Automatic Control Systems

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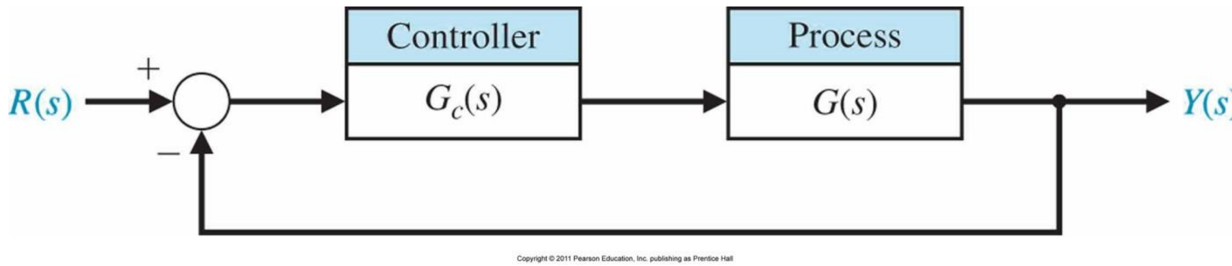
- **Reading: Chapter 7**
  - Section 7.6

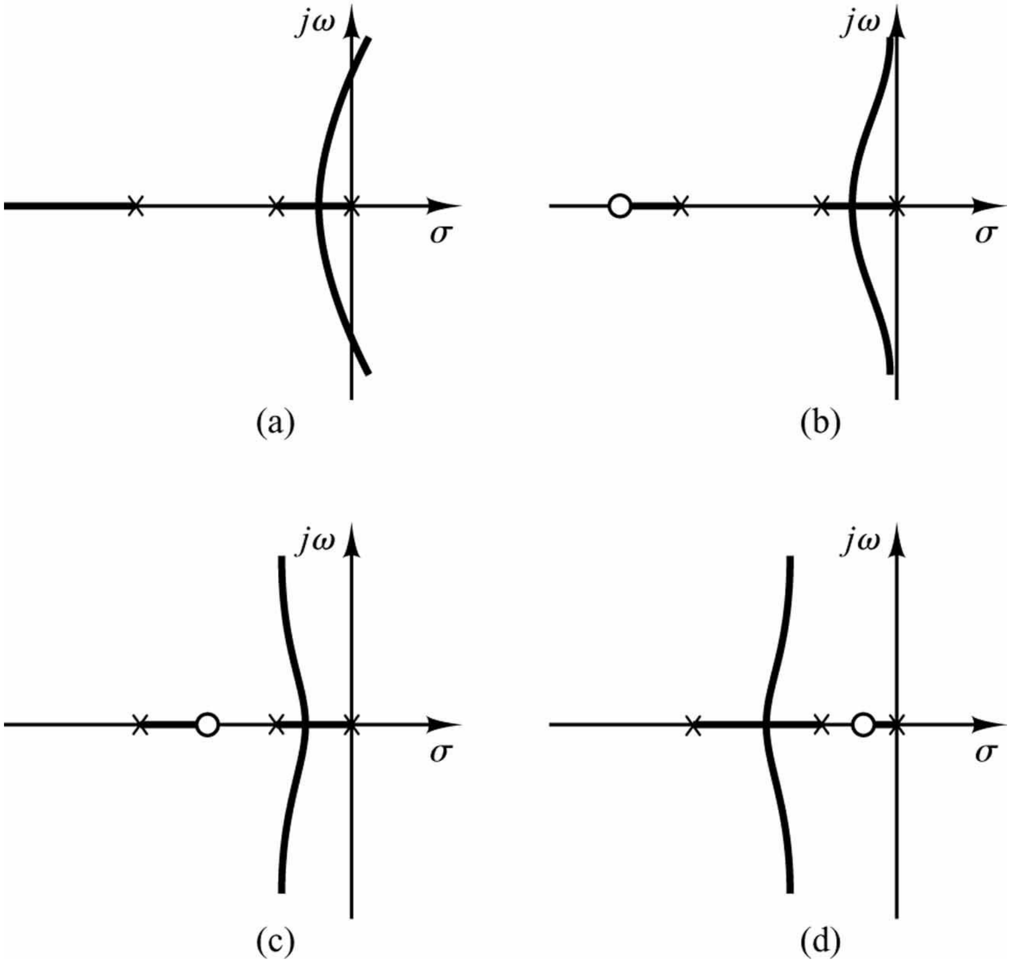
PID Controller



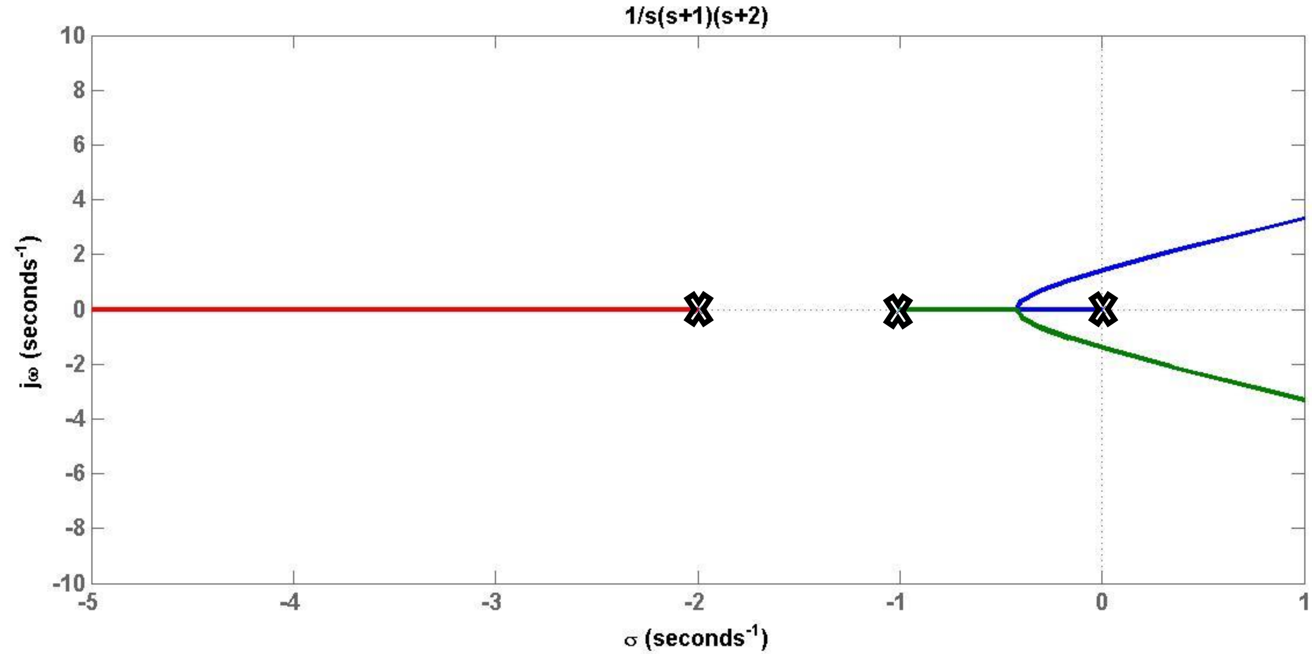
- A controller adds **poles and/or zeros** to the system in order to improve its response (e.g., steady state and transient response)

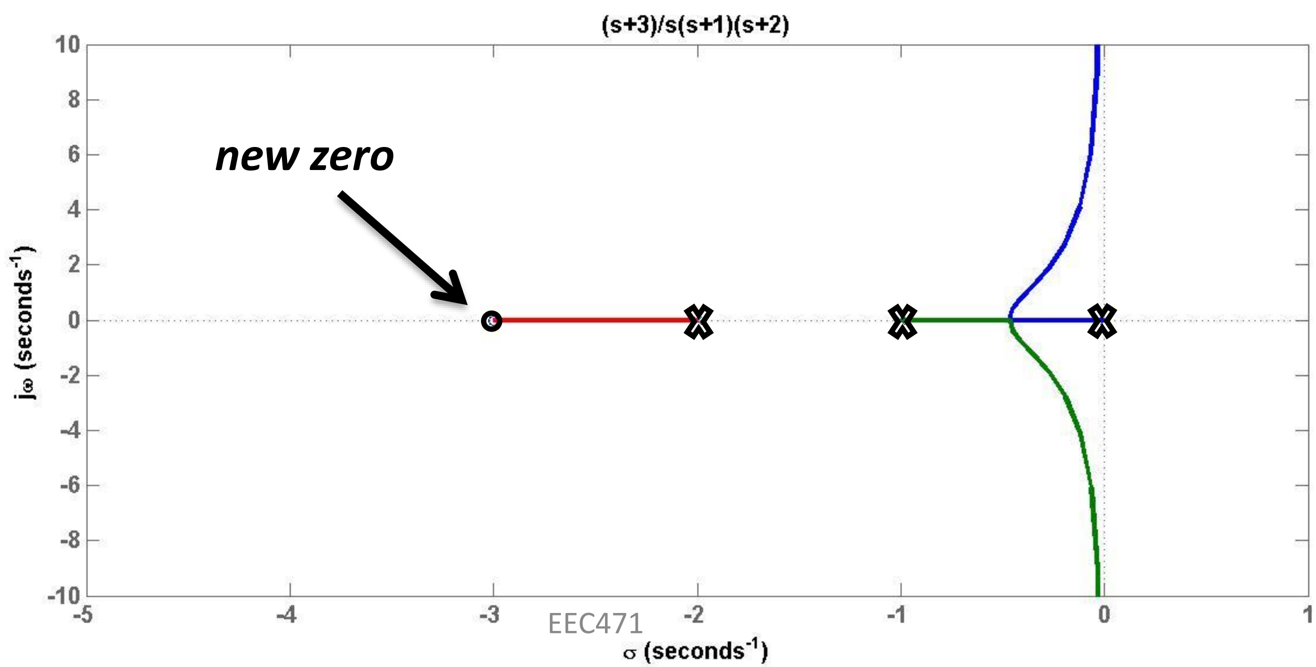
- **PD**: proportional plus derivative
- **PI**: proportional plus integral
- **PID**: proportional integral derivative

- ***Adding left-half plane zeros to the function G(s)H(s) generally has the effect of moving and bending the root loci toward the zero which makes the system more stable.***

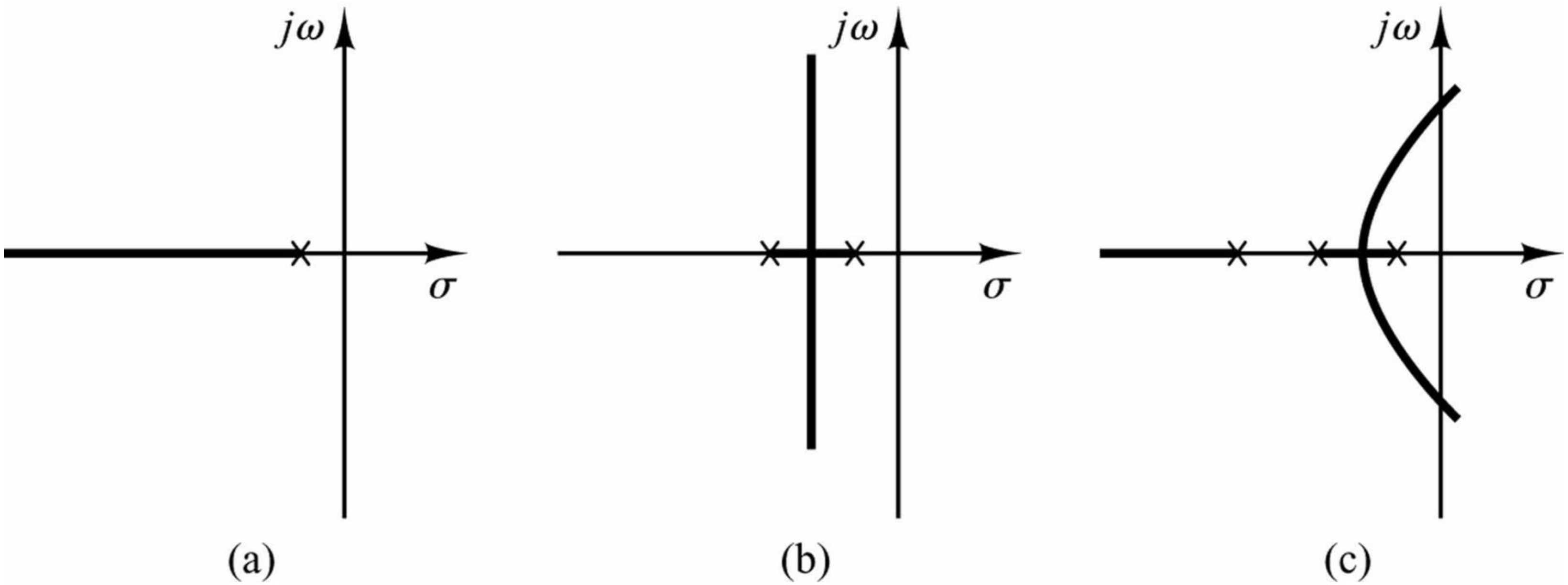


- ***What is the effect on the transient response?***

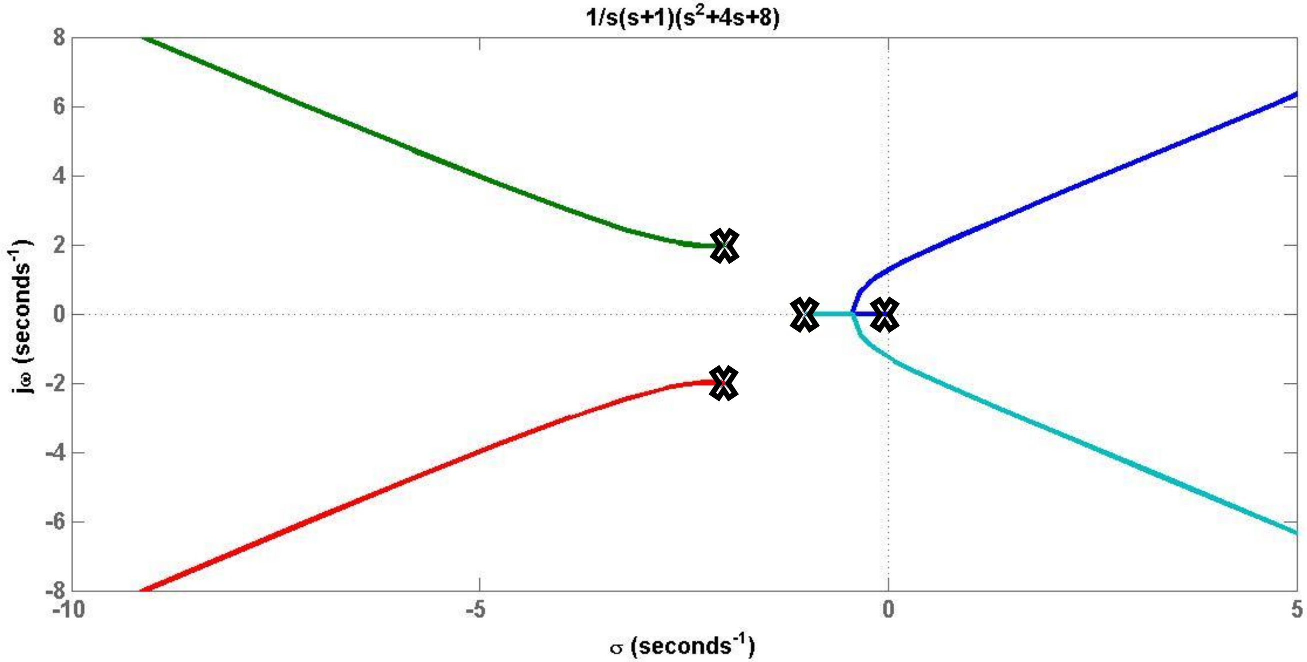

1/s(s+1)(s+2)
jω (seconds⁻¹)
σ (seconds⁻¹)

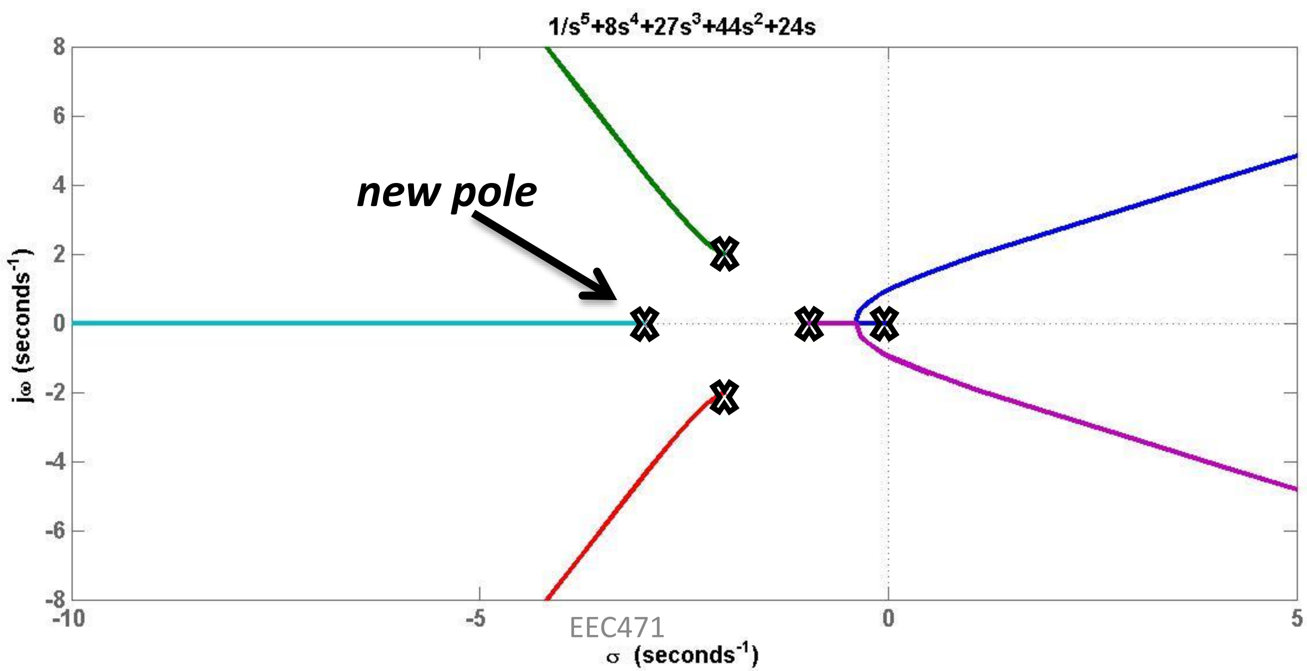

(s+3)/s(s+1)(s+2)
new zero
jω (seconds⁻¹)
σ (seconds⁻¹)

- ***Adding a pole to G(s)H(s) has the effect of pushing the root loci toward the right half s-plane – makes the system less stable***

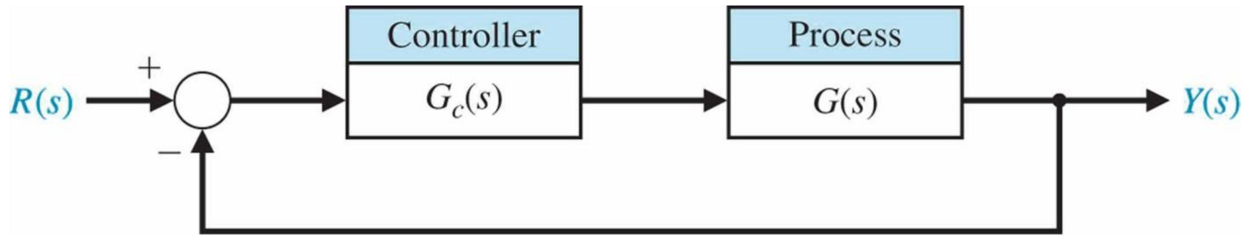


- ***What is the effect on the transient response?***


1/s(s+1)(s2+4s+8)
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
jω (seconds-1)
σ (seconds-1)


1/s5+8s4+27s3+44s2+24s
new pole
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
jω (seconds-1)
σ (seconds-1)

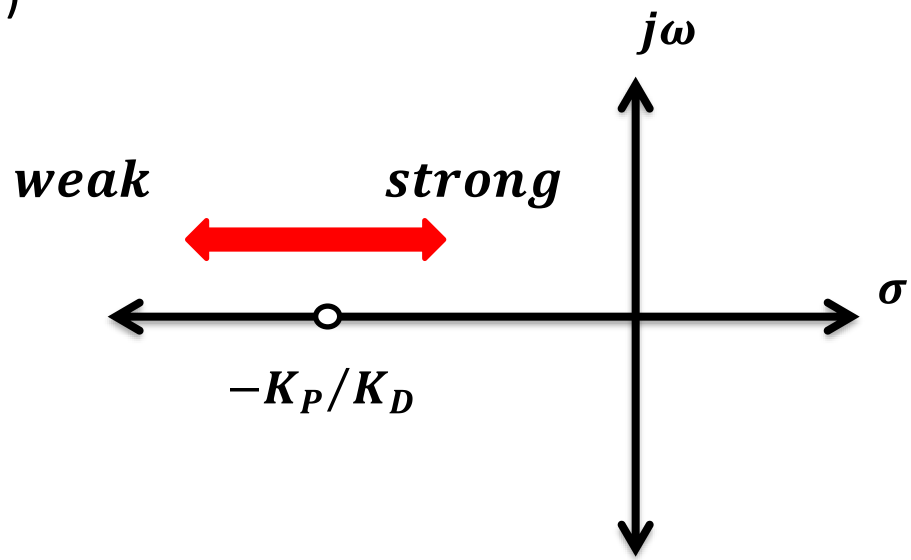
PD controller



- proportional plus **derivative** (**PD**)

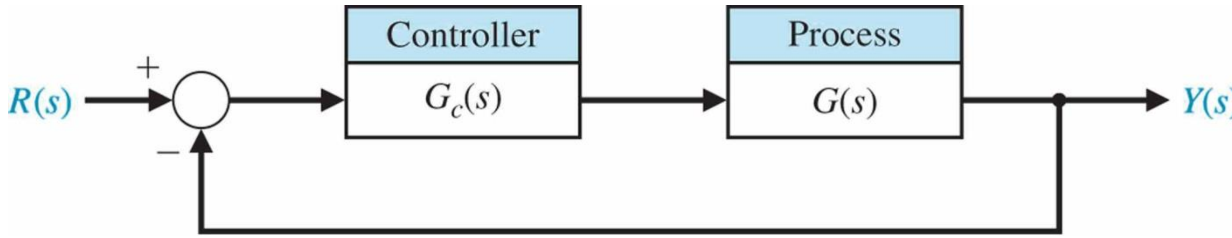
$$G_c(s) = K_P + K_D s$$

$$G_c(s) = K_D(s + K_P/K_D)$$



- one zero at $s = -K_P/K_D$
- acts as a ***high-pass filter***
- ***has no effect on the steady state error (why??)***

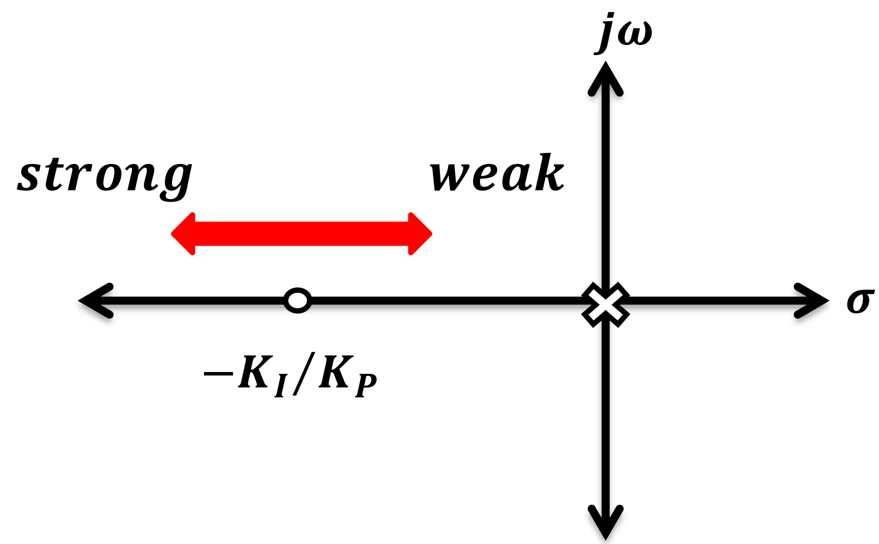
PI controller



- proportional plus **integral** (**PI**)

$$G_c(s) = K_P + \frac{K_I}{s}$$

$$G_c(s) = \frac{K_P(s + K_I/K_P)}{s}$$



- one **pole at the origin** and a **zero at $s = -K_I/K_P$**
- acts as a ***low-pass filter***
- ***Improves the steady state error***

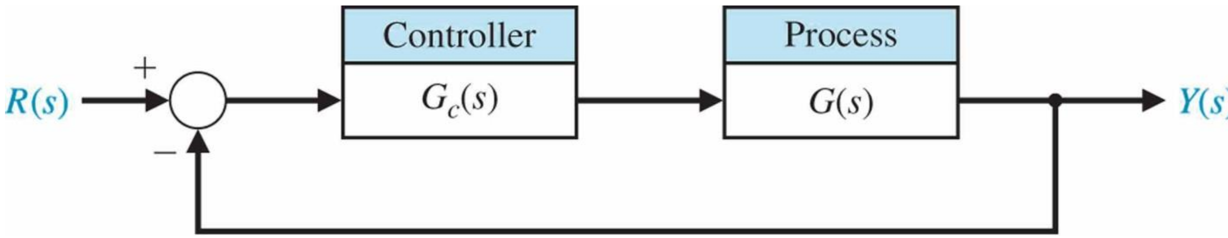
Steady state error

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Input: Ramp, At, A/s^2	Input: Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \dfrac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\dfrac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\dfrac{A}{K_a}$

- ***the pole at the origin improves*** the **steady state error (why?)**
- ***the pole at the origin increases the system type***

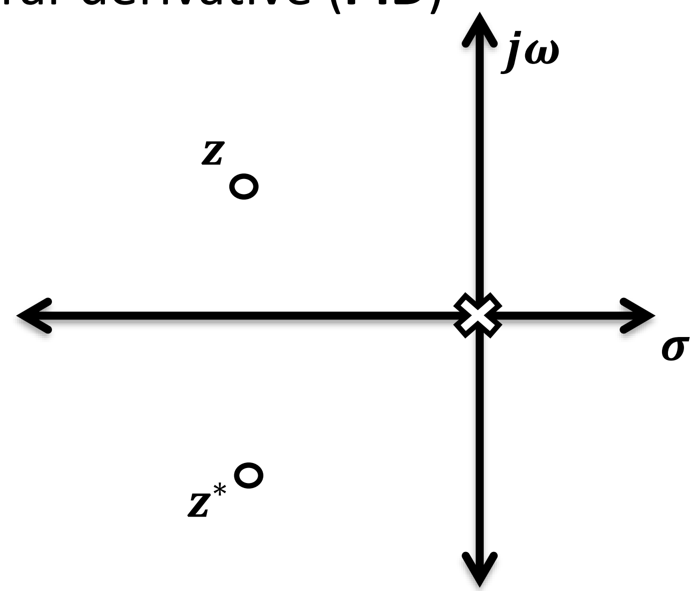
design using a PID controller



- Three-terms controller: proportional-Integral-derivative (**PID**)

$$\boxed{G_c(s) = K_P + K_D s + \frac{K_I}{s}}$$

$$\boxed{G_c(s) = \frac{K_D\left(s^2 + K_P/K_D\, s + K_I/K_D\right)}{s}}$$



- one pole at the origin and two zeros (need not to be complex zeros)
- *the pole at the origin* ***improves*** *the* ***steady state error***

How to tune the PID controller coefficients K_D, K_P & K_I?

- **method 1**: analytical method
  - **Zeigler-Nichols** PID tuning method
  - Used as **an initial** PID tuning, then refinement can be used.

- **method 2**: Manual PID tuning
  - Root locus & root contour
  - use common sense
  - trial and error

Closed-loop Ziegler-Nichols PID tuning method

$$\boxed{G_c(s) = K_P + K_D s + K_I/s}$$

- Set $K_I = 0$ & $K_D = 0$
- Increase K_P till it reaches the **boundary of instability**, $K_p = \boldsymbol{K_U}$ (**the ultimate gain)**
- The period of the sustained oscillation is called the **ultimate period** $\boldsymbol{T_U}$
- Once $\boldsymbol{K_U}$ **and** $\boldsymbol{T_U}$ are known, PID coefficients can be found using the table

Table 7.7 Ziegler-Nichols PID Tuning Using Ultimate Gain, K_U, and Oscillation Period, P_U

Ziegler-Nichols PID Controller Gain Tuning Using Closed-loop Concepts

Controller Type	K_P	K_I	K_D
Proportional (P) $G_c(s) = K_P$	$0.5K_U$	–	–
Proportional-plus-integral (PI) $G_c(s) = K_P + \dfrac{K_I}{s}$	$0.45K_U$	$\dfrac{0.54K_U}{T_U}$	–
Proportional-plus-integral-plus-derivative (PID) $G_c(s) = K_P + \dfrac{K_I}{s} + K_D s$	$0.6K_U$	$\dfrac{1.2K_U}{T_U}$	$\dfrac{0.6K_U T_U}{8}$

example: determine the PID controller using Ziegler-Nichols tuning method for the system

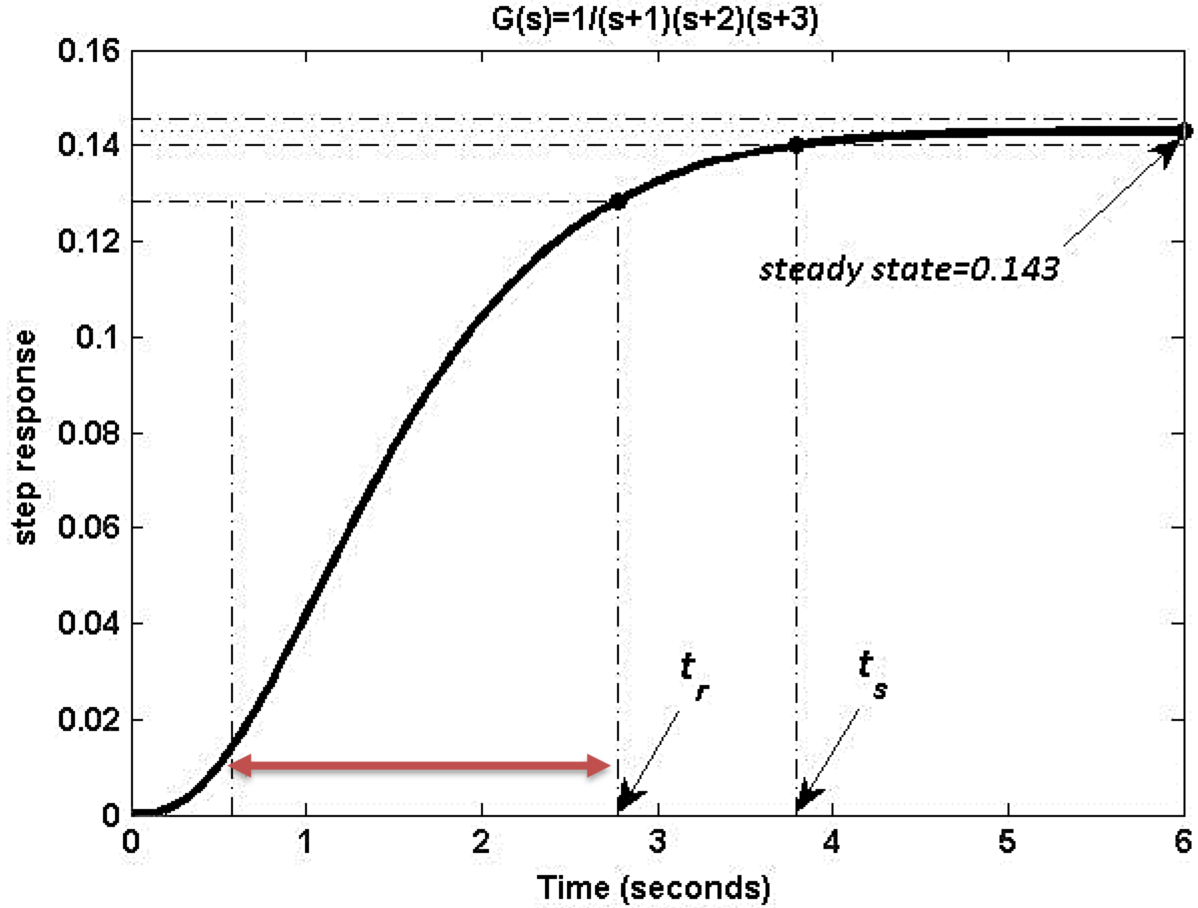
$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

- **Steady state error for *K=1*:**

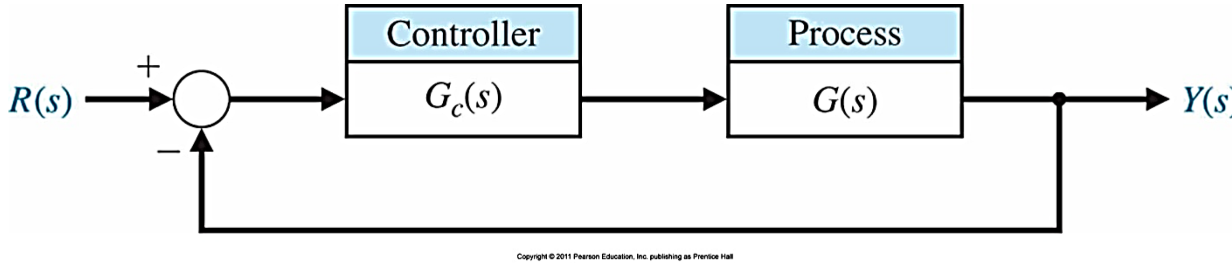
$$E_{ss} = 1/(1 + K_{Pos})$$

$$\boxed{K_{Pos} = \lim_{s \to 0} G(s) = 1/6}$$

$$E_{ss} = \frac{6}{7} = 1 - 0.143 \neq 0$$



- **Ziegler-Nichols tuning method**



$$\boxed{G_c(s)G(s) = \frac{K_p + K_D s + K_I/s}{(s+1)(s+2)(s+3)}}$$

- **Set $K_D = K_I = 0$**

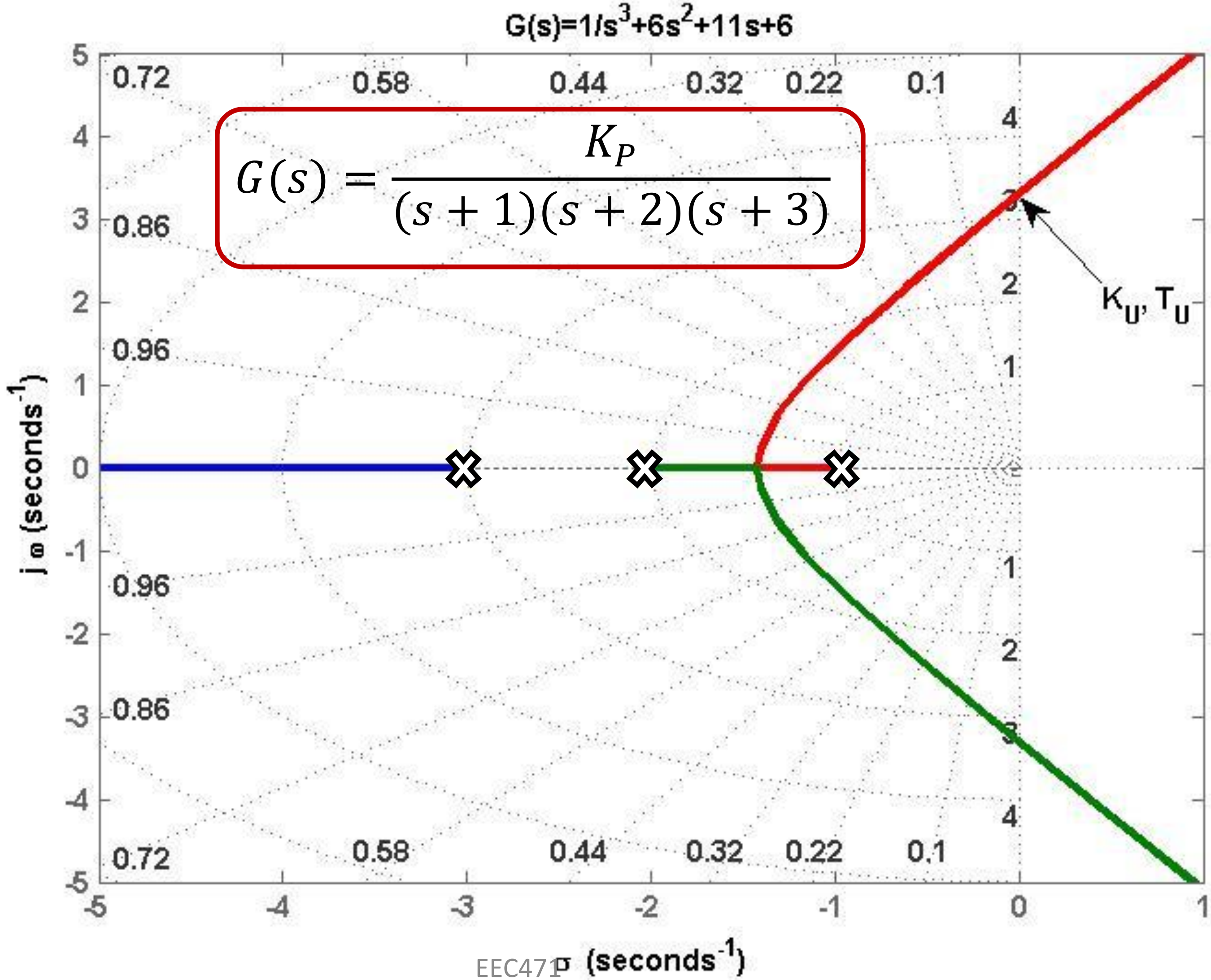
- **Determine the ultimate gain K_U & the ultimate period T_U.**

- **From the Ziegler-Nichols table, determine the PID coefficients K_P, K_D & K_I**

Define $\boldsymbol{K_U}$, the **ultimate gain**, the proportional gain at the **border of instability**

- $\boldsymbol{K_U = 60}$

(why?)

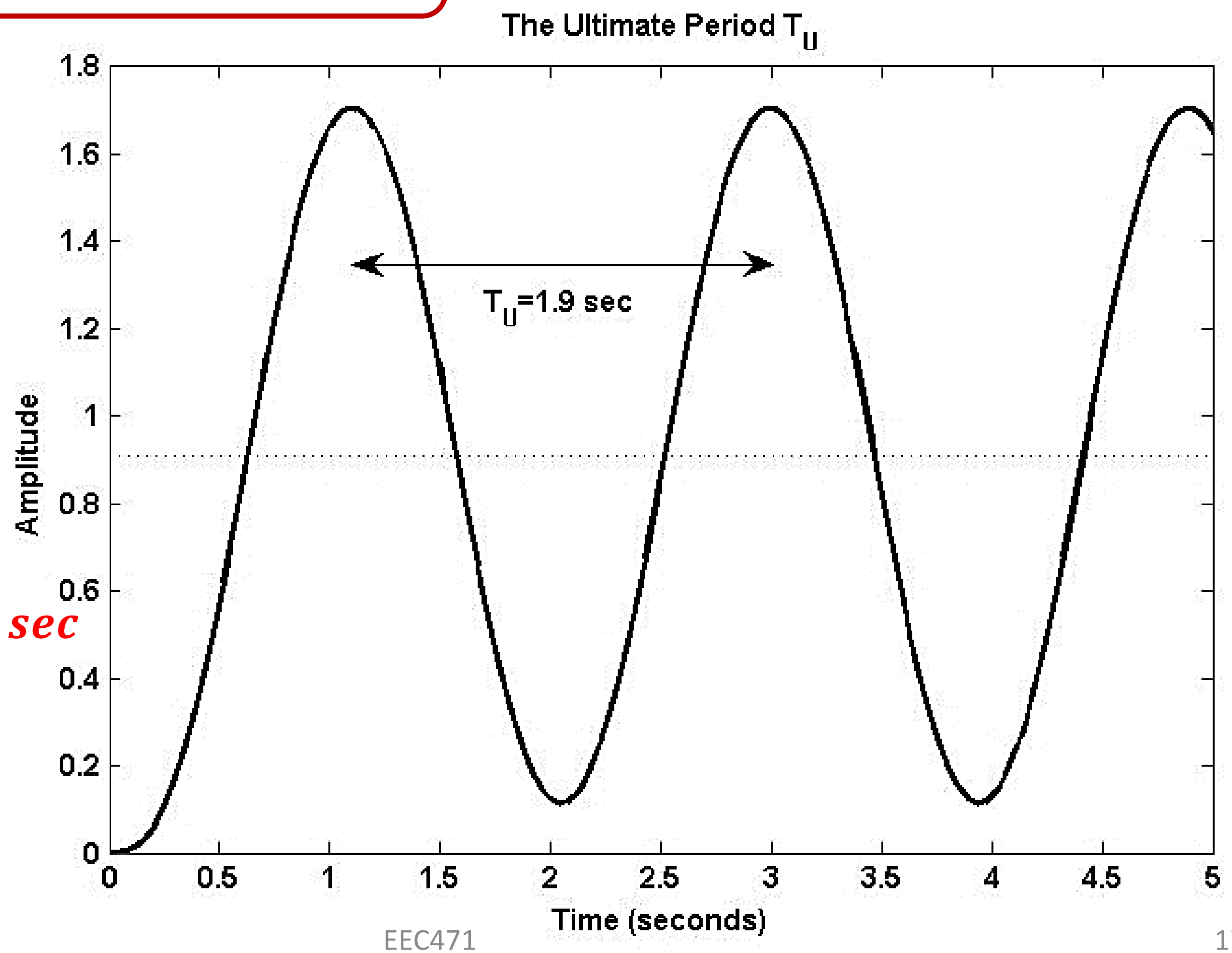


Define T_U, the **ultimate period,** the period of the sustained oscillations **at the border of instability**

$$\boxed{G(s) = \frac{K_P}{(s+1)(s+2)(s+3)}}$$

$T_U = 1.9\ sec$

(why?)



$\omega_n \approx 3.3\ rad/sec$

$\omega_n = \dfrac{2\pi}{T_U} \rightarrow T_U = 1.9\ sec$

Determination of K_U&T_U Analytically

- $\boldsymbol{K_U}$: The characteristic equation

$$1 + GH(s) = 0$$
$$(s+1)(s+2)(s+3) + K = 0$$
$$s^3 + 6s^2 + 11s + (6+K) = 0$$

Using Routh-Hurwitz

$$\boxed{66 = (6+K) \rightarrow K_U = 60}$$

- $\boldsymbol{T_U}$: $s = j\omega$ @ $K = K_U$

$$-j\omega^3 - 6\omega^2 + j11\omega + (6+K_U) = 0$$
$$\text{IMAGINRY} = 0 \Longrightarrow (11 - \omega^2) = 0$$
$$\omega = \sqrt{11} = 2\pi/T_U$$

$$\boxed{T_U = 1.9\ sec}$$

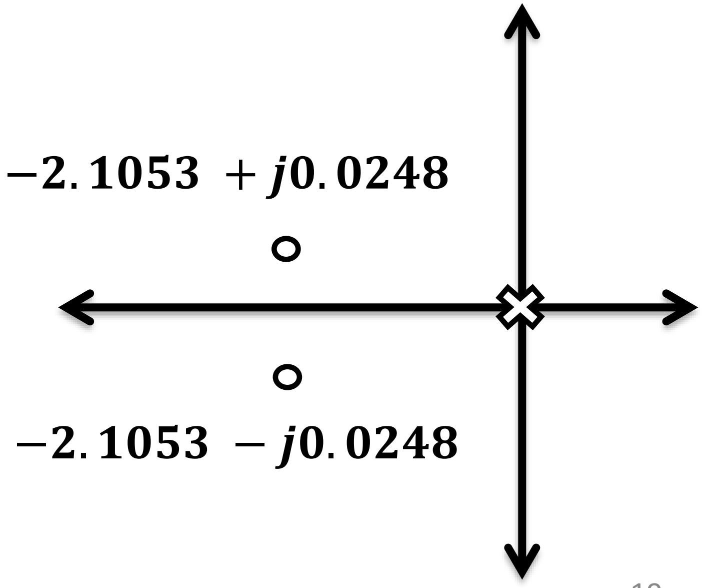
Using **Ziegler-Nichols table**

- $K_p = 0.6K_U = 36$
- $K_I = 1.2\, K_U/T_U = 37.9$
- $K_D = 0.6\, K_U T_U/8 = 8.55$

PID controller

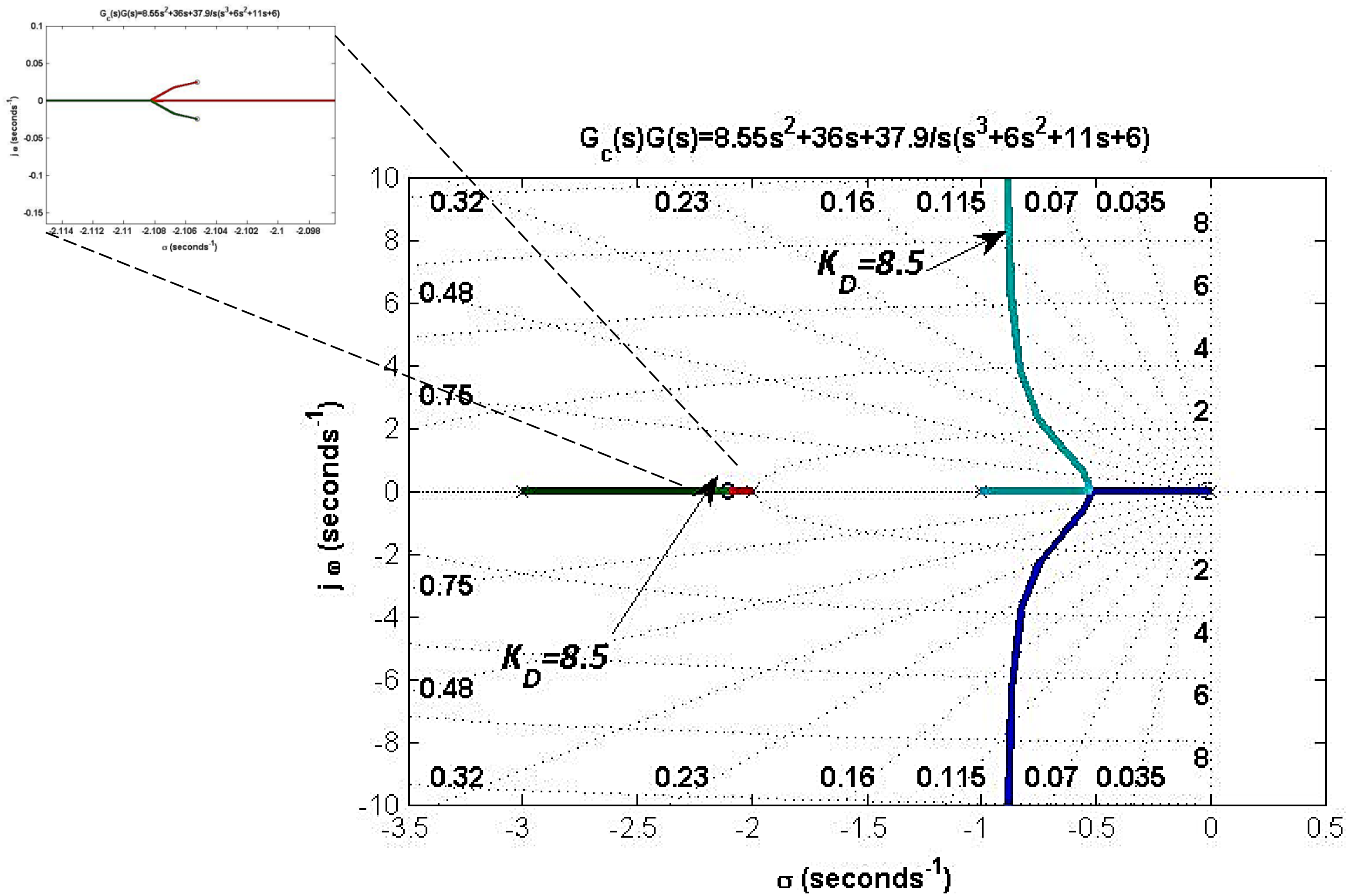
$$G_c(s) = K_P + K_D s + K_I/s$$

- $G_c(s) = 36 + 8.55s + 37.9/s$



- The compensated system

$$G_c(s)G(s) = \frac{8.55s^2 + 36s + 37.9}{s(s^3 + 6s^2 + 11s + 6)}$$

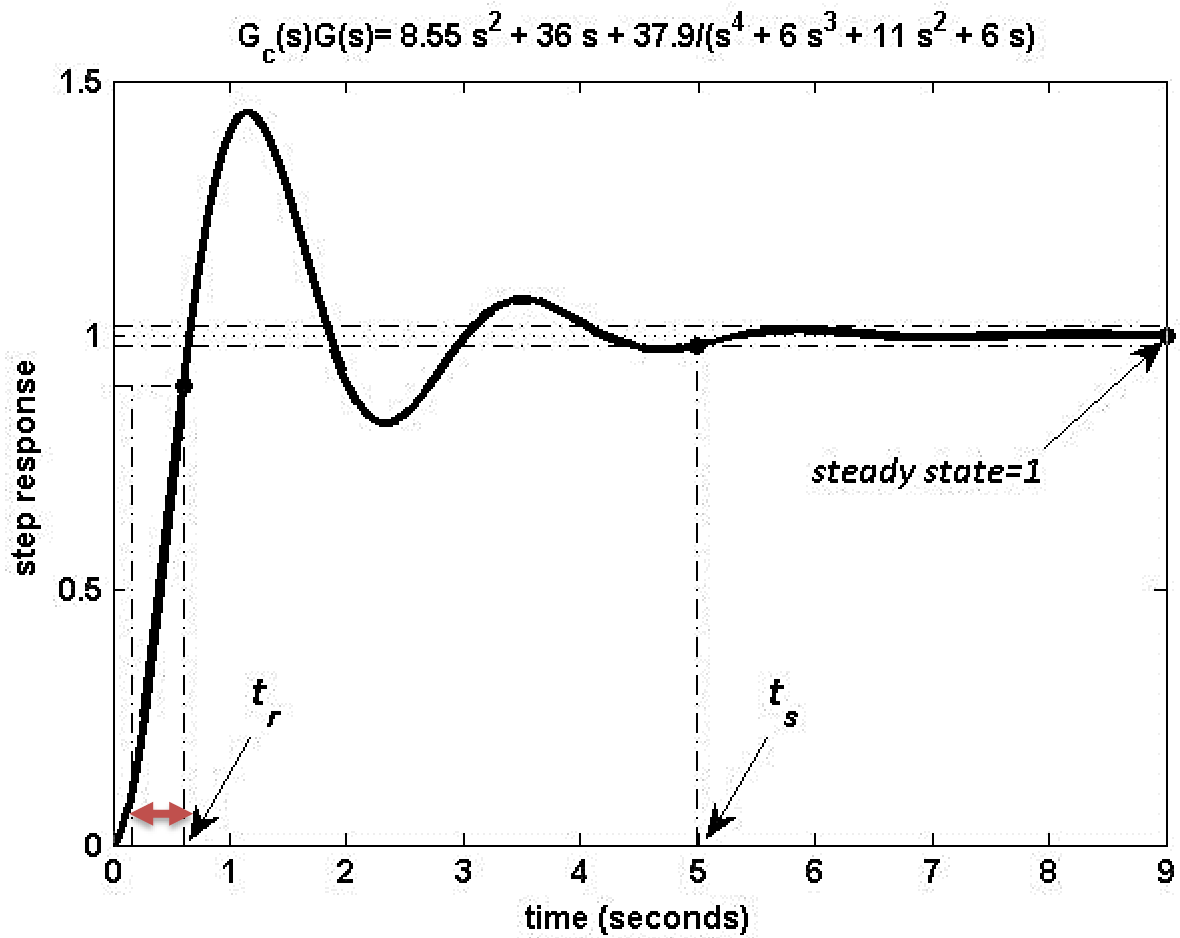


Step response of the compensated system

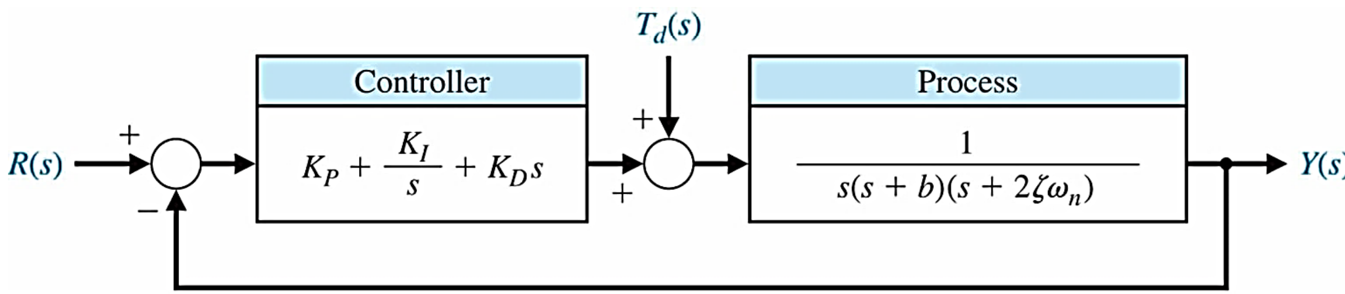
- Steady state error has been improved !!

- $K_P = \infty$

- $\boxed{E_{ss} = 0}$

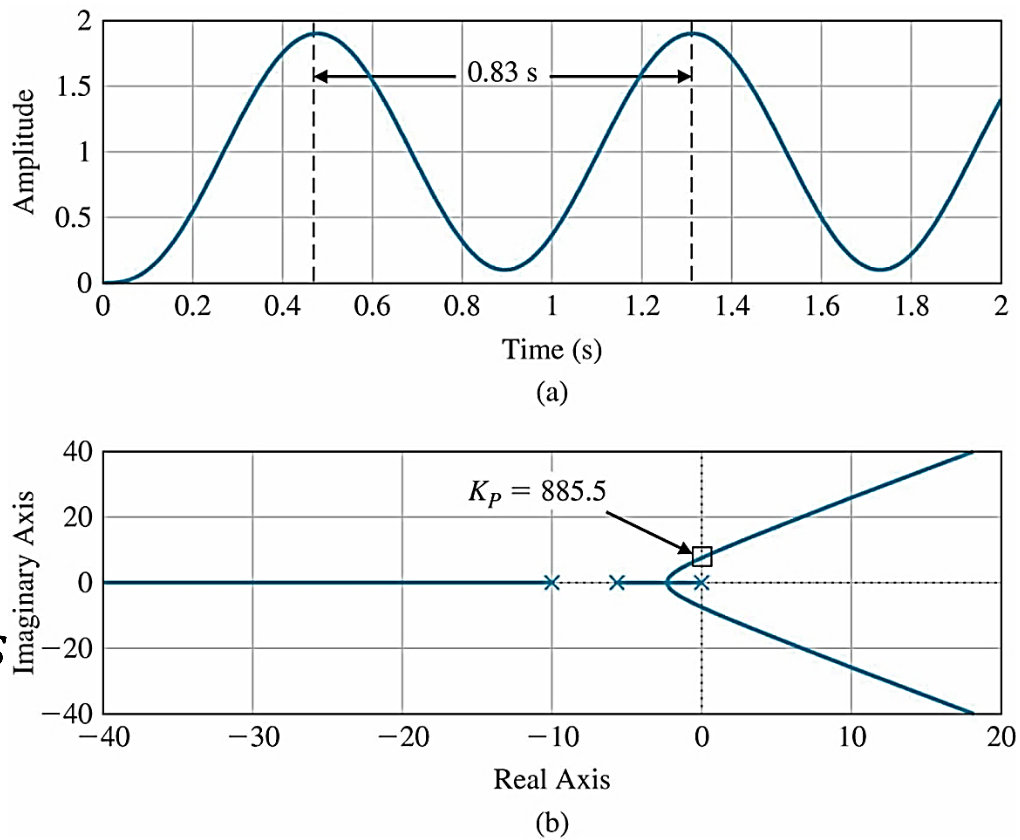


- ***example***



- $b = 10, \xi = 0.707, \omega_n = 4$

$$\boxed{G(s) = \frac{1}{s(s+10)(s+5.66)}}$$



- **Ultimate gain** $K_U = 885.5$

(How?)

- **Ultimate period** $T_U = 0.83\ s$

(How?)

Determination of K_U & T_U Analytically

- $\boldsymbol{K_U}$: The characteristic equation

$$1 + GH(s) = 0$$
$$s(s+10)(s+5.66) + K = 0$$
$$s^3 + 15.66s^2 + 56.6s + K = 0$$

$$\boxed{K_U = 15.66 \times 56.6 = 886.3}$$

- $\boldsymbol{T_U}$: $s = j\omega$ @ $K = K_U$

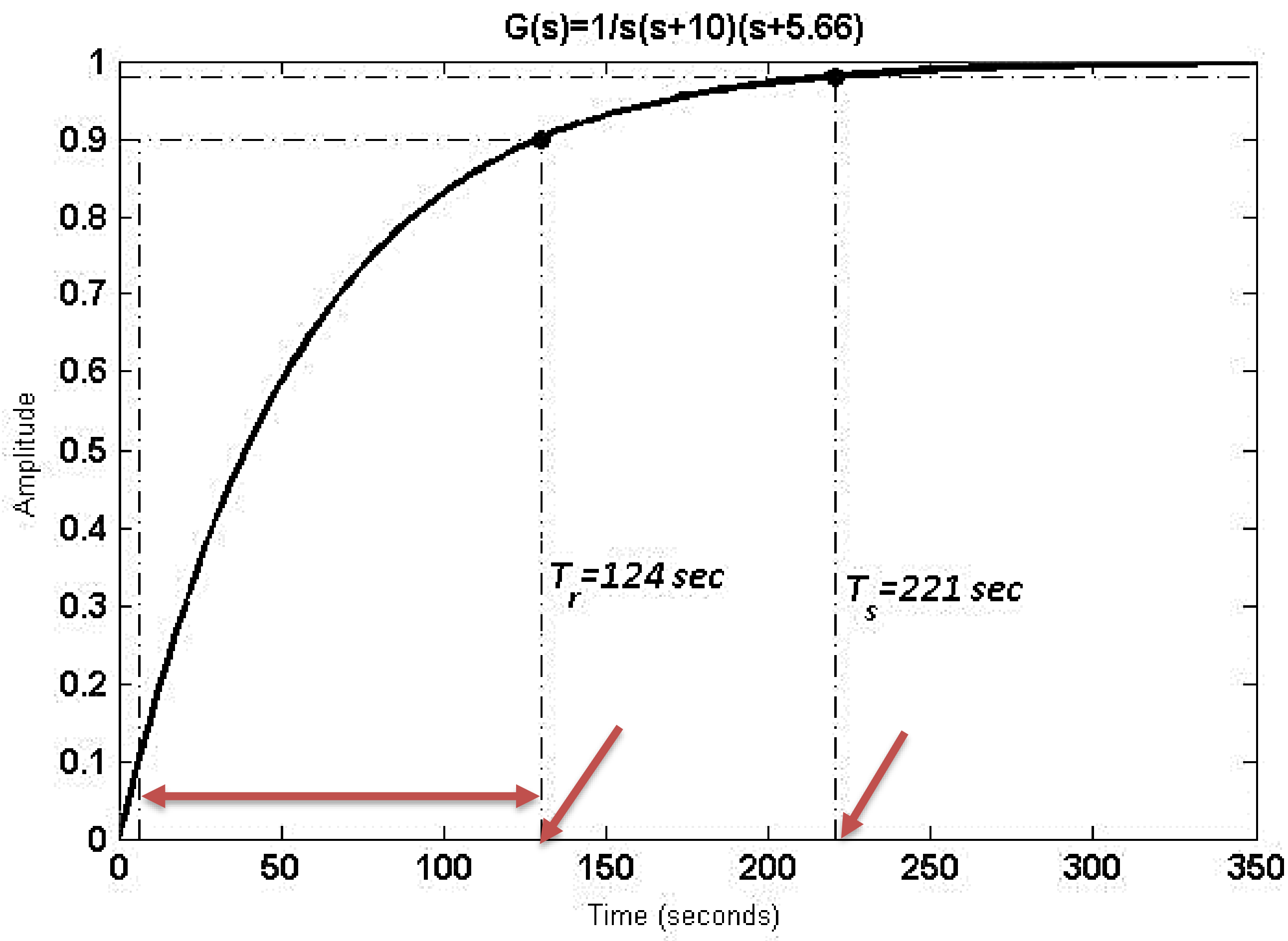
$$j\omega(j\omega + 10)(j\omega + 5.66) + K_U = 0$$
$$\text{IMAGINRY} = 0 \Longrightarrow (56.6 - \omega^2) = 0$$
$$\omega = \sqrt{56.6} = 2\pi/T_U$$

$$\boxed{T_U = 0.83\ sec}$$

- **Step response** of the **uncompensated** system

$$\boxed{G(s) = \frac{1}{s(s+10)(s+5.66)}}$$

- $E_{ss} = 0$



- Using Ziegler-Nichols table

$$K_P = 0.6K_U = 531.3$$

$$K_I = \frac{1.2K_U}{T_U} = 1280.2$$

$$K_D = \frac{0.6K_U T_U}{8} = 55.1$$

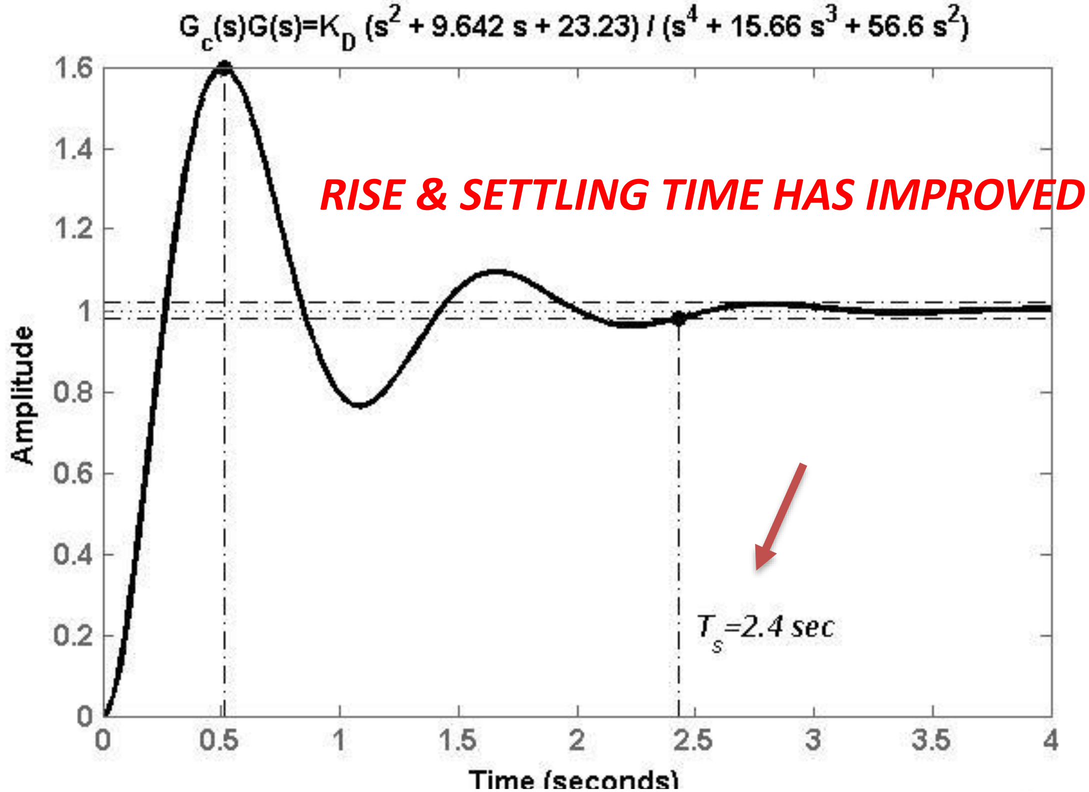


Table 7.7 Ziegler-Nichols PID Tuning Using Ultimate Gain, K_U, and Oscillation Period, P_U

Ziegler-Nichols PID Controller Gain Tuning Using Closed-loop Concepts

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Proportional-plus-integral (PI) $G_c(s) = K_P + \frac{K_I}{s}$	$0.45K_U$	$\frac{0.54K_U}{T_U}$	–
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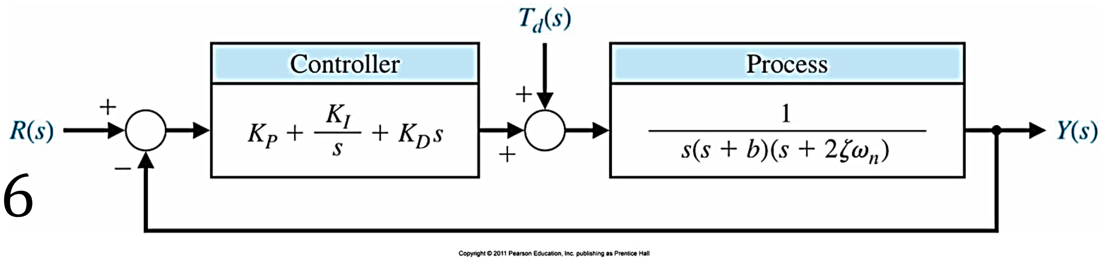
Steady state error – ramp input $\left(\frac{A}{s^2}\right)$

- **Before PID**



$$K_v = \lim_{s \to 0} sG(s) = 1/56.6$$

$$e_{ss} = A/K_v = 56.6 \times A \neq 0$$

$$\boxed{G(s) = \frac{1}{s(s+10)(s+5.66)}}$$

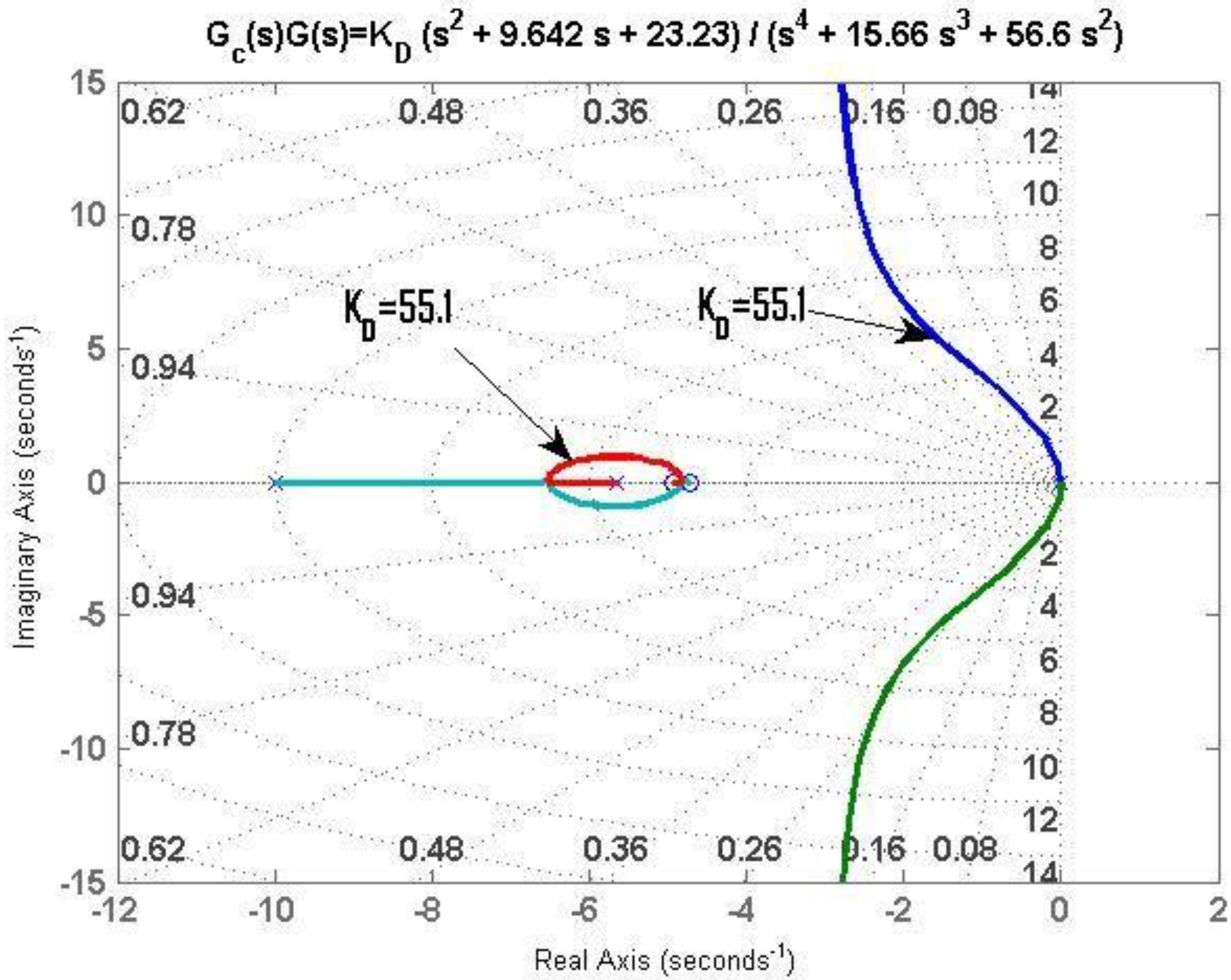
- **After PID**

$$K_v = \lim_{s \to 0} sG_c(s)G(s) = \infty$$

$$e_{ss} = A/K_v = zero$$

Root locus of the compensated system

$$G_c(s)G(s) = \frac{55.1(s^2+9.642s+23.23)}{s^2(s+10\ )(s+5.66)}$$



Automatic Control Systems

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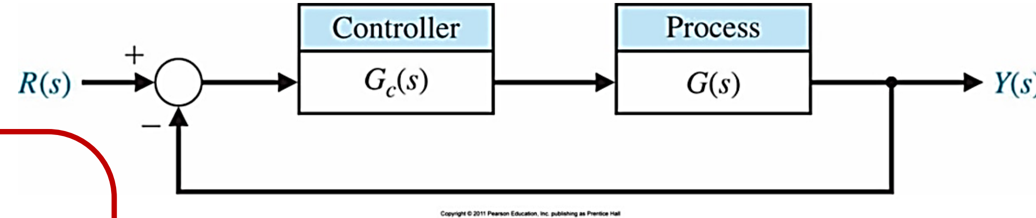
- **Reading: Chapter 7**
  - Section 7.6

How to tune the PID controller coefficients K_D, K_P & K_I?

- **method 2**: Manual PID tuning
  - Root locus & root contour
  - use common sense
  - trial and error

- ***example***



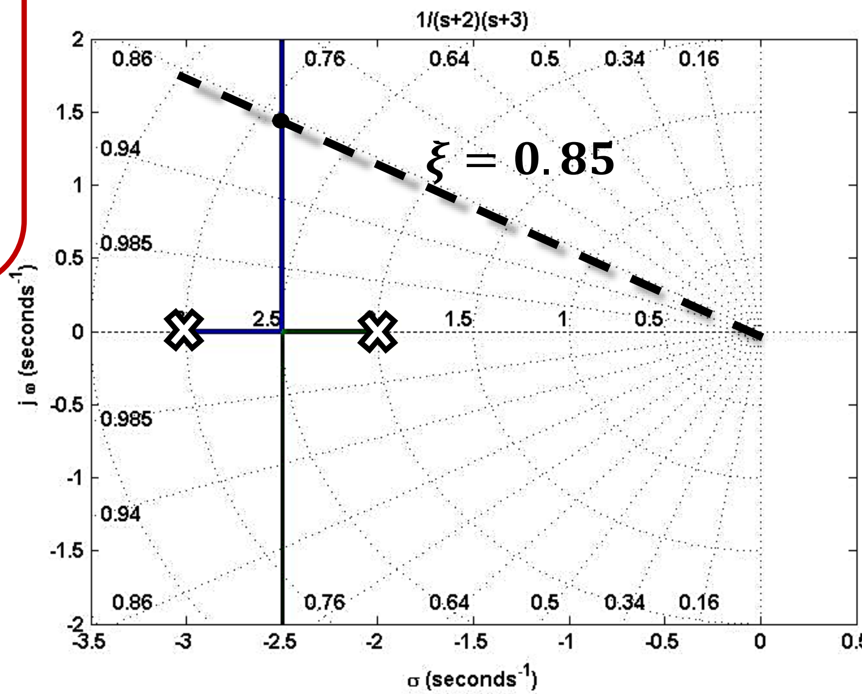
$G(s) = K/(s+2)(s+3)$

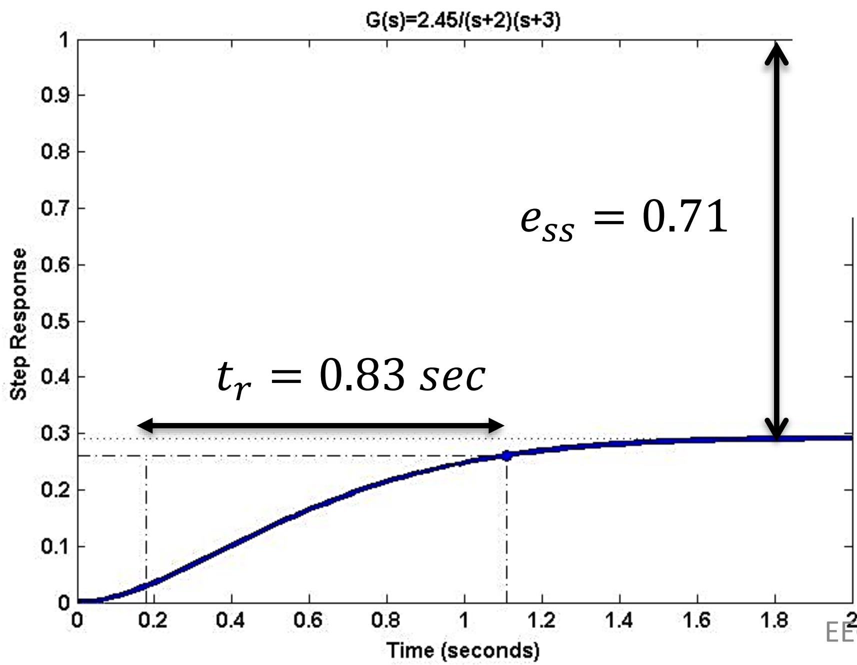
For damping ratio $\xi = 0.85$

- *Find steady state error for step input?*
- *Determine the rise time?*

for $\xi = 0.85 \rightarrow \theta = 33.9^0$,

we have $K = 2.45, \omega_n = 2.9$ **??**





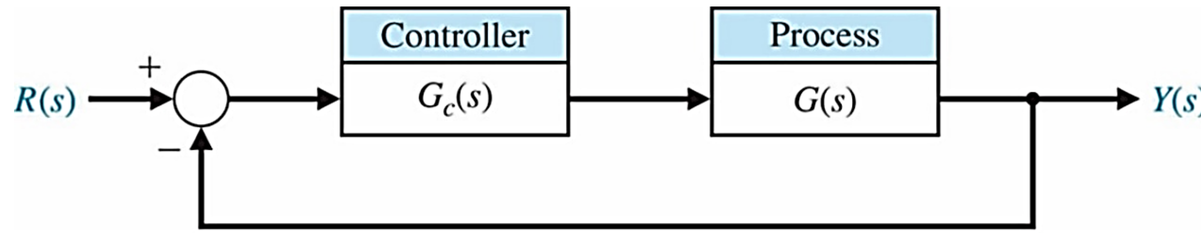
- steady state error due to step input

$$K_p = \lim_{s \to 0} G(s) = 2.45/6 = 0.408$$

$$e_{ss} = 1/(1 + K_p) = 0.71$$

rise time $t_r = \dfrac{2.16\,\xi + 0.6}{\omega_n} = 0.83\ sec$

- Required design specifications



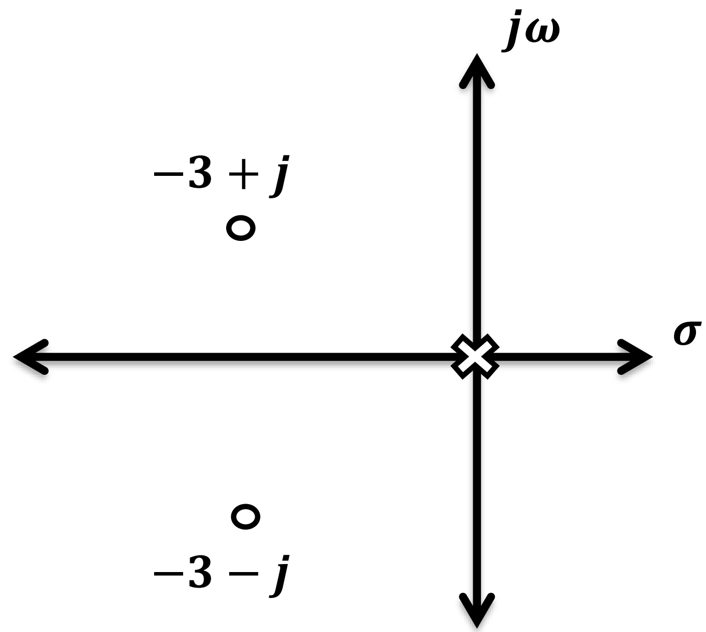
- ***DS1*: *zero steady state error* due to unit step input**
  - *requires a **pole at the origin***
- ***DS2*: *Improve the* rise time**

<u>*Solution:*</u>

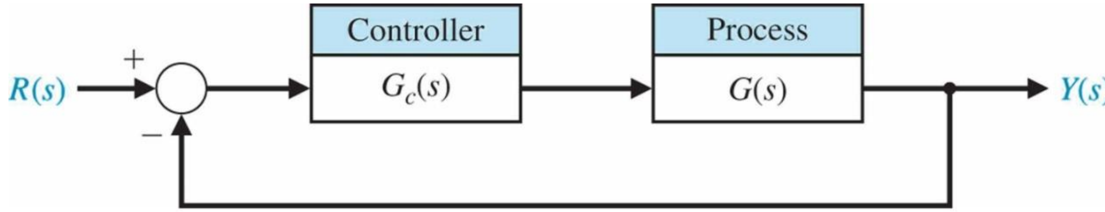
- Consider the **PID** controller

$$\boxed{G_c(s) = \frac{K_D(s^2 + 6s + 10)}{s}}$$

- $K_P/K_D = 6$
- $K_I/K_D = 10$



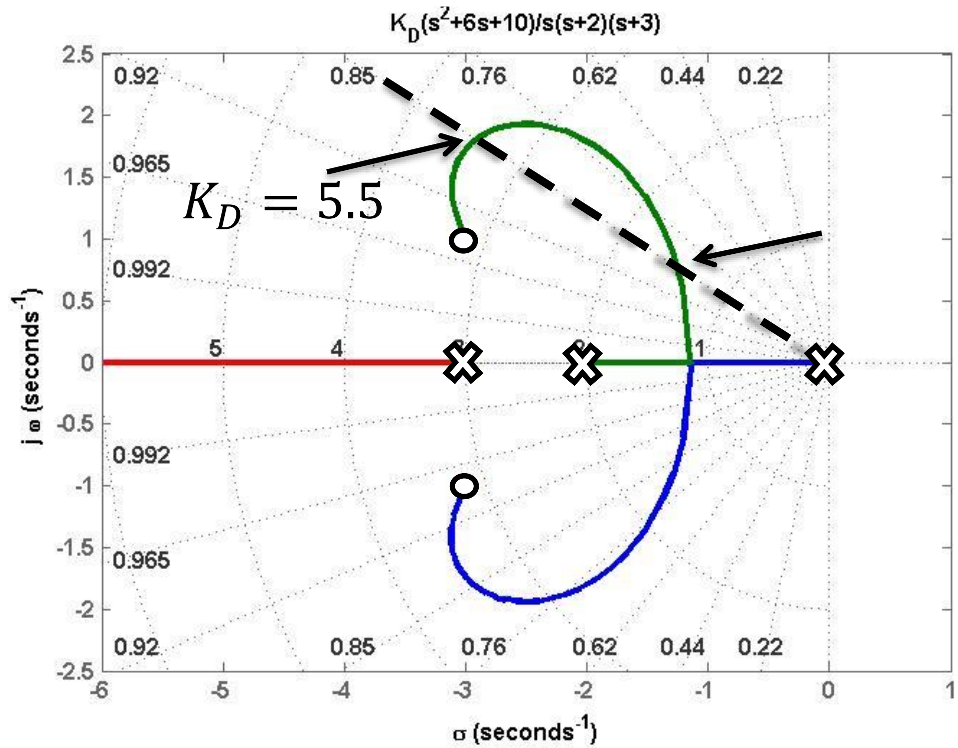
- **With PID controller**

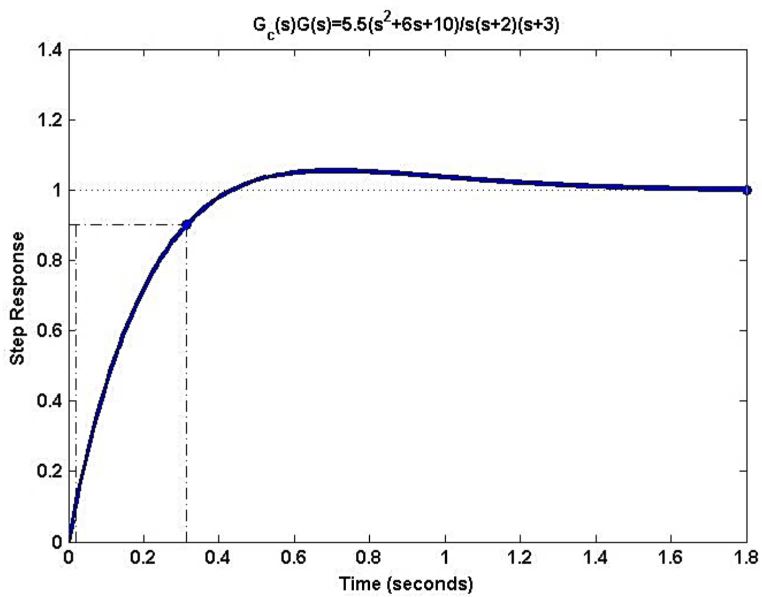


$$G_c(s) = \frac{K_D(s^2+6s+10)}{s}$$

$$\boxed{G_c(s)G(s) = \frac{K_D(s^2+6s+10)}{s(s+2)(s+3)}}$$

- **Which point to choose?**





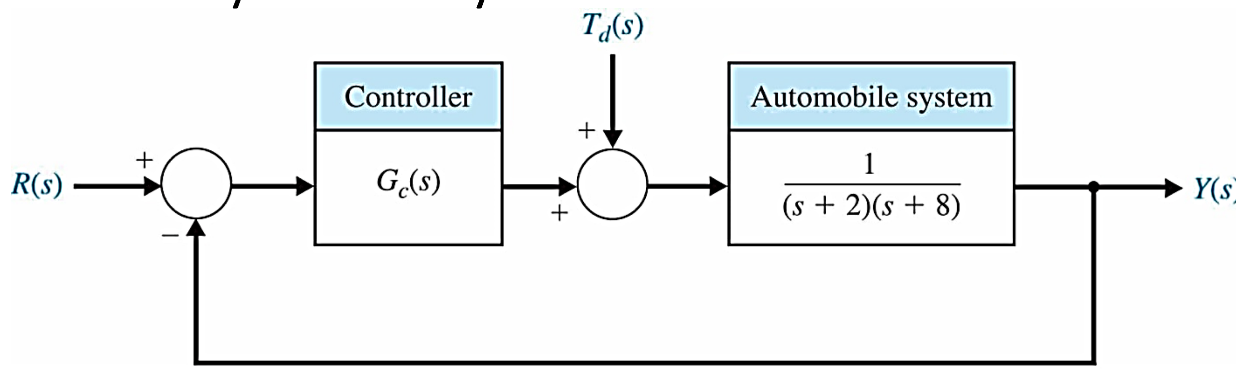
for $\xi = \mathbf{0.85}$, we have $\boldsymbol{K_D} = \mathbf{5.5}$

- the steady state error $\boldsymbol{e_{ss}}(\infty) = \mathbf{0}$ ***(why?)***

- rise time $\boldsymbol{t_r} = \mathbf{0.295}\ seconds$

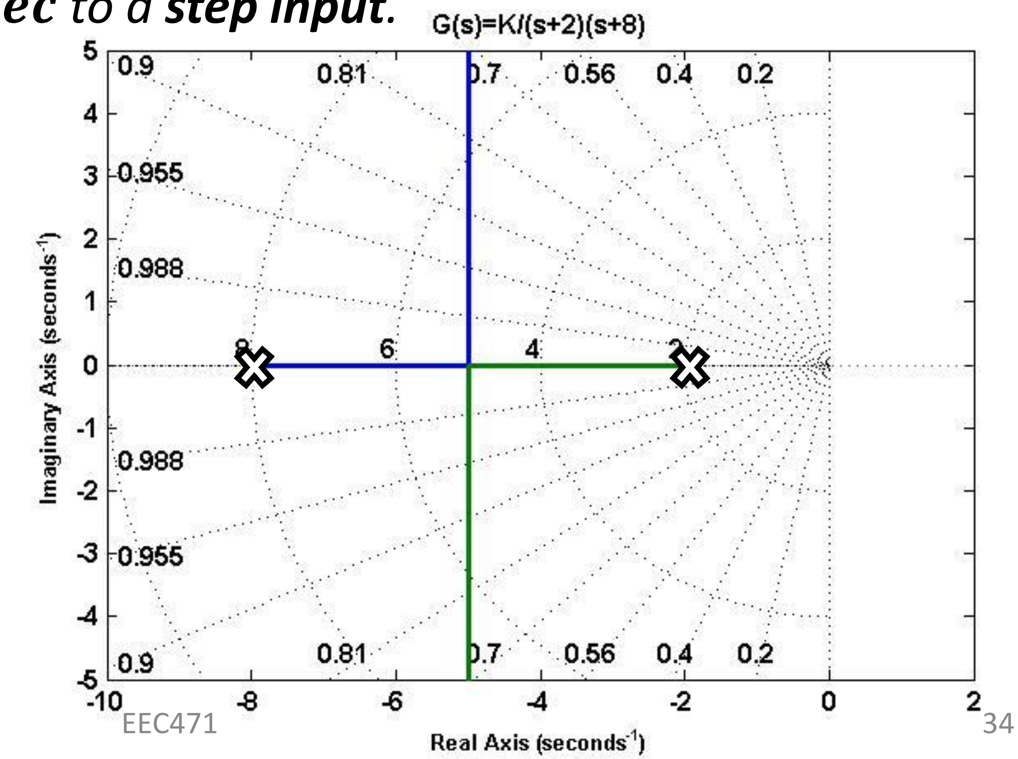
- rise time has been improved ***(why?)***

- ***example*** “vehicle velocity control system”



***Design specifications*:**

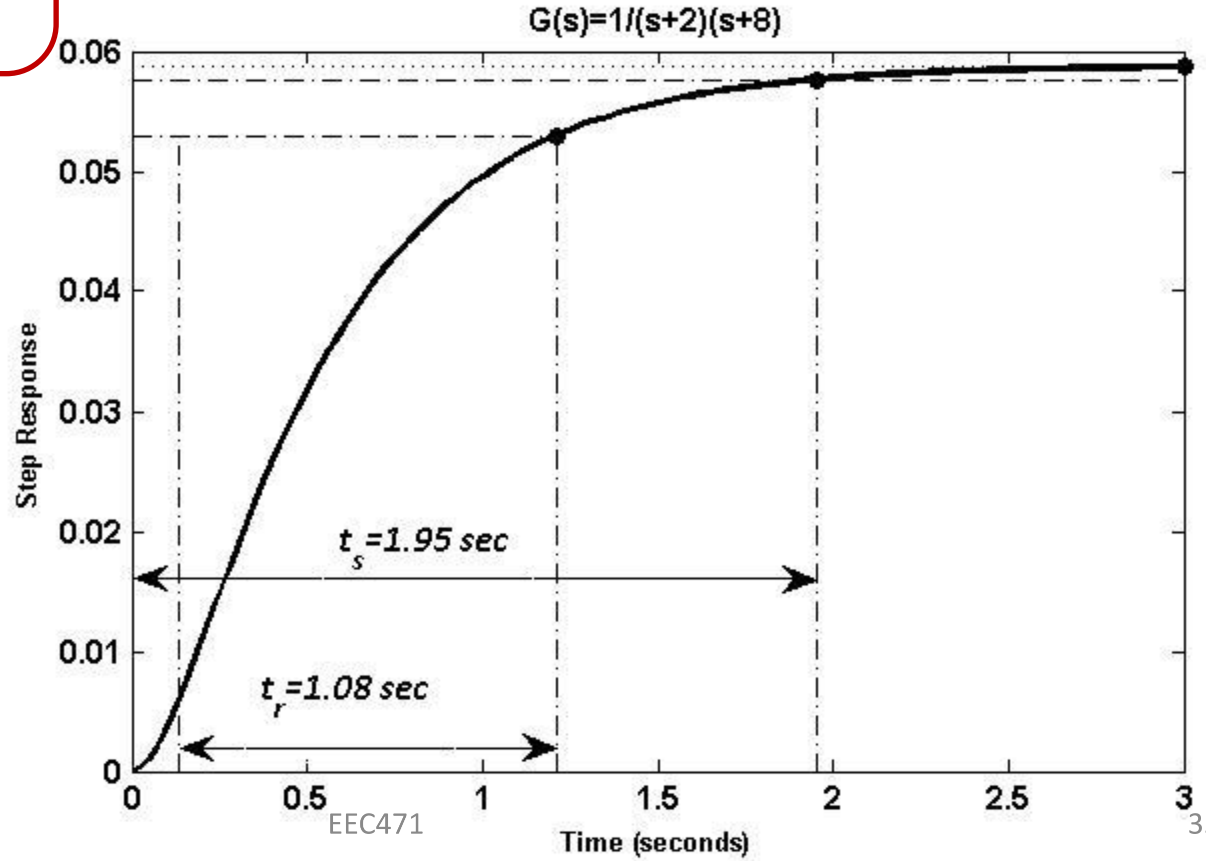
- ***DS1: zero steady state error** for **step input***
- ***DS2**: percent **overshoot** $< 5\%$ to a **step input***
- ***DS3**: 2% **settling time** $< 1.5\ sec$ to a **step input**.*



- *Steady state error for step input*

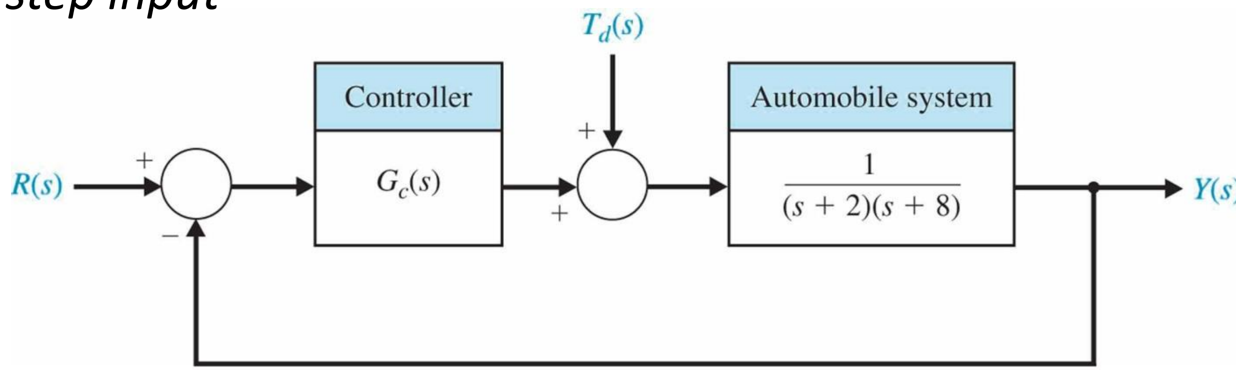
$$e_{ss} = 1/(1+K_p)\,, \qquad K_p = \lim_{s\to 0} G(s) = 1/16\ , \qquad \boxed{e_{ss} = 16/17 \neq 0}$$

- *Settling time = 1.95 sec*
- *Rise time = 1.08 sec*
- *Overshoot = 0 % ??*



- ***DS1**: zero state error for step input*

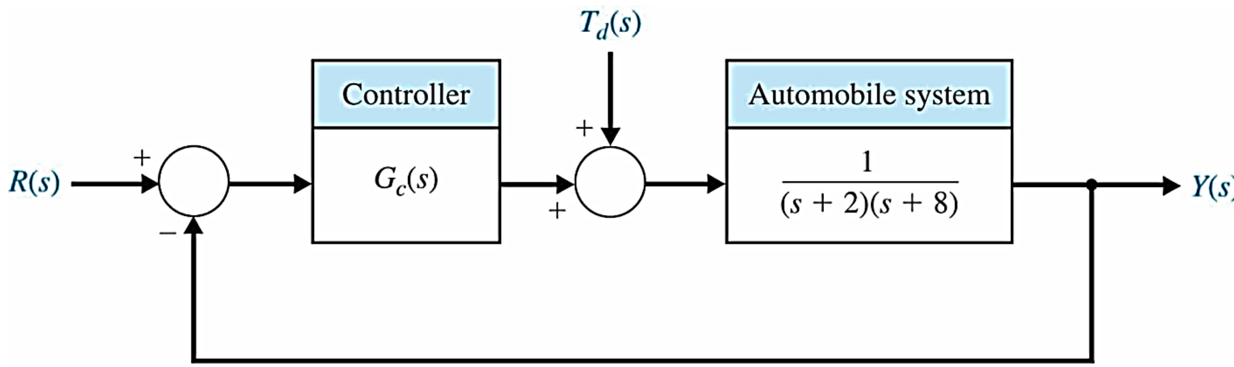
$$G(s) = \frac{1}{(s+2)(s+8)}$$



- Open-loop transfer function $G(s)$ is of **type zero**
- For zero steady state error to a step input we need **type one**
- The controller $G_c(s)$ should have, at least, **one pole at the origin**

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \frac{A}{1+K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\frac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

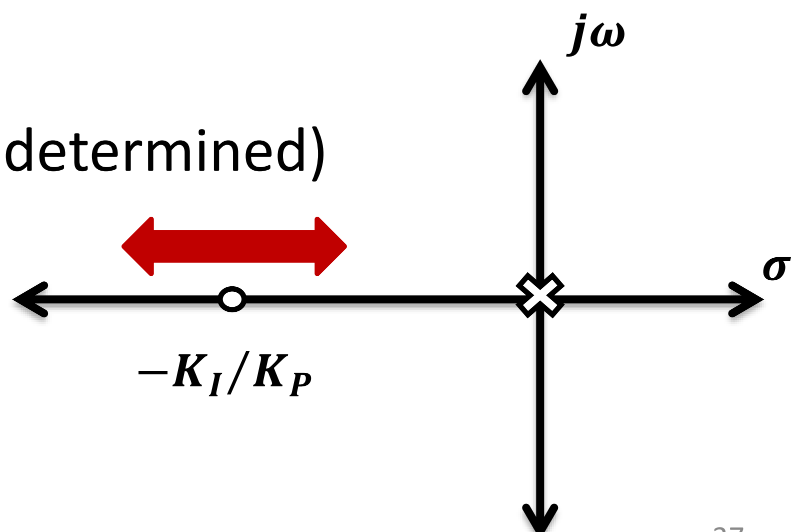


- Consider the **PI** controller:

$$\boxed{G_c(s) = \frac{K_P s + K_I}{s}}$$

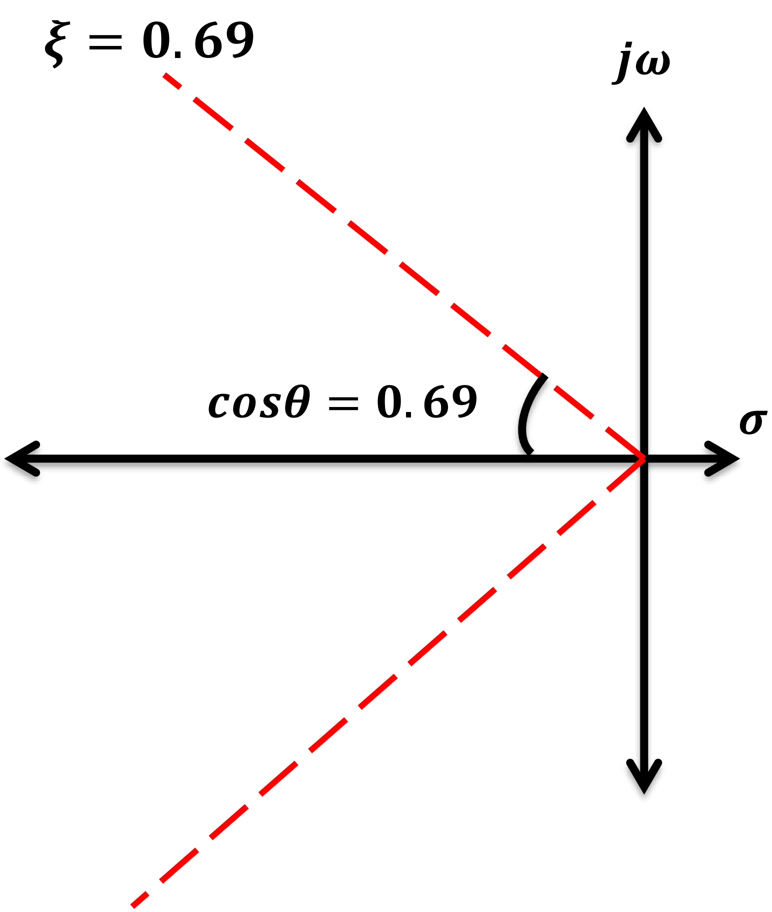
✓ has **one pole at the origin**

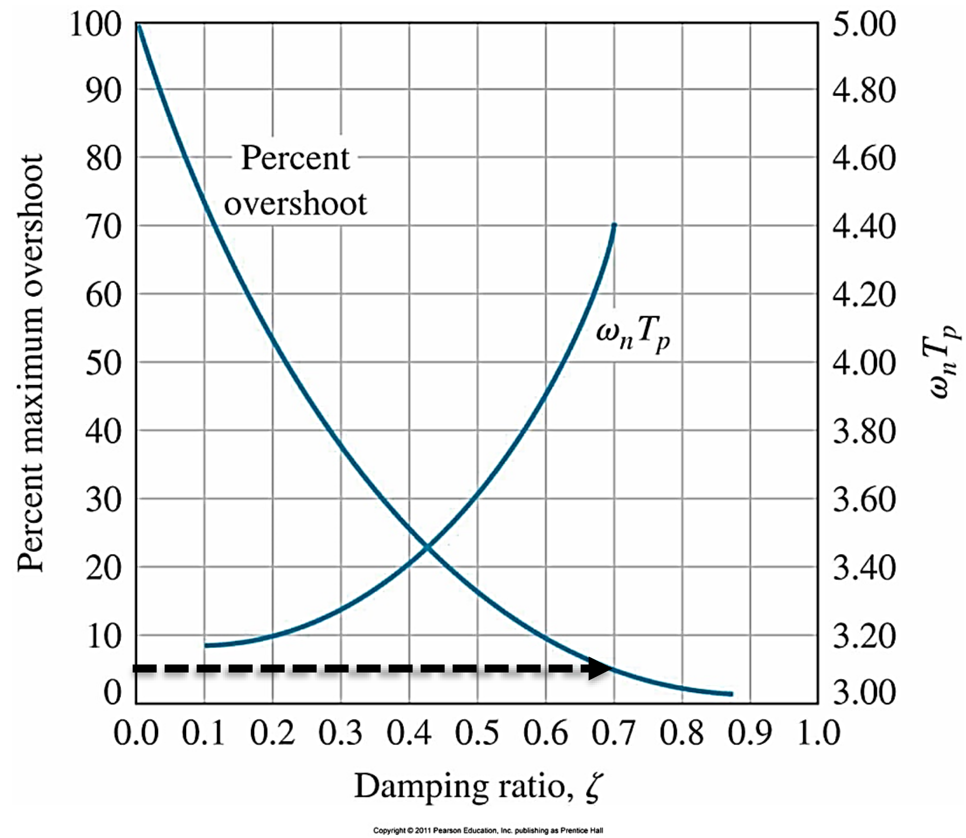
✓ has a **zero** at $\boldsymbol{s = -K_I/K_P}$ (to be determined)



- ***DS2**: percent overshoot less than 5% to a step input*

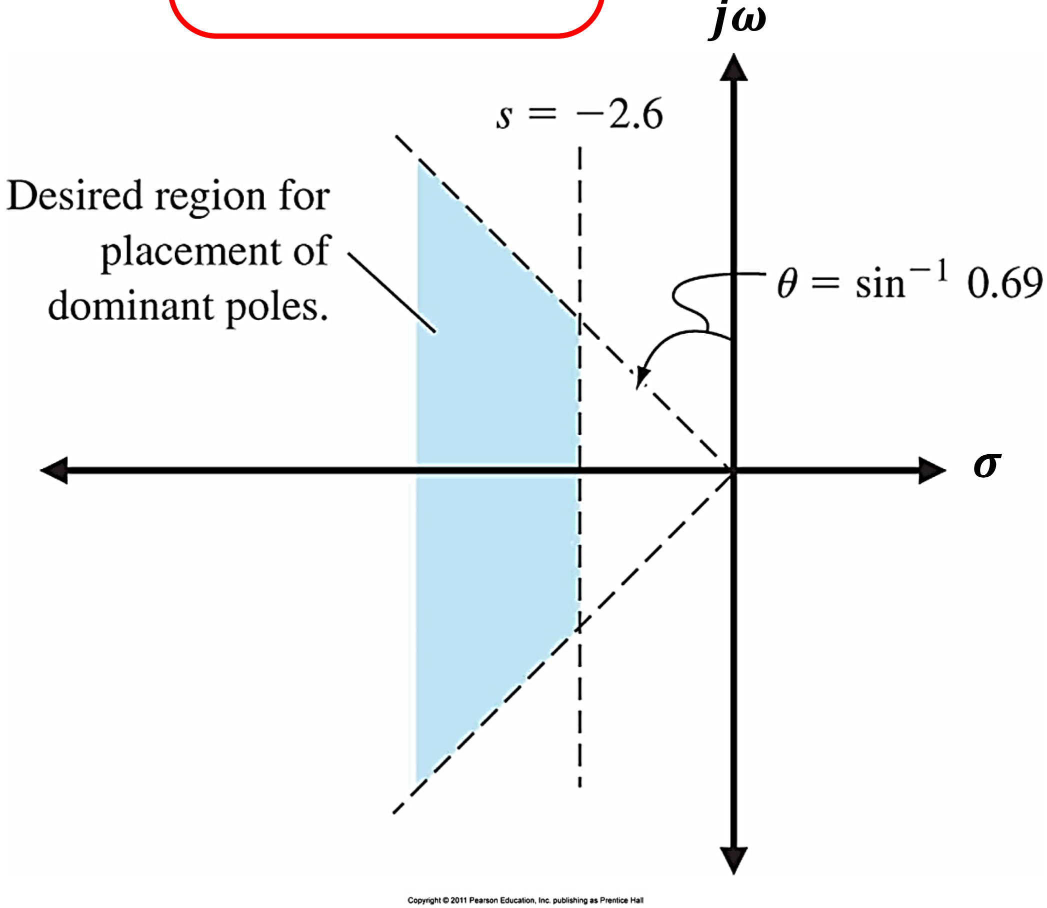
$$\boxed{\begin{aligned} &P.O. \le 5\% \\ &\Rightarrow \zeta \ge 0.69 \end{aligned}}$$





- ***DS3***: *2% settling time less than 1.5 sec to a step input.*

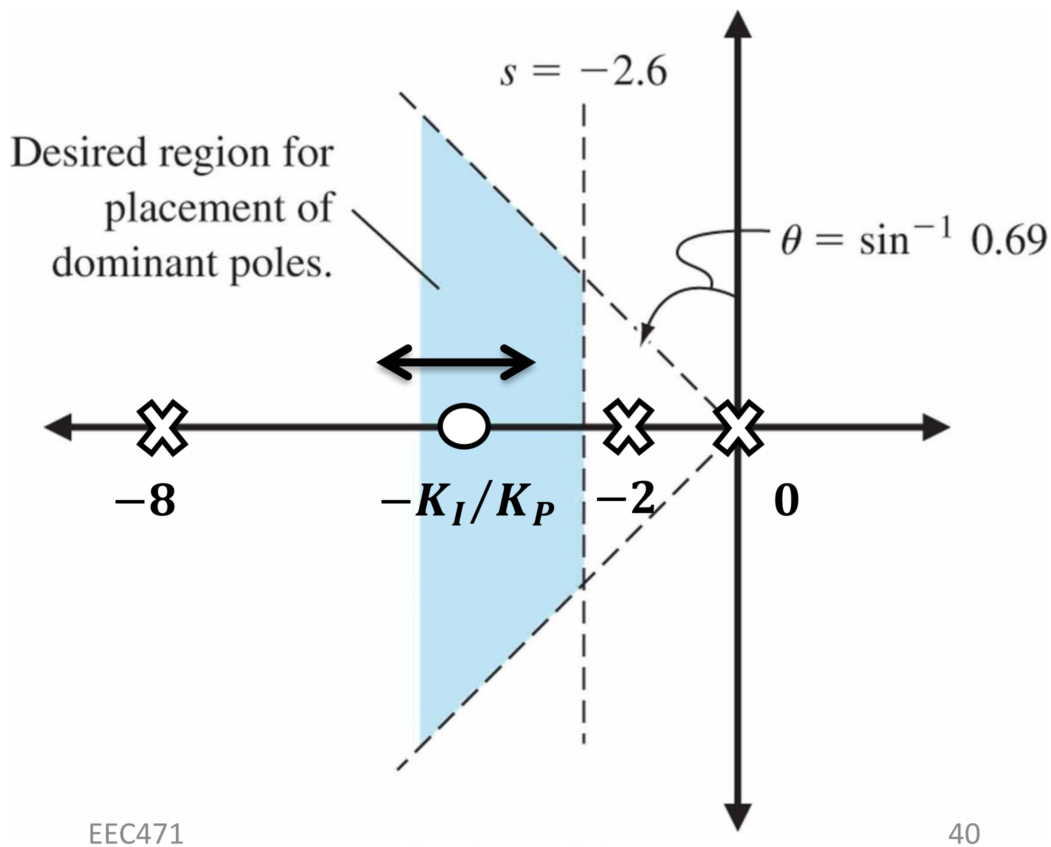
$$T_s \approx \frac{4}{\sigma} \le 1.5$$
$$\Rightarrow \sigma = \zeta\omega_n \ge 2.6$$



Drawing the root locus

- The root locus has **two asymptotes** with angles $\pm 90^o$
- Condition for stability !!

$$G_c(s)G(s) = \frac{K_P(s + K_I/K_P)}{s(s+2)(s+8)}$$



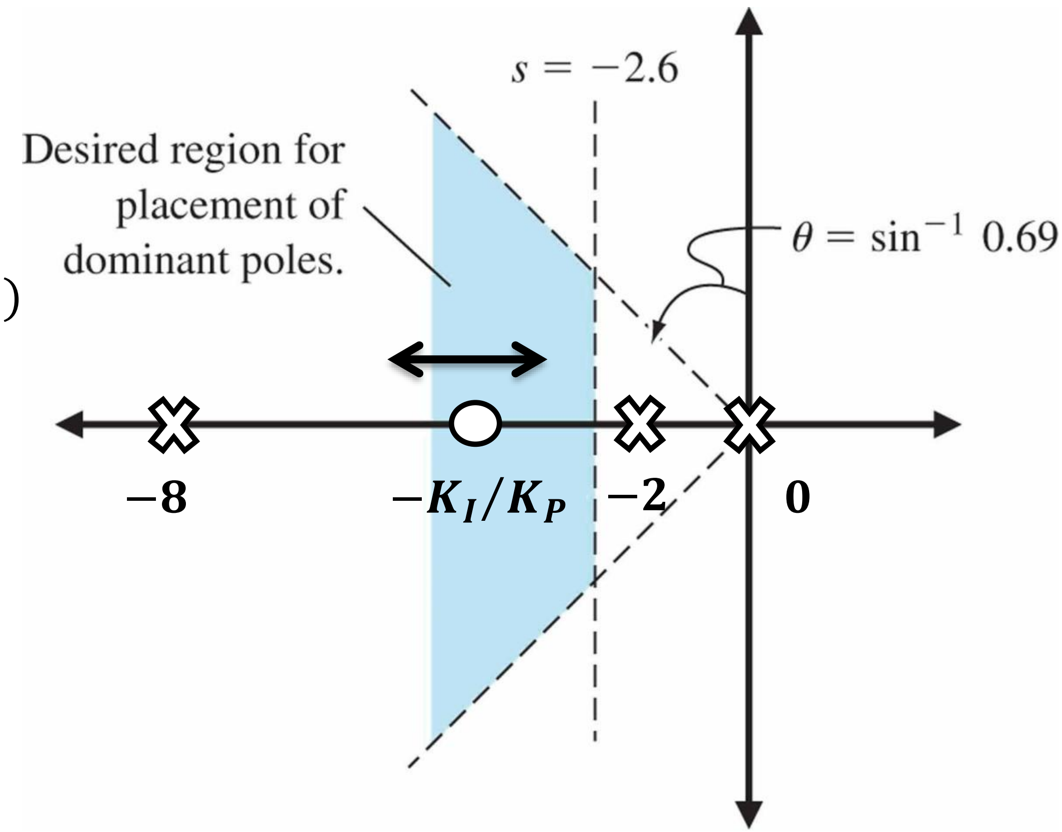
- intersection of the asymptotes

$$\sigma_A = \frac{(0-2-8)-(-K_I/K_P)}{3-1}$$

$$\boxed{\sigma_A = -5 + \frac{1}{2}\frac{K_I}{K_P}}$$

- from DS3

$$\sigma_A < -2.6 \Rightarrow \boxed{\frac{K_I}{K_P} < 4.7} \dashrightarrow (1)$$



	condition
1	$K_I/K_P < 4.7$

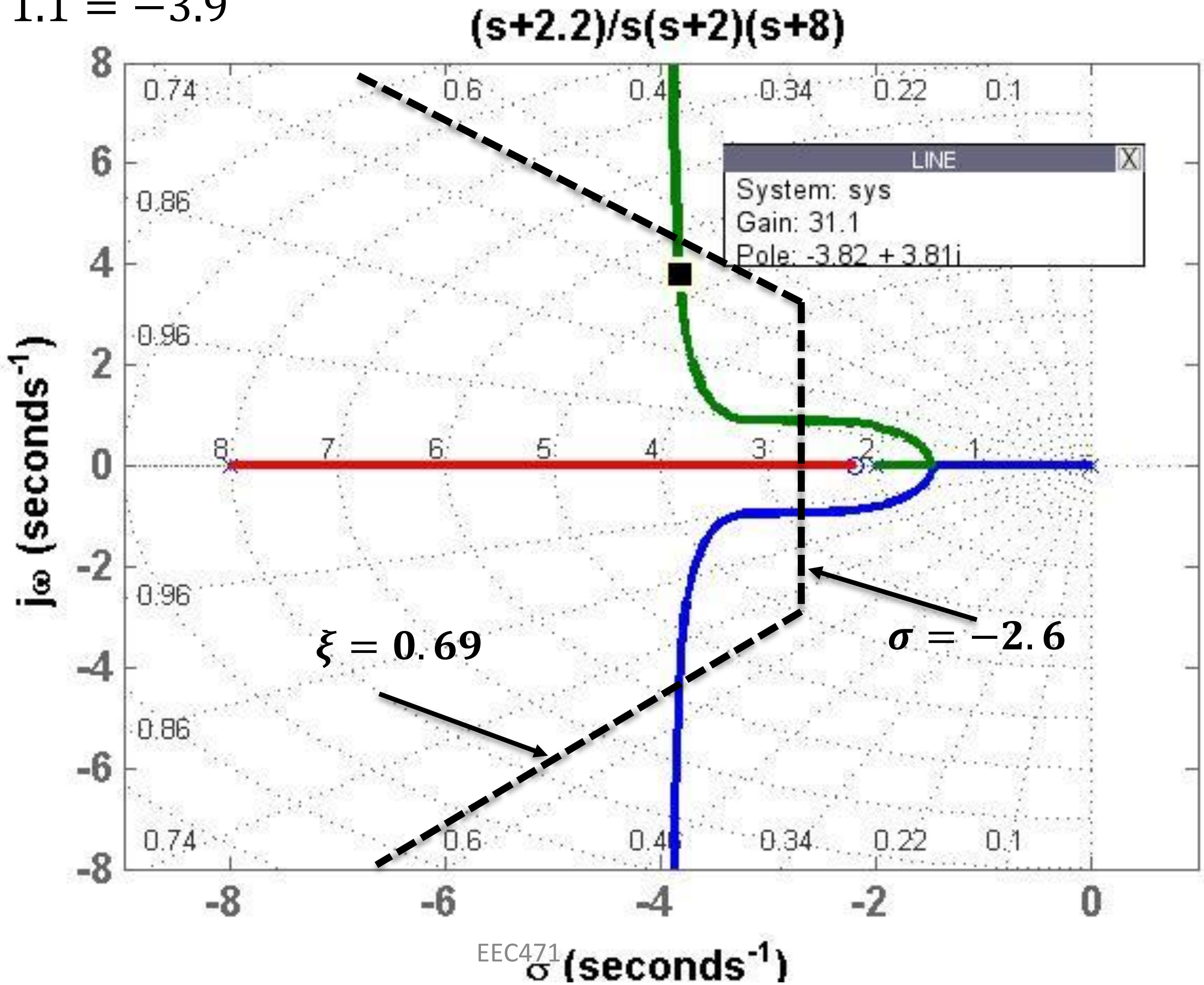
- **Choose**

$$\boxed{\frac{K_I}{K_P} = 2.2 < 4.7}$$

- $\sigma_A = -5 + 1.1 = -3.9$

The characteristic equation becomes

$$\boxed{1 + K_P \frac{(s+2.2)}{s(s+2)(s+8)} = 0}$$



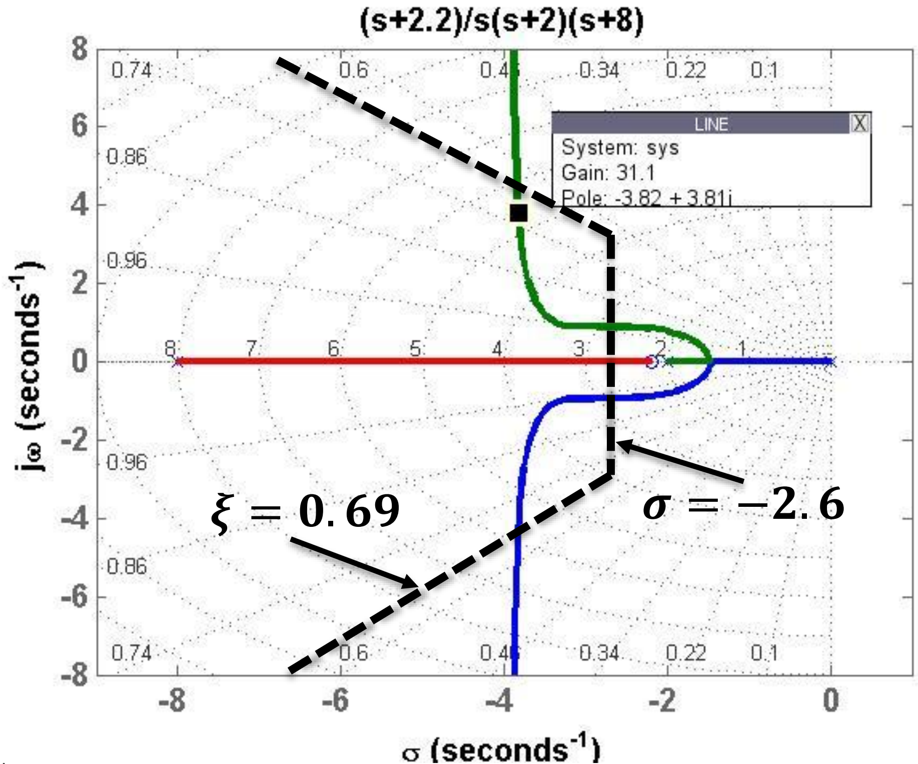
let

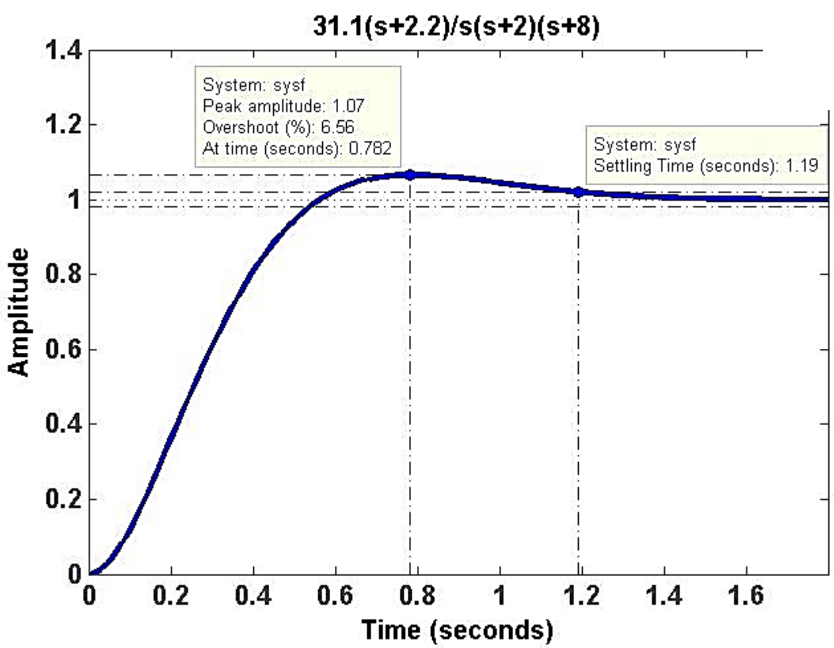
$$\boxed{\frac{K_I}{K_P} = 2.2 < 4.7}$$

Choose

$$\boxed{K_P = 31.1 \Rightarrow K_I = 68.4}$$

(OK)





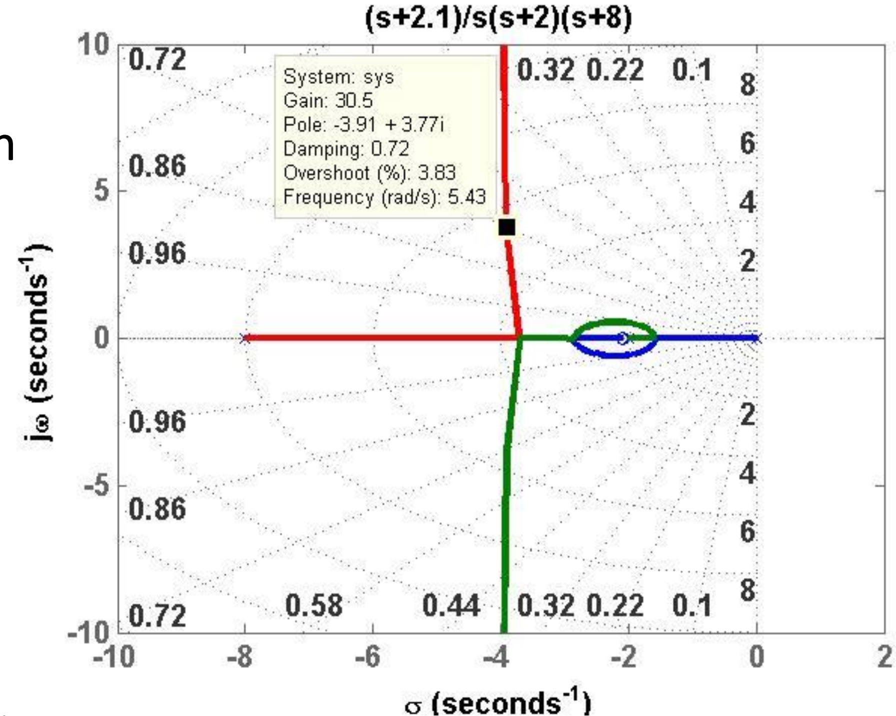
$$\zeta = 0.709 > 0.69$$
$$\zeta\omega_n = 3.82 > 2.6$$
$$P.O. = 6.5\% > 5\%$$
$$t_s = 1.19 < 1.5$$

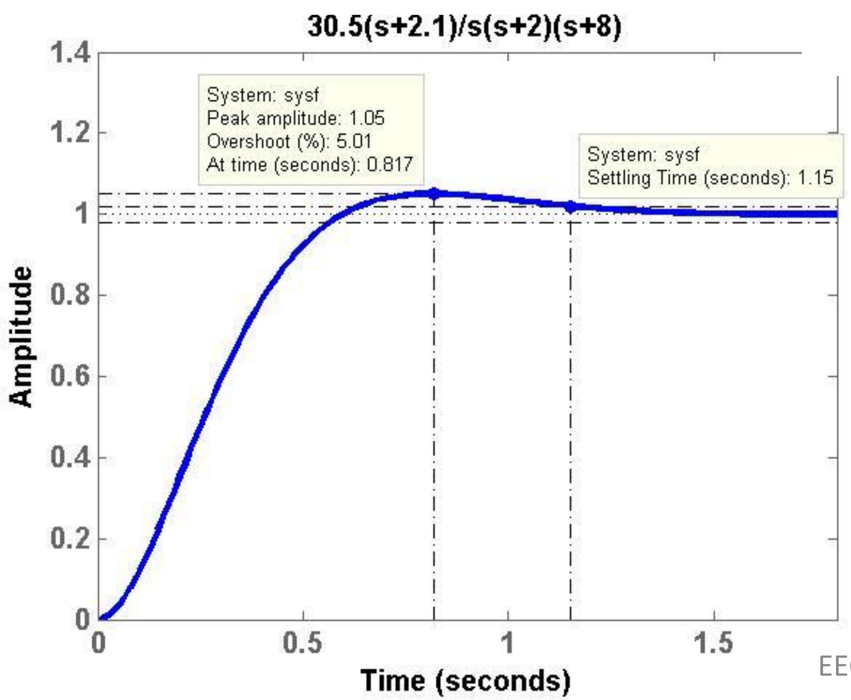
- Let $\boxed{\dfrac{K_I}{K_P} = 2.1 < 4.7}$
- The closed-loop characteristic equation

$$\boxed{1 + K_P \frac{(s + 2.1)}{s(s + 2)(s + 8)} = 0}$$

- choose $\boxed{K_P = 30.5 \Rightarrow K_I = 64.05}$

(OK)





$$\boxed{\begin{gathered} \zeta = 0.72 > 0.69 \\ \zeta\omega_n = 3.91 > 2.6 \\ P.O. = 5.01\% \approx 5\% \\ t_s = 1.15 < 1.5 \end{gathered}}$$