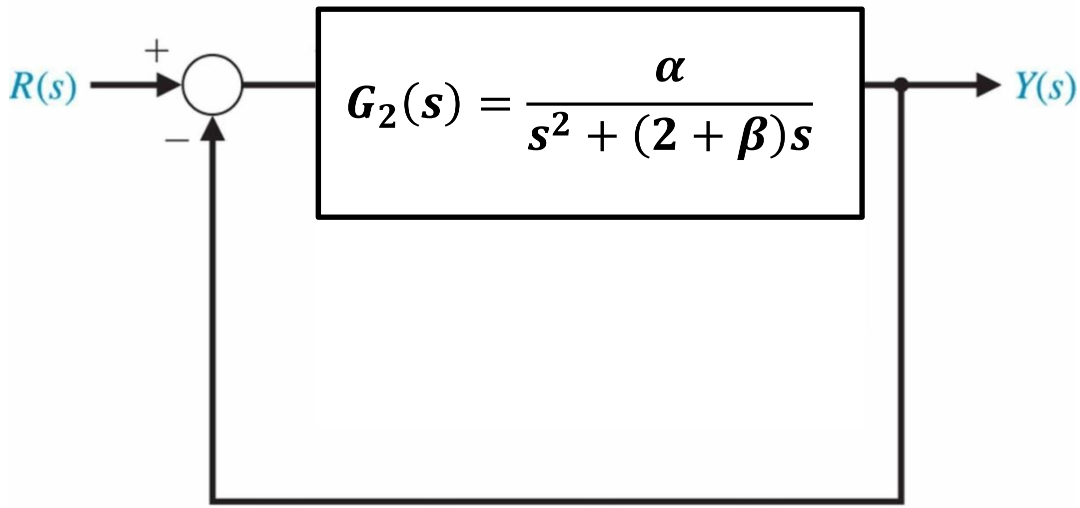
$$\frac{Y(s)}{R(s)} = \frac{G_2(s)}{1+G_2(s)}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{\alpha}{s^2+(2+\beta)s+\alpha}}$$



- The transfer function:

$$\boxed{\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{G_2(s)}{1+G_2(s)}\\ \frac{Y(s)}{R(s)} &= \frac{\alpha}{s^2+(2+\beta)s+\alpha}\end{aligned}}$$



- The characteristic equation

$$\boxed{s^2+(2+\beta)s+\alpha=0}$$

- $\boldsymbol{RL_1}$ for parameter $0 \le \alpha < \infty$ with $\beta = 0$

$$\boxed{1+\frac{\alpha}{s(s+2)}=0}$$

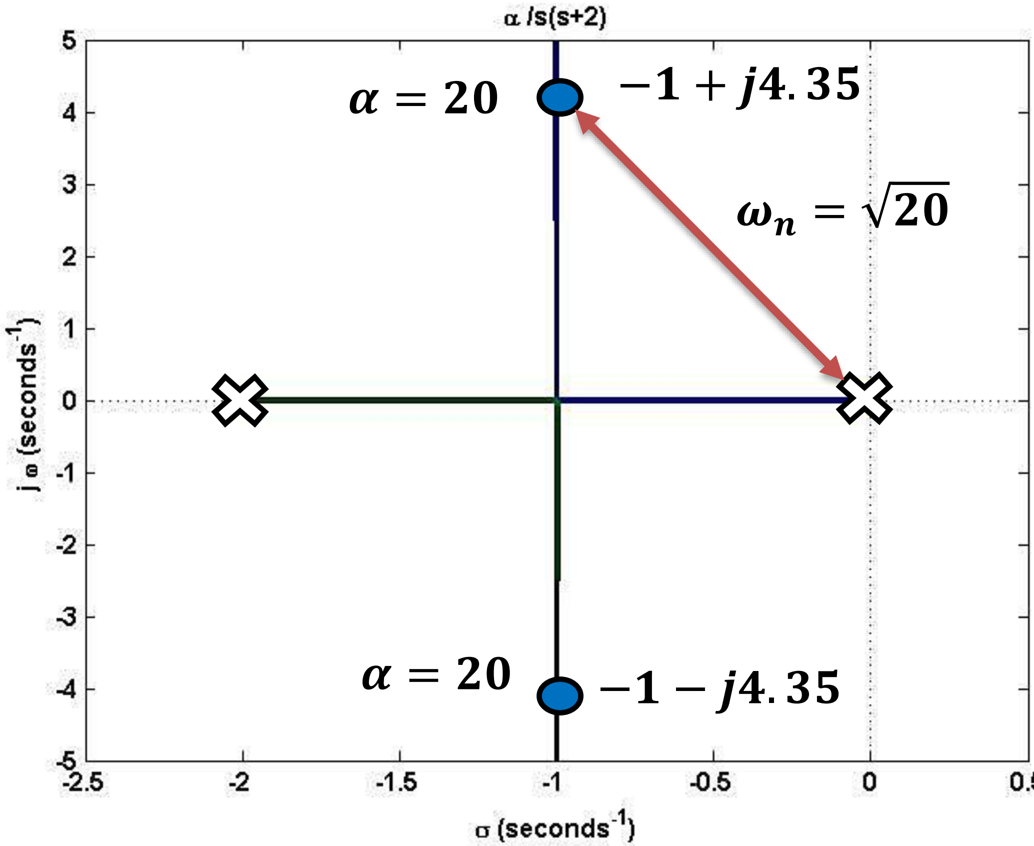
- $\boldsymbol{RL_2}$ for parameter β, with $\alpha = \alpha^*$ fixed

$$\boxed{1+\frac{\beta s}{s^2+2s+\alpha^*}=0}$$

$\boldsymbol{RL_1}$ with $\boldsymbol{\beta = 0}$ and parameter $\boldsymbol{0 \leq \alpha < \infty}$

- The characteristic equation with α is a parameter

$$\boxed{1 + \frac{\alpha}{s(s+2)} = 0}$$



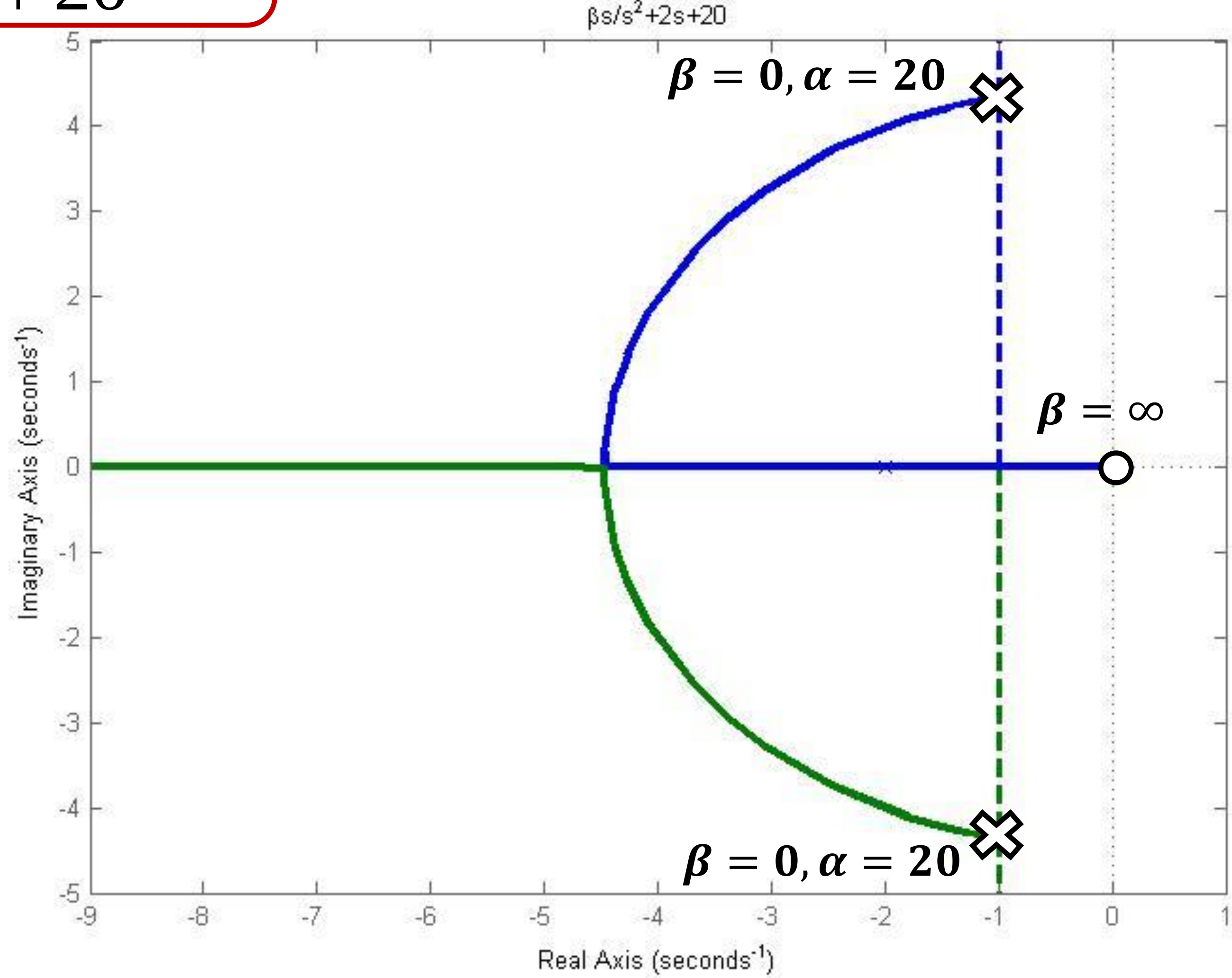
- Choose $\boldsymbol{\alpha}$ such that the natural frequency $\boldsymbol{\omega_n = \sqrt{20}}$

- Then $\boldsymbol{\alpha = \alpha^* = 20}$ **(why??)**

- The damping coefficient $\xi = 0.223 < 0.707$ **(why??)**

RL_2 with $0 \le \beta < \infty$ parameter and $\alpha = \alpha^* = 20$

$$1 + \frac{\beta s}{s^2 + 2s + 20} = 0$$



1. Damping ratio of dominant roots ≥ 0.707

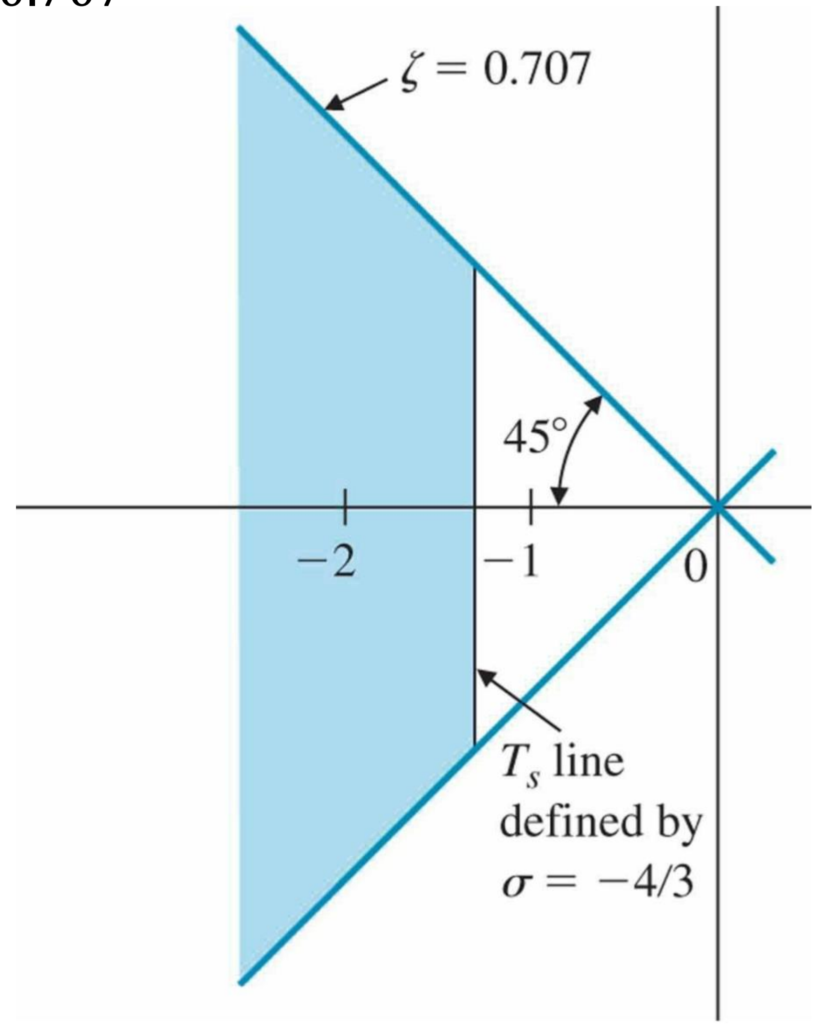
2. 2% settling time $\leq 3\ sec$.

$$T_s = \frac{4}{\sigma} \leq 3$$

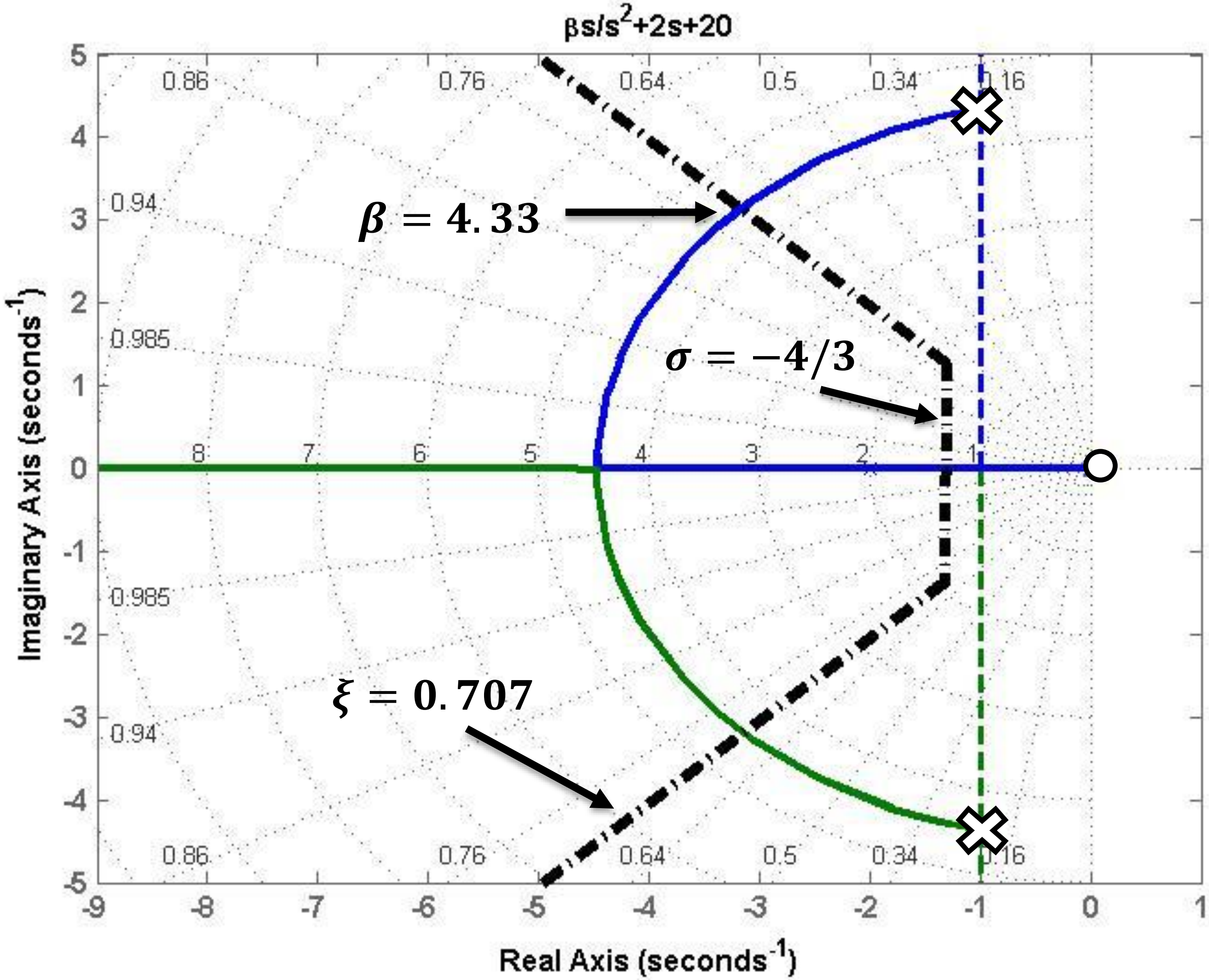
$$\Rightarrow \sigma \geq \frac{4}{3}$$

$$\Rightarrow \boxed{-\sigma \leq \frac{-4}{3}}$$

- all roots must lie within the shaded area of the left-half plane



- choose $\beta = K_1 K_2 = 4.3$ for $\xi = 0.707$



- $K_1 = \alpha = 20,$

- $K_2 = \beta / K_1 = 0.215$

- 2% settling time $= 4/3.15 = 1.27 < 3 sec$

- Natural frequency

$\omega_n = \sqrt{20}$