Math for CS 2015/2019 solutions to "In-Class Problems Week 14, Wed. (Session 35)"

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Contents

1 Problem 1

1.1 (a)

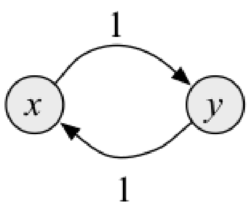


Figure 1

Find a stationary distribution for the random walk graph in Figure 1.

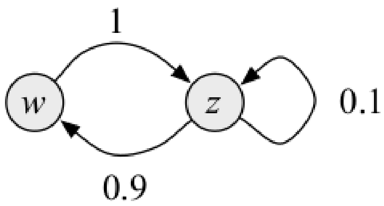
Proof. □

1.2 (b)

Explain why a long random walk starting at node x in Figure 1 will not converge to a stationary distribution. Characterize which starting distributions will converge to the stationary one.

Proof. □

1.3 (c)



Find a stationary distribution for the random walk graph in this figure.

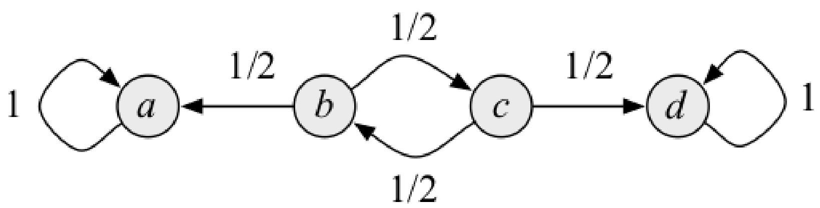
Proof. □

1.4 (d)

If you start at node w above and take a (long) random walk, does the distribution over nodes ever get close to the stationary distribution? You needn't prove anything here, just write out a few steps and see what's happening.

Proof. □

1.5 (e)



Explain why the random walk graph in this figure has an uncountable number of stationary distributions.

Proof. □

1.6 (f)

If you start at node b in the last figure and take a long random walk, the probability you are at node d will be close to what fraction? Explain.

Proof. □

1.7 (g)

Give an example of a random walk graph that is not strongly connected but has a unique stationary distribution. Hint: There is a trivial example.

Proof. □

2 Problem 2

Prove that for finite random walk graphs, the uniform distribution is stationary if and only the probabilities of the edges coming into each vertex always sum to 1, namely

$$\sum_{u \in \text{into}(v)} p(u, v) = 1$$

where $\text{into}(w) ::= \{v \mid \langle v \to w \rangle \text{ is an edge}\}$.

Proof. □

3 Problem 3

A Google-graph is a random-walk graph such that every edge leaving any given vertex has the same probability. That is, the probability of each edge $\langle v \to w \rangle$ is 1= outdeg.v/.

A digraph is symmetric if, whenever $\langle v \to w \rangle$ is an edge, so is $\langle w \to v \rangle$. Given any finite, symmetric Google-graph, let $d(v) ::= \text{outdeg}(v)/e$ where e is the total number of edges in the graph.

3.1 (a)

If d was used for webpage ranking, how could you hack this to give your page a high rank? ...and explain informally why this wouldn't work for "real" page rank using digraphs?

Proof. □

3.2 (b)

Show that d is a stationary distribution.

Proof. □