4.1/

The derivative

1. **Derivative:**

2. **The forward/backward difference formula:**

   > Derivative formulat [at $x = x_0$]
   >
   > This formula is known as the forward-difference formula if
   > and the backward-difference formula if

   > **Error Bound:**

3. **The $(n+1)$-point formula to approximate $f'(x_j)$:**

   - Derivation:

4. **Three-point Formula:**

for each $j = 0, 1, 2$, where the notation ζ_j indicates that this point depends on x_j.

- Derivation:

Three-Point Formulas

1. **Equally Spaced nodes:**

> The formulas from Eq. (4.3) become especially useful if
>
> $x_1 =$
>
> $x_2 =$

2. **Three-Point Endpoint Formula:**

> The approximation in Eq. (4.4) is useful at

> **Errors:** the errors in both Eq. (4.4) and Eq. (4.5) are

3. **Three-Point Midpoint Formula:**

> **Errors:**
>
> > This is because Eq. (4.5) uses data on and Eq. (4.4) uses data
> >
> > Note also that f needs to be evaluated at in Eq. (4.5), whereas in Eq. (4.4) it

Five-Point Formulas

1. **What?**

2. **Why?**

3. **Error:**

   The error term for these formulas is

4. **Five-Point Midpoint Formula:**

> **Used** for approximation at

5. **Five-Point Endpoint Formula:**

> **Used** for approximation at

> ***Left-endpoint*** *approximations* are found using this formula with and ***right-endpoint*** *approximations* with .

> The five-point endpoint formula is particularly useful for

Approximating Higher Derivatives

1. **Approximations to Second Derivatives:**

2. **Second Derivative Midpoint Formula:**

   - **Derivation:**

> **Error Bound:** If $f^{(4)}$ is on it is , and the approximation is .

Round-Off Error Instability

1. **Form of Error:**
   - We assume that our computations actually use the values
   - which are related to the true values by:

   > &

2. **The total error:**

> It is due to

3. **Error Bound:**
   - **ASSUMPTION:**
   - **ERROR:**

4. **Reducing Truncation Error:**
   - **How?**

- **Effect of reducing h:**

5. **Conclusion:**

4.2/

Extrapolation

1. **What?**
   - Extrapolation is used to
   - Extrapolation can be applied whenever
   - Suppose that for each number $h \neq 0$ we have a formula $N_1(h)$ that approximates an unknown constant , and that the truncation error involved with the approximation has the form,

- The **truncation error** is

  and, in general,

- The object of extrapolation is

2. **Why?**
   1.
   2.

3. **The $\mathcal{O}(h)$ formula for approximating M:**

> **The First Formula:**

> **The Second Formula:**

4. **The $\mathcal{O}(h^2)$ approximation formula for M:**

5. **When to apply Extrapolation?**

6. **The $\mathcal{O}(h^4)$ formula for approximating M:**

7. **The $\mathcal{O}(h^6)$ formula for approximating M:**

8. **The $\mathcal{O}(h^{2j})$ formula for approximating M:**

9. **The Order the Approximations Generated:**

Deriving n-point Formulas with Extrapolation

1. **Deriving Five-point Formula:**

4.3/

Numerical Quadrature

1. **What?**

2. **How?**

3. **Based on?:**

4. **Method:**

5. **The Quadrature Formula:**

6. **The Error:**

The Trapezoidal Rule

1. **What?**

2. **The Trapezoidal Rule:**

   - **Derivation:**

3. **Error:**

Simpson's Rule

1. **What?**

2. **Simpson's Rule:**

    - **Derivation:**

3. **Error:**

Measuring Precision

1. **What?**

2. **Precision [degree of accuracy]:**

3. **Precision of Quadrature Formulas:**
    - The degree of precision of a quadrature formula is
    - The Trapezoidal and Simpson's rules are examples of
    - **Types of Newton-Cotes formulas:**

Closed Newton-Cotes Formulas

1. **What?**

- **It is called closed because:**

2. **Form of the Formula:**

> where,

3. **The Error Analysis:**

4. **Degree of Preceision:**
   - **Even-n:** the degree of precision is
   - **Odd-n:** the degree of precision is
5. **Closed Form Formulas:**
   - $n = 1$: rule
   - $n = 2$: rule

- $n = 3$: **rule**

- **n = 4:**

Open Newton-Cotes Formulas

1. **What?**
    - They
    - They use
    - This implies that
    - Open formulas contain
2. **Form of the Formula:**

> where,

3. **The Error Analysis:**

4. **Degree of Preceision:**
   - **Even-n:**
   - **Odd-n:**
5. **Open Form Formulas:**
   - $n = 0$**:** [PUT NAME HERE]
   - $n = 1$**:**
   - $n = 2$**:**
   - **n = 3:**

4.4/

Composite Rules

1. **What?**

2. **Why?**
    1.
    2.
    3.

3. **Notice:**

Composite Simpson’s rule

1. **Composite Simpson’s rule:**

2. **Error in Comoposite Simpson's Rule:**

> **Error:**

3. **Theorem [Rule and Error]:**

4. **Algorithm:**

Composite Newton-Cotes Rules

1. **Composite Trapezoidal rule:**

2. **Composite Midpoint rule:**

Round-Off Error Stability

1. **Stability Property:**

2. **Proof:**

4.5/

Main Idea

1. **What?**

2. **Why?**

3. **Error in Composite Trapezoidal rule:**

> This implies that

4. **Extrapolation Formula:**

> Extrapolation then is used to produce approximations by

> and according to this table,

> Calculate the Romberg table this way:

5. **Algorithm:**

4.6/

Main Idea

1. **What?**
    1.
    2.

2. **Why?**

3. **Approximation Formula:**
   $\int_a^b f(x)dx =$

   - **Derivation:**

4. **Error Bound:**
   - **Error relative to *Composite Approximations*:**
   - **Error relative to *True Value*:**
   - **ERROR DERIVATION:**

> This implies

5. **Procedure:**

> When the approximations in (4.38)

> Then we use the error estimation procedure to

> If the approximation on one of the subintervals fails to be within the tolerance
> , then

6. **Algorithm:**

7. **Derivation:**

4.7/

Main Idea

1. **What?**
    1.
    2.
    3.

    - **To Measure Accuracy:**

    - The **Coefficients** $c_1, c_2, ..., c_n$ in the approximation formula are

      ,

      and,

      The **Nodes** $x_1, x_2, ..., x_n$ are restricted by/to

      This gives,

      The number of Parameters to choose is

    - If the coefficients of a polynomial are considered parameters,

      This, then, is

2. **Why?**

Legendre Polynomials

1. **What?**

2. **Why?**

3. **Properties:**
    1.
    2.
    3. The roots of these polynomials are:
        -
        -
        -
        -
        -

4. **The first Legendre Polynomials:**

   $P_0(x) =$, $P_1(x) =$ $P_2(x) =$,
   $P_3(x) =$, $P_4(x) =$

5. **Determining the nodes:**

- **PROOF:**

> The nodes $x_1, x_2, \ldots, x_n$ needed to

Gaussian Quadrature on Arbitrary Intervals

1. **What?**

2. **The Change of Variables:**

3. **Gaussian quadrature [arbitrary interval]:**

4.8/

Approximating Double Integral

1. **What?**

2. **Why?**

3. **Comoposite Trapezoidal Rule for Double Integral:**
   $\iint_R f(x,y)\,dA \ =$

   - **DERIVATION:**

4. **Comoposite Simpsons' Rule for Double Integral:**
   - **Rule:**

- **Error:**

- **Derivation:**

Gaussian Quadrature for Double Integral Approximation

1. **What?**

2. **Why?**

3. **Example:**

Non-Rectangular Regions

1. **What?**

   Form:

   or,

2. **How?**
    - We use
    - Step Size:
        - **x:**
        - **y:**
3. **Simpsons' Rule for Non-Rect Regions:**

4. **Simpsons' Double Integral [Algorithm]:**

5. **Gaussian Double Integral [Algorithm]:**

Triple Integral Approximation

1. **What?**
    - **On what?**
    - **Form:**

2. **Gaussian Triple Integral [Algorithm]:**