Automatic Control Systems

Lag-lead

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Frequency Domain Representations

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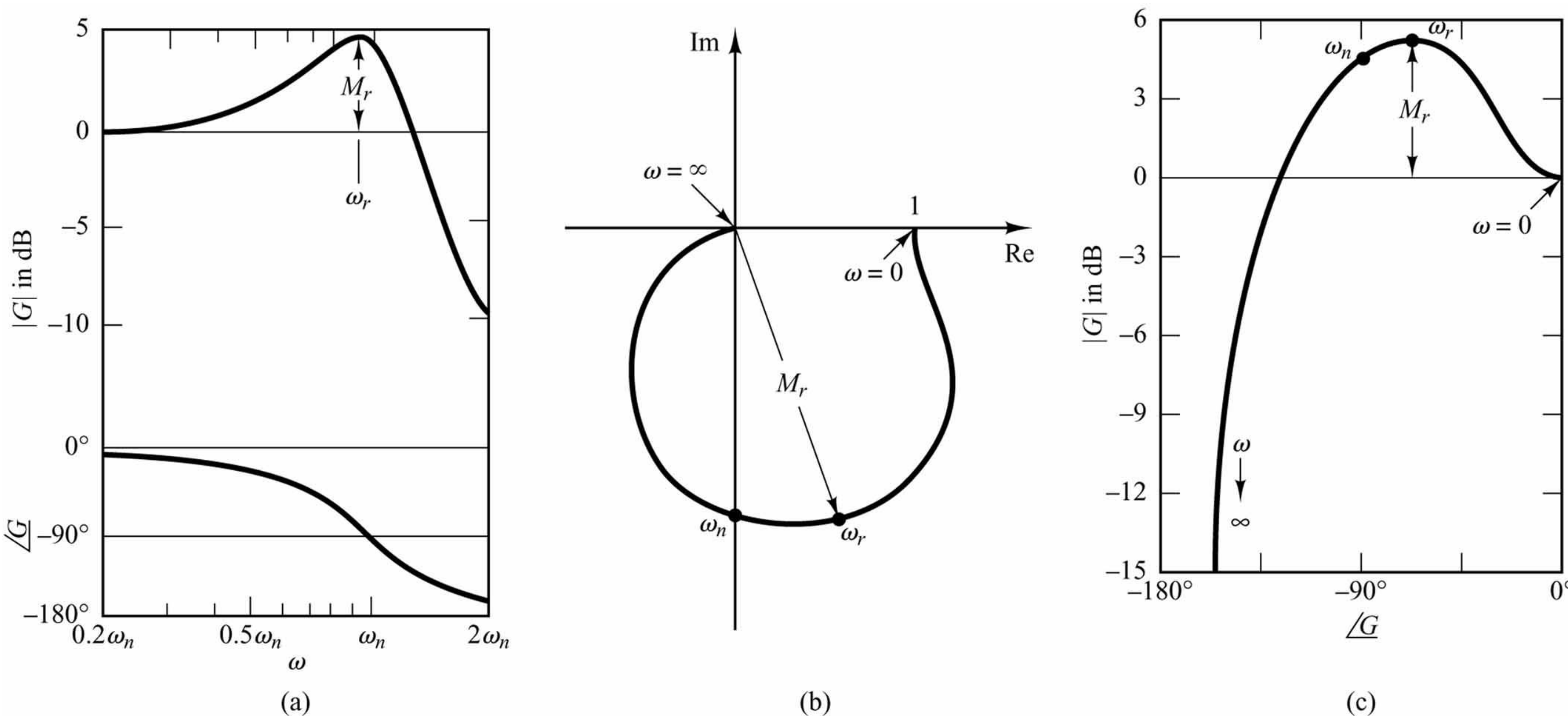
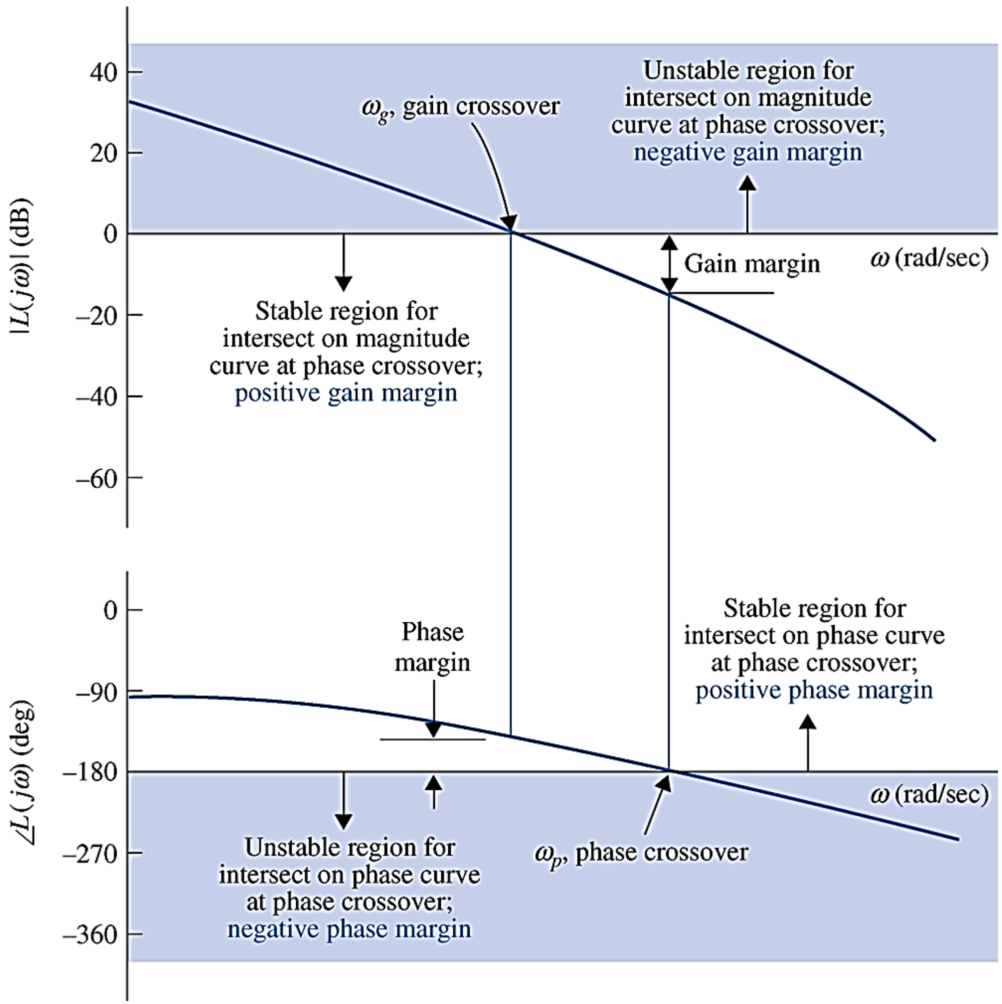


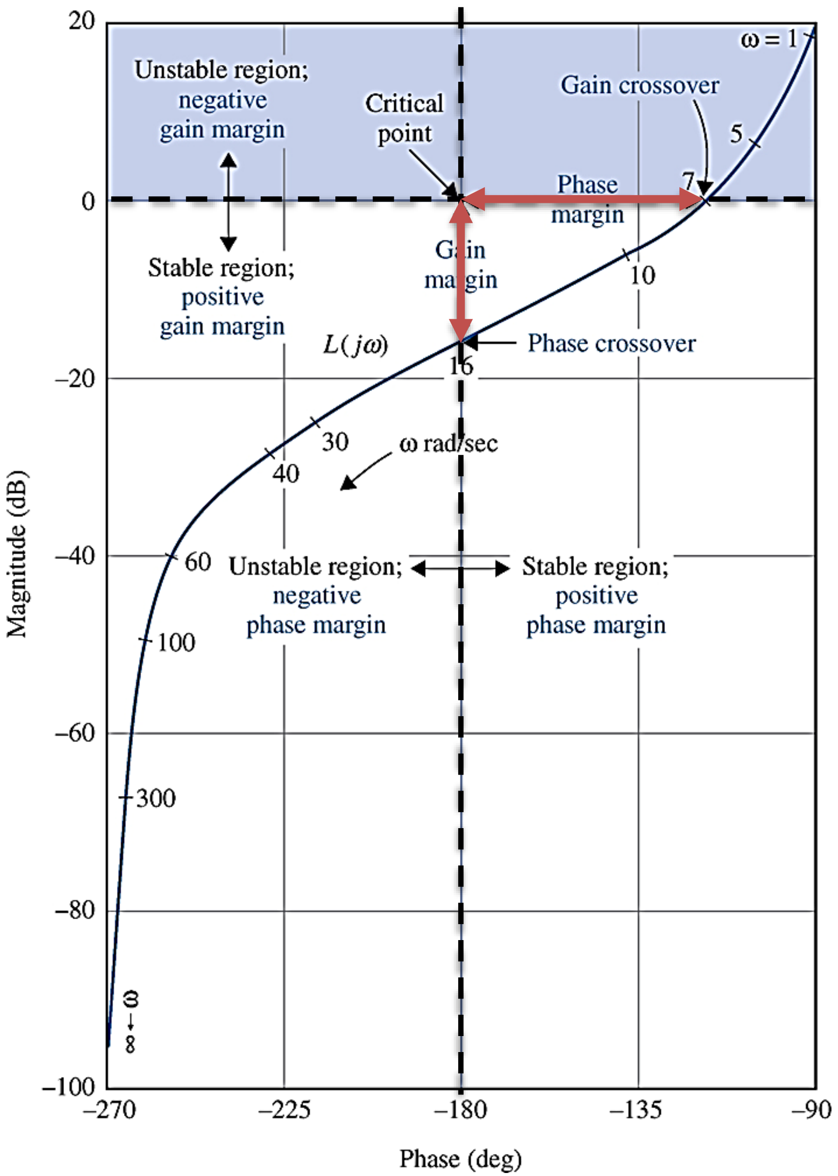
Logarithmic plots (Bode Plots)

Polar plots (Nyquist Diagrams)

Log Magnitude-Phase (Nichols Charts)

Stability analysis with the log-magnitude and phase diagram





Stability analysis with the log-magnitude and phase diagram

- This is **open-loop** representation

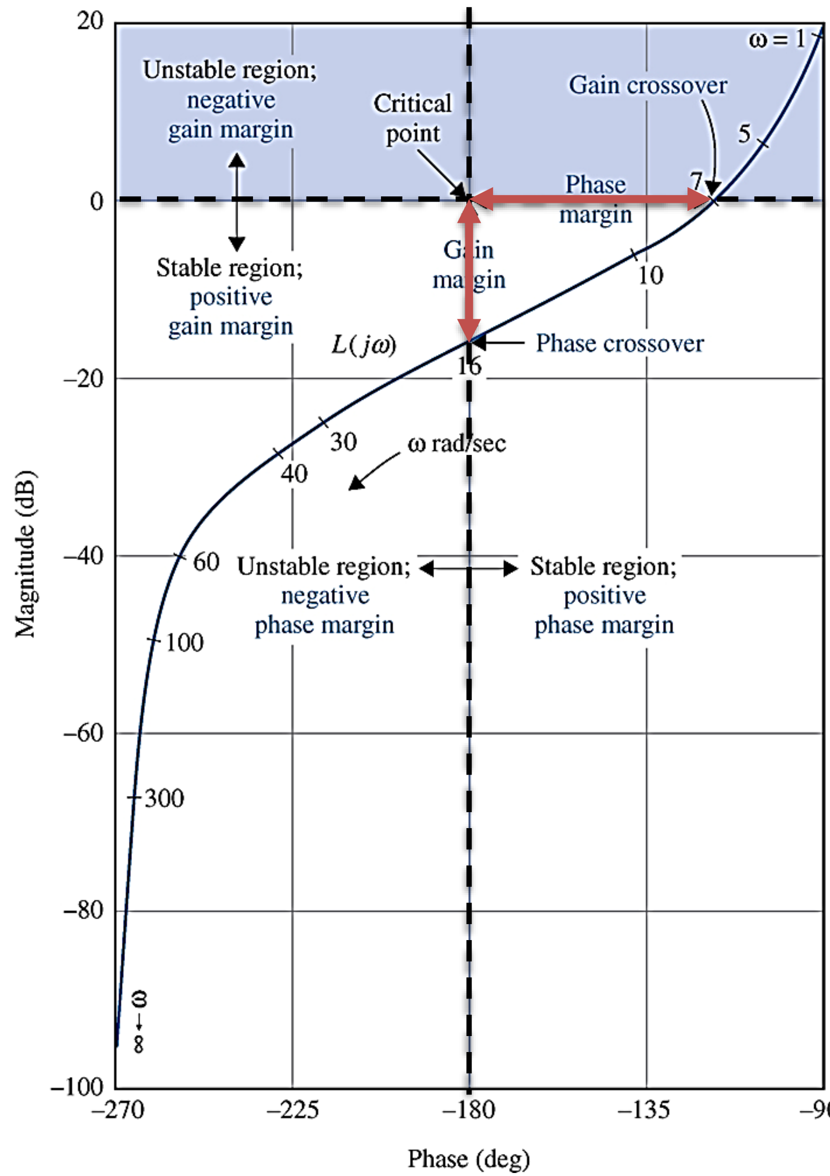
- The critical point at $(\mathbf{0}\ \boldsymbol{dB}, \mathbf{-180^o})$

The **upper-right** corner is the **low frequency**, the **lower-left** is the **high frequency**

- The system is stable if, in the direction of increasing frequency, the curve intersects the $0\ dB$ line before the -180^0 line

- Changing the gain $\boldsymbol{K}$ shifts the curve **vertically**

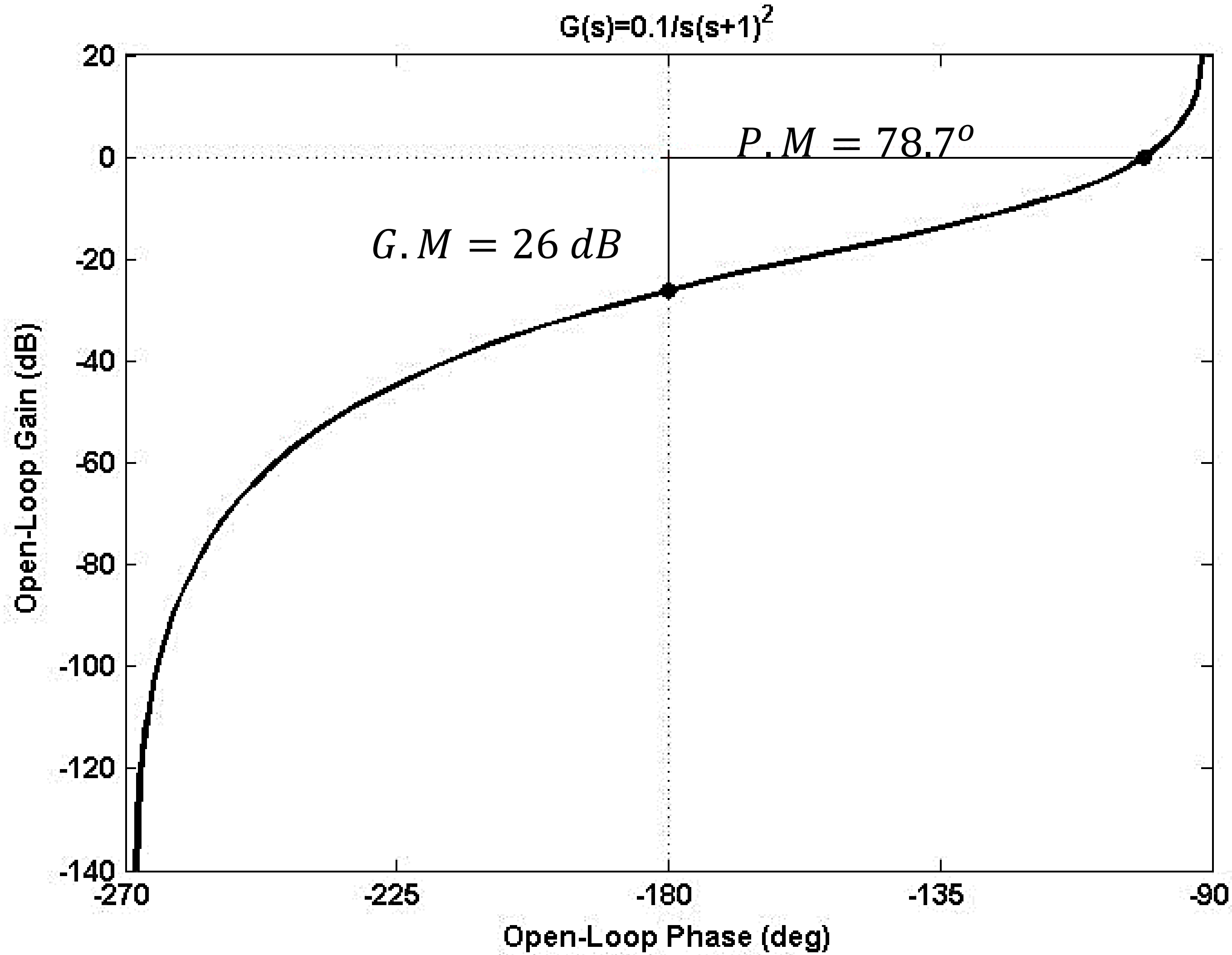
- Changing the **phase** $\boldsymbol{\phi}$ shifts the curve **horizontally**



- ***example***

$$G(s) = \frac{0.1}{s(s+1)^2}$$

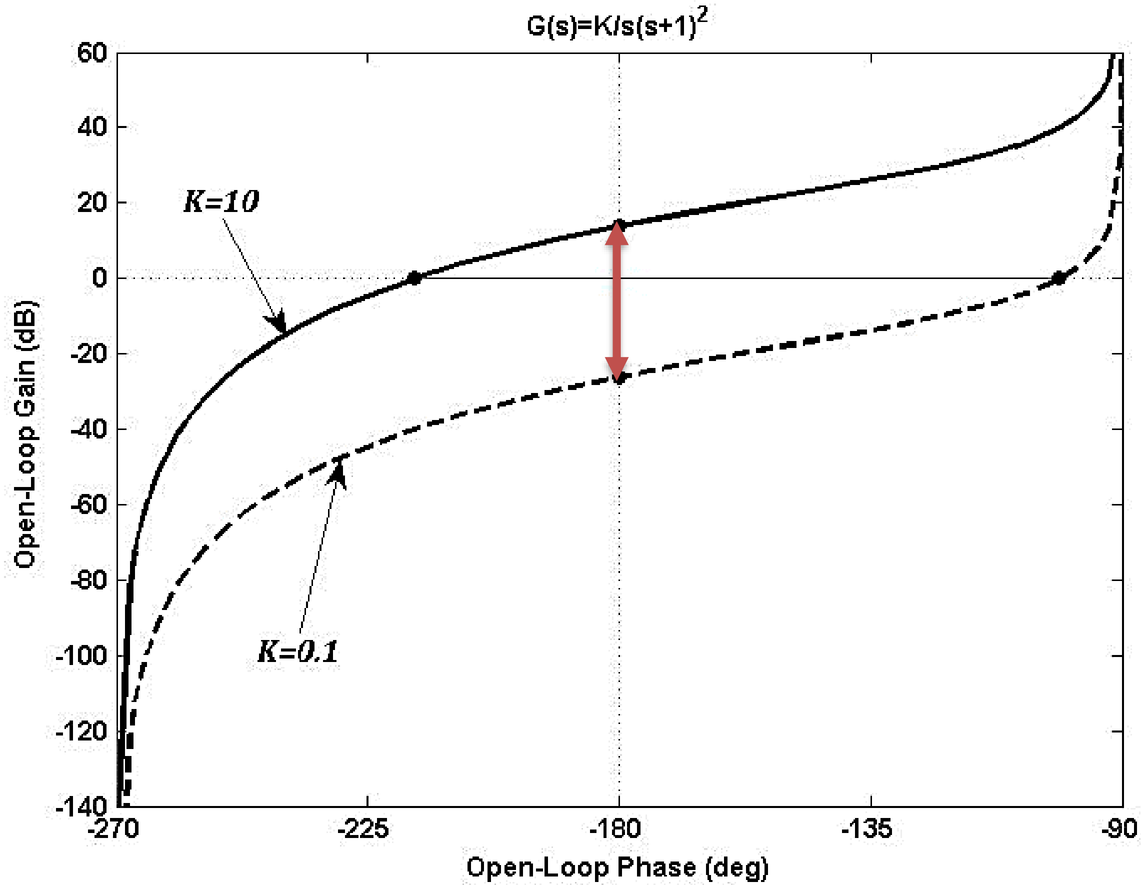
- The system is stable.

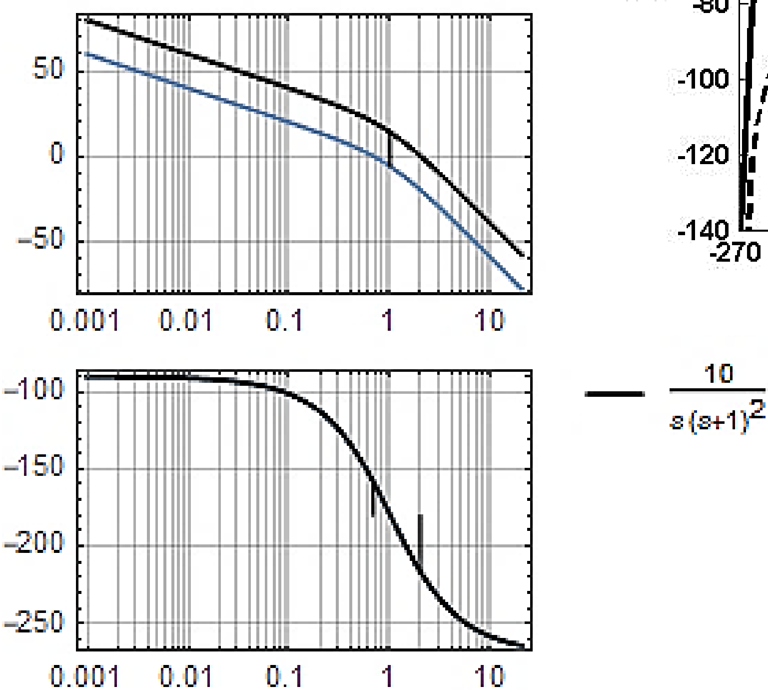


- Increasing the gain shifts the curve vertically

$$G(s) = \frac{K}{s(s+1)^2}$$

K	stability
$K = 0.1$	stable
$K = 10$	unstable





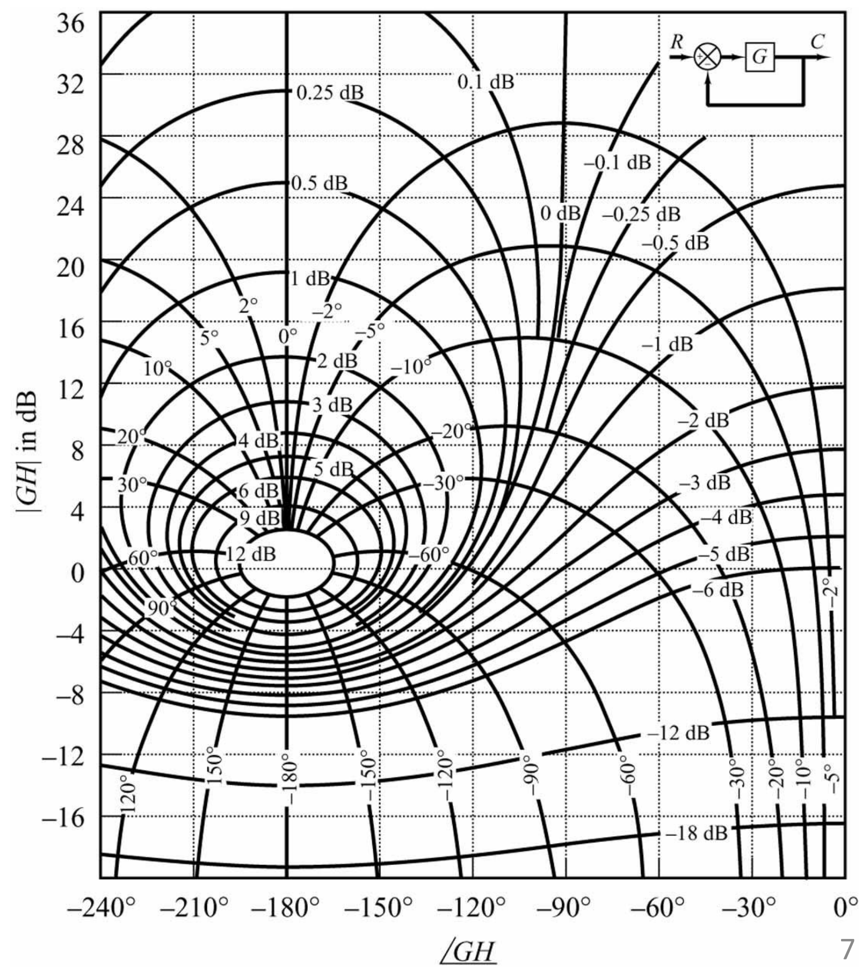
The Nichols chart

- Constant $\boldsymbol{M}$**-circles and** $\boldsymbol{N}$**-circles** in the **magnitude-phase plane**

- The intersects between the constant M-loci and the $G(j\omega)$ trajectory gives the value of **closed-loop** $|M(j\omega)|$

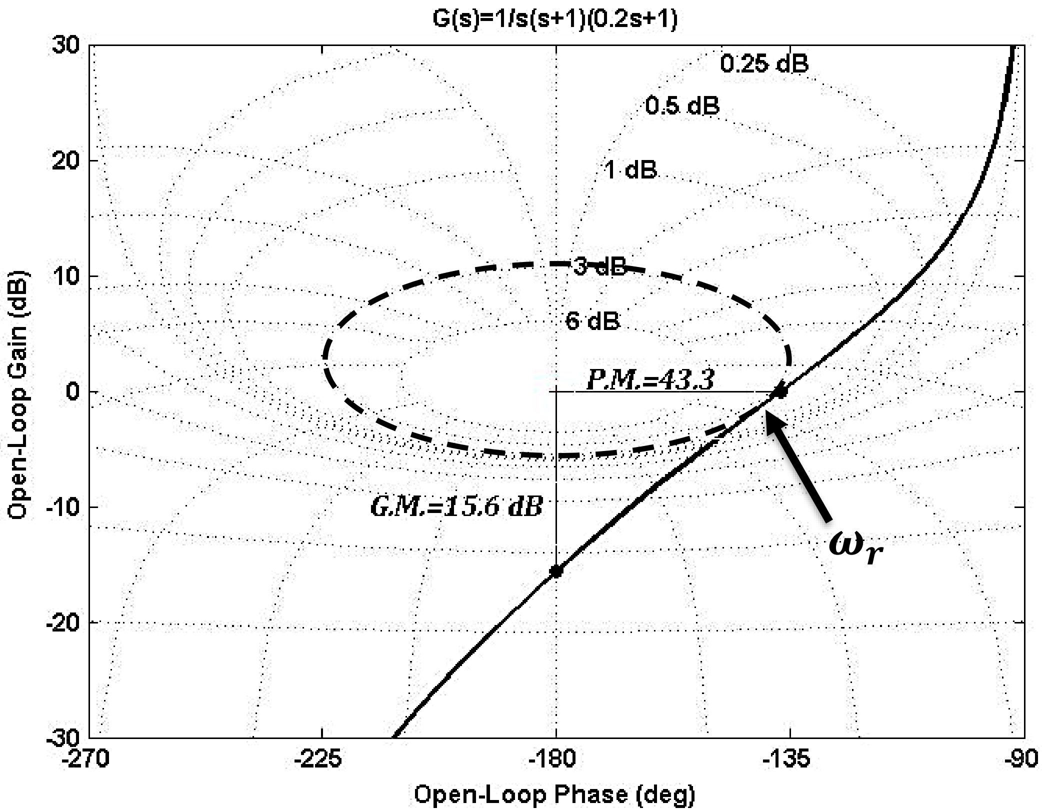
- The **resonance peak** M_r and the **resonant frequency** ω_r are found by locating the **smallest** $\boldsymbol{M}$**-locus that is tangent** to the $G(j\omega)$ trajectory

- The **bandwidth** is the frequency at which $G(j\omega)$ curve intersects $\boldsymbol{M = 0.707}$ **locus**

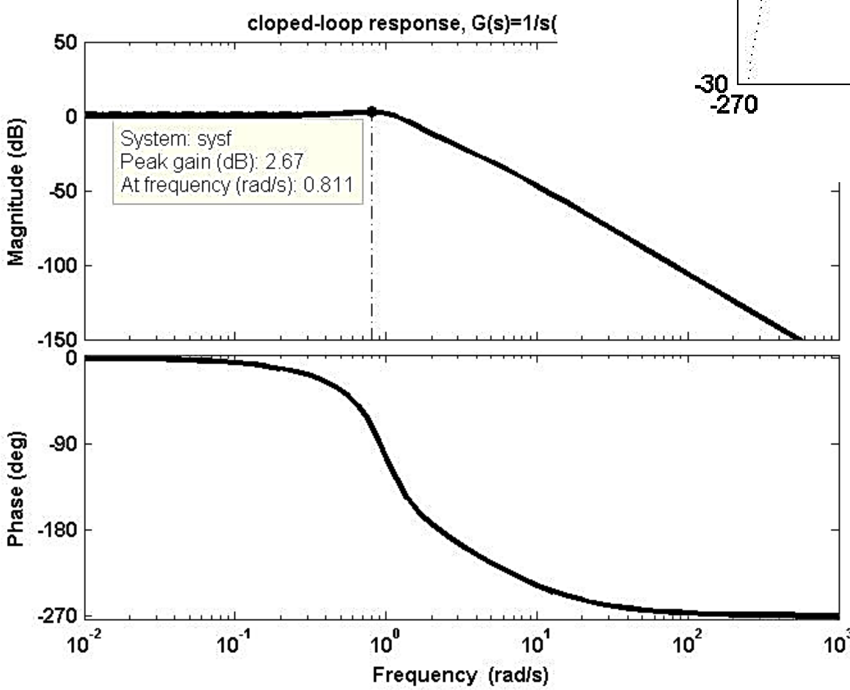


- ***example***

$$\boxed{G(s)=\frac{1}{s(s+1)(0.2s+1)}}$$



- **Closed-loop response**



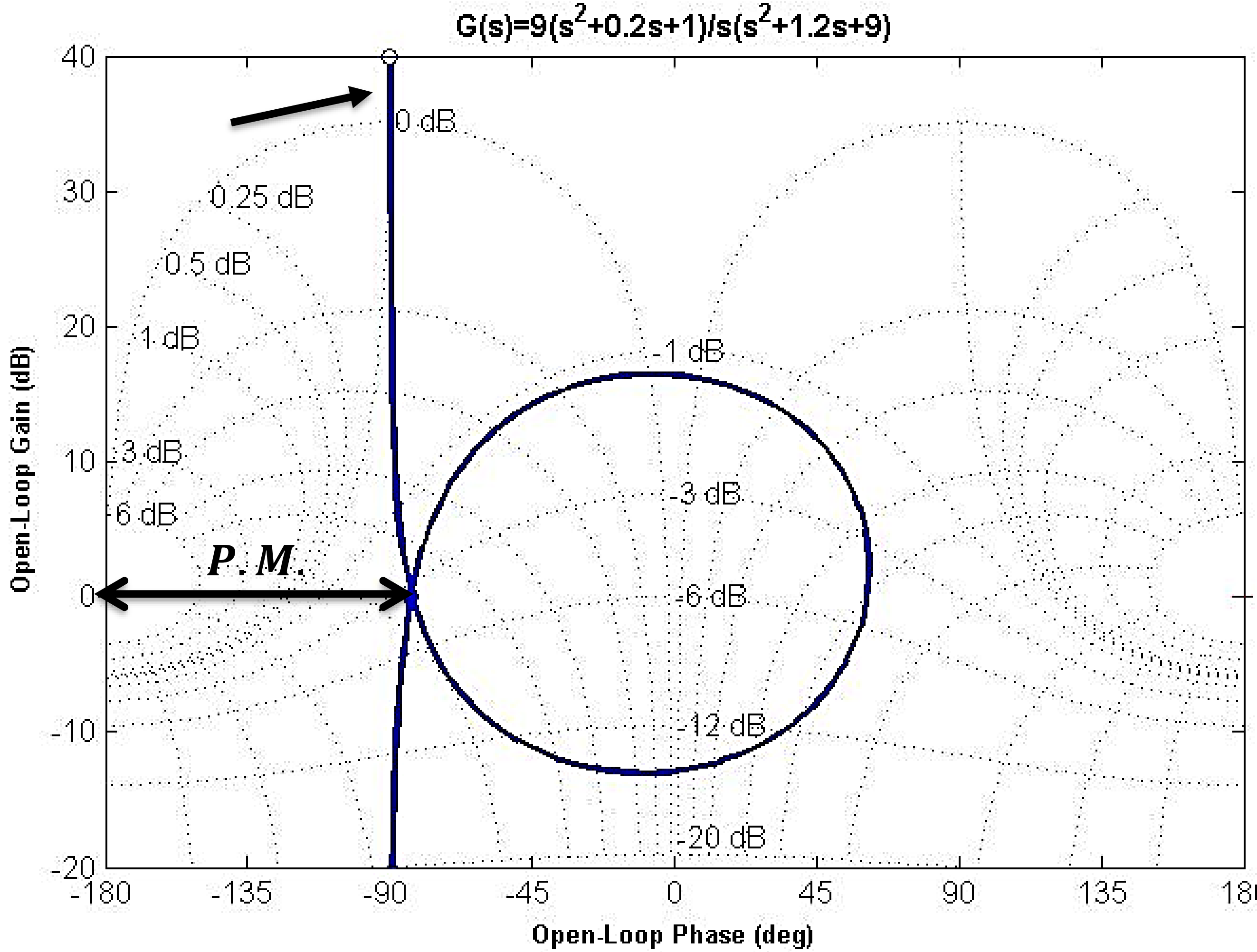
M_r	$2.67\ dB$
ω_r	$0.811\ rad/sec$
ω_B	$1.33\ rad/sec$

- ***example***

$$G(s) = \frac{9(s^2 + 0.2s + 1)}{s(s^2 + 1.2s + 9)}$$

- $G.M = \infty$?

- $M_r = 0\ dB$



b=9*[1 0.2 1]; a=[1 1.2 9 0]; sys=tf(b,a); nichols(sys)

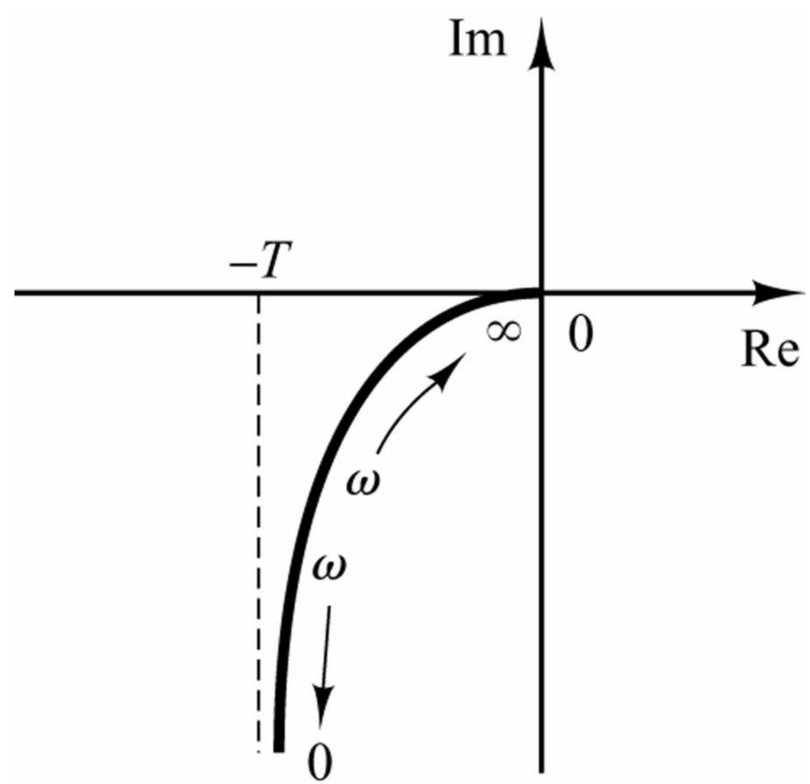
Frequency domain design of control systems

- The **low-frequency** region of the open-loop locus indicates the **steady state** behavior of the closed-loop system

- The **medium-frequency** region of the locus indicates the **relative stability**

- The **high-frequency** region

indicates the **transient response**



- **example:**

$$\boxed{G(s) = \frac{K}{s(1+s)(1+0.0125s)}}$$

- We need the steady state error for a **unit ramp** must be less than 0.01
- We need to realize a resonant peak $M_p = 1.25$

$$e_{ss} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{K} \le 0.01$$

$$\boxed{K > 100}$$

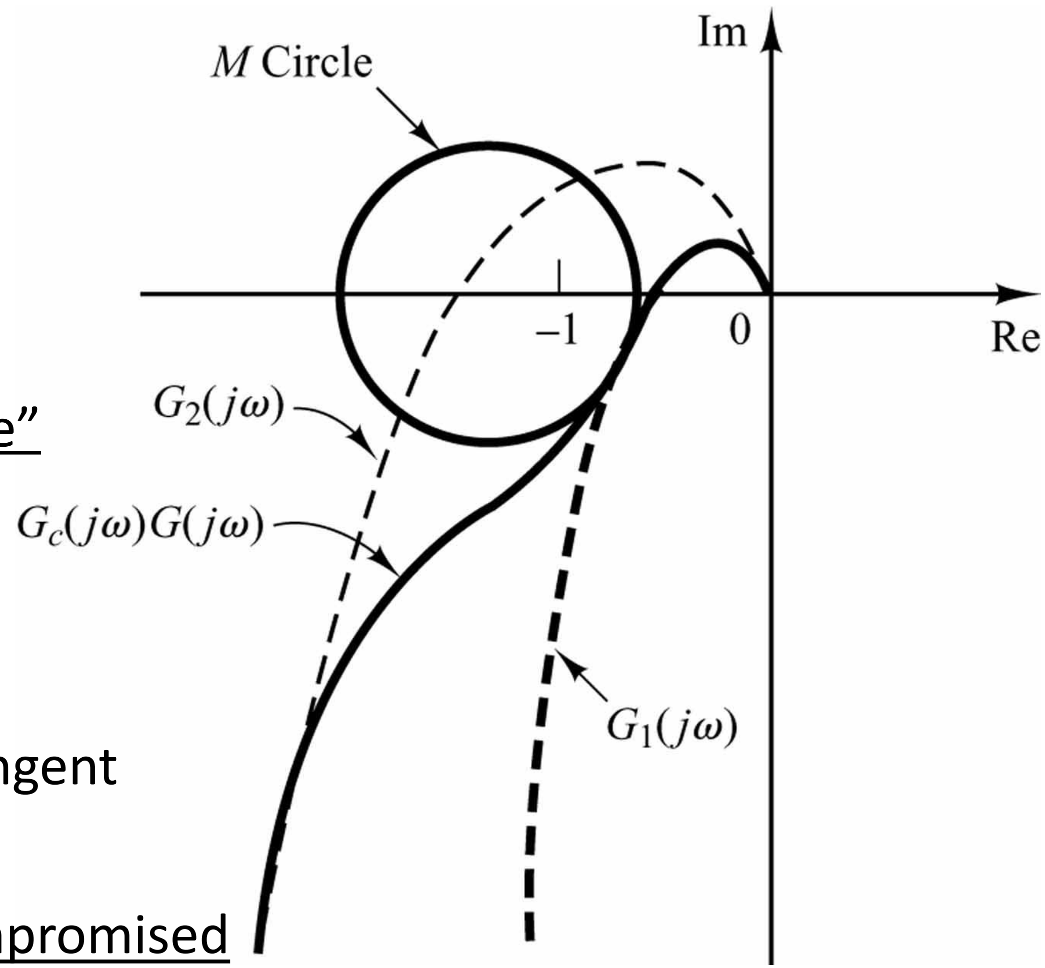
for $\underline{K > 100 \Rightarrow \; G_2(j\omega)}$ <u>"unstable"</u>

for stability, we need $\boxed{K < 81}$

- In order for the curve to be tangent

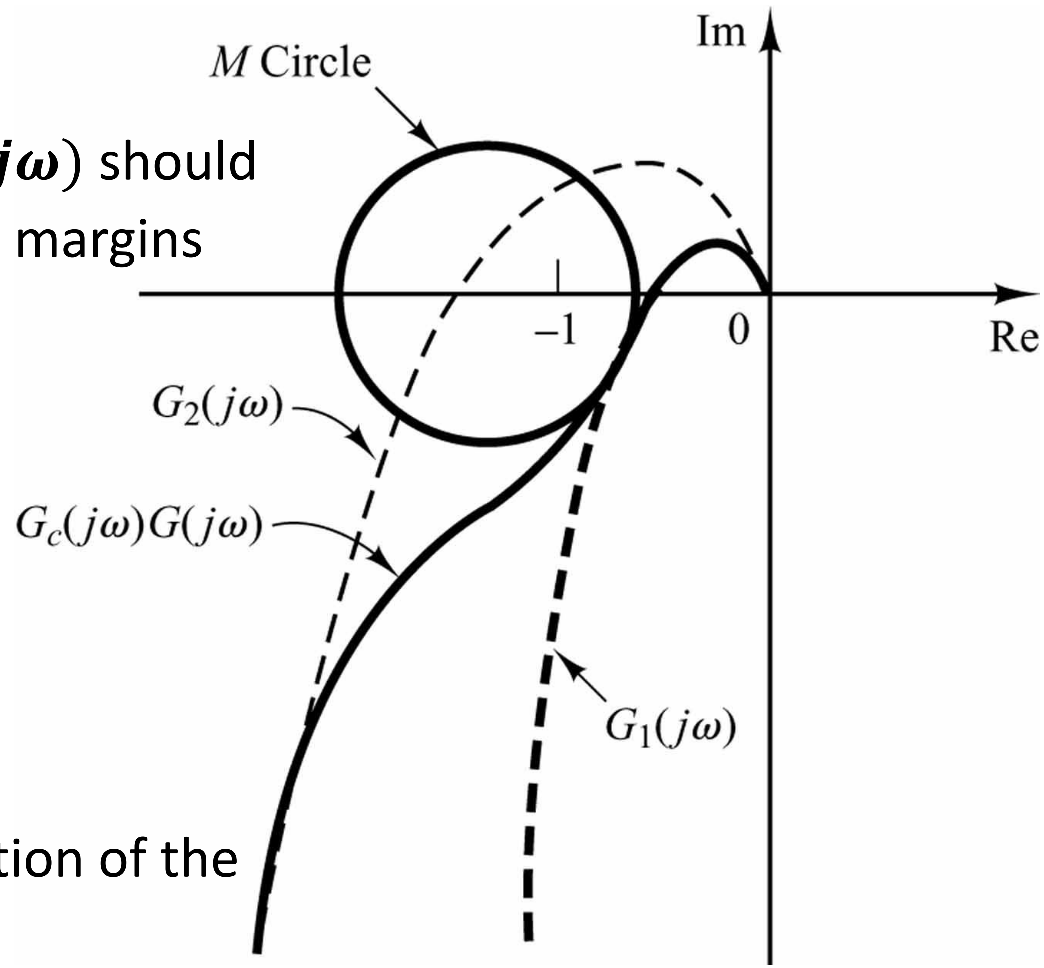
to $M = 1.25 \;\; \Longrightarrow \boxed{K = 1.2}$

then <u>the steady state error is compromised</u>



- we need to obtain a **compromise** between **steady state accuracy and relative stability.**

- This is obtained by **reshaping** the open-loop frequency response plot.



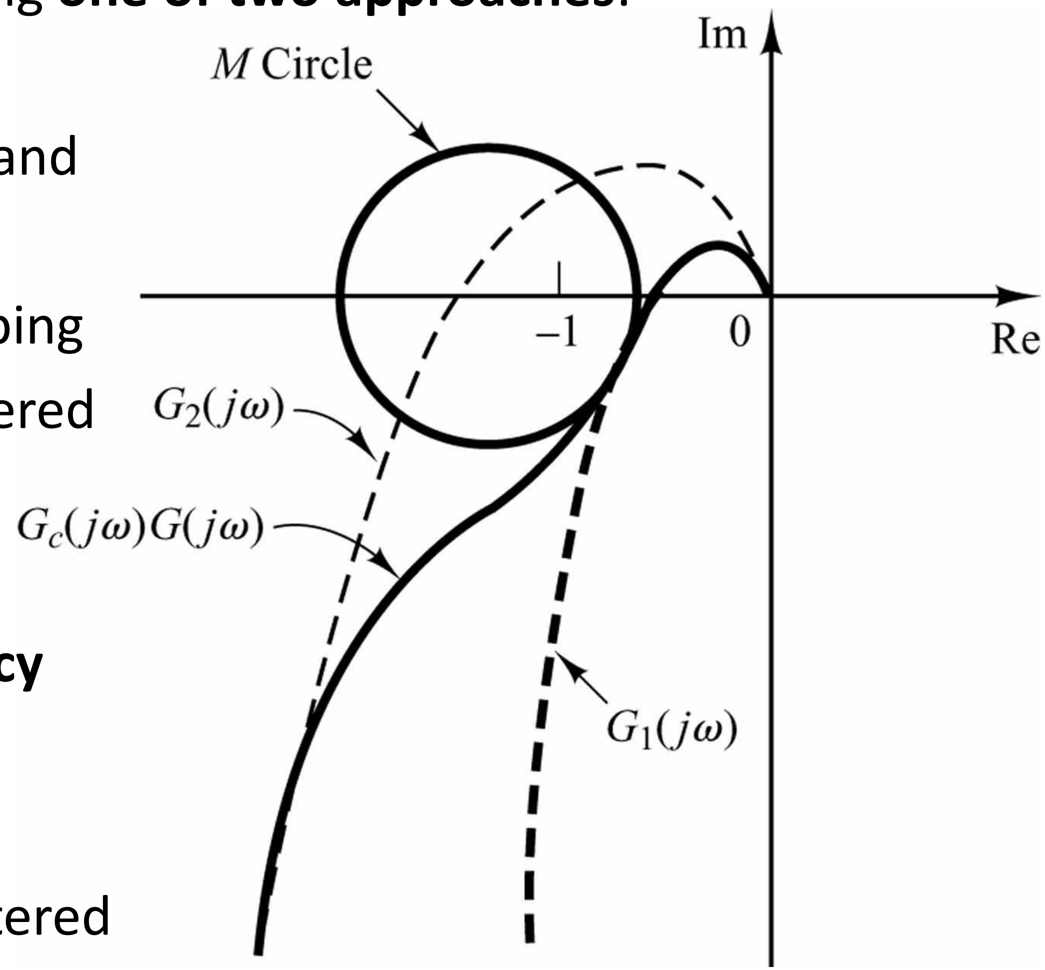
- The reshaped locus $\boldsymbol{G_c(j\omega)G(j\omega)}$ should have a reasonable phase and gain margins or should be tangent to a certain $\boldsymbol{M}$ circle

- Reshaping is performed using **compensation**

- Reshaping is performed such that the **high-frequency** portion of the locus follows the $\boldsymbol{G_1(j\omega)}$ while the **low-frequency** portions follows the $\boldsymbol{G_2(j\omega)}$locus

- then the Nyquist plot should be reshaped to follow $\boldsymbol{G_1(j\omega)}$ in the **high-frequency** region

- this can be performed in using **one of two approaches**:

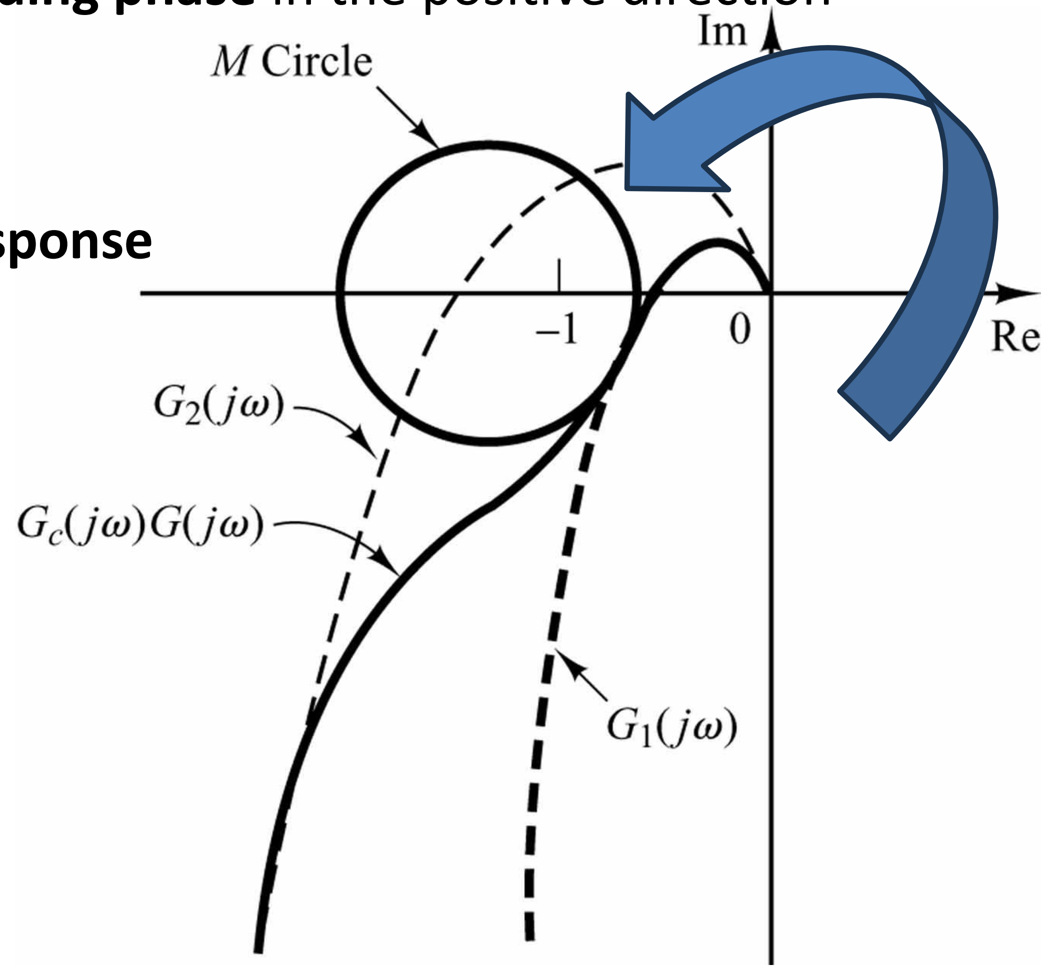


1. Starting with $G_2(j\omega)$ locus and reshaping the locus in the **high-frequency** region and keeping the low-frequency region unaltered

2. Starting with $G_1(j\omega)$ locus and reshaping the **low-frequency** portion to obtain a ramp-error constant of 100 while keeping the high-frequency region unaltered

Phase-lead compensation

- the **high-frequency** portion of $G_2(j\omega)$ is rotated in the **counter-clockwise** direction, i.e., by **adding phase** in the positive direction



- Lead compensation yields an **improvement in the transient response** while and a **small change in steady state accuracy**

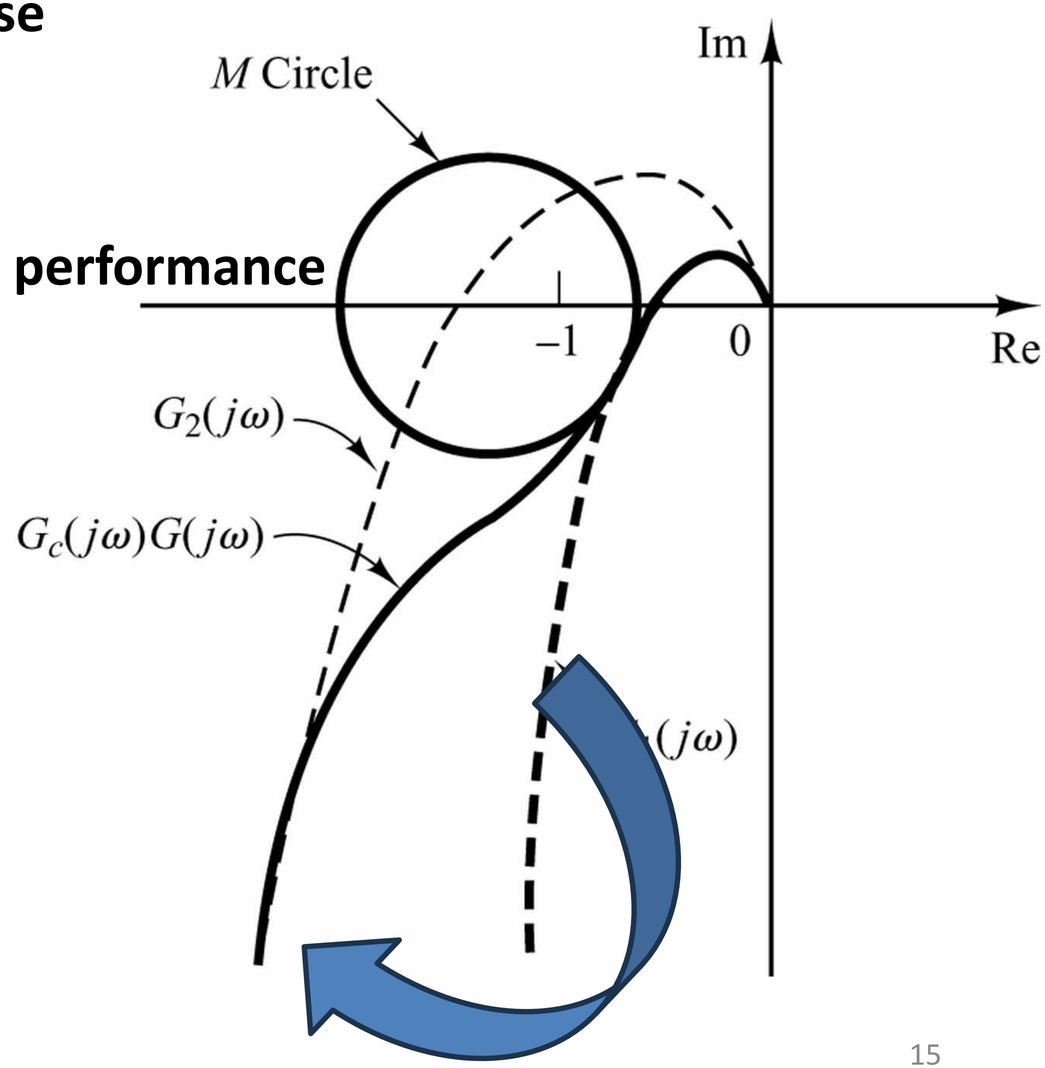
- lead compensation **increases** the system order by **one-degree** (unless cancellation occurs)

Phase-lag compensation

- the **low-frequency** portion of $\boldsymbol{G_1(j\omega)}$ is shifted in the **clockwise** direction, i.e., by **subtracting phase**

- Lag compensation yields an **improvement in the steady-state performance** at the expense of **increasing the transient response time**

- lag compensation **increases** the system order by **one-degree** (unless cancellation occurs)



Lag-lead compensation

- Combines the characteristics of both lag and lead compensation

- The lag-lead compensation increases the system order by **two-degrees** (unless cancellation occurs), which increases system complexity.

- Rarely used

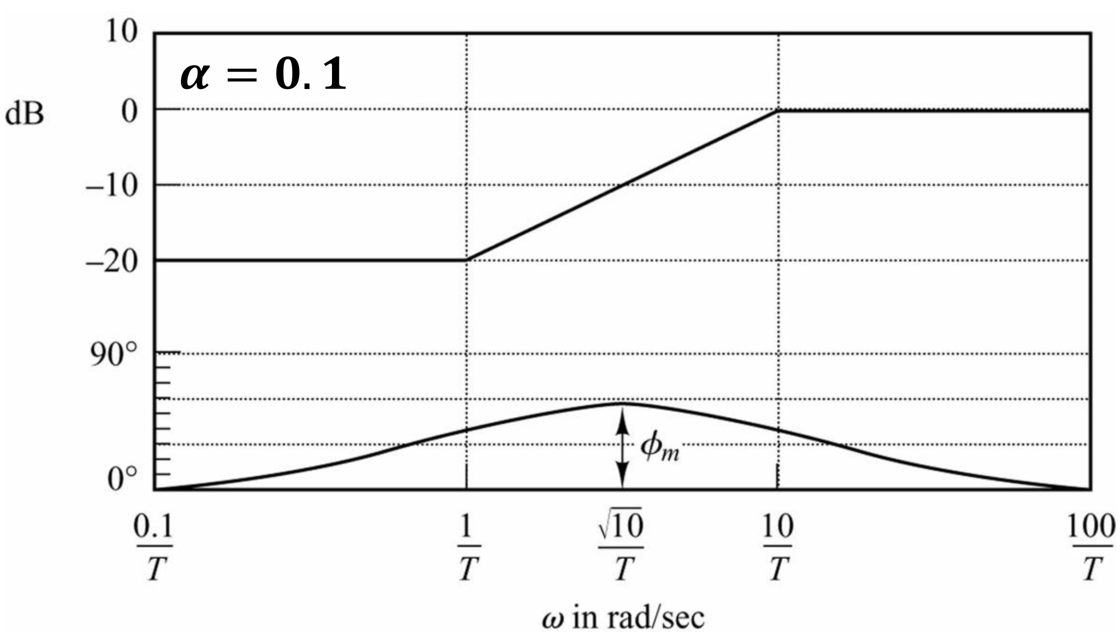
Phase-lead compensation

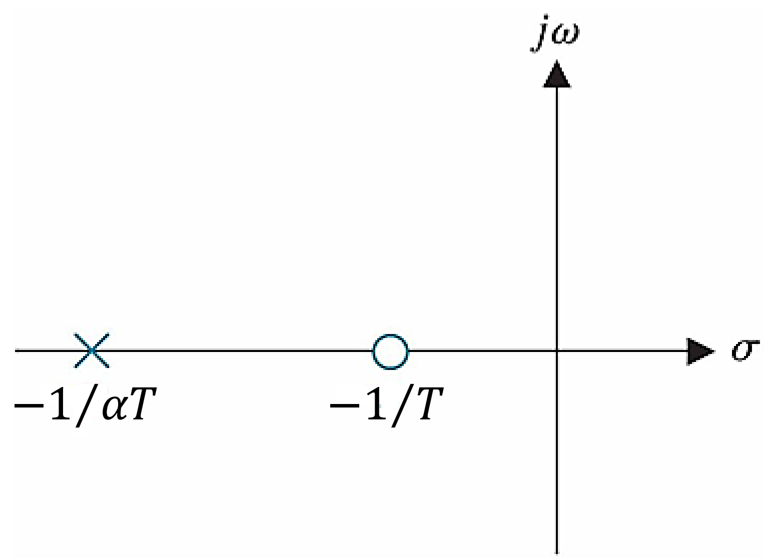
- Phase-lead adds a **zero**
- Phase lead is a **high-pass filter (PD)**
- may amplify high-frequency noise

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1} = K \frac{Ts+1}{\alpha Ts+1}$$

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \qquad (0<\alpha<1)$$

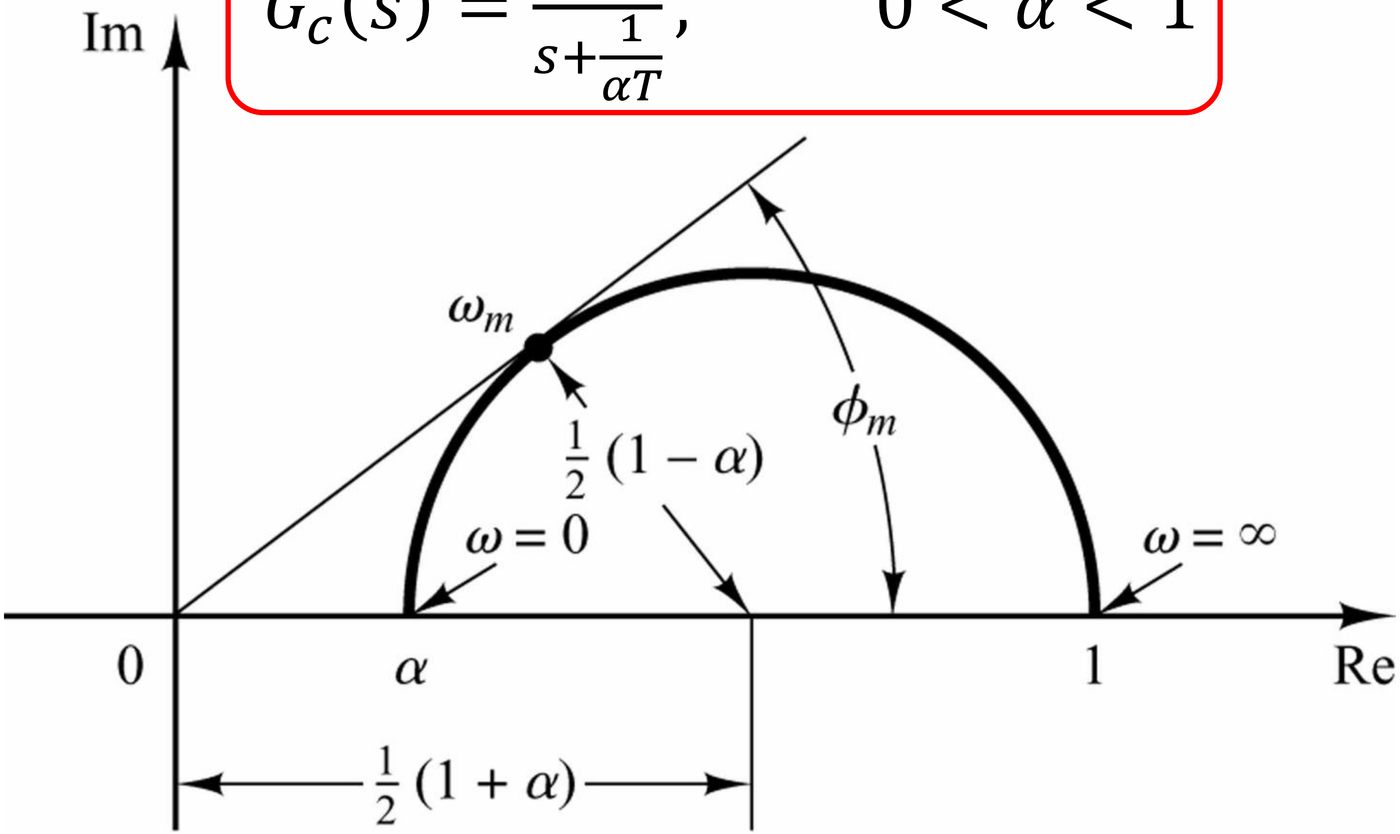
- The <u>**zero is closer to the origin**</u> than the pole ***(strong zero)***





- Polar plot of the phase lead compensator,

$$G_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \qquad 0 < \alpha < 1$$



- The maximum phase ϕ_m the compensator can provide is given by

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \qquad (0 < \alpha < 1)$$

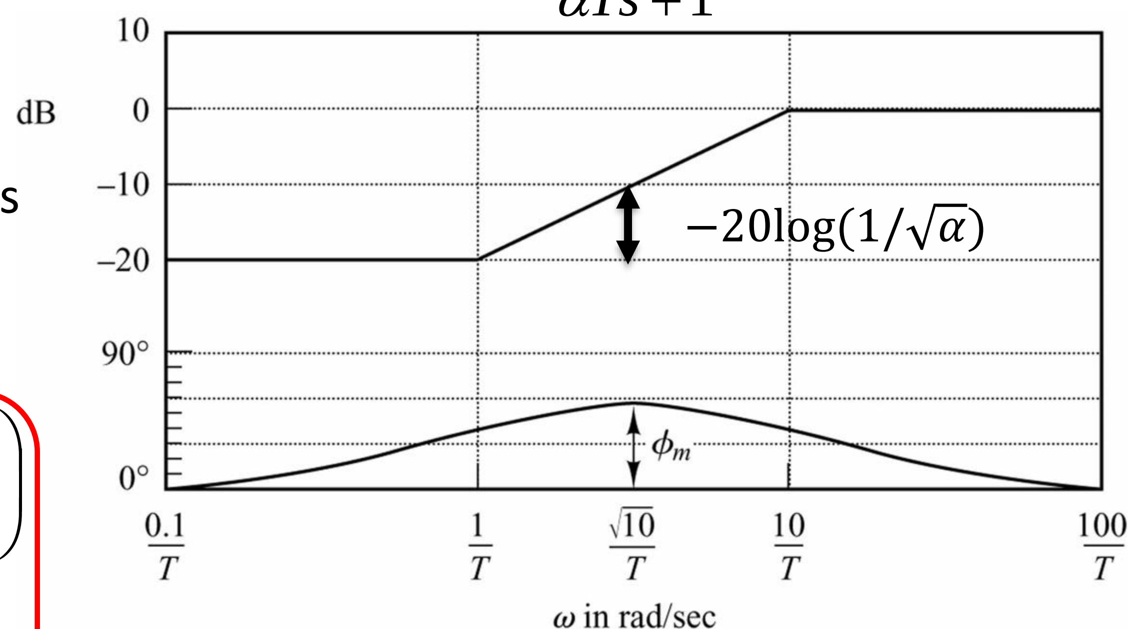
- The minimum value of the attenuation factor $\alpha \cong 0.05$

- The **maximum phase-lead** that the compensator can provides is $\boldsymbol{\phi_m \cong 65^o}$

$$\boxed{\sin\phi_m = \frac{1-\alpha}{1+\alpha} \qquad (0<\alpha<1)}$$

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} \qquad (0<\alpha<1)$$

- the maximum phase happens at ω_m:

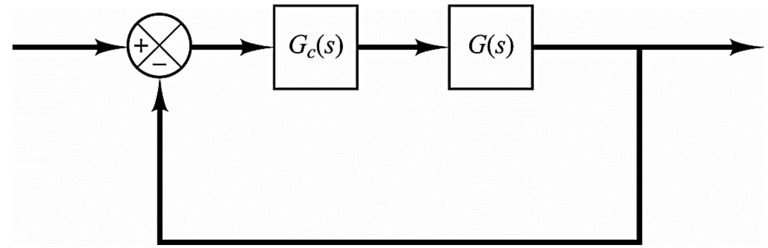


$$\log\omega_m = \frac{1}{2}\left(\log\frac{1}{T}+\log\frac{1}{\alpha T}\right)$$

$$\Rightarrow \omega_m = \frac{1}{T\sqrt{\alpha}}$$

Design steps using the phase-lead compensator

$$\boxed{G_c(s)G(s) = \frac{Ts+1}{\alpha Ts+1} KG(s)} \qquad (0<\alpha<1)$$

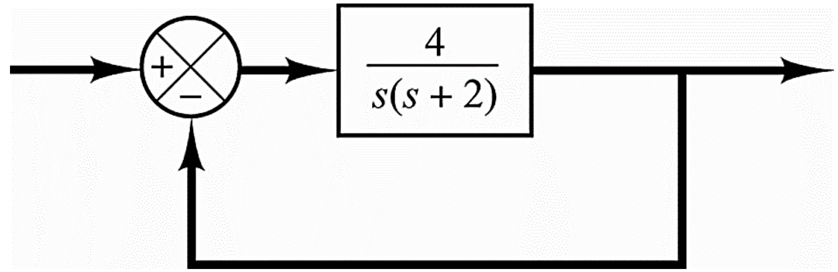


1. Determine the value of $\boldsymbol{K}$ for a specific **steady state error.**
2. Evaluate the **phase margin** from Bode plot using the determined gain.
3. Determine the **necessary phase-lead angle to be added** to the system.
4. Add an additional $\mathbf{5^o}$**to** $\mathbf{12^o}$to be added to the required phase.
5. Determine the **attenuation factor** $\boldsymbol{\alpha}$ from $\boxed{\sin\phi_m = \frac{1-\alpha}{1+\alpha}}$ $\quad (0<\alpha<1)$
6. the new gain crossover frequency $\omega_m = 1/T\sqrt{\alpha}$ happens at the magnitude of the uncompensated system $\boldsymbol{KG(s) = -20log\left(1/\sqrt{\alpha}\right)}$**.**
7. Determine the corner frequencies:

   zero @ $\boldsymbol{\omega = 1/T = \omega_m\sqrt{\alpha},}$ **pole @** $\boldsymbol{\omega = 1/\alpha T = \omega_m/\sqrt{\alpha}}$
8. Determine the constant $\boldsymbol{K_c = K/\alpha}$

- ***Example:***

$$\boxed{G(s) = \frac{4}{s(s+2)}}$$



- **Design specs:**

1. *velocity error constant* $K_v = 20 \ \ sec^{-1}$
2. $P.M. \geq \ 50^o, G.M > 10\ dB$

1. The velocity error constant

$$G_c(s)G(s) = \frac{Ts+1}{\alpha Ts+1}\frac{4K}{s(s+2)}$$

$$K_v = \lim_{s \to 0} sG_c(s)G(s) = 2K = 20$$

$$\Rightarrow K = 10 \ \Rightarrow \boxed{KG(s) = \frac{40}{s(s+2)}}$$

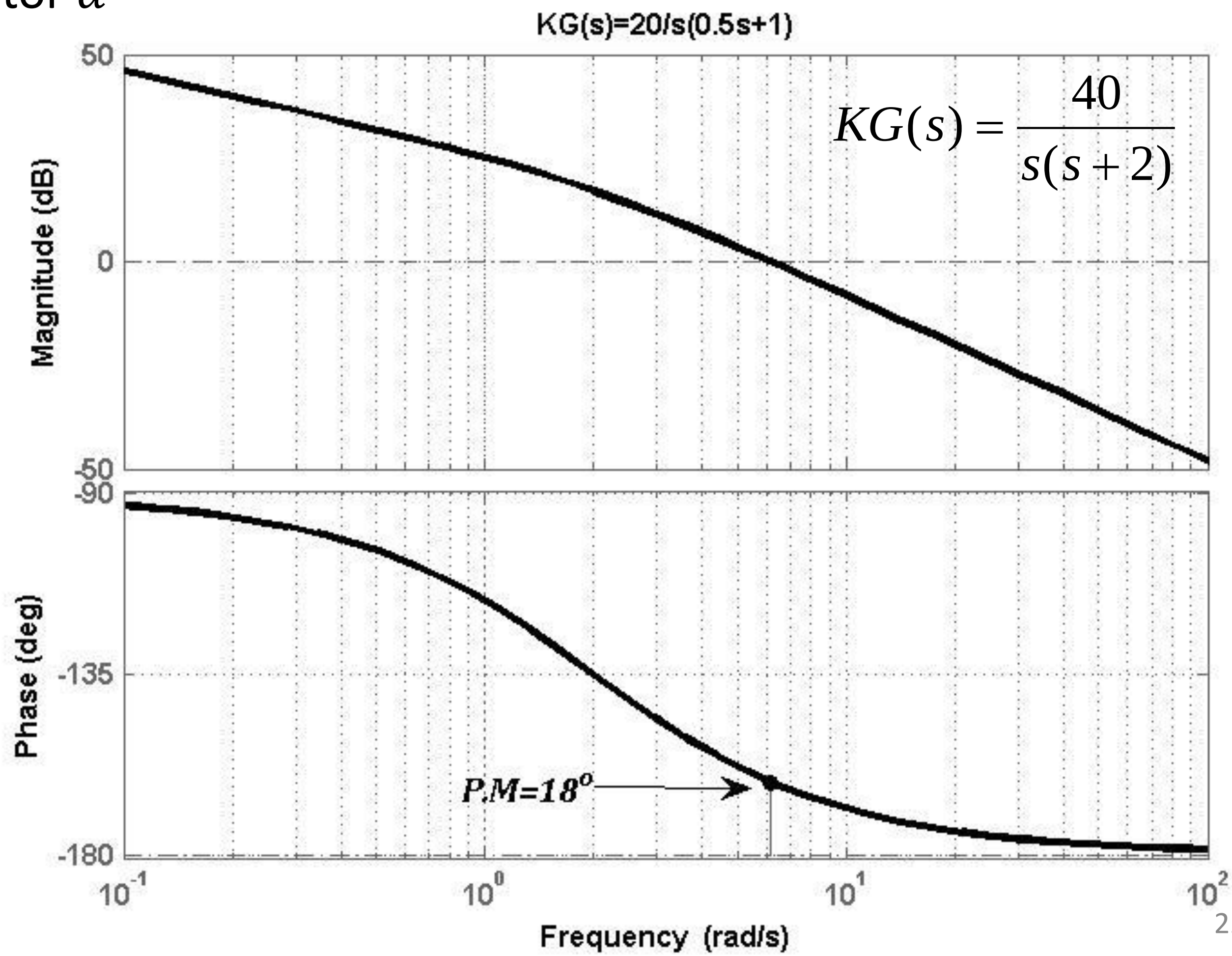
- The phase margin from Bode-plot $P.M. = 18^o$

- The maximum phase to be added $\boxed{\phi_m = (50^o - 18^o) + 5^o = 37^o}$

- the attenuation factor α

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

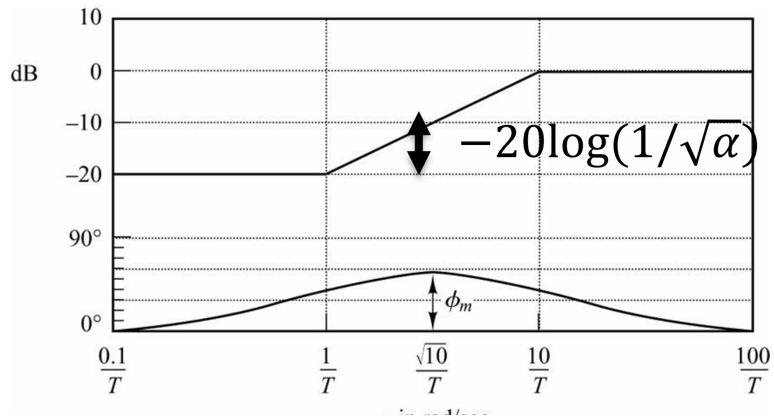
$$\Longrightarrow \boxed{\alpha = 0.24}$$



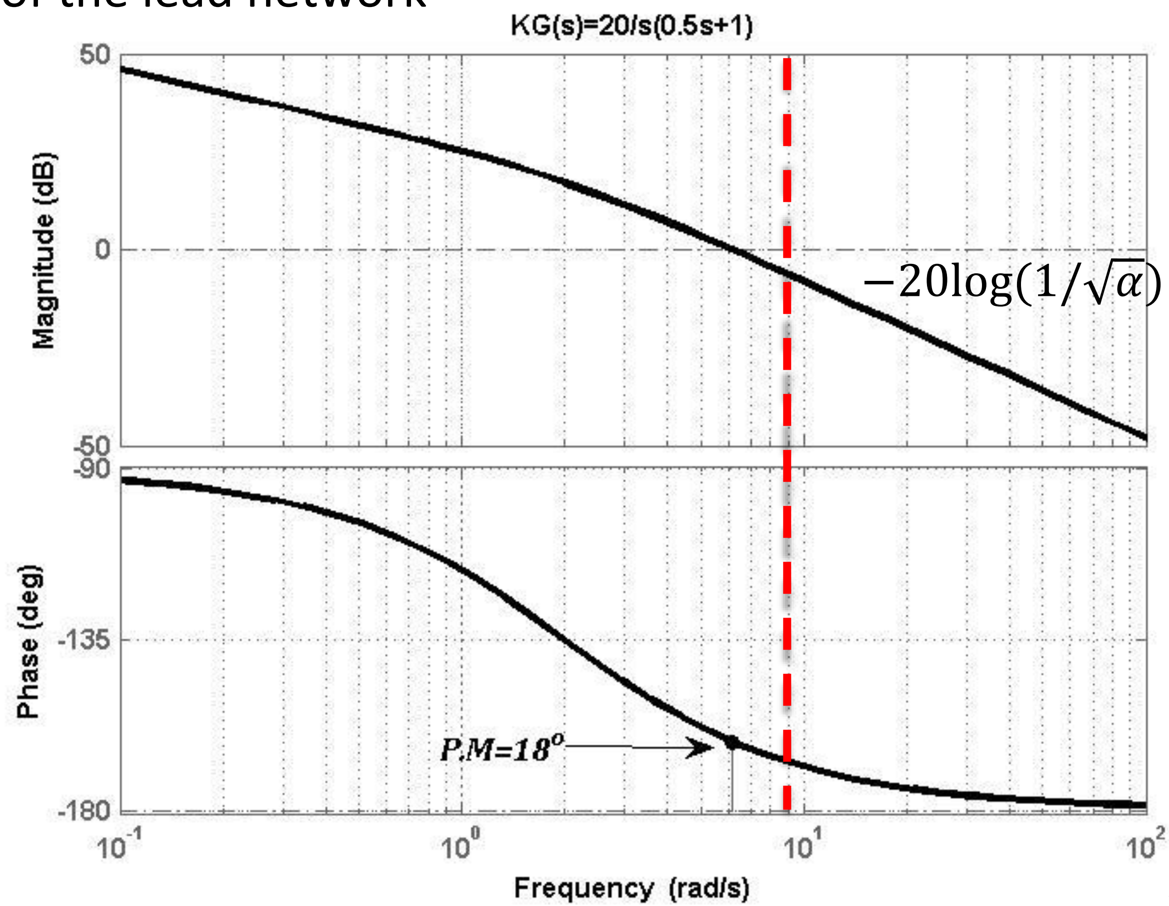
- the **new gain crossover** frequency

$$\boxed{-20log(1/\sqrt{\alpha}) = -6.2 \text{ dB},}$$

happens at $\omega_m = 1/T\sqrt{\alpha} = 9\ rad/sec$



- The pole and the zero of the lead network



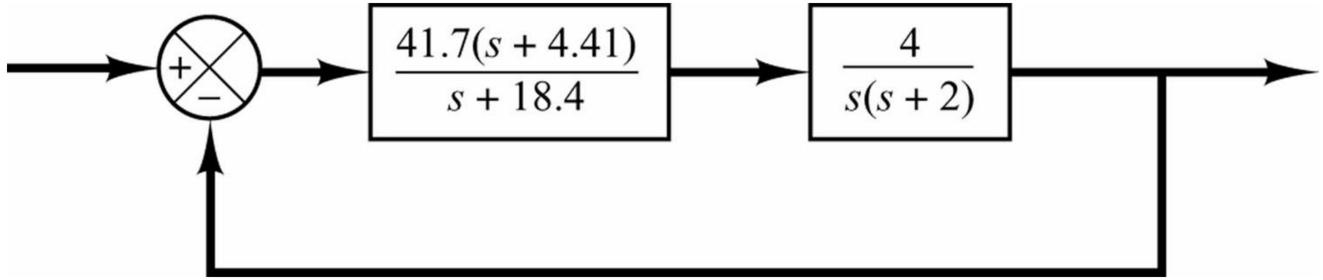
- **Zero:**

$$\boxed{\begin{aligned} &@\ 1/T = \omega_m\sqrt{\alpha} \\ &= 4.41\ \ rad/sec \end{aligned}}$$

- **Pole:**

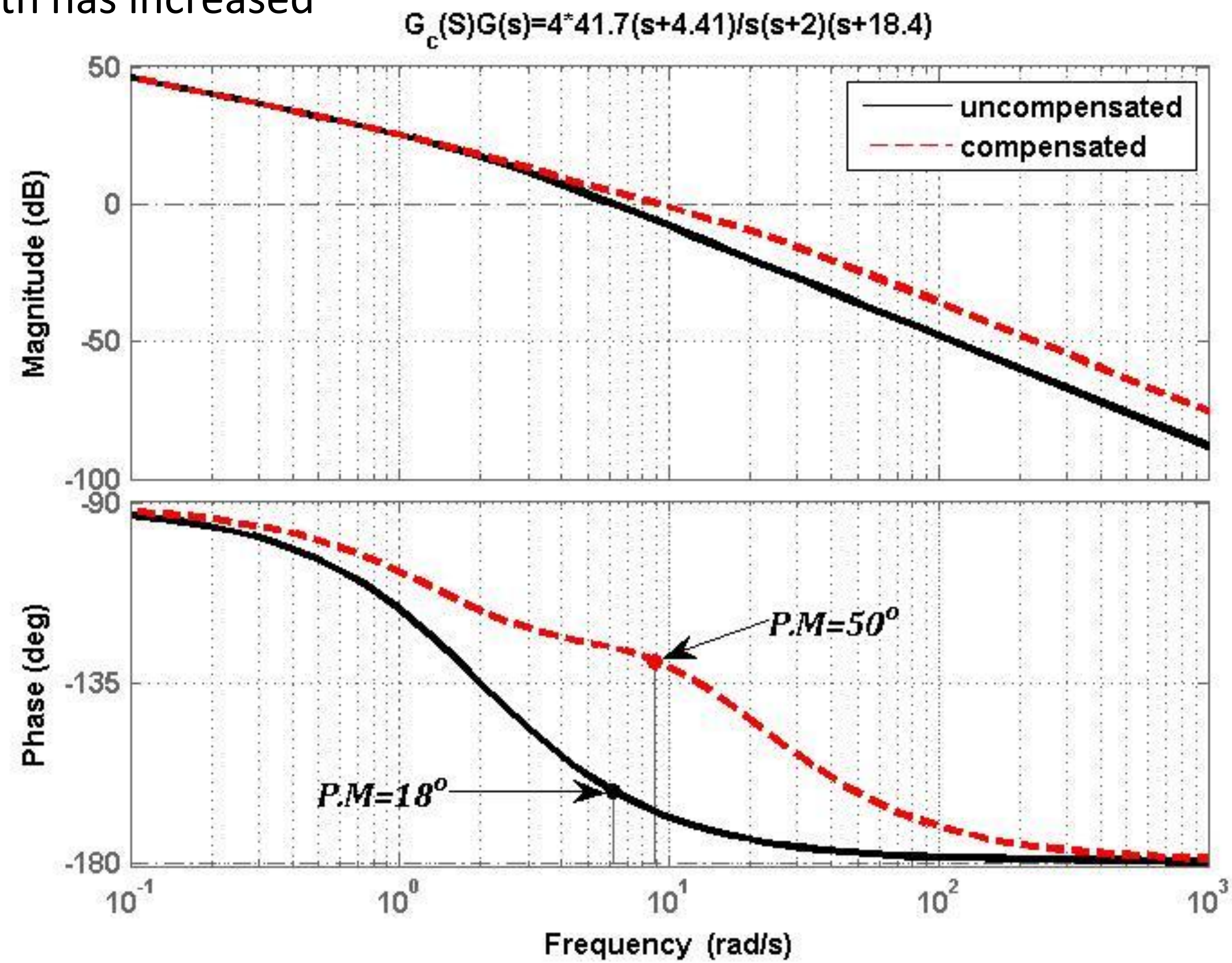
$$\boxed{\begin{aligned} &@\ 1/\alpha T = \omega_m/\sqrt{\alpha} \\ &= 18.4\ rad/sec \end{aligned}}$$

- **Bode-diagram**



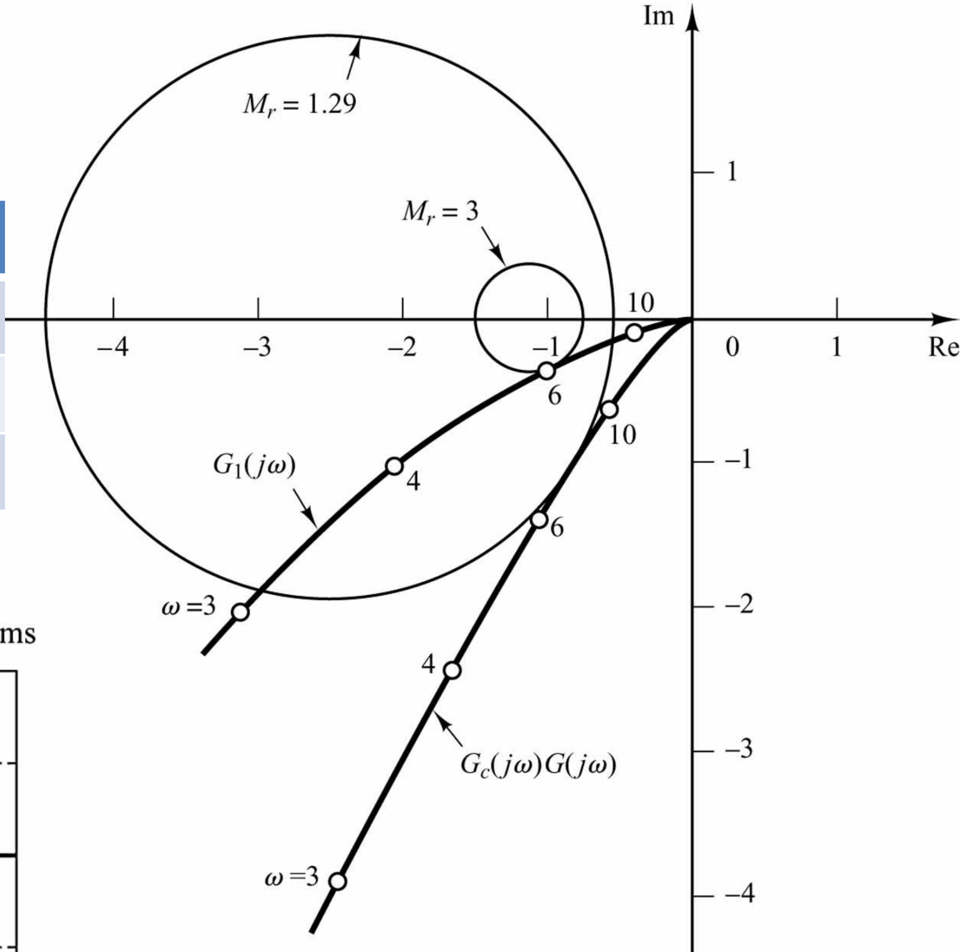
- The bandwidth has increased

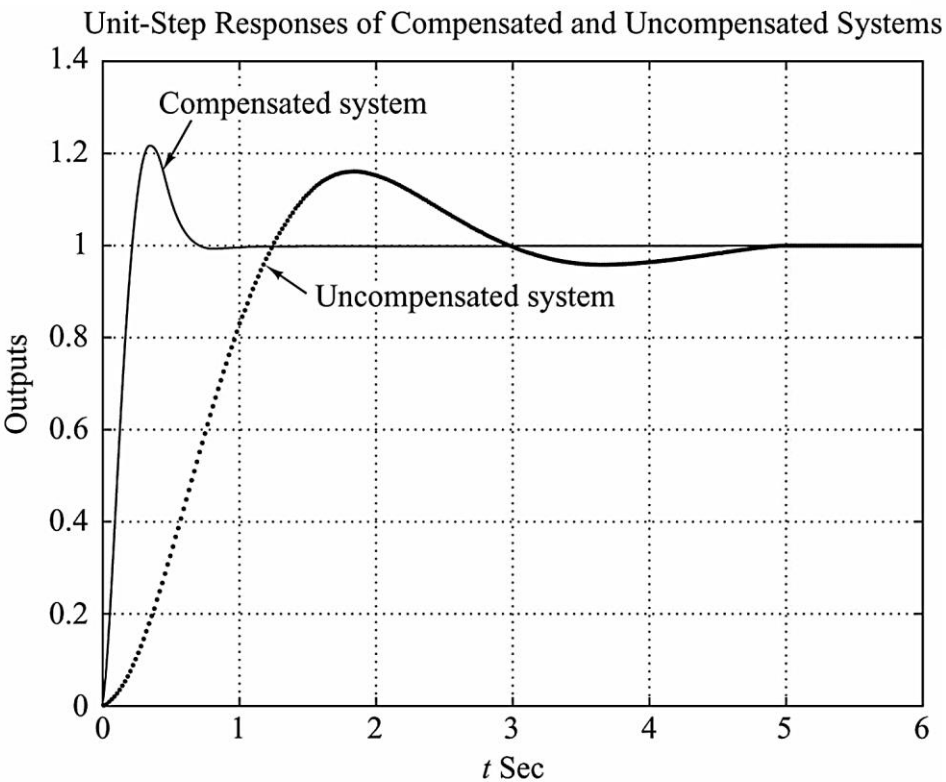
$$P.M. = 50^o$$
$$G.M. = \infty$$



Nyquist plot:

	uncompensated	compensated
M_r	3	1.29
ω_r	6 rad/sec	7 rad/sec
ω_B	6.3 rad/sec	9 rad/sec



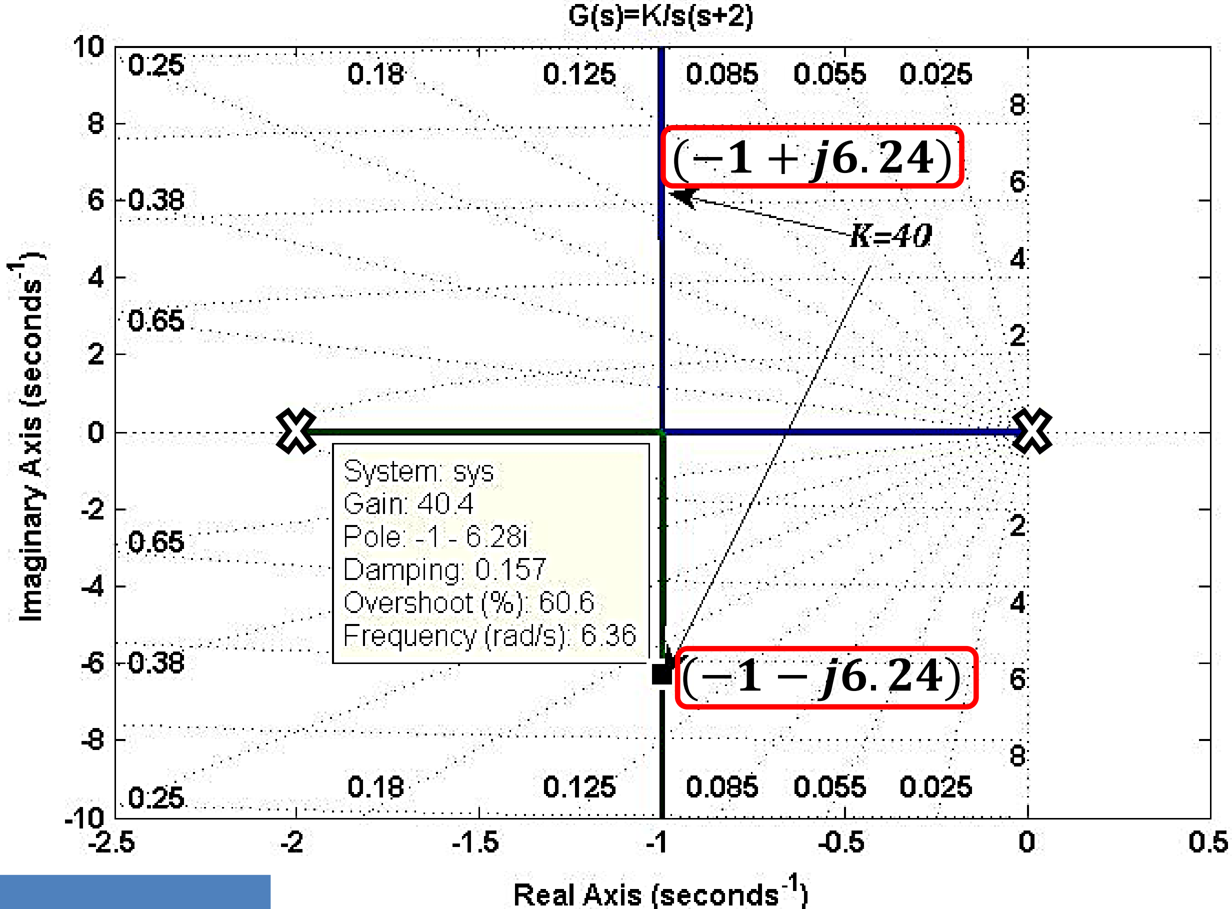


$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 2s + 4}$$

$$\frac{C(s)}{R(s)} = \frac{166.8s + 735.588}{s^3 + 20.4s^2 + 203.6s + 735.588}$$

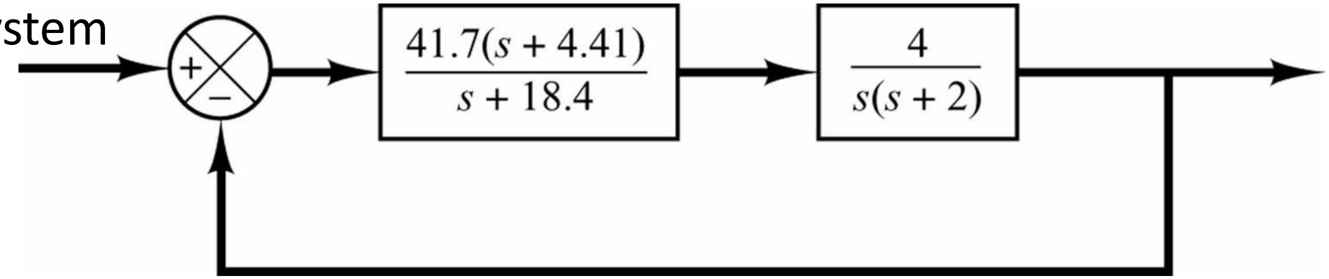
- The original system that satisfies the steady state error

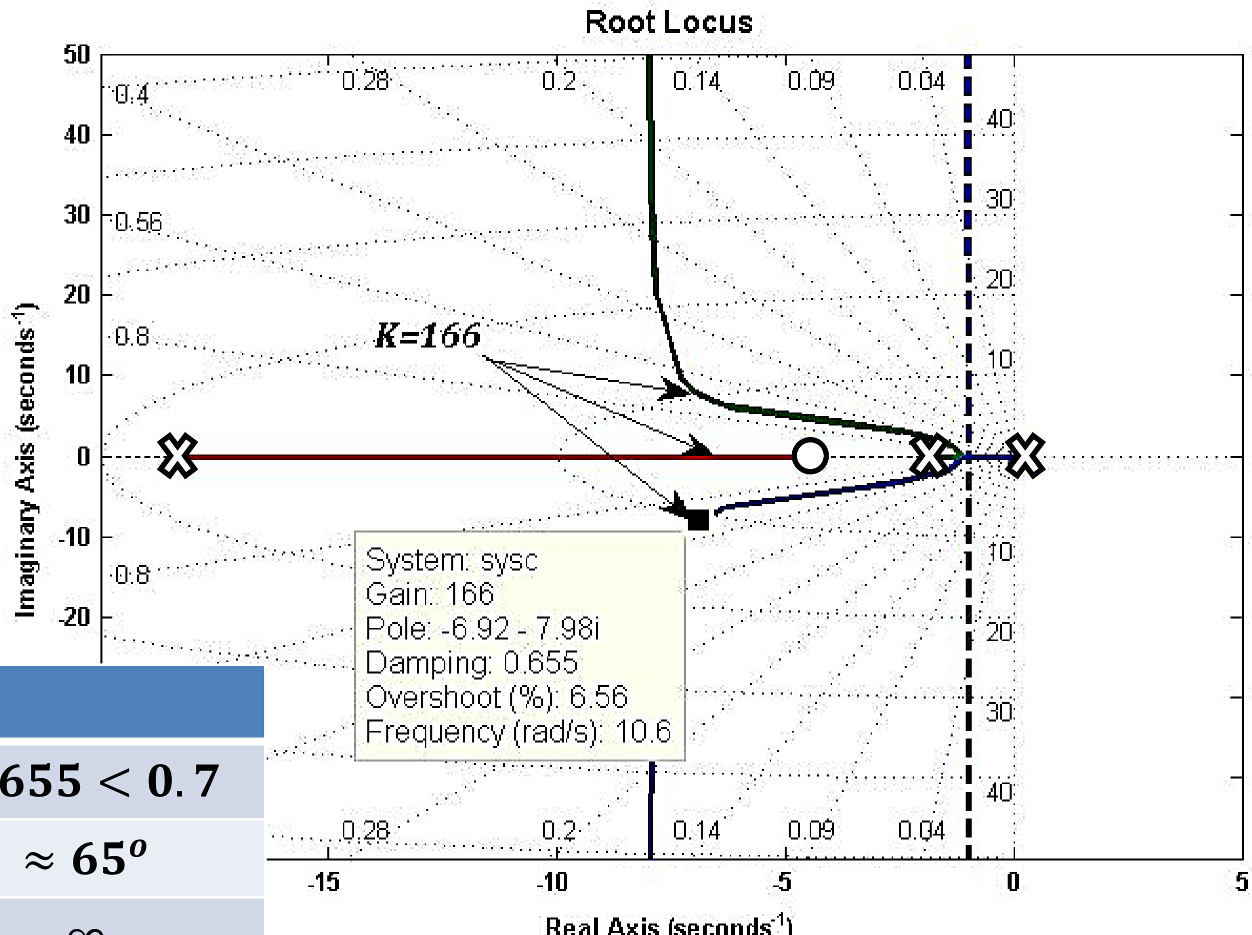
$$KG(s) = \frac{40}{s(s+2)}$$



ξ	$0.157 < 0.7$
$P.M. \cong 100\xi$	$\approx 16^o$
$G.M.$	∞
$B.W.$	$6.3\ rad/sec$

- The compensated system

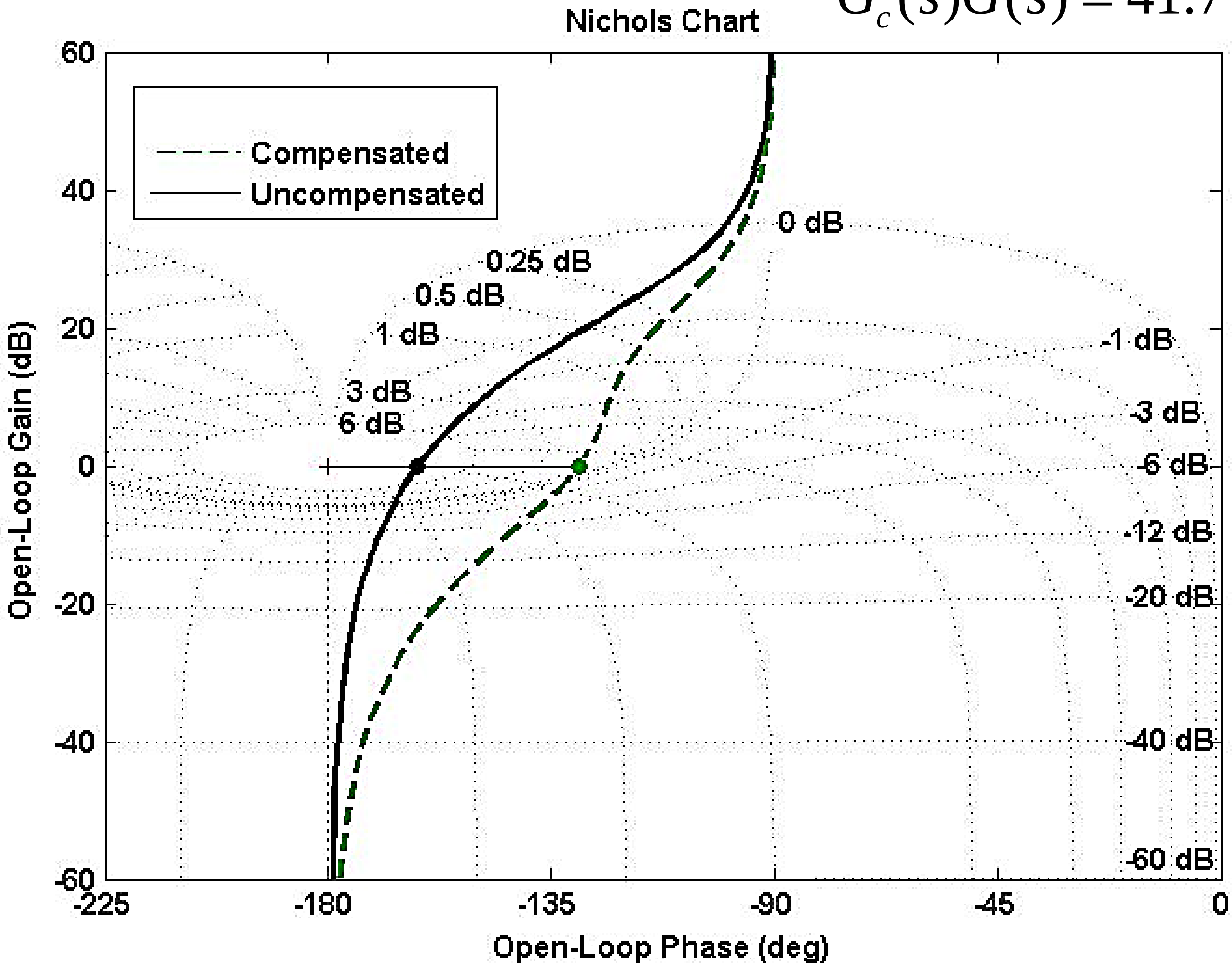




ξ	$0.655 < 0.7$
$P.M. \cong 100\xi$	$\approx 65^o$
$G.M.$	∞
$B.W.$	$10.6\, rad/sec$

- ***Nichols chart***

$$G_c(s)G(s) = 41.7\frac{(s+4.41)}{(s+18.4)}\frac{4}{s(s+2)}$$



	uncompensated	compensated
$P.M.$	$18^o\ @\omega = 6.17\ rad/sec$	$50.5^o\ @\omega = 8.89\ rad/sec$
$G.M.$	∞	∞