

2.1/

Bisection Technique

1. **What?**

2. **Why?**

3. **Method:**

4. **Algorithm:**

5. **Drawbacks:**

6. **Stopping Criteria:**

The best criterion is:

7. Convergence:

It:

8. Rate of Convergence \ Error Bound:

9. The problem of Percision:

We use,

10. The Signum Function:

We use,

2.2/

Fixed-Point Problems

1. Fixed Point:

2. Root-finding problems and Fixed-point problems:

Root Finding and Fixed-point problems are

3. Why?:

4. Existence and Uniqueness of a Fixed Point.:

Fixed-Point Iteration

1. Approximating Fixed-Points:

2. Algorithm:

3. Convergence:

- **Fixed-Point Theorem:**
- **Error bound in using p_n for p :**

Notice:

4. Using Fixed-Points:

Question:

Answer:

5. Newton's Method as a Fixed-Point Problem:

2.3/

Newton's Method

1. What?:

- **Newton's (or the Newton-Raphson) method is:**

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2. Derivation:

3. **Algorithm:**

4. **Stopping Criteria:**

Convergence using Newton's Method

1. **Convergence Theorem:**

Theorem:

The crucial assumption is

Theorem 2.6 states that,

(1)

(2)

The Secant Method

1. What?

In Newton's Method

We approximate $f'(p_n - 1)$ as:

To produce:

2. Why?

:

Frequently,



Note:

3. **Algorithm:**

4. **Convergence Speed:**

The Method of False Position

1. **What?**

2. **Why?**

3. **Method:**

4. Algorithm:

2.4/

Order of Convergence

1. Order of Convergence:

2. Important, Two cases of order:

3. An arbitrary technique that generates a convergent sequences does so only linearly:

Theorem 2.8 implies

4. **Conditions to ensure Quadratic Convergence:**

5. **Theorems 2.8 and 2.9 imply:**

(i)

(ii)

6. **Newtons' Method Convergence Rate:**

Multiple Roots

1. **Problem:**

2. **Zeros and their Multiplicity:**

3. Identifying Simple Zeros:

- **Theorem:**

- **Generalization of Theorem 2.11:**

The result in Theorem 2.12 implies

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4. Why Simple Zeros:

Example:

5. Handling the problem of multiple roots:

- We
- We define as:

- **Derivation:**

- **Properties:**

-

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2.5/

Aitken's Δ^2 Method

1. **What?**

2. **Why?**

3. Derivation:

4. Del [Forward Difference]:

5. \hat{p}_n [Formula]:

6. Generating the Sequence [Formula]:

Steffensen's Method

1. What?:

2. Zeros and their Multiplicity:

3. Difference from Aitken's method:

- **Aitken's method:**

- **Steffensen's method:**

Notice

4. **Algorithm:**

5. **Convergence of Steffensen's Method:**


2.6/

Algebraic Polynomials

1. **Fundamental Theorem of Algebra:**

2. Existence of Roots:

3. Polynomial Equivalence:



This result implies

Horner's Method

1. What?

2. Why?

3. Horner's Method:

4. **Algorithm:**

5. **Horner's Derivatives:**

6. **Deflation:**

7. **MatLab Implementation:**

Complex Zeros: Müller's Method

1. **What?**

- **It is a:**

- Müller's method uses

2. **Why?**

1. **First:**

2. **Second:**

If the initial approximation is a real number,

3. **Complex Roots:**

4. **Algorithm:**

5. **Calculations and Evaluations:**

Müller's method can: