Q) QR decomp. of $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$.

Sol. Since $A = QR$.

& $R = Q^{-1}A$ or $Q^{T}A$.

But we don't know Q or Q^{T}. So firstly, lets find Q, i.e the "orthogonal matrix".

Applying Gram Schmidth of A, we let,

$$u_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$u_2 = v_2 - \text{Proj}_{u_1} v_2$$

$$= \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \frac{4}{5}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 4/5 \\ 8/5 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} 1 \\ 6/5 \\ -3/5 \end{pmatrix}$$

Now, $\|u_1\| = \sqrt{(0)^2 + (1)^2 + (2)^2} = \sqrt{5}$

$\|u_2\| = \sqrt{(1)^2 + (6/5)^2 + (-3/5)^2} = \frac{\sqrt{70}}{5}$

Hence, $Q = \begin{bmatrix} 0 & 5/\sqrt{70} \\ 1/\sqrt{5} & 6/\sqrt{70} \\ 2/\sqrt{5} & -3/\sqrt{70} \end{bmatrix}$

$Q^T = \begin{bmatrix} 0 & 1/\sqrt{5} & 2/\sqrt{5} \\ 5/\sqrt{70} & 6/\sqrt{70} & -3/\sqrt{70} \end{bmatrix}$

Now, $R = Q^T A$

$= \begin{bmatrix} 0 & 1/\sqrt{5} & 2/\sqrt{5} \\ 5/\sqrt{70} & 6/\sqrt{70} & -3/\sqrt{70} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$

$R = \begin{bmatrix} \sqrt{5} & 4/\sqrt{5} \\ 0 & \sqrt{70}/5 \end{bmatrix}$ or $\begin{bmatrix} \sqrt{5} & 4/\sqrt{5} \\ 0 & 14/\sqrt{70} \end{bmatrix}$