Automatic Control Systems

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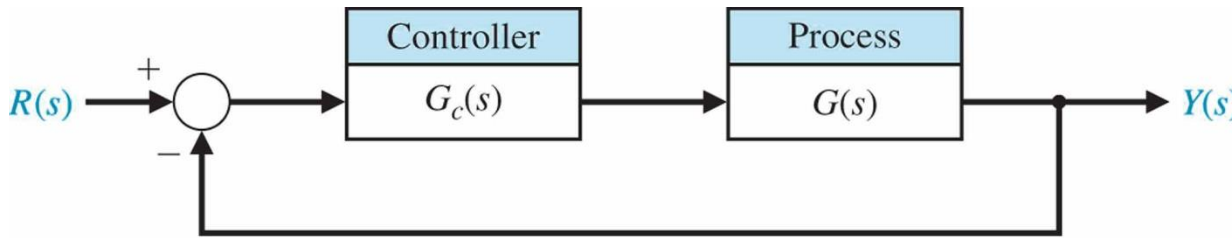
- **Reading: Chapter 7**
  - Section 7.6

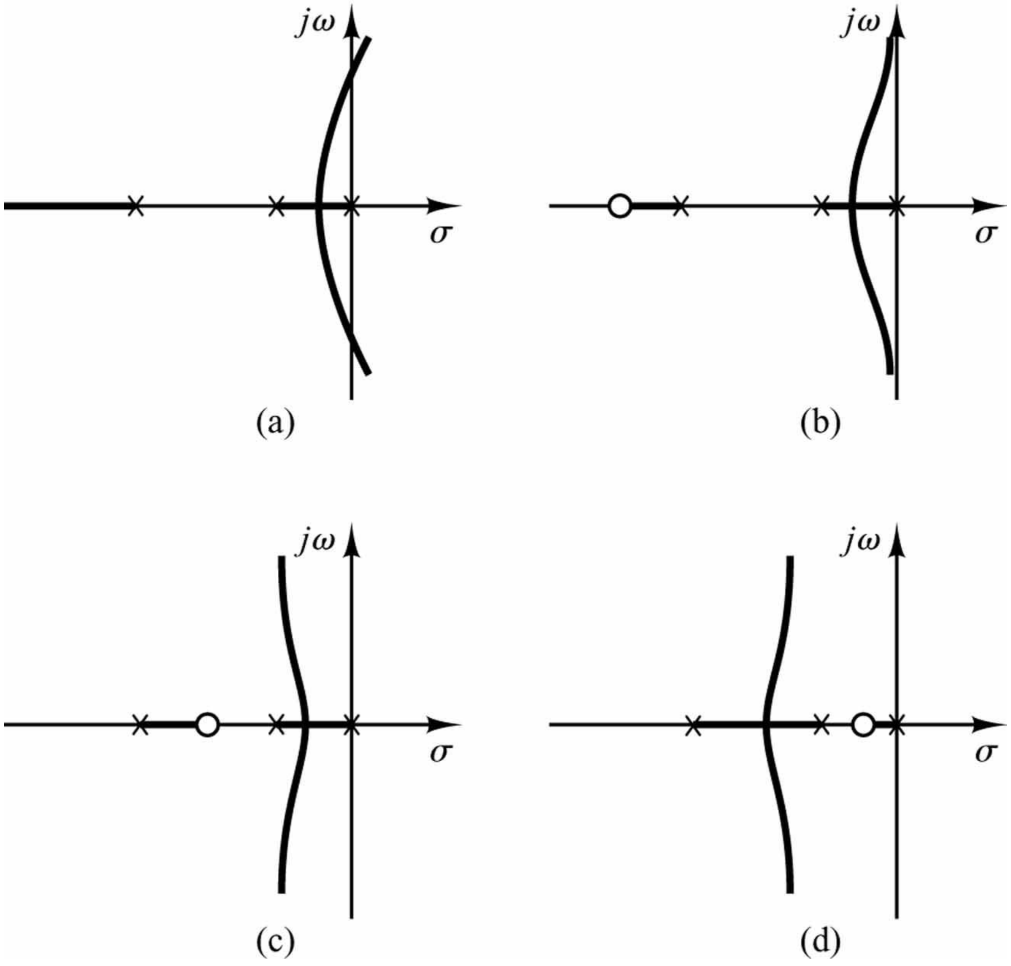
PID Controller



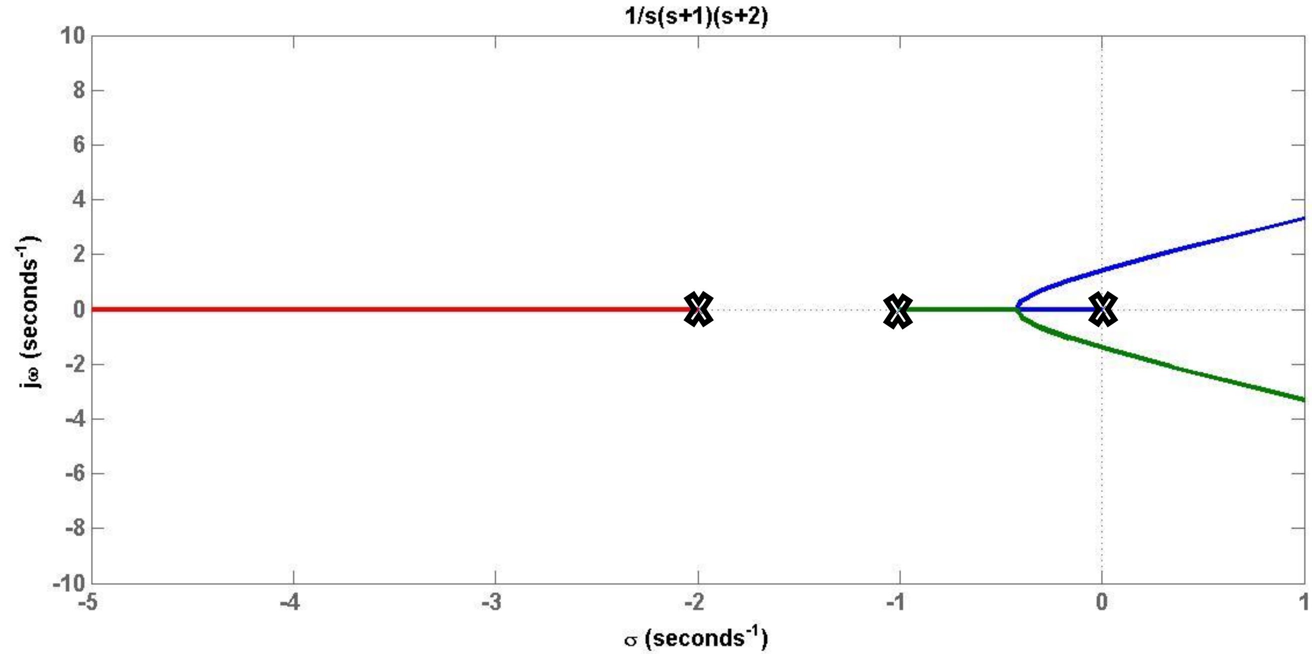
- A controller adds **poles and/or zeros** to the system in order to improve its response (e.g., steady state and transient response)

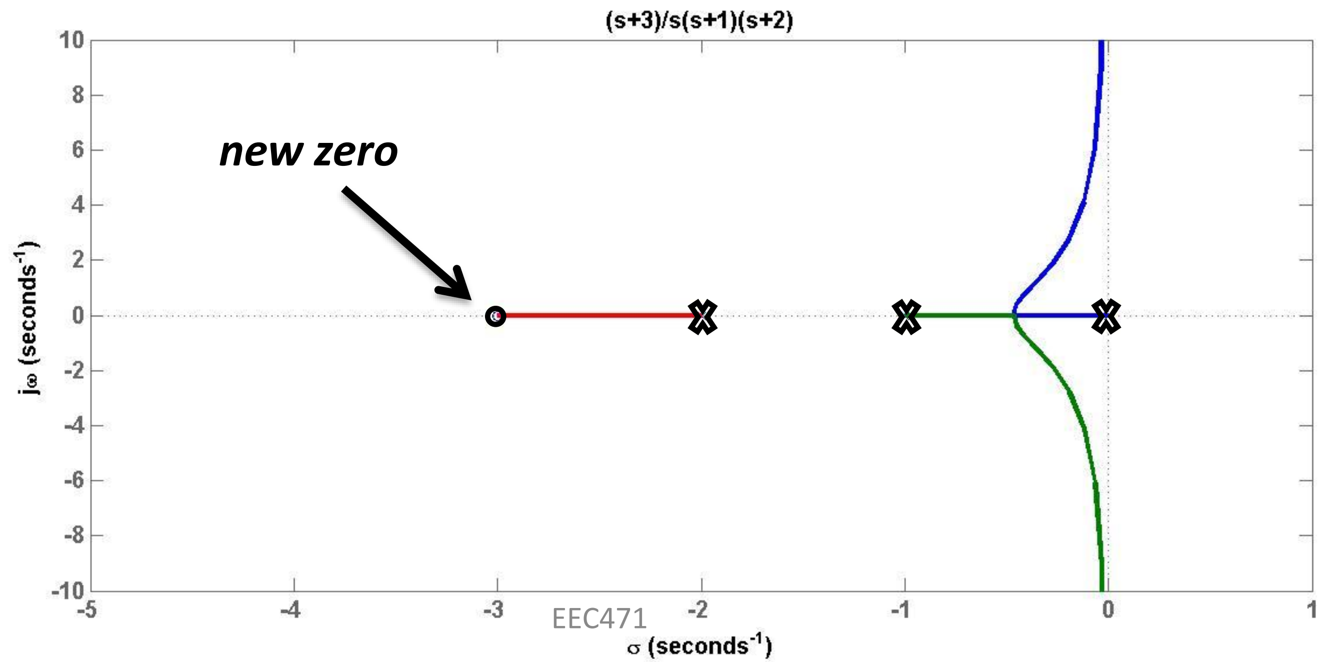
- **PD**: proportional plus derivative
- **PI**: proportional plus integral
- **PID**: proportional integral derivative

- ***Adding <u>left-half plane zeros</u> to the function G(s)H(s) generally has the effect of moving and bending the root loci toward the zero which makes the system <u>more stable.</u>***

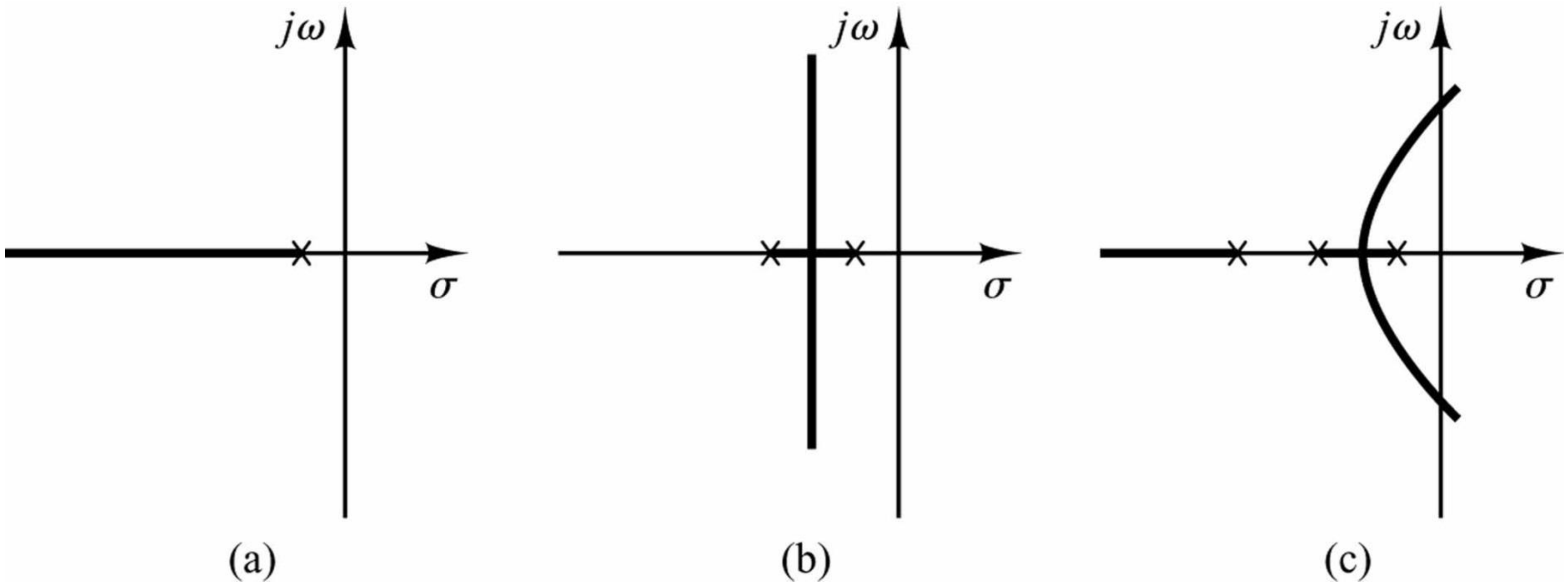


- ***What is the effect on the transient response?***

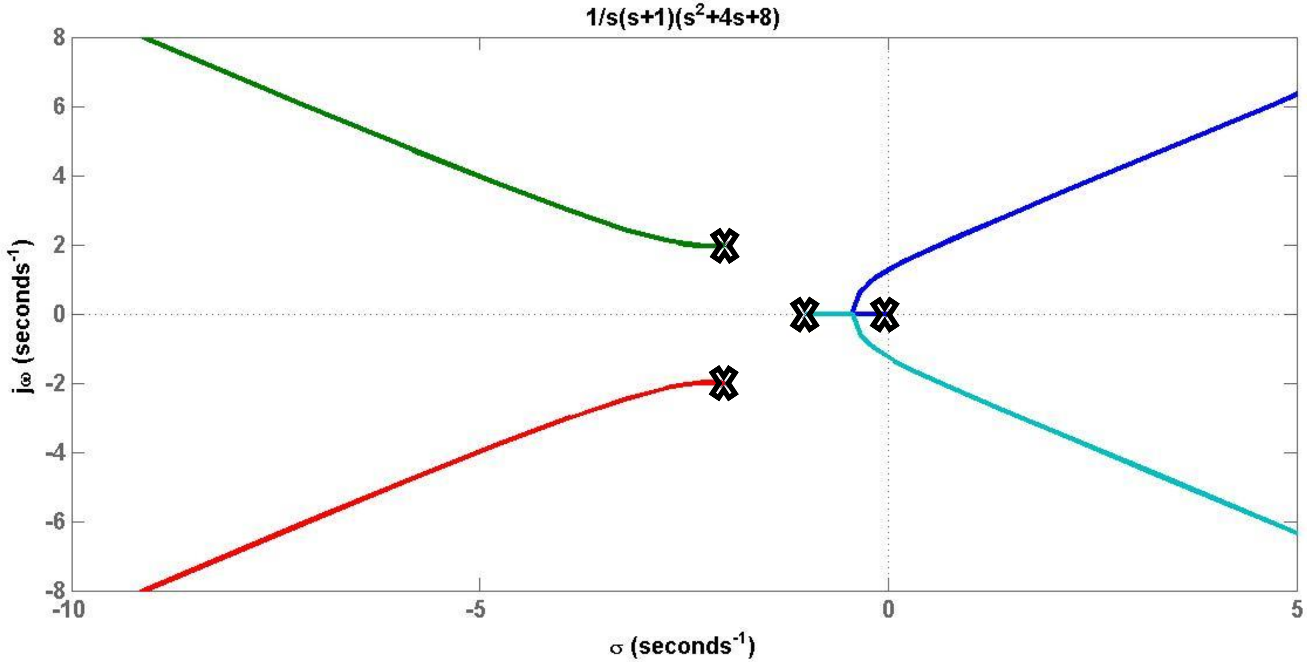

1/s(s+1)(s+2)
jω (seconds^-1)
σ (seconds^-1)

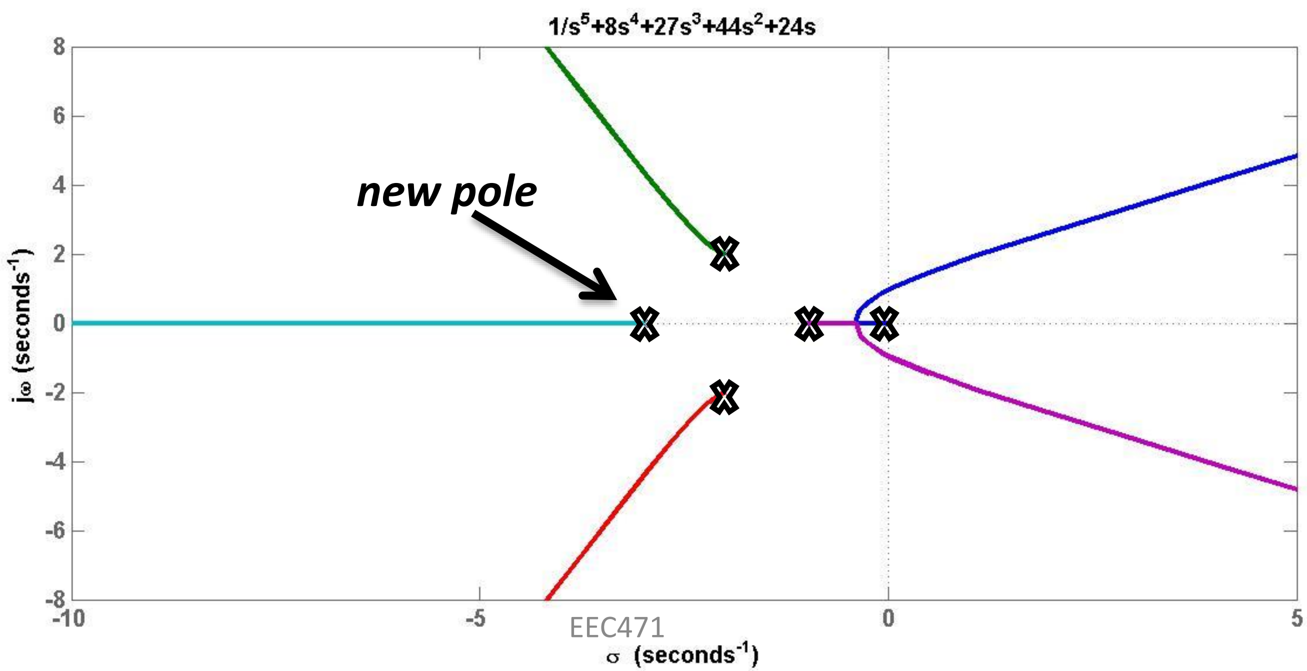

(s+3)/s(s+1)(s+2)
new zero
jω (seconds^-1)
σ (seconds^-1)

- ***Adding a pole to G(s)H(s) has the effect of pushing the root loci toward the right half s-plane – makes the system <u>less stable</u>***

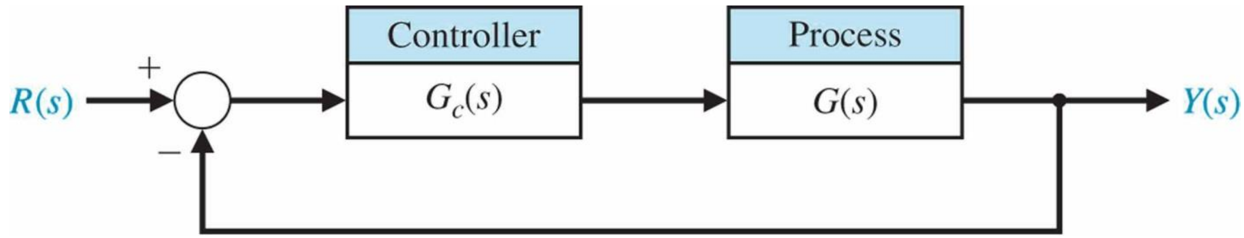


- ***What is the effect on the transient response?***


1/s(s+1)(s2+4s+8)
jω (seconds-1)
σ (seconds-1)


1/s5+8s4+27s3+44s2+24s
new pole
jω (seconds-1)
σ (seconds-1)

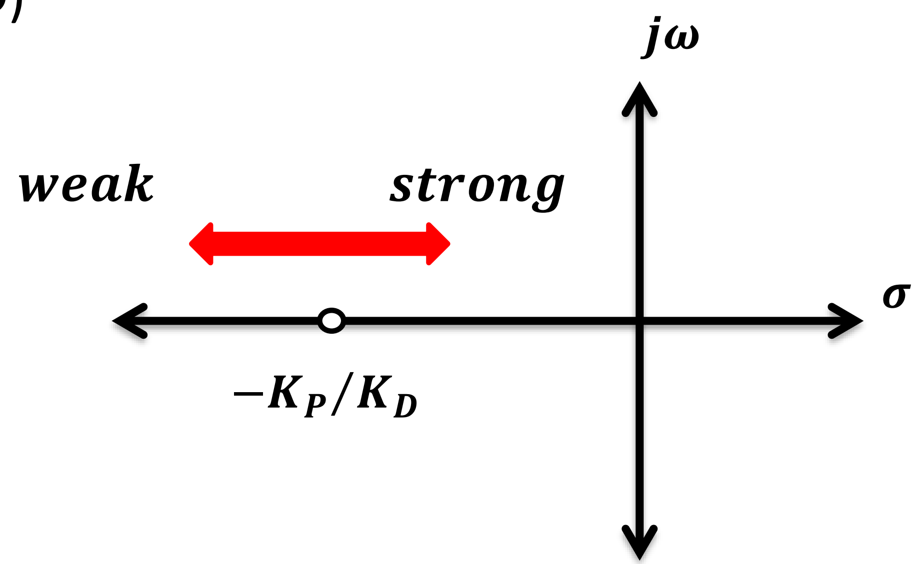
PD controller



- proportional plus **derivative** (**PD**)

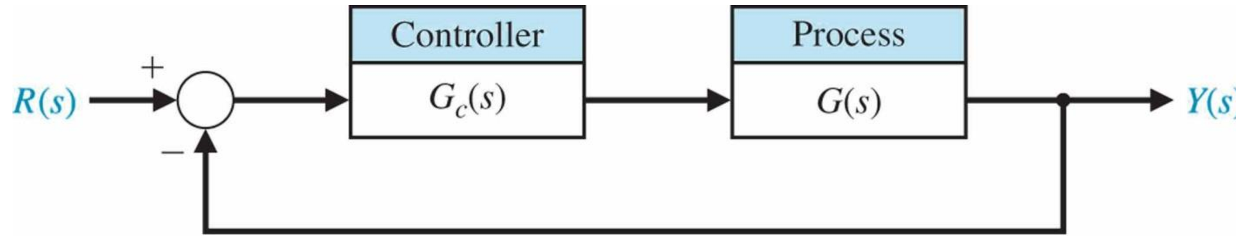
$$G_c(s) = K_P + K_D s$$

$$G_c(s) = K_D(s + K_P/K_D)$$



- one zero at $s = -K_P/K_D$
- acts as a ***high-pass filter***
- ***has no effect on the steady state error (why??)***

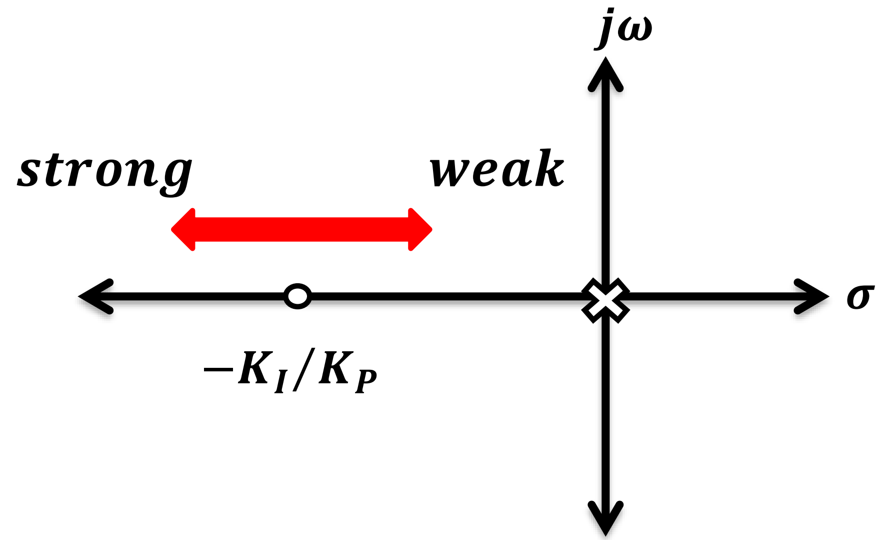
PI controller



- proportional plus **integral** (**PI**)

$$G_c(s) = K_P + \frac{K_I}{s}$$

$$G_c(s) = \frac{K_P(s + K_I/K_P)}{s}$$



- one **pole at the origin** and a **zero at $s = -K_I/K_P$**
- acts as a ***low-pass filter***
- ***Improves the steady state error***

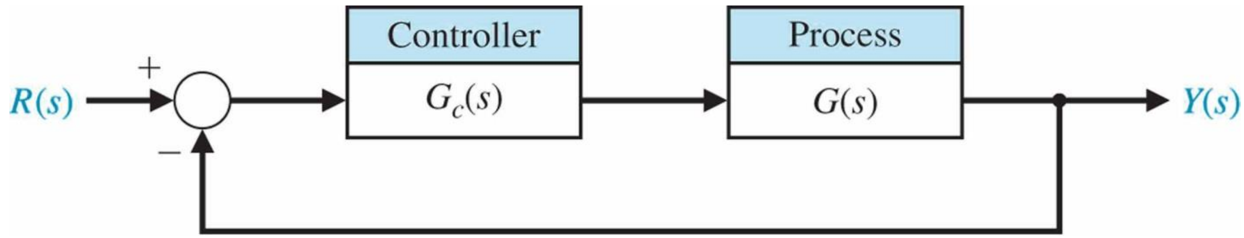
Steady state error

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \frac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\frac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

- ***the pole at the origin improves*** the **steady state error (why?)**
- ***the pole at the origin increases the system type***

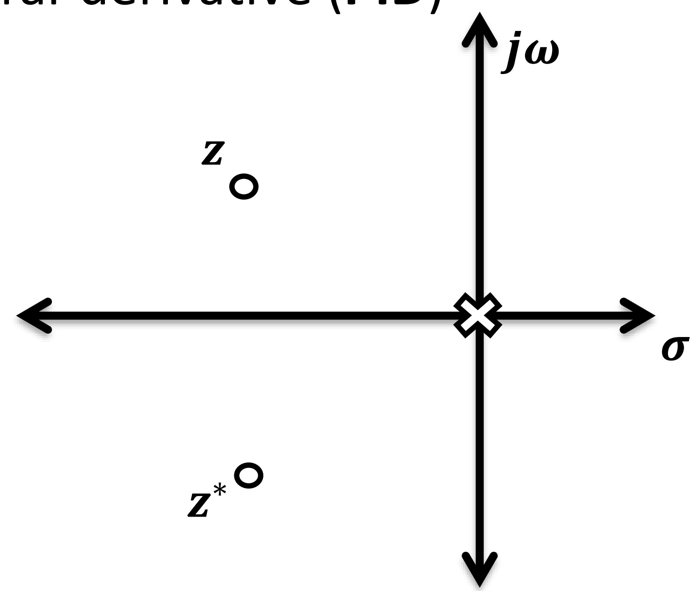
design using a PID controller



- Three-terms controller: proportional-Integral-derivative (**PID**)

$$\boxed{G_c(s) = K_P + K_D s + \frac{K_I}{s}}$$

$$\boxed{G_c(s) = \frac{K_D\left(s^2 + K_P/K_D\, s + K_I/K_D\right)}{s}}$$



- one pole at the origin and two zeros (need not to be complex zeros)
- *the pole at the origin **improves** the **steady state error***

How to tune the PID controller coefficients $K_D, K_P \,\&\, K_I$?

- **method 1**: analytical method
  - **Zeigler-Nichols** PID tuning method
  - Used as **an initial** PID tuning, then refinement can be used.

- **method 2**: Manual PID tuning
  - Root locus & root contour
  - use common sense
  - trial and error

Closed-loop Ziegler-Nichols PID tuning method

$$G_c(s) = K_P + K_D s + K_I/s$$

- Set $K_I = 0$ & $K_D = 0$
- Increase K_P till it reaches the **boundary of instability**, $K_p = \boldsymbol{K_U}$ **(the ultimate gain)**
- The period of the sustained oscillation is called the **ultimate period** $\boldsymbol{T_U}$
- Once $\boldsymbol{K_U}$ **and** $\boldsymbol{T_U}$ are known, PID coefficients can be found using the table

Table 7.7 Ziegler-Nichols PID Tuning Using Ultimate Gain, K_U, and Oscillation Period, P_U

Ziegler-Nichols PID Controller Gain Tuning Using Closed-loop Concepts

Controller Type	K_P	K_I	K_D
Proportional (P) $G_c(s) = K_P$	$0.5K_U$	–	–
Proportional-plus-integral (PI) $G_c(s) = K_P + \frac{K_I}{s}$	$0.45K_U$	$\frac{0.54K_U}{T_U}$	–
Proportional-plus-integral-plus-derivative (PID) $G_c(s) = K_P + \frac{K_I}{s} + K_D s$	$0.6K_U$	$\frac{1.2K_U}{T_U}$	$\frac{0.6K_U T_U}{8}$

example: determine the PID controller using Ziegler-Nichols tuning method for the system

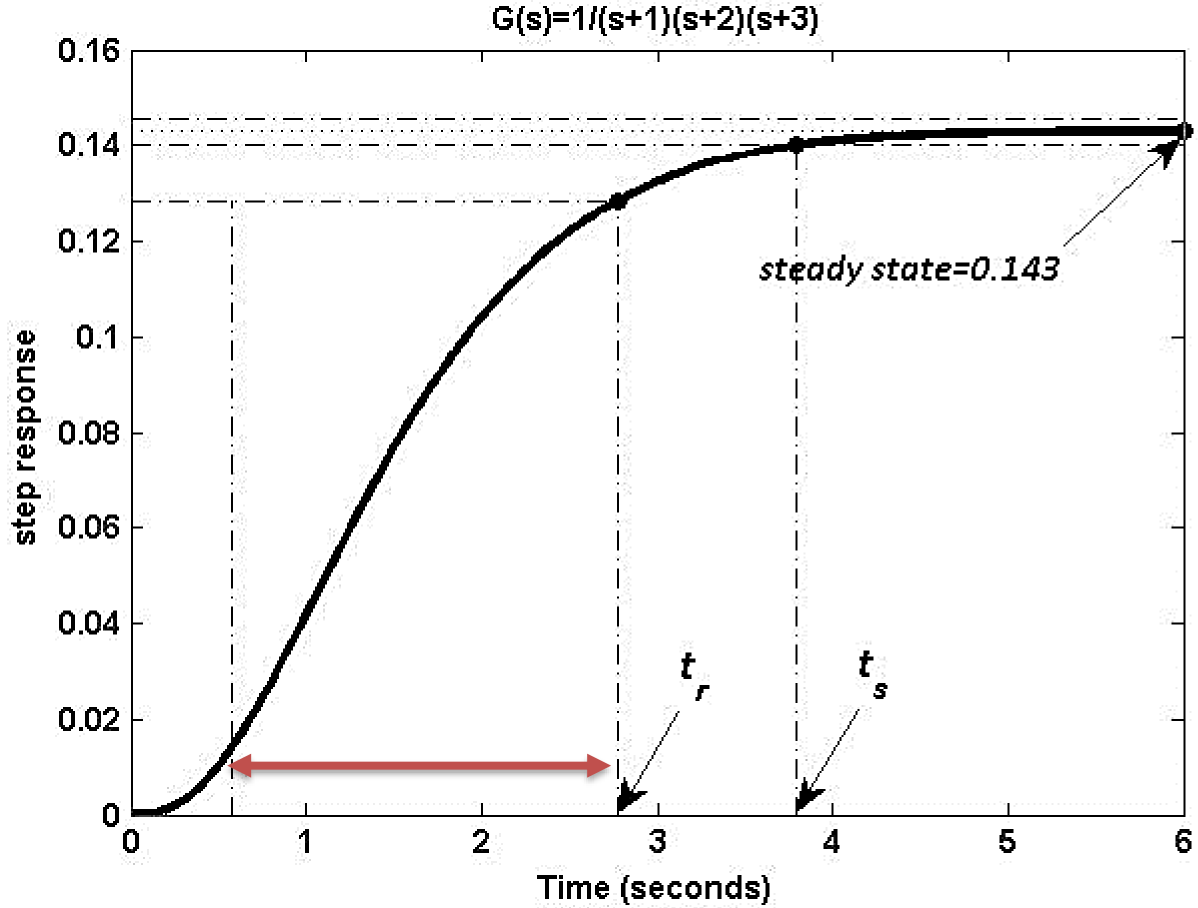
$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

- **Steady state error for *K=1*:**

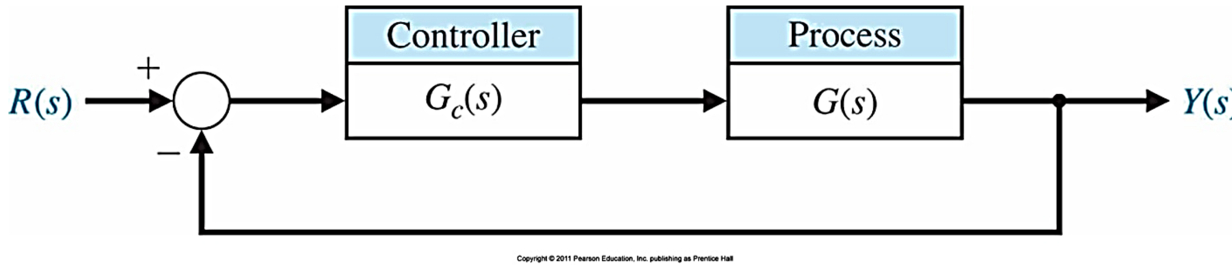
$$E_{ss} = 1/(1 + K_{Pos})$$

$$\boxed{K_{Pos} = \lim_{s \to 0} G(s) = 1/6}$$

$$E_{ss} = \frac{6}{7} = 1 - 0.143 \neq 0$$



- **Ziegler-Nichols tuning method**



$$\boxed{G_c(s)G(s) = \frac{K_p + K_D s + K_I/s}{(s+1)(s+2)(s+3)}}$$

- **Set $\boldsymbol{K_D = K_I = 0}$**

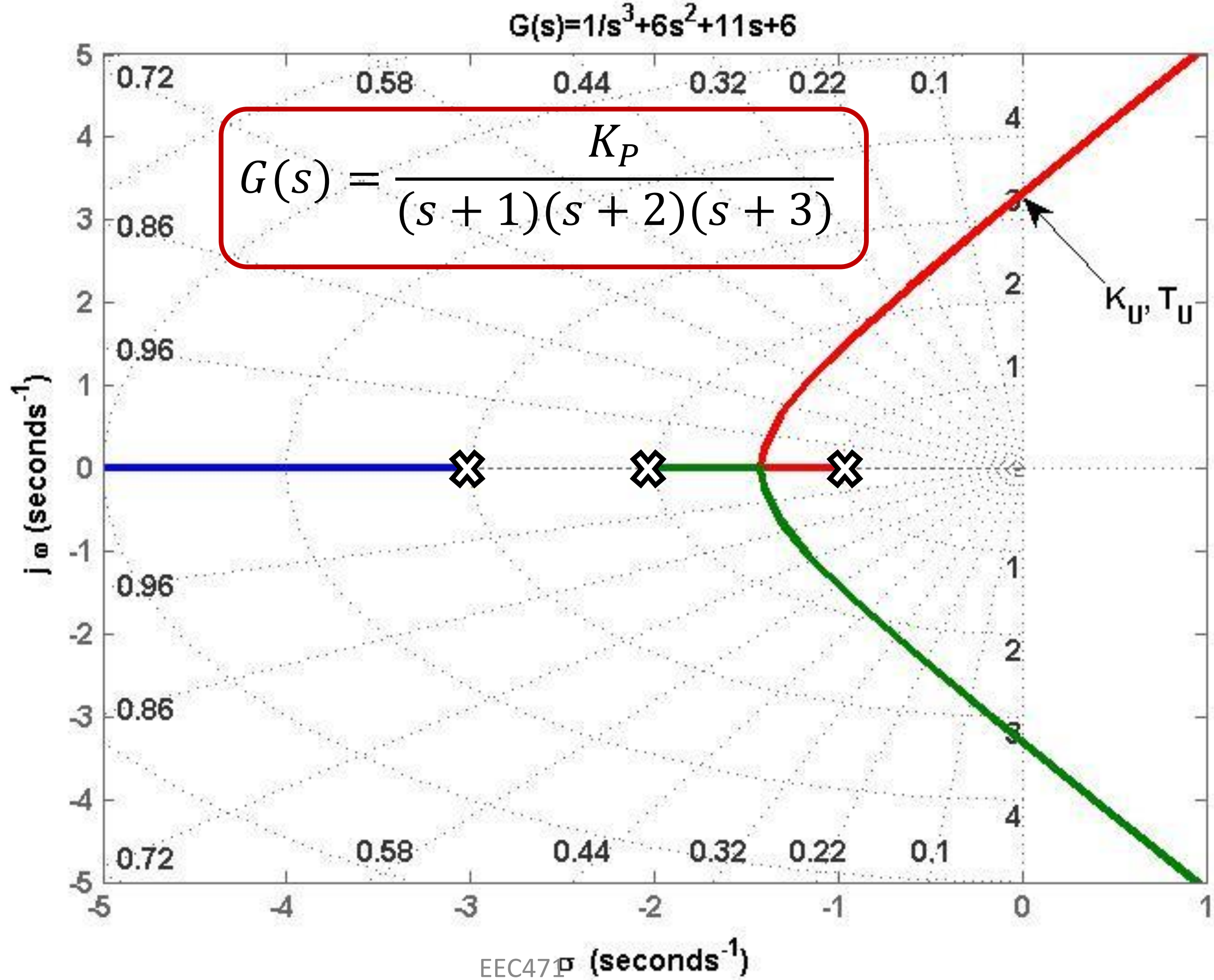
- **Determine the ultimate gain $\boldsymbol{K_U}$ & the ultimate period $\boldsymbol{T_U}$.**

- **From the Ziegler-Nichols table, determine the PID coefficients $\boldsymbol{K_P, K_D}$ & $\boldsymbol{K_I}$**

Define K_U, the **ultimate gain**, the proportional gain at the **border of instability**

- $K_U = 60$

(why?)



Define T_U, the **ultimate period**, the period of the sustained oscillations **at the border of instability**

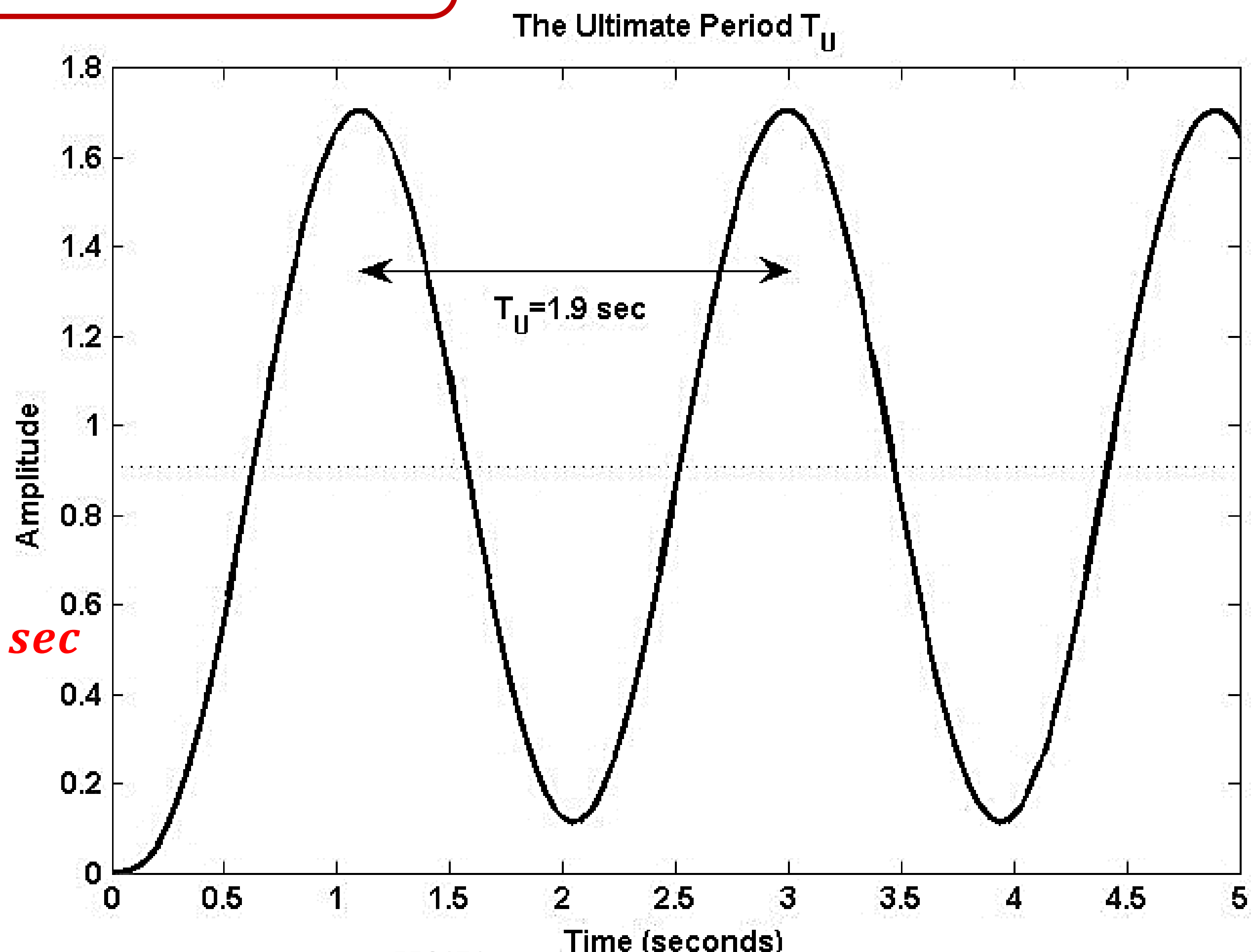
$$\boxed{G(s) = \frac{K_P}{(s+1)(s+2)(s+3)}}$$

$\boldsymbol{T_U = 1.9\ sec}$

(why?)

$\boldsymbol{\omega_n \approx 3.3\ rad/sec}$

$$\boldsymbol{\omega_n = \frac{2\pi}{T_U} \rightarrow T_U = 1.9\ sec}$$



Determination of K_U&T_U Analytically

- $\boldsymbol{K_U}$: The characteristic equation

$$1 + GH(s) = 0$$
$$(s+1)(s+2)(s+3) + K = 0$$
$$s^3 + 6s^2 + 11s + (6+K) = 0$$

Using Routh-Hurwitz

$$\boxed{66 = (6+K) \rightarrow K_U = 60}$$

- $\boldsymbol{T_U}$: $s = j\omega \; @ \; K = K_U$

$$-j\omega^3 - 6\omega^2 + j11\omega + (6+K_U) = 0$$
$$\text{IMAGINRY} = 0 \Longrightarrow (11 - \omega^2) = 0$$
$$\omega = \sqrt{11} = 2\pi/T_U$$

$$\boxed{T_U = 1.9\; sec}$$

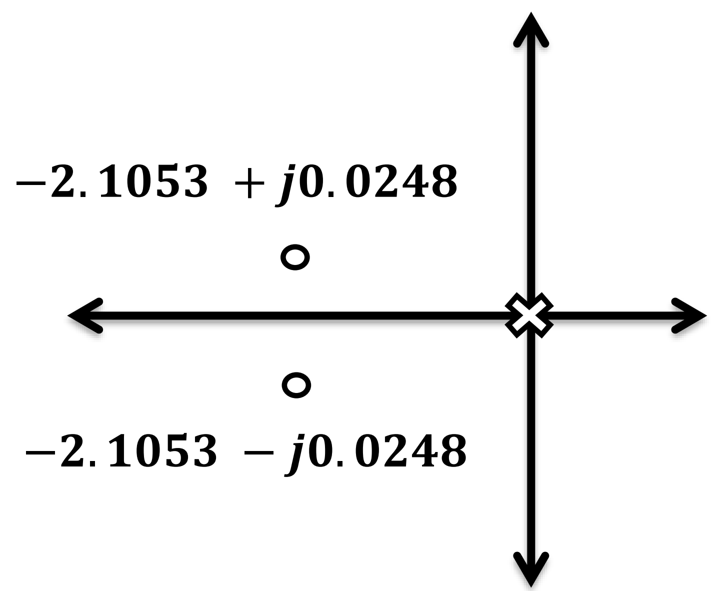
Using **Ziegler-Nichols table**

- $K_p = 0.6K_U = 36$
- $K_I = 1.2\,K_U/T_U = 37.9$
- $K_D = 0.6\,K_U T_U/8 = 8.55$

PID controller

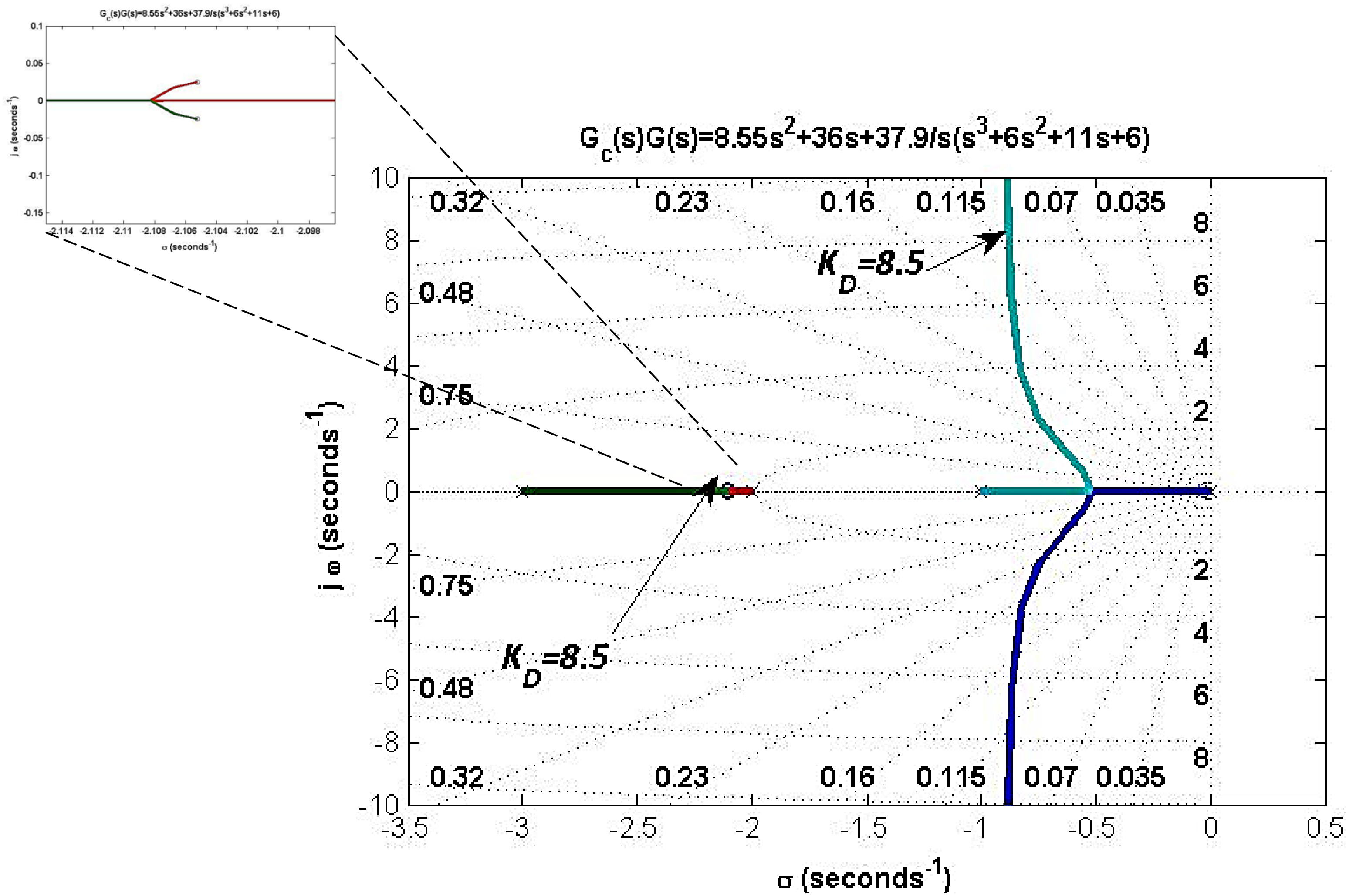
$$G_c(s) = K_P + K_D s + K_I/s$$

- $G_c(s) = 36 + 8.55s + 37.9/s$



- The compensated system

$$G_c(s)G(s) = \frac{8.55s^2 + 36s + 37.9}{s(s^3 + 6s^2 + 11s + 6)}$$

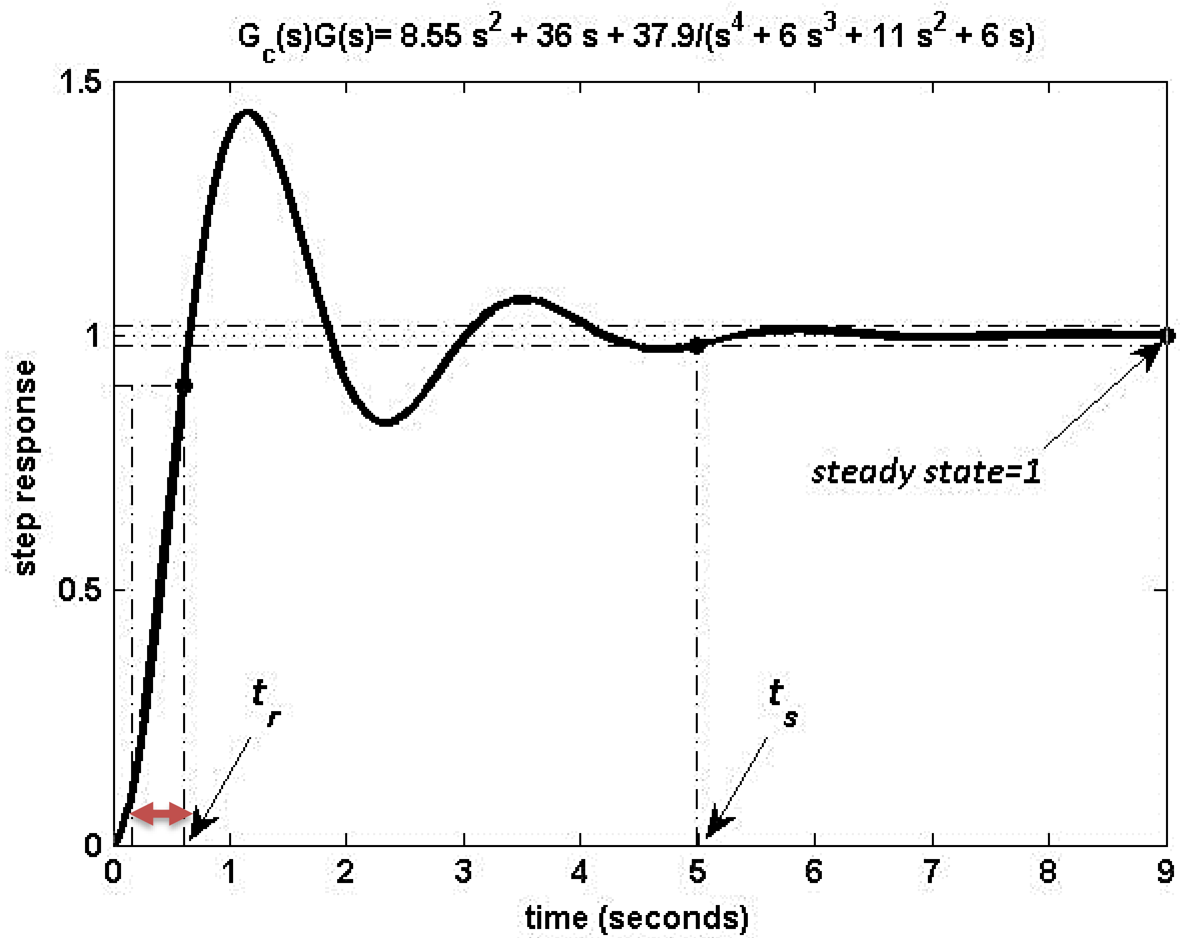


Step response of the compensated system

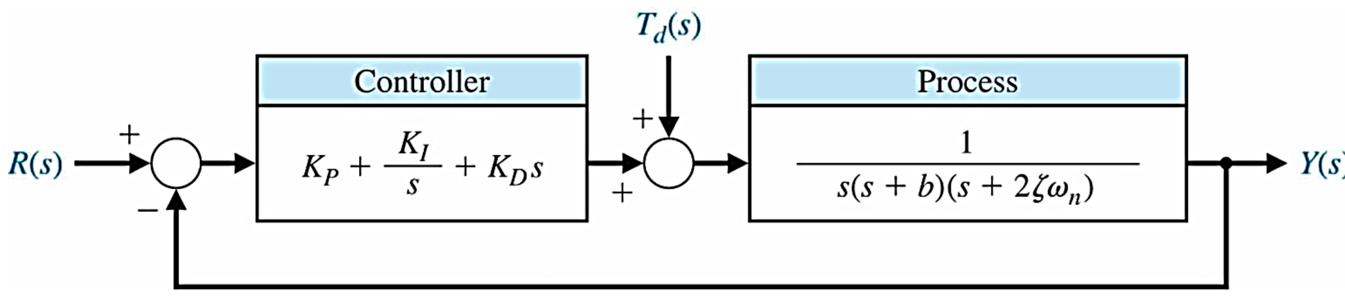
- Steady state error has been improved !!

- $K_P = \infty$

- $\boxed{E_{ss} = 0}$



- ***example***



- $b = 10, \xi = 0.707, \omega_n = 4$

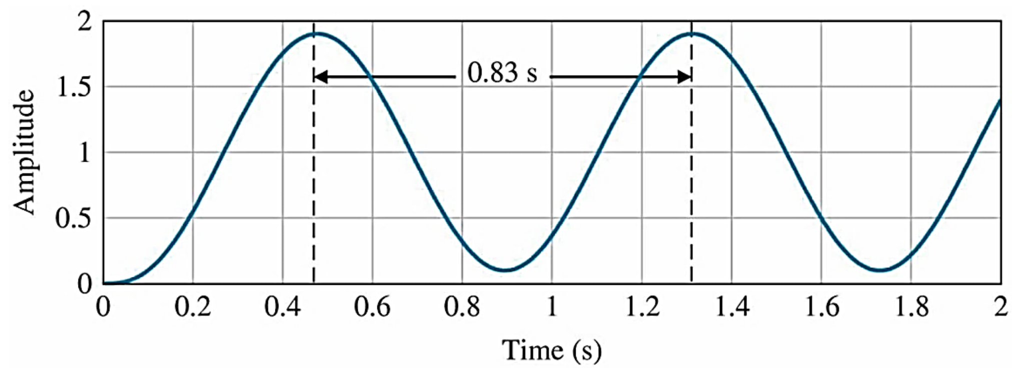
$$\boxed{G(s) = \frac{1}{s(s+10)(s+5.66)}}$$

- **Ultimate gain** $K_U = 885.5$

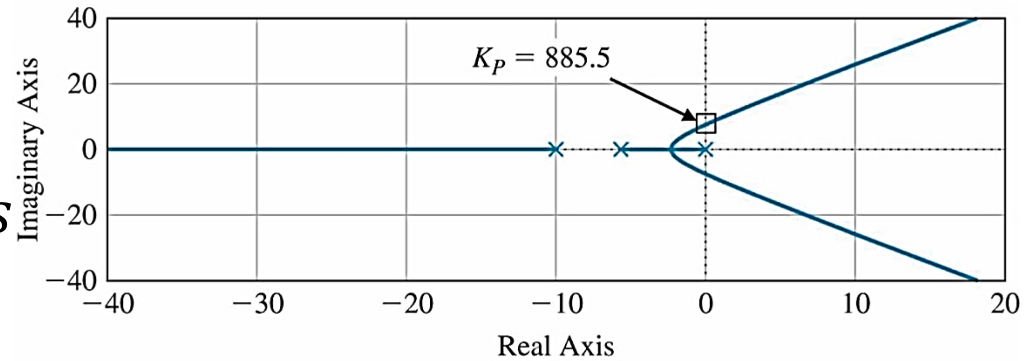
(How?)

- **Ultimate period** $T_U = 0.83\ s$

(How?)



(a)



(b)

Determination of K_U&T_U Analytically

- $\boldsymbol{K_U}$: The characteristic equation

$$1 + GH(s) = 0$$
$$s(s + 10)(s + 5.66) + K = 0$$
$$s^3 + 15.66s^2 + 56.6s + K = 0$$

$$\boxed{K_U = 15.66 \times 56.6 = 886.3}$$

- $\boldsymbol{T_U}$: $s = j\omega$ @ $K = K_U$

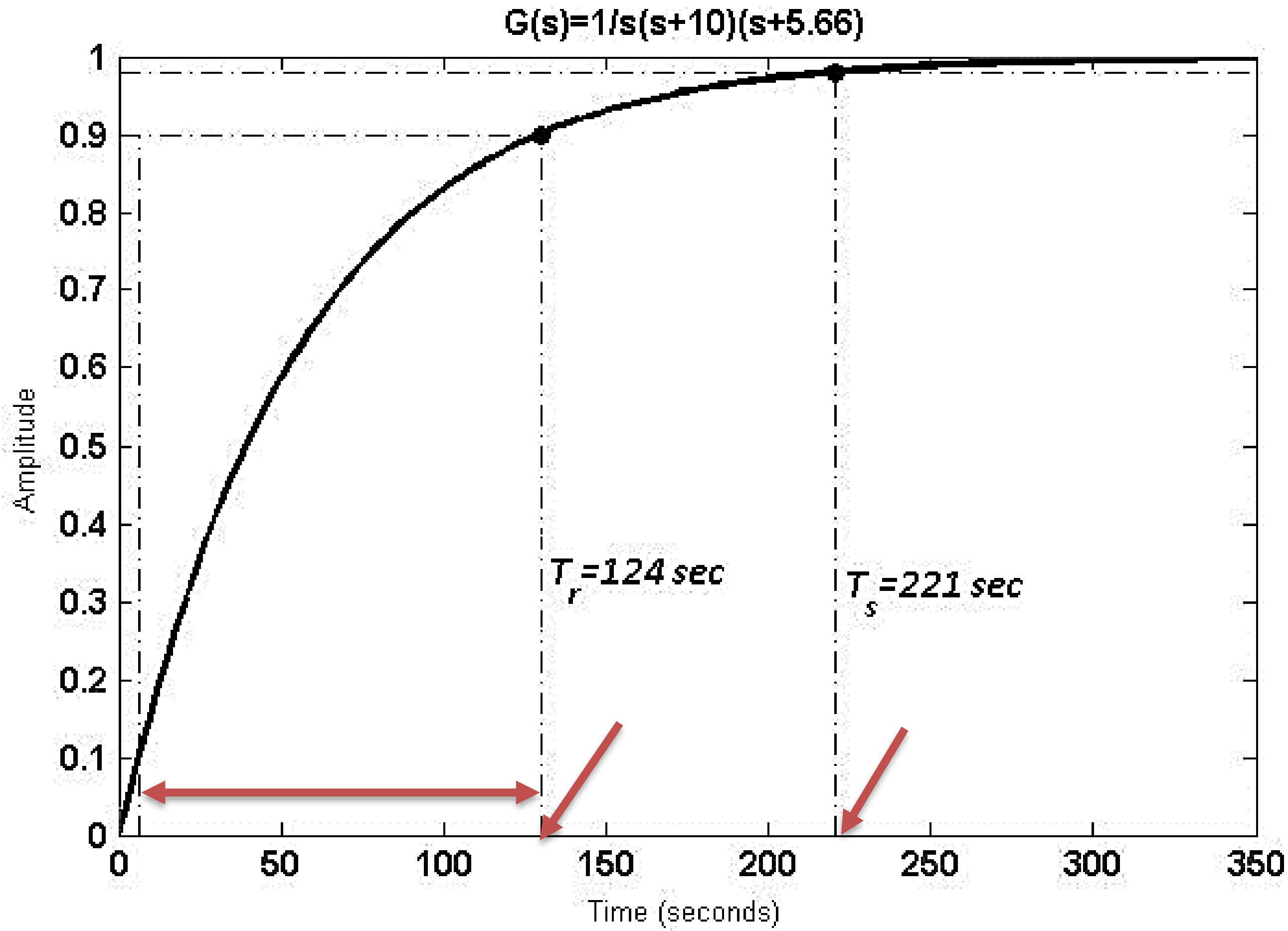
$$j\omega(j\omega + 10)(j\omega + 5.66) + K_U = 0$$
$$\text{IMAGINRY} = 0 \Longrightarrow (56.6 - \omega^2) = 0$$
$$\omega = \sqrt{56.6} = 2\pi/T_U$$

$$\boxed{T_U = 0.83\ sec}$$

- **Step response** of the **uncompensated** system

$$G(s) = \frac{1}{s(s+10)(s+5.66)}$$

- $E_{ss} = 0$

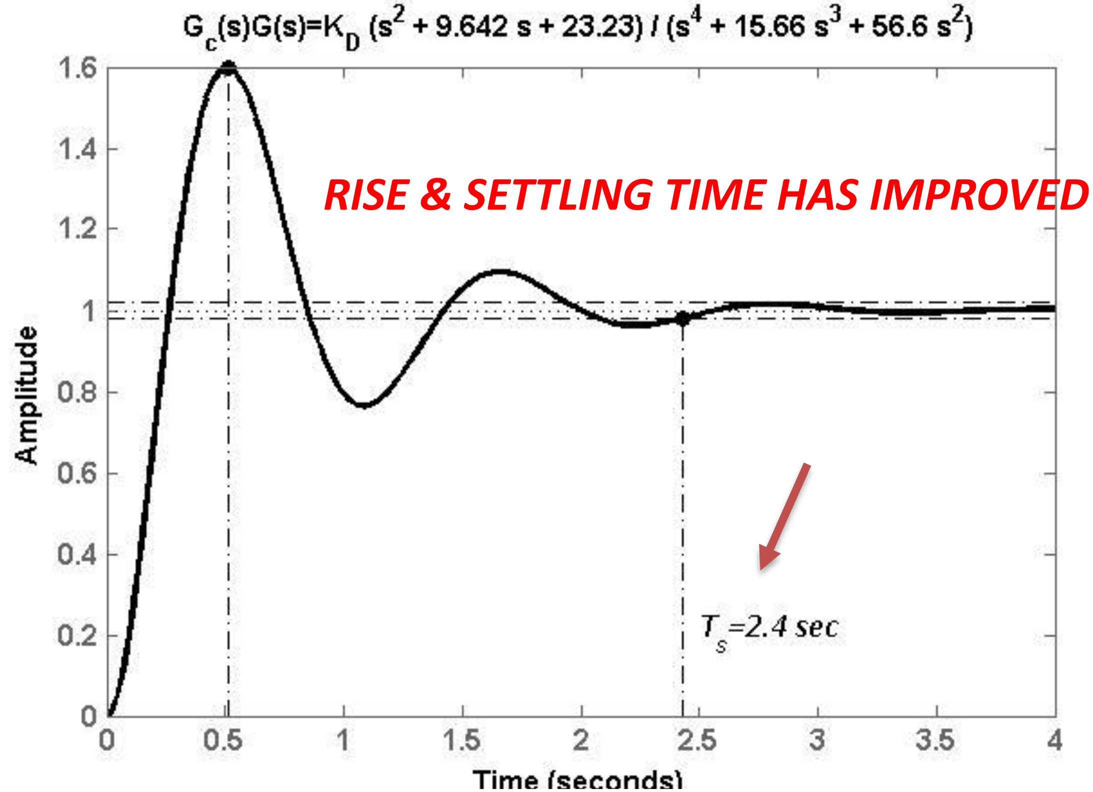


- Using Ziegler-Nichols table

$$K_P = 0.6K_U = 531.3$$

$$K_I = \frac{1.2K_U}{T_U} = 1280.2$$

$$K_D = \frac{0.6K_U T_U}{8} = 55.1$$



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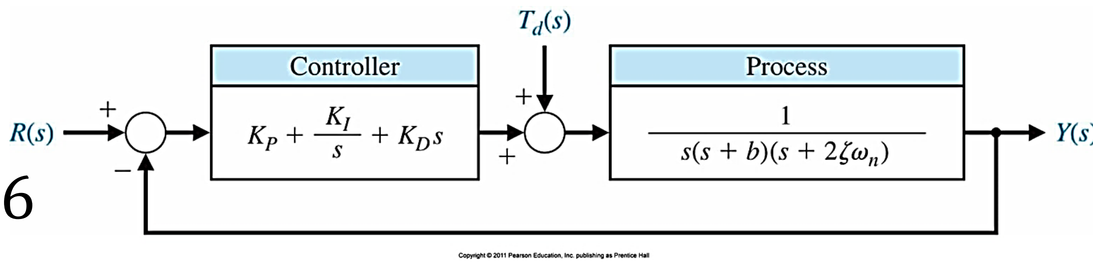
Steady state error – ramp input $\left(\frac{A}{s^2}\right)$

- **Before PID**



$$K_v = \lim_{s \to 0} sG(s) = 1/56.6$$

$$e_{ss} = A/K_v = 56.6\ \times A \neq 0$$

$$\boxed{G(s) = \frac{1}{s(s+10)(s+5.66)}}$$

- **After PID**

$$K_v = \lim_{s \to 0} sG_c(s)G(s) = \infty$$

$$e_{ss} = A/K_v = zero$$

Root locus of the compensated system

$$G_c(s)G(s) = \frac{55.1\left(s^2+9.642s+23.23\right)}{s^2(s+10\ )(s+5.66)}$$