1.1/

Limits and Continuity

1. **Limit [of Function]:**

2. **Continuity:**

3. **Limit [of Sequence]:**

4. **Convergence and Continuity, Correspondance:**

Differentiability

1. **Differentiablity:**

2. **Differentiablity and Continuity, Correspondance:**

3. **Rolle's Theorem:**

4. **Generalized Rolle's Theorem:**

5. **Mean Value Theorem:**

    - **Proof**.

6. **Extreme Value Theorem:**

7. **Intermediate Value Theorem:**

Integration

1. **The Riemann Integral:**
    - **Or, for equally spaced intervals:**
2. **Integrability and Continuity, Correspondance:**
3. **Weighted Mean Value Theorem for Integrals:**

- **Implications:**
- **When $g(x) \equiv 1$, Theorem 1.13 is:**
- **It gives:**

Taylor Polynomials and Series

1. **Taylor's Theorem:**
    - $P_n(x)$**:** is called the for about .
    - $R_n(x)$**:** is called the (or) associated with $P_n(x)$.
    - Since the number $\xi(x)$ in the truncation error $R_n(x)$, it is a function of
    - Taylor's Theorem, **only**, **ensures** that , and that its value , and **not**
2. **Polynomials:**
    - **Taylor's Polynomial:** The polynomial definied by
    - **Maclaurin Polynomial:**

3. **Series:**
    - **Taylor's Series:**
    - **Maclaurin Series:**
4. **Truncation Error:**
    - **Refers to:**

1.2/

Binary Machine Numbers

1. **Representing Real Numbers:**

    A (binary digit) representation is used for a real number.

    > - The **first bit** is
    > - Followed by:
    > - and a , called the
    > - The base for the exponent is

2. **Floating-Point Number Form:**

3. **Smallest Normalized positive Number:**
    - **When:**
    - **Equivalent to:**
4. **Largest Normalized positive Number:**
    - **When:**
    - **Equivalent to:**
5. **UnderFlow:**
    - **When numbers occurring in calculations have**
6. **OverFlow:**
    - **When numbers occurring in calculations have**
7. **Representing the Zero:**
    - There are Representations of the number zero:

Decimal Machine Numbers

1. **What?**

2. **(k-digit) Decimal Machine Numbers:**

3. **Normalized Form:**

4. **Floating-Point Form of a Decimal Machine Number:**
    - The floating-point form of y, denoted $f_l(y)$, is obtained by:

5. **Termination:**

    There are two common ways of performing this termination:

    1. **:**

        > This produces the floating-point form:

    2. **:** which

        > This produces the floating-point form:
        >
        > > For rounding, when $d_{k+1} \geq 5$, we
        > >
        > > When $d_{k+1} < 5$, we
        > >
        > > If we round down, then $\delta_i =$
        > >
        > > However, if we round up,

6. **Approximation Errors:**

- **The Absolute Error:** .
- **The Relative Error:** .

7. **Significant Digits:**

8. **Error in using Floating-Point Repr.:**
    - **Chopping:**

      The Relative Error =

      The Machine Repr. [for k decimial digits] =

      .

      $\Longrightarrow$

      Bound $\Longrightarrow$.

    - **Rounding:**

      > In a similar manner, a bound for the relative error when using k-digit rounding arithmetic is

      Bound $\Longrightarrow$.

9. **Distribution of Numbers:**

   The number of decimal machine numbers in is for

Finite-Digit Arithmetic

1. **Values:**

$$x =$$

$$y =$$

2. **Operations:**

3. **Error-producing Calculations:**
    - First:
    - Second:

4. **Avoiding Round-Off Error:**
    - First:
    - Second:

$$\implies \quad x_1 = \quad , \quad x_2 =$$

Nested Arithmetic

1. **What?**

> Remember that chopping (or rounding) is performed:

-

> Polynomials should always be expressed , becasue,

2. **Why?**

1.3/

Main Idea

1. **Algorithm:**

Characterizing Algorithms

1. **Stability:**
    - **Stable Algorithm:**
    - **Conditionally Stable Algorithm:**
2. **Error Growth:**
3. **Stability and Error-Growth:**
    - **Stable Algorithm:**
    - **UnStable Algorithm:**

Rates of Convergence

1. **Rate of Convergence:**

> $\beta_n =$, for

2. **Big-Oh Notation:**