<u>Masters theorem</u>

Masters theorem for decreasing function: -

Recurrence relation form: : $T(n) = a * \ T(n-1) + f(n)$, assume that a > 0 and b > 0 and $f(n)$ = O(N^k) and $k \geq 0$

we have three cases:-

- Case1: if a < 1 ➔ $O(f(n))$
- Case2: if a = 1 ➔$O(n * f(n))$
- Case3: if a > 1 ➔$O(n^k \ a^{\frac{n}{b}})$

Examples: -

1. $T(n) = 1$

$F(n) = O(1) = O(n^0), k = 0$

$$a = 0\ , b = 1, since\ a = 0\ then\ order\ of\ recurrence\ realtion = O\big(f(n)\big)$$
$$= O(1)$$

2. $T(n) = T(n-1) + n$

$f(n) = O(n) = O(n^1), k = 1$

$$a = 1, b = 1, since\ a = 1\ then\ order\ of\ the\ recurrence\ raltion = O(n * n)$$

3. $T(n) = 2T(n-2) + 1$

$f(n) = O(1) = O(n^0), k = 0$

$$a = 2, b = 2, since\ a > 1\ then\ order\ of\ the\ recurrence\ relation$$
$$= O\left(n^0 * 2^{\frac{n}{2}}\right) = O(2^{\frac{n}{2}})$$

Masters theorem for dividing function: -

Recurrence relation form: $T(n) = a * T\left(\frac{n}{b}\right) + f(n)$ $assume\ that\ a \geq 1\ and\ b > 1\ and\ f(n) = O(n^k \log^p n)$

We have three cases: -

- Case1: $if\ \log_b a > k\ then\ order\ of\ recurrence\ relation = O(n^{\log_b a})$

- Case2: $if\ \ \log_b a = k$ and
  $p > -1\ then\ order\ of\ recurrence\ relation = O(n^k \log^{p+1} n)$
  $p = -1\ then\ order\ of\ recurrence\ relation = O(n^k \log(\log(n)))$
  $p < -1\ then\ order\ of\ recurrence\ relation = O(n^k)$

- Case3: $if\ \ \log_b a < k\ and$
  $p \geq 0\ then\ order\ of\ recurrence\ relation = O(n^k \log^p n)$
  $p < 0\ then\ order\ of\ recurrence\ relation = O(n^k)$

Examples: -

1. $T(n) = 2\,T\left(\frac{n}{2}\right) + 1$

$a = 2, b = 2\ , \log_b a = 1$

$f(n) = O(1) = O(n^0 \log^0 n), k = 0, p = 0$

$since\ \ \log_b a > k\ then\ order\ of\ recurrence\ relation = O\left(n^{\log_b a}\right) = O(n^2)$

2. $T(n) = T\left(\frac{n}{2}\right) + 1$

$a = 1, b = 2\ , \log_b a = 0$

$f(n) = O(1) = O(n^0 \log^0 n), k = 0, p = 0$

$$since\ \log_b a = k\ and\ p > -1\ then\ order\ of\ recurrence\ relation = O(n^k \log^{p+1} n) = O(\log n)$$

3. $T(n) = 2\,T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2, b = 2\,, \log_b a = 1$

$f(n) = O(n \log^{-1} n), k = 1, p = -1$

$$since \log_b a = k\ and\ p = \ -1\ then\ order\ of\ recurrence\ relation = O(n \log(\log(n)))$$

4. $T(n) = 2\,T\left(\frac{n}{2}\right) + \frac{n}{\log^2 n}$

$a = 2, b = 2\,, \log_b a = 1$

$f(n) = O(n \log^{-2} n), k = 1, p = -2$

$$since \log_b a = k\ and\ p < \ -1\ then\ order\ of\ recurrence\ relation = O(n\ )$$

5. $T(n) = T\left(\frac{n}{2}\right) + n^3$

$a = 1, b = 2\,, \log_b a = 0$

$f(n) = O(n^3 \log^0 n), k = 3, p = 0$

$$since \log_b a < k\ and\ p \geq \ 0\ then\ order\ of\ recurrence\ relation = O(n^3\ )$$

6. $T(n) = 2\,T\left(\frac{n}{2}\right) + n^3 logn$

$a = 2, b = 2\,, \log_b a = 1$

$f(n) = O(n^3 \log^1 n), k = 3, p = 1$

$$since \log_b a < k\ and\ p \geq \ 1\ then\ order\ of\ recurrence\ relation = O(n^3\ \mathrm{logn}\ )$$

7. $T(n) = 2\,T\left(\frac{n}{2}\right) + \frac{n^3}{\log^2 n}$

$a = 2, b = 2\,, \log_b a = 1$

$f(n) = O(n^3 \log^{-2} n), k = 3, p = -2$

$$since \log_b a < k\ and\ p < \ 0\ then\ order\ of\ recurrence\ relation = O(n^3\ )$$