Chapter 1 Solutions, Susanna Epp Discrete Math 5th Edition

https://github.com/spamegg1

April 19, 2023

Contents

1 Exercise Set 1.1 **2**
- 1.1 Problem 1 2
  - 1.1.1 (a) 2
  - 1.1.2 (b) 2
- 1.2 Problem 2 2
  - 1.2.1 (a) 2
  - 1.2.2 (b) 2
- 1.3 Problem 3 3
  - 1.3.1 (a) 3
  - 1.3.2 (b) 3
- 1.4 Problem 4 3
  - 1.4.1 (a) 3
  - 1.4.2 (b) 3
- 1.5 Problem 5 3
  - 1.5.1 (a) 3
  - 1.5.2 (b) 4
  - 1.5.3 (c) 4
- 1.6 Problem 6 4
  - 1.6.1 (a) 4
  - 1.6.2 (b) 4
  - 1.6.3 (c) 4
- 1.7 Problem 7 4
  - 1.7.1 (a) 4
  - 1.7.2 (b) 5
  - 1.7.3 (c) 5
  - 1.7.4 (d) 5
- 1.8 Problem 5
  - 1.8.1 (a) 5
  - 1.8.2 (b) 5

1.9 Problem 5
1.9.1 (a) 5
1.9.2 (b) 5

1 Exercise Set 1.1

In each of 1–6, fill in the blanks using a variable or variables to rewrite the given statement.

1.1 Problem 1

Is there a real number whose square is -1?

1.1.1 (a)

Is there a real number x such that ____ ?

Proof. Is there a real number x such that $\underline{x^2 = -1}$? □

1.1.2 (b)

Does there exist ____ such that $x^2 = -1$?

Proof. Does there exist <u>a real number x</u> such that $x^2 = -1$? □

1.2 Problem 2

Is there an integer that has a remainder of 2 when it is divided by 5 and a remainder of 3 when it is divided by 6?

Note: There are integers with this property. Can you think of one?

1.2.1 (a)

Is there an integer n such that n has ____ ?

Proof. Is there an integer n such that n has <u>a remainder of 2 when it is divided by 5 and a remainder of 3 when it is divided by 6</u>? □

1.2.2 (b)

Does there exist ____ such that if n is divided by 5 the remainder is 2 and if ____ ?

Proof. Does there exist <u>an integer n</u> such that if n is divided by 5 the remainder is 2 and if <u>n is divided by 6 the remainder is 3</u>? □

1.3 Problem 3

Given any two distinct real numbers, there is a real number in between them.

1.3.1 (a)

Given any two distinct real numbers a and b, there is a real number c such that c is ____ .

Proof. Given any two distinct real numbers a and b, there is a real number c such that c is between a and b. □

1.3.2 (b)

For any two ____ , ____ such that c is between a and b.

Proof. For any two distinct real numbers a and b, there exists a real number c such that c is between a and b. □

1.4 Problem 4

Given any real number, there is a real number that is greater.

1.4.1 (a)

Given any real number r, there is ____ s such that s is ____ .

Proof. Given any real number r, there is a real number s such that s is greater than r. □

1.4.2 (b)

For any ____ , ____ such that $s > r$.

Proof. For any real number r, there exists a real number s such that $s > r$. □

1.5 Problem 5

The reciprocal of any positive real number is positive.

1.5.1 (a)

Given any positive real number r, the reciprocal of ____ .

Proof. Given any positive real number r, the reciprocal of r is positive. □

1.5.2 (b)

For any real number r, if r is ____ , then ____ .

Proof. For any real number r, if r is <u>positive</u>, then <u>$1/r$ is positive</u>. ☐

1.5.3 (c)

If a real number r ____ , then ____ .

Proof. If a real number r <u>is positive</u>, then <u>$1/r$ is positive</u>. ☐

1.6 Problem 6

The cube root of any negative real number is negative.

1.6.1 (a)

Given any negative real number s, the cube root of ____ .

Proof. Given any negative real number s, the cube root of <u>s is negative</u>. ☐

1.6.2 (b)

For any real number s, if s is ____ , then ____ .

Proof. For any real number s, if s is <u>negative</u>, then <u>$\sqrt[3]{s}$ is negative</u>. ☐

1.6.3 (c)

If a real number s ____ , then ____ .

Proof. If a real number s <u>is negative</u>, then <u>$\sqrt[3]{s}$ is negative</u>. ☐

1.7 Problem 7

Rewrite the following statements less formally, without using variables. Determine, as best as you can, whether the statements are true or false.

1.7.1 (a)

There are real numbers u and v with the property that $u + v < u - v$.

Proof. Rewrite: There are real numbers such that their sum is less than their difference.

True: 0 and -1 have this property: $-1 = 0 + (-1) < 0 - (-1) = 1$ ☐

1.7.2 (b)

There is a real number x such that $x^2 < x$.

Proof. Rewrite: there is a real number whose square is less than itself.

True: $1/2$ has this property: $\frac{1}{4} = \left(\frac{1}{2}\right)^2 < \frac{1}{2}$ □

1.7.3 (c)

For every positive integer n, $n^2 \geq n$.

Proof. Rewrite: The square of every positive integer is greater than or equal to itself.

True: if we look at the first few examples it holds: $1^2 = 1 \geq 1$, $2^2 = 4 \geq 2$, $3^2 = 9 \geq 3$ and so on. This is however not a proof. Later we'll learn methods to prove this for all positive integers. □

1.7.4 (d)

For all real numbers a and b, $|a + b| \leq |a| + |b|$.

Proof. Rewrite: for all two real numbers, the absolute value of their sum is less than or equal to the sum of their absolute values.

True: this is known as the Triangle Inequality and it will be proved later. □

1.8 Problem

1.8.1 (a)

Proof. □

1.8.2 (b)

Proof. □

1.9 Problem

1.9.1 (a)

Proof. □

1.9.2 (b)

Proof. □