

3.2/

Neville's Method

1. **What?**

2. **Why?**

3. **The lagrange Polynomial of the point x_{m_i} :**

4. **Method to recursively generate Lagrange polynomial:**

- **Method:**

- **Examples:**

- **Generated according to the following Table:**

5. Notation and subscripts:

- Proceeding down the table corresponds to
- Proceeding to the right corresponds to
- **To avoid the multiple subscripts**, we

: $Q_{i,j} =$

6. Algorithm:

7. Stopping Criterion:

- Criterion:
- If the inequality is true, $Q_{i,i}$ is
- If the inequality is false,

3.3/

Divided Differences

1. What?

2. Form of the Polynomial:

- $P_n(x) =$

- **Evaluated at x_0 :**

- **Evaluated at x_1 :**

- \implies

3. The divided differences:

- **The *zeroth* divided difference** of the function f with respect to x_i :

- **Denoted:**

- **Defined:**

- $f[x_i] =$

- The remaining divided differences are defined:

- **The *first* divided difference** of f with respect to x_i and x_{i+1} :

- **Denoted:**

- **Defined:**

- **The *second* divided difference** of f with respect to x_i , x_{i+1} and x_{i+2} :

- **Denoted:**

- **Defined:**

- The ***K*th divided difference** of f with respect to $x_i, x_{i+1}, \dots, x_{i+k-1}, x_{i+k}$:

- Denoted:

- Defined:

- The process ends with

- The ***n*th divided difference** of f with respect to $x_i, x_{i+1}, \dots, x_{i+k-1}, x_{i+k}$:

- Denoted:

- Defined:

4. The Interpolating Polynomial:

$$P_n(x) =$$

5. Newton's Divided Difference:

$$P_n(x) =$$

The value of $f[x_0, x_1, \dots, x_k]$ is

6. Generation of Divided Differences:

7. Algorithm:

Forward Differences

1. Forward Difference:

2. The divided differences (with del notation):

and in general,

3. Newton Forward-Difference Formula:

Backward Differences

1. Backward Difference:

2. The divided differences:

and in general,

Consequently, the Interpolating Polynomial \

If we extend the binomial coefficient notation to

then \

3. Newton Backward-Difference Formula:

Centered Differences

1. What?

2. Why?

3. Stirling's Formula:

- If $n = 2m + 1$ is odd:

- If $n = 2m$ is even: [we use the same formula but delete the last line]

4. Table of Entries: