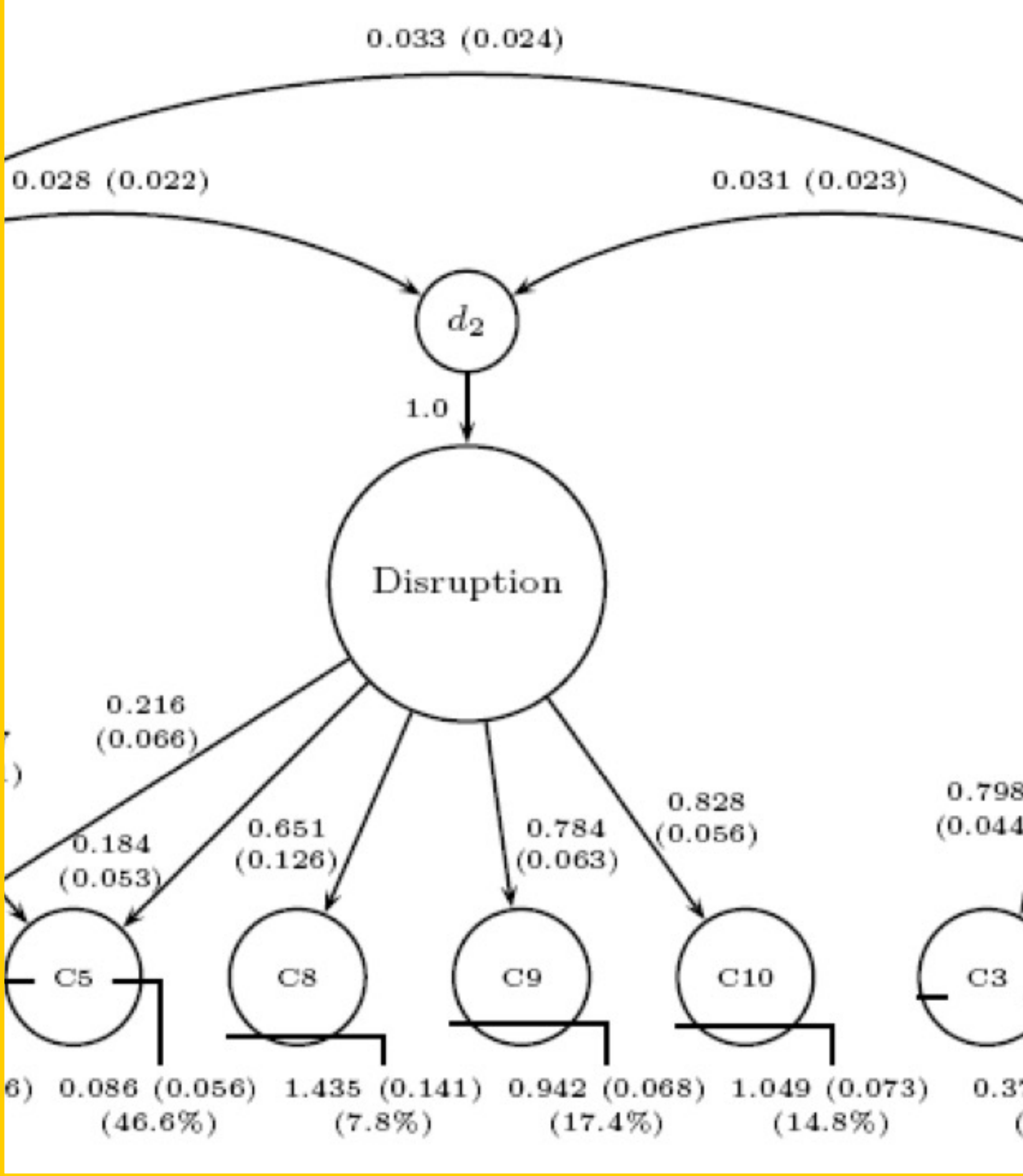
Diagnostic Structural Models: Examining the Distribution of Attributes



Lecture Overview

- This lecture will provide an understanding of structural models used in DCMs
  - What they are: estimates of how attributes are distributed in a sample of examinees
  - Differing types of structural models
    - Mplus: log-linear structural models
    - FlexMIRT: saturated log-linear structural models and higher order models
    - BayesNets – Discussed in a future lecture
    - Other methods not readily available in commercial software

Notation Used Throughout

- **Attributes**: $a = 1, \dots, A$

- **Respondents**: $r = 1, \dots, R$

- **Attribute Profiles**: $\boldsymbol{\alpha}_r = [\alpha_{r1}, \alpha_{r2}, \dots, \alpha_{rA}]$
  - Each attribute α_{ra} today is defined as being 0 or 1: $\alpha_{ra} \in \{0,1\}$

- **Latent Classes**: $c = 1, \dots, C$
  - We have $C = 2^A$ latent classes – one for each possible attribute profile
  - An attribute profile is a specific permutation of all A attributes

- **Items**: $i = 1, \dots, I$
  - Restricted to dichotomous item responses (either 0 or 1): $Y_{ri} \in \{0,1\}$

- **Q-matrix**: Elements q_{ia} are indicators an item i measures attribute a
  - q_{ia} is either 0 (does not measure a) or 1 (measures a): $q_{ia} \in \{0,1\}$

STRUCTURAL MODELS

DCM Structural Models

- Throughout this class, attribute profile base-rates have been mentioned as being influential in DCMs
  - Part of respondent diagnoses: the attribute "base rates"
  - Describes distribution of attribute profiles in a sample
    - Proportion of masters for any given attribute
    - Correlation of attributes

- The base-rates represent the probability any respondent has a given attribute profile

- For a test measuring A attributes, 2^A profiles are possible
  - The structural model provides the probability for each profile

DCMs are Constrained Latent Class Models

- Previously we’ve learned how different DCMs provide different parameterizations of the measurement component of the model
  - The LCDM – and which attributes are specified in the q-matrix
- In this lecture we’ll learn about the parameterization of the structural component of DCMs
  - Choice of structural model not dependent on the measurement component

Observed Data: Probability of observing examinee *r*’s vector of item responses to all *I* items

Measurement Component:
Product of Conditional Item Response Probabilities (Item Responses are Independent)

$$P(\mathbf{Y}_r = \mathbf{y}_r) = \sum_{c=1}^{C} \nu_c \prod_{i=1}^{I} \pi_{ic}^{y_{ri}} (1 - \pi_{ic})^{1-y_{ri}}$$

Structural component:
Proportion of examinees in each class

DCM Structural Models – Defined

- The parameter vector for the structural model is ν_c
- Each attribute profile $\boldsymbol{\alpha}_c$ has one
- ν_c is the base-rate probability of attribute profile c:

$$\nu_c = P(\boldsymbol{\alpha}_c)$$

- The example estimates of ν_c for a 4-attribute Q-matrix are shown on the next slide

c	ν_c	α_1	α_2	α_3	α_4
1	.212	0	0	0	0
2	.070	0	0	0	1
3	.056	0	0	1	0
4	.084	0	0	1	1
5	.038	0	1	0	0
6	.026	0	1	0	1
7	.050	0	1	1	0
8	.153	0	1	1	1
9	.002	1	0	0	0
10	.003	1	0	0	1
11	.002	1	0	1	0
12	.017	1	0	1	1
13	.003	1	1	0	0
14	.011	1	1	0	1
15	.017	1	1	1	0
16	.255	1	1	1	1

Interpreting the Structural Model

- Because of numerous ν_c parameters, interpretation can difficult

- Structural model can be useful for detecting attribute hierarchies

- Often, the ν_c parameters are re-expressed as:
  - The marginal probability an attribute is mastered in the population
  - The correlation between any two attributes

SAS Structural Model Summary

- SAS can be used to compute summaries of the structural model parameters

```
DATA structural;
INPUT class eta alpha1-alpha4;
DATALINES;
1  .21247 0 0 0 0
2  .06991 0 0 0 1
3  .05575 0 0 1 0
4  .08358 0 0 1 1
5  .03801 0 1 0 0
6  .02582 0 1 0 1
7  .04954 0 1 1 0
8  .15337 0 1 1 1
9  .00226 1 0 0 0
10 .00354 1 0 0 1
11 .00234 1 0 1 0
12 .01671 1 0 1 1
13 .00337 1 1 0 0
14 .01089 1 1 0 1
15 .01733 1 1 1 0
16 .25512 1 1 1 1
;
RUN;

PROC FREQ DATA=structural;
WEIGHT eta;
TABLE alpha1-alpha4;
TABLE alpha1*alpha2 alpha1*alpha3 alpha1*alpha4
      alpha2*alpha3 alpha2*alpha4 alpha3*alpha4 / PLCORR;
RUN;
```

R Structural Model Summary

- Using the psych package, we can calculate tetrachoric correlations using R:

```r
if (!require(psych)) install.packages("psych")
library(psych)


structuralModel = read.csv(file = "structuralModelEstimates.csv")

# summarizing via tetrachoric correlations:

tetrachoric(x = structuralModel[,paste0("Att", 1:4)], weight = structuralModel$Prob)
```

SAS Structural Model Summary

- For each attribute, marginally:

Proportion of Masters

alpha1	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	0.68845	68.84	0.68845	68.84
1	0.31156	31.16	1.00001	100.00

alpha2	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	0.44656	44.66	0.44656	44.66
1	0.55345	55.34	1.00001	100.00

alpha3	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	0.36627	36.63	0.36627	36.63
1	0.63374	63.37	1.00001	100.00

alpha4	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	0.38107	38.11	0.38107	38.11
1	0.61894	61.89	1.00001	100.00

SAS Structural Model Summary

- For each pair of attributes:

Tetrachoric Correlation

Statistics for Table of alpha1 by alpha2

Statistic	Value	ASE
Gamma	0.8961	0.6958
Kendall's Tau-b	0.4963	0.7247
Stuart's Tau-c	0.4571	0.7520
Somers' D C|R	0.5328	0.7618
Somers' D R|C	0.4624	0.7542
Pearson Correlation	0.4963	0.7247
Spearman Correlation	0.4963	0.7247
Tetrachoric Correlation	0.7809	0.8990
Lambda Asymmetric C|R	0.3470	1.5014
Lambda Asymmetric R|C	0.0641	2.3100
Lambda Symmetric	0.2308	1.6109
Uncertainty Coefficient C|R	0.2053	0.6539
Uncertainty Coefficient R|C	0.2275	0.7020
Uncertainty Coefficient Symmetric	0.2159	0.6733

R Structural Model Summary

```
> tetrachoric(x = structuralModel[,paste0("Att", 1:4)], weight = structuralModel$Prob)

Call: tetrachoric(x = structuralModel[, paste0("Att", 1:4)], weight = structuralModel$Prob)
tetrachoric correlation
     Att1 Att2 Att3 Att4
Att1 1.00
Att2 0.78 1.00
Att3 0.73 0.74 1.00
Att4 0.70 0.63 0.71 1.00

 with tau of
 Att1  Att2  Att3  Att4
 0.49 -0.13 -0.34 -0.30
```

Attribute Summary

- For the DTMR data, we have the following summary of attribute summary information

Attribute	Prop. Master		Tetrachoric Correlations			
1. Referent Unit	.312					
2. Partitioning/Iterating	.553		.781			
3. Appropriateness	.634		.723	.740		
4. Multiplicative Comparison	.619		.703	.626	.711	

- Such information is helpful in determining nature of attributes in a population of interest
  - Analogous to information about latent variables in CFA/MIRT

Differing Structural Models

- The structural model of a DCM has the potential to have an overwhelming number of parameters
  - For A attributes, total estimated: 2^A-1
    - All must sum to 1
    - ***Saturated model***

- Multiple structural models exist
  - All reduce the number of parameters
  - All use categorical data analysis techniques to model ν_c

- Analogous to latent variable covariance structure in structural equation modeling
  - Distribution of attributes is categorical, not continuous
  - Can help to determine nature of attribute relationships

Types of Structural Models

- **Log-linear model**
  - Models the natural logarithm of ν_c by the attributes in each profile
  - Allows for varying levels of complexity
    - Most: Saturated Model – full set of parameters
    - Least: Independent Attributes Model – no parameters
  - Implemented in Mplus and main focus of discussion today

- **Tetrachoric correlation model**
  - Provides an item factor model for latent attributes
  - Uses only bivariate information for pairs of attributes
  - Allows for covariance structures to be estimated
  - Not available in any software packages (see Templin & Henson, 2006)

- **Hierarchical factors model**
  - Special case of tetrachoric correlation model (see de la Torre & Douglas, 2004)

- **Mixture models**
  - Henson and Templin (2005): used to evaluate types of pathological gamblers
  - Also given by von Davier (2008)

- **Bayesian Networks**

- All of these except BayesNets are described in Chapter 8 of Rupp et al. (2010)

LOG-LINEAR STRUCTURAL MODELS

The Logic Behind Log-Linear Models

- Log-linear models take the set of probabilities from the structural model and re-express them on the log scale

$$\mu_c = \log \nu_c$$

- Re-expression on the log scale is convenient as these terms can now be modeled (predicted) by other features in the model
  - The attributes themselves
  - Covariates (if any)
- Because of the re-expression, redundant terms can be removed from the model
  - Simplifying estimation, improving parsimony
- In a structural model, there are 2^A probabilities…
  …but they all add up to 1.0
  - Therefore there can only be at most $2^A - 1$ log-linear model parameters

Log-Linear Structural Models

- The log-linear structural model is the easiest to implement with Mplus
  - Due to its availability in Mplus (called a latent variable mean)

- μ_c is the natural logarithm of ν_c:
$$\mu_c = \log \nu_c$$

- We can convert from μ_c back to probabilities:
$$\nu_c = \frac{\exp(\mu_c)}{\sum_{i=1}^{2^A} \exp(\mu_i)}$$

DTMR Latent Variable Means

- Mplus fixes the value of the last class "mean" to zero…
- So the rest are in reference to this last class

Categorical Latent Variables

Means

C#1	-0.183	0.142	-1.288	0.198
C#2	-1.295	0.259	-5.006	0.000
C#3	-1.521	0.341	-4.455	0.000
C#4	-1.116	0.307	-3.637	0.000
C#5	-1.904	0.314	-6.058	0.000
C#6	-2.290	0.439	-5.213	0.000
C#7	-1.639	0.307	-5.347	0.000
C#8	-0.509	0.226	-2.257	0.024
C#9	-4.725	0.783	-6.036	0.000
C#10	-4.277	0.680	-6.288	0.000
C#11	-4.690	0.700	-6.704	0.000
C#12	-2.726	0.537	-5.076	0.000
C#13	-4.327	0.674	-6.422	0.000
C#14	-3.154	0.550	-5.737	0.000
C#15	-2.689	0.381	-7.049	0.000

- We can overcome this by subtracting off what the last class would have been from every cell

Log-Linear Model Set Up

- It is important to remember that the structural model is a re-expression of the probability of any examinee having a given attribute pattern
  - The "saturated" log-linear model has as many parameters as possible ($2^A - 1$ for a test measuring A attributes)

- In our example, we have 4 attributes (16 probabilities, 15 of which must be estimated)
  - Mplus fixes the value of the last class to zero

- Our parameterization (and the Mplus implementation) will reflect this constraint
  - We will therefore omit what we will learn to be an intercept

Log-Linear Structural Model Notation

- Like the LCDM, the log-linear structural model parameters have several subscripts:

$$\gamma_{\boxed{e,}\boxed{(a_1,\dots)}}$$

- Subscript #1 – *e*: the level of the effect
  - 0 would be the intercept – but we won't have one
  - 1 is the main effect
  - 2 is the two-way interaction
  - 3 is the three-way interaction
- Subscript #2 – (a_1,…): the attributes the effect applies to
  - Same number of attributes listed as number in Subscript #2

Log-Linear Model for μ_c

- The structural model then uses an ANOVA-like model to predict the value of μ_c as a function of the attributes that are defined in attribute profile *c*
  - Shown for 4-attribute model (used in the DTMR)
  - Includes main effects, 2-way, 3-way, and 4-way interactions

Main effects

- The general model is given by:

$$
\begin{aligned}
\mu_c = {} & \gamma_{1,(1)}(\alpha_{c1}) + \gamma_{1,(2)}(\alpha_{c2}) + \gamma_{1,(3)}(\alpha_{c3}) + \gamma_{1,(4)}(\alpha_{c4}) \\
& + \gamma_{2,(1,2)}(\alpha_{c1})(\alpha_{c2}) + \gamma_{2,(1,3)}(\alpha_{c1})(\alpha_{c3}) + \gamma_{2,(1,4)}(\alpha_{c1})(\alpha_{c4}) \\
& + \gamma_{2,(2,3)}(\alpha_{c2})(\alpha_{c3}) + \gamma_{2,(2,4)}(\alpha_{c2})(\alpha_{c4}) + \gamma_{2,(3,4)}(\alpha_{c3})(\alpha_{c4}) \\
& + \gamma_{3,(1,2,3)}(\alpha_{c1})(\alpha_{c2})(\alpha_{c3}) + \gamma_{3,(1,2,4)}(\alpha_{c1})(\alpha_{c2})(\alpha_{c4}) \\
& + \gamma_{3,(2,3,4)}(\alpha_{c2})(\alpha_{c3})(\alpha_{c4}) + \gamma_{4,(1,2,3,4)}(\alpha_{c1})(\alpha_{c2})(\alpha_{c3})(\alpha_{c4})
\end{aligned}
$$

Log-Linear Model Explained

- Because not all attribute profiles include all attributes, only some terms get used to predict each value of μ_c

- For profile 1: $\boldsymbol{\alpha}_1 = [\alpha_{11} = 0; \alpha_{12} = 0; \alpha_{13} = 0; \alpha_{14} = 0]$:

$$\begin{gathered}
\mu_1 = \gamma_{1,(1)}(0) + \gamma_{1,(2)}(0) + \gamma_{1,(3)}(0) + \gamma_{1,(4)}(0) \\
+\gamma_{2,(1,2)}(0)(0) + \gamma_{2,(1,3)}(0)(0) + \gamma_{2,(1,4)}(0)(0) \\
+\gamma_{2,(2,3)}(0)(0) + \gamma_{2,(2,4)}(0)(0) + \gamma_{2,(3,4)}(0)(0) \\
+\gamma_{3,(1,2,3)}(0)(0)(0) + \gamma_{3,(1,2,4)}(0)(0)(0) \\
+\gamma_{3,(2,3,4)}(0)(0)(0) + \gamma_{4,(1,2,3,4)}(0)(0)(0)(0)
\end{gathered}$$

  - As all attributes are zero, the predicted value of $\mu_1 = 0$

- Although this may seem counter-intuitive, this is our constraint
  - We only get 15 parameters, not 16
  - The value of μ_1 is relative – the probability ν_1 depends on the other terms in the model

Log-Linear Model Explained

- For profile 2: $\boldsymbol{\alpha}_2 = [\alpha_{21} = 0; \alpha_{22} = 0; \alpha_{23} = 0; \alpha_{24} = 1]$:

$$\mu_2 = \gamma_{1,(1)}(0) + \gamma_{1,(2)}(0) + \gamma_{1,(3)}(0) + \boxed{\gamma_{1,(4)}}(1)$$
$$+\gamma_{2,(1,2)}(0)(0) + \gamma_{2,(1,3)}(0)(0) + \gamma_{2,(1,4)}(0)(1)$$
$$+\gamma_{2,(2,3)}(0)(0) + \gamma_{2,(2,4)}(0)(1) + \gamma_{2,(3,4)}(0)(1)$$
$$+\gamma_{3,(1,2,3)}(0)(0)(0) + \gamma_{3,(1,2,4)}(0)(0)(1) + \gamma_{3(1,3,4)}(0)(0)(1)$$
$$+\gamma_{3,(2,3,4)}(0)(0)(1) + \gamma_{4,(1,2,3,4)}(0)(0)(0)(1)$$

  - The main effect of attribute 4 only applies

$$\mu_4 = \gamma_{1,(4)}$$

Log-Linear Model Explained

- For profile 6: $\boldsymbol{\alpha}_6 = [\alpha_{61} = 0; \alpha_{62} = 1; \alpha_{63} = 0;\ \alpha_{64} = 1]$:

$$
\begin{gathered}
\mu_6 = \gamma_{1,(1)}(0) + \boxed{\gamma_{1,(2)}(1)} + \gamma_{1,(3)}(0) + \boxed{\gamma_{1,(4)}(1)} \\
+\gamma_{2,(1,2)}(0)(1) + \gamma_{2,(1,3)}(0)(0) + \gamma_{2,(1,4)}(0)(1) \\
+\gamma_{2,(2,3)}(1)(0) + \boxed{\gamma_{2,(2,4)}(1)(1)} + \gamma_{2,(3,4)}(0)(1) \\
+\gamma_{3,(1,2,3)}(0)(1)(0) + \gamma_{3,(1,2,4)}(0)(1)(1) + \gamma_{(1,3,4)}(0)(0)(1) \\
+\gamma_{3,(2,3,4)}(1)(0)(1) + \gamma_{4,(1,2,3,4)}(0)(1)(0)(1)
\end{gathered}
$$

  - The main effects of attribute 2 and attribute 4, and interaction between attributes 2 and 4 apply

$$\mu_6 = \gamma_{1,(2)} + \gamma_{1,(4)} + \gamma_{2,(2,4)}$$

Log-Linear Model Explained

- For profile 16: $\boldsymbol{\alpha}_{16} = [\alpha_{16,1} = 1; \alpha_{16,2} = 1; \alpha_{16,3} = 1; \alpha_{16,4} = 1]$:

$$\begin{gathered}\mu_{16} = \gamma_{1,(1)}(1) + \gamma_{1,(2)}(1) + \gamma_{1,(3)}(1) + \gamma_{1,(4)}(1) \\ +\gamma_{2,(1,2)}(1)(1) + \gamma_{2,(1,3)}(1)(1) + \gamma_{2,(1,4)}(1)(1) \\ +\gamma_{2,(2,3)}(1)(1) + \gamma_{2,(2,4)}(1)(1) + \gamma_{2,(3,4)}(1)(1) \\ +\gamma_{3,(1,2,3)}(1)(1)(1) + \gamma_{3,(1,2,4)}(1)(1)(1) \\ +\gamma_{3,(2,3,4)}(1)(1)(1) + \gamma_{4,(1,2,3,4)}(1)(1)(1)(1)\end{gathered}$$

  - All parameters apply

$$\begin{gathered}\mu_{16} = \\ \gamma_{1,(1)} + \gamma_{1,(2)} + \gamma_{1,(3)} + \gamma_{1,(4)} + \gamma_{2,(1,2)} + \gamma_{2,(1,3)} + \gamma_{2,(1,4)} \\ +\gamma_{2,(2,3)} + \gamma_{2,(2,4)} + \gamma_{2,(3,4)} + \gamma_{3,(1,2,3)} + \gamma_{3,(1,2,4)} + \gamma_{3,(2,3,4)} + \gamma_{4,(1,2,3,4)}\end{gathered}$$

Interpretations of Model Parameters

- The log-linear model with ALL main effects and interactions is statistically equivalent to the saturated structural model
- Two-way interactions are analogous to bivariate correlations in categorical models
  - Higher-level interactions represent higher level of characteristics of attribute distribution (i.e., skewness, kurtosis, etc…)
- Models without interactions imply uncorrelated attributes
  - Main effects are essentially attribute base-rates
- Models without main effects or interactions assume all attribute profiles are equally likely
- Higher order interactions can be removed if not significantly different from zero

Log-Linear Model for ECPE Data (Templin & Hoffman, 2013)

- To demonstrate the log-linear model, we again present our an example with 3 attributes (so 8 classes)
  - Full model (all parameters)

Parameter	Estimate	SE	p-value
γ_0	-0.139	0.112	0.216
$\gamma_{1,(1)}$	-3.539	0.888	0.000
$\gamma_{1,(2)}$	-3.228	1.378	0.019
$\gamma_{1,(3)}$	-0.846	0.220	0.000
$\gamma_{2,(1,2)}$	3.438	1.913	0.072
$\gamma_{2,(1,3)}$	1.578	1.079	0.144
$\gamma_{2,(2,3)}$	3.533	1.270	0.005
$\gamma_{3,(1,2,3)}$	-0.797	2.087	0.703

Reductions in the Structural Model

- Because the three-way interaction was not significant, we can remove that parameter from the model without greatly affecting model fit
  - New results:

Parameter	Estimate	SE	p-value
γ_0	-0.159	0.103	0.122
$\gamma_{1,(1)}$	-3.357	0.524	0.000
$\gamma_{1,(2)}$	-2.909	0.722	0.000
$\gamma_{1,(3)}$	-0.836	0.214	0.000
$\gamma_{2,(1,2)}$	2.785	0.608	0.000
$\gamma_{2,(1,3)}$	1.255	0.583	0.031
$\gamma_{2,(2,3)}$	3.222	0.638	0.000

New Results for Attribute Probabilities

- The reduced model only slightly modifies the attribute probabilities:

c	*Original* η_c	*New* η_c
1	0.30	0.30
2	0.13	0.13
3	0.01	0.02
4	0.18	0.18
5	0.01	0.01
6	0.02	0.02
7	0.01	0.01
8	0.34	0.34

TETRACHORIC STRUCTURAL MODELS

Tetrachoric Structural Models

- Because most summary information is given about attributes and pairs of attributes, tetrachoric models have been developed

- Such models use the tetrachoric correlation between attributes as a model for the probability for each attribute pattern

- Available in FlexMIRT, CDM package, and GDINA package

Defining Tetrachoric Correlations

- The tetrachoric correlation is a measure of the association between two binary variables (X, Y)
- The correlation comes from mapping the binary variables $(\tilde{X}, \tilde{Y})$ onto two "underlying" continuous variables
- Each of the continuous variables is bisected by a threshold (τ_X, τ_Y) which transforms the continuous response into a categorical outcome
- The distribution of the underlying continuous variables is

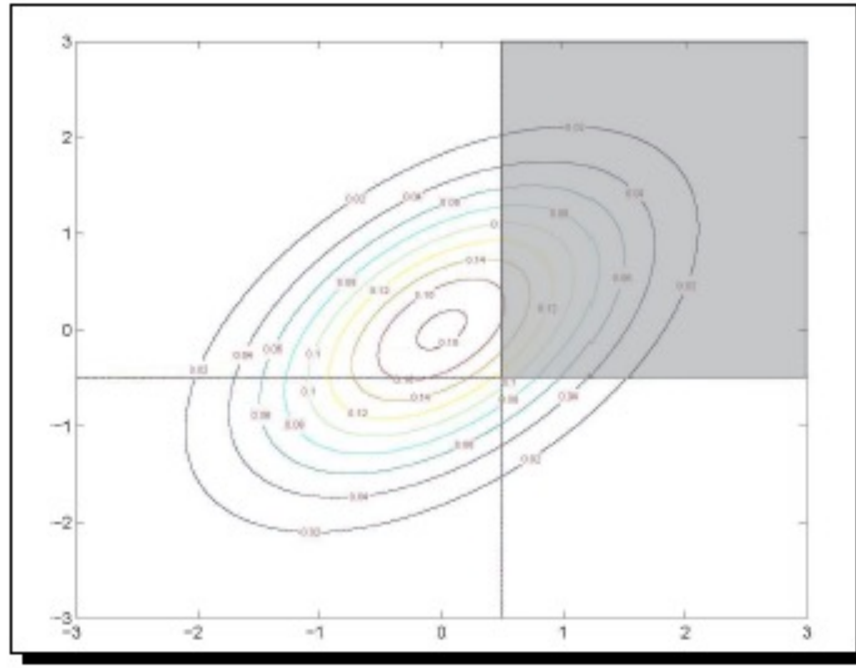
$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix} \sim N\left(\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\Xi} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

- ρ is the tetrachoric correlation coefficient

Tetrachoric Correlation Explained

Bivariate Contingency Table:

	X = 0	X = 1	
Y = 0	0.27	0.04	0.31
Y = 1	0.42	0.27	0.69
	0.69	0.31	1.00



$$P(X=1, Y=1) = P(\tilde{X} \geq \tau_X, \tilde{Y} \geq \tau_Y) =$$

$$\int_{\tau_X}^{\infty} \int_{\tau_Y}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\tilde{X}^2 - 2\rho\tilde{X}\tilde{Y} + \tilde{Y}^2\right) d\tilde{Y}\, d\tilde{X}$$

Technical Specifics: Multivariate Attributes

- The tetrachoric models assume use the following function to model the probability of an attribute profile:

$$\eta_c = \int_{a_1}^{b_1}\int_{a_2}^{b_2}\cdots\int_{a_A}^{b_A} \boxed{\frac{1}{(2\pi)^{A/2}\left|\boxed{\mathbf{\Xi}}\right|^{1/2}}\exp\left[-\frac{1}{2}\tilde{\boldsymbol{\alpha}}'\mathbf{\Xi}^{-1}\tilde{\boldsymbol{\alpha}}\right]}\,\partial\tilde{\alpha}_A \ldots \partial\tilde{\alpha}_2\partial\tilde{\alpha}_1$$

Tetrachoric Correlation Matrix

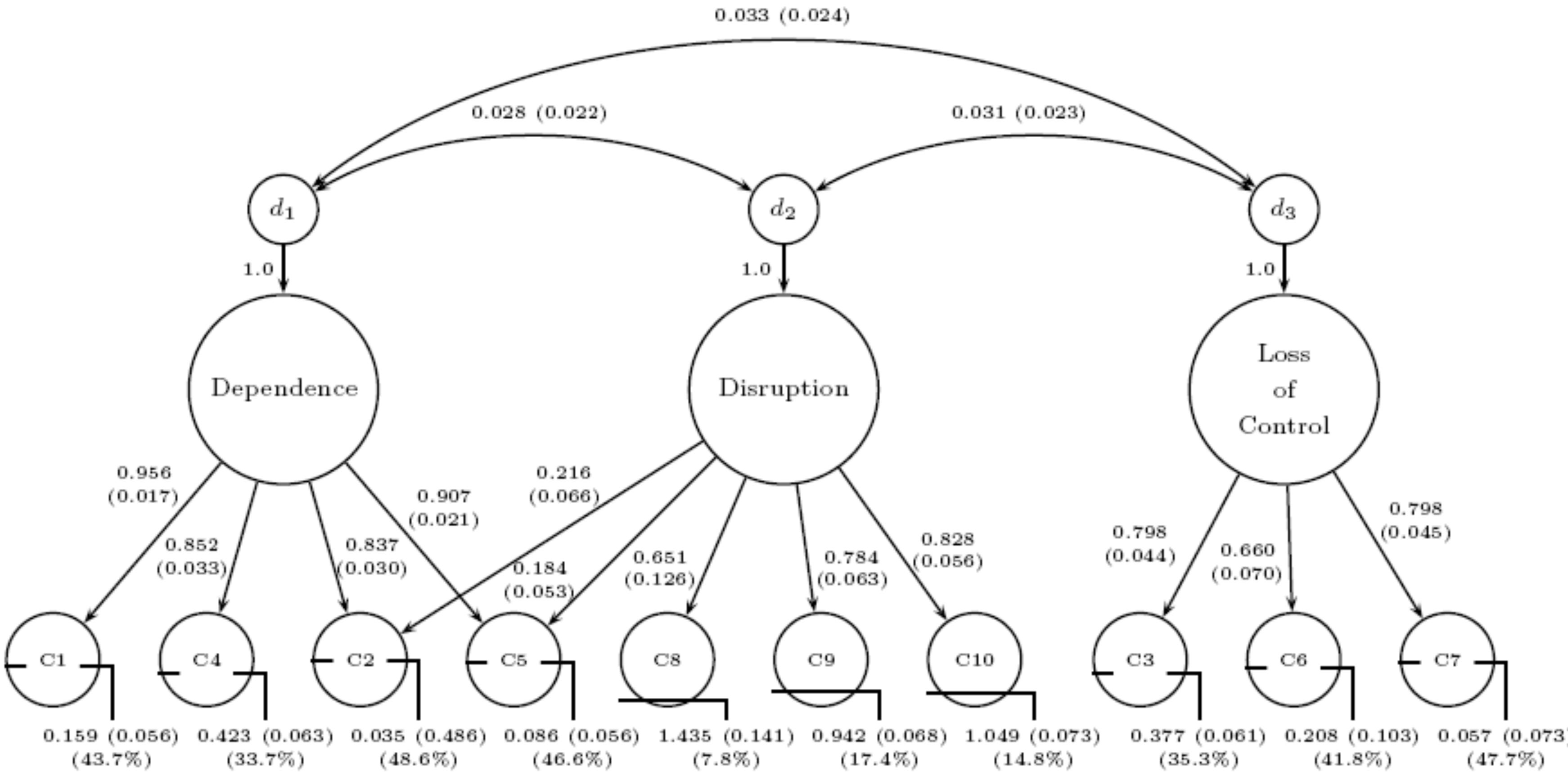
Multivariate Normal Density

- Where:

$$a_a = \begin{cases} \kappa_a & if \quad \alpha_{ca} = 1 \\ -\infty & if \quad \alpha_{ca} = 0 \end{cases} \qquad b_a = \begin{cases} \infty & if \quad \alpha_{cb} = 1 \\ \kappa_a & if \quad \alpha_{cb} = 0 \end{cases}$$

Structured Matrices

- Placing a structure on the Ξ tetrachoric correlation matrix expands the model to mimic SEM (Templin & Henson, 2006)



CONCLUDING REMARKS

Take-home Points

- DCM Structural Models describe the distribution of attributes
  - Means
  - Correlations
  - Overall structure

- Log-linear structural models are implemented in Mplus
  - Provide great flexibility in terms of number of parameters
  - Allow for ability to detect higher order structures
    - Attribute hierarchies
  - Allow for potential to model attributes using covariates