Daniyal Faraz 22I-1096 Discrete assignment 2. Section B.

Ans 1a) ~~$\exists x \in$ tools, $\sim C(x)$~~ $\exists x, \sim C(x)$

b) $\forall x \in$ tools, $T(x) \rightarrow C(x) \wedge E(x)$

c) ~~$\forall x, C(x) \wedge E(x)$~~ $\neg \exists x, \sim C(x) \wedge \sim E(x)$

d) $\neg \exists x, C(x) \wedge E(x)$

e) $\exists x, T(x) \wedge (\sim C(x) \wedge E(x))$

Ans 2 a) $\forall x, y \in Z, x<0 \wedge y<0 \rightarrow x*y > 0$

b) $\forall x, y \in Z, x>0 \wedge y>0 \rightarrow$ Average$(x,y) > 0$

c) $\forall x, y \in Z, \exists z \in Z$, (isNeg(x) $\wedge$ isNeg(y)) $\wedge$ $(x-y=z \wedge z>0)$

d) ~~$\forall x, y \in Z$, abs(sum(x,y)) <= abs(x) + abs(y)~~.

$\hookrightarrow \forall x, y \in Z$, $s = $ sum$(x,y) \rightarrow$ abs(s) <= sum(abs(x), abs(y))

Ans 3)

1) False, because for ~~every number there exits a number which is less than that by~~ 1 There is no positive integer y such that $y+2=1$

2) True, because for every number in integers. There exist an integer which is 1 less than the number itself.

3) False, because if we chose x as 4 then all y where $y \neq 3$ does not satisfy the equation.

4) True, because according to closure property of addition and subtraction of integers the difference of two integers is a integer so there exists z.

Ans 4)

1) $\forall n \in Z$, isOdd(n) $\rightarrow$ $3n+15$ is even.

if n is odd then n can be written as 2(some integer)+1

so $n = 2k+1$ where k is an integer.

$3n+15 = 3(2k+1)+15$ by substitution

$= 6k+18 = 2(3k+6)$

as k is a integer and by closure property of product and sum $3k+6$ is integer. $\therefore$ let $t = 3k+6$ then

$2(3k+6) = 2t$ and so it is proved that $3n+15$ is even by the definition of even numbers.

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2) $\forall m \in Z$, isEven(m) $\rightarrow$ $3m+15$ is odd.
if m is even then m can be written as
2 (some integers) then $m = 2k$ where k = some integer.
then $3m+15$
$= 3(2k)+15$ by substitution
$= 6k+15$
$= 6k+14+1$ by addition property
$= 2(3k+7)+1$ as $3k+7$ is an integer by closure
property of addition and product of integers so let $t = 3k+7$
$= 2(3k+7)+1 = 2t+1$. hence proved by the
definition odd that for even integer m $3m+15$ is odd.

Ans 3) using integer $n=3$ we then can see that
$2(3)^2 - 5(3)+2 = 5$ hence proved that there
exists an integer such that $2n^2-5n+2$ is prime

4) $\forall n \in Z$, isOdd $\Leftrightarrow$ isOdd(square(n))
take any odd integer n so n can be written as
$2k+1$ where k is an integer by definition of odd numbers.
then $n^2 = (2k+1)^2 = 4k^2+4k+1$ by substitution
$= 2(2k^2+2k)+1$ and $2k^2+2k$ = some integer
because of closure property of addition and product of
integers. let $t = 2k^2+2k$ then
$2(2k^2+2k)+1 = 2t+1$ hence n^2 is odd for
odd integer n.
For the converse where if n^2 is odd then n is odd we try to
prove its contrapositive where
if n is even then n^2 is even. suppose n is even
then $n = 2k$ where k is integer. Thus $n^2 = (2k)^2 = 4k^2$
$= 2(2k^2)$ as $2k^2$ is an integer ~~at~~ by definition
of closure of product of integers. let $t = 2k^2$ so
$n^2 = 2t$. Thus it is proved that if n^2 is
odd then n is odd. so the biconditional is proved.

Ans 5)

Ans 1) $\exists x\,(Triangle(x) \wedge \forall y\,(circle(y) \rightarrow isAbove(x,y)))$

$\forall x\,(\sim Triangle(x) \vee \exists y\,(circle(y) \wedge \sim isAbove(x,y)))$

2) $\forall x\,(isObject(x) \rightarrow \exists y\,(isObject(y) \wedge IsEqual(x,y) \rightarrow \sim sameColor(x,y)))$

$\exists x\,(isObject(x) \wedge \forall y\,(\sim isObject(y) \vee (IsEqual(x,y) \wedge sameColor(x,y)))$

3) $\forall x\,(circle(x) \rightarrow \forall y\,(Triangle(y) \rightarrow isToRight(x,y)))$

$\exists x\,(Circle(x) \wedge \exists y\,(Triangle(y) \wedge \sim isToRight(x,y)))$

4) $\exists x\,(circle(x) \wedge \exists y\,(Triangle(y) \wedge sameColor(x,y)))$

$\forall x\,(\sim circle(x) \vee \forall y\,(\sim Triangle(y) \vee \sim sameColor(x,y)))$

5) $\exists y\,(object(y) \wedge \forall x\,(object(x) \rightarrow (isEqual(x,y) \rightarrow \sim sameColor(x,y))))$

$\forall y\,(\sim object(y) \vee \exists x\,(object(x) \wedge (isEqual(x,y)) \wedge sameColor(x,y))))$

Ans 6)

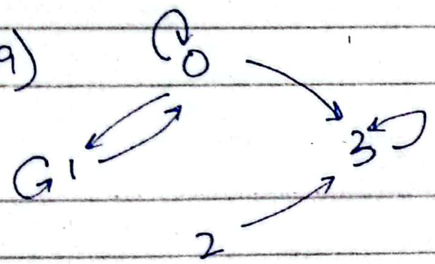
$\forall a,b,c \in Z \quad a|b \wedge b|c \rightarrow a|c$

if $a|b$ then b can be written as $a \times$ some integer.
so $b = as$ where s is an integer. Also same applies to
$b|c$ so $c = br$ where ~~a~~ r is some integer.
now $b = as$ and $c = br$ substituting b in second
gives ~~c = ars~~ $c = asr$. by the closure property of
product of integers ~~sr a~~ product of s and r is integer
let $t = sr$ then
$c = at$ $\therefore$ c can be written as $a \times$ (some integer)
$\therefore$ it is proved that a divides c.

Ans 7) The number of binary relations on set A having n elements can be obtained as follows:
By definition a relation is a subset of $A \times A$ and if it is a binary relation then that subset it can have at most 2 elements.
so number of elements of $A \times A = n \times n = n^2$
all possible subsets of $A \times A$ = power set of $A \times A$
= and number of elements in $A \times A$ is n^2 and so number of elements in power set ~~are~~ of $A \times A$ are 2^{n^2} hence shown.

Ans 8) R =
((2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8))

Ans 9)

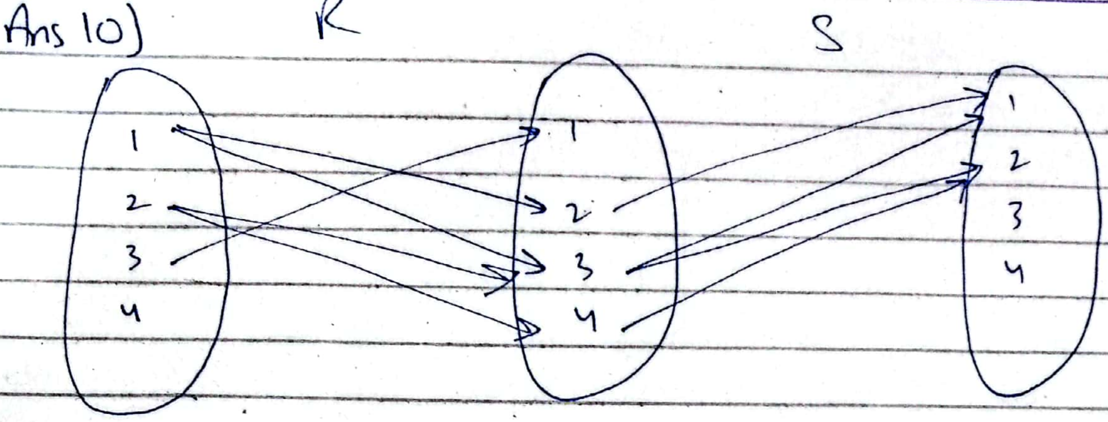


a) R is not reflexive because there is not ~~an~~ a relation that maps 2 to itself. 2 does not have a edge going from itself to itself.

b) R is not symmetric. Because there is edge going from 2 to 3 but not 3 to 2.

c) It is transitive

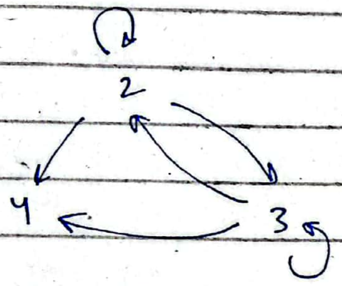
Ans 10)



$S \circ R = \{(1,1), (2,1), (2,2), (1,2)\}$

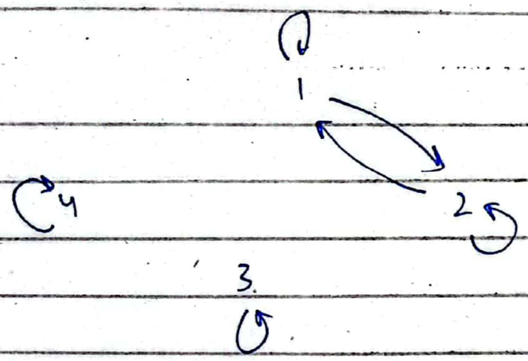
Ans 11)

a)



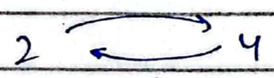
not reflexive
not symmetric, antisymmetric
transitive yes.

b)



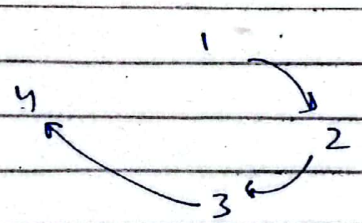
is reflexive
is symmetric, not antisymmetric
is transitive

c)



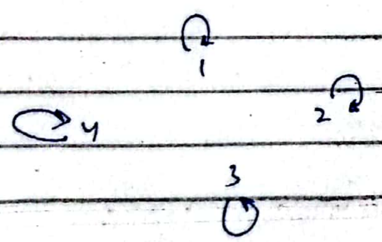
not reflexive
is symmetric, not antisymmetric
not transitive

d)



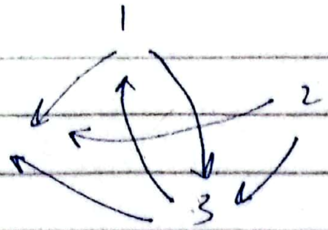
not reflexive
not symmetric, is antisymmetric
not transitive

e)



is reflexive
is symmetric, not antisymmetric
is transitive

f)



not reflexive
not symmetric
not transitive
is antisymmetric

Ans 12 a) True, every real number has a square.

b) False, this is not true for negative real numbers.

c) True, this is true for every y where $x = 0$.

d) True, addition of real numbers is associative. $2+3 = 3+2 = 5$

e) False, when $x = 0$ then anything multiplied with it is 0.

f) False, if $y = 0$ then xy is 0 regardless what x is.

g) True, $x+y=1 \rightarrow$ ~~$x=1-y$ and~~ $y = 1-x$ and for all real numbers x this property of existence of y holds.

h) False, no such x, y exists because the solution of this system is inconsistent.

i) False, this system is only valid for $x=1$ and where $y=1$. for all other x, it fails.

j) True, using closure property of addition and division of real numbers the expression $(x+y)/2$ is also a real number hence, z exists.