جامعة الإسكندرية
ALEXANDRIA UNIVERSITY
كلية الهندسة
Faculty of Engineering

CAR TRUNCK LINKAGE

MECHANICS OF MACHINERY I

TEAM 20

TEAM WORK

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Table of Contents

Abstract

This project **system**". The objective is to simulate the trunk motion using a theoretical mechanism, develop a **3D CAD model in SolidWorks**, perform kinematic analysis using **Analytical solution** which we have studied in our academic course and using **MATLAB**, and build a functional **Physical Prototype** to validate the design. The results demonstrate a practical application of mechanism theory.

Introduction

The 4-bar linkage is one of the simplest and most widely used mechanical systems for motion transmission. In automotive design, it is used for controlled opening and closing of trunk doors. This project explores the application of the 4-bar mechanism in designing a functional and smooth-operating **trunk linkage.**

Objective

- To design a 4-bar mechanism to simulate the motion of a car trunk.
- To perform kinematic analysis using Analytical solution and MATLAB.
- To create a CAD model using SolidWorks.
- To build a working physical prototype for demonstration.

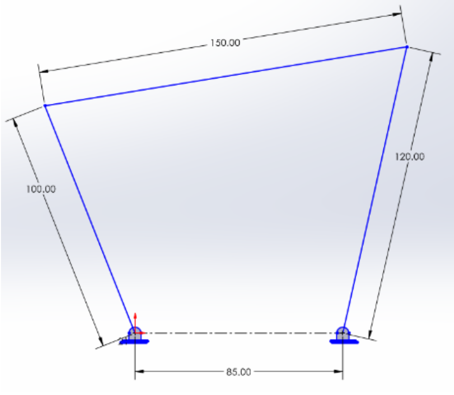
Theory and Background

A 4-bar mechanism consists of four rigid links connected by pivot joints. In this project, one link is fixed (Ground), one is the input crank, one is the connecting rod, and the last is the output link.

Key Equations Used:

- Position Analysis
- Velocity Analysis
- Acceleration Analysis
- Force and Torque Analysis

Project Process

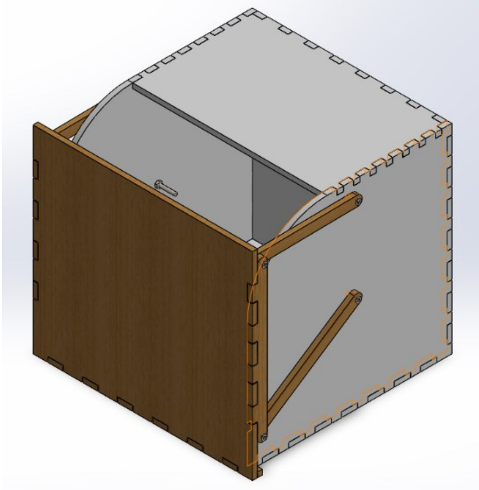


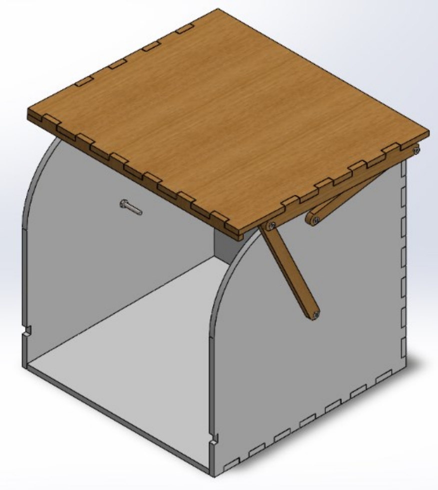
1. Mechanism Configuration

- Ground Link (Fixed): 85 mm
- Input Crank: 120 mm
- Connecting Rod Link: 150 mm
- Output Link: 100 mm

2. CAD Modeling

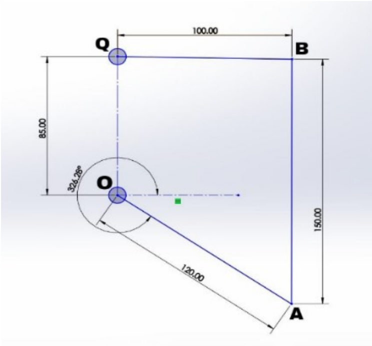
- Modeled using SolidWorks 2024
- Full assembly with motion simulation
- Interference and clearance checks completed

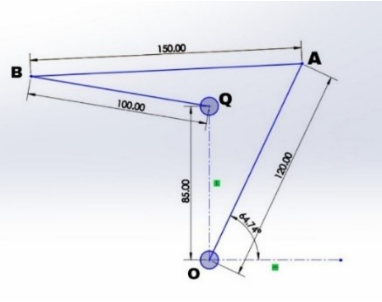




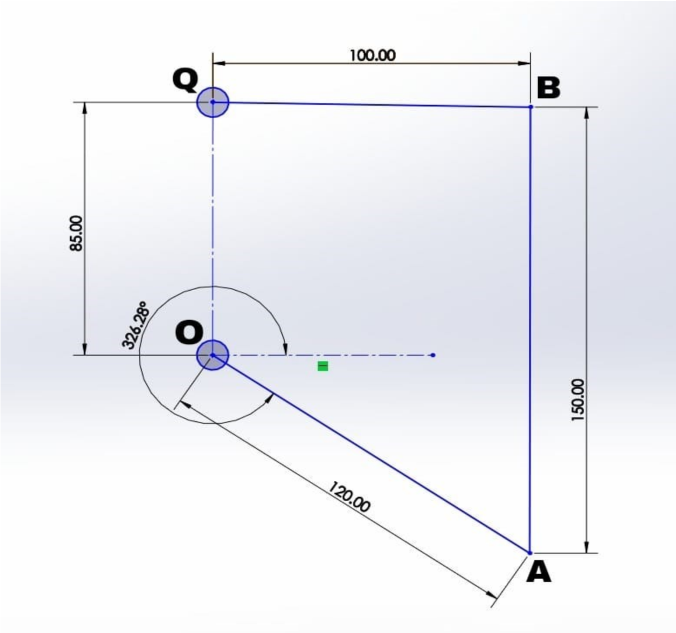
3. Analytical solution

- **Position, velocity,** and **acceleration** analyses were conducted for the **ERP (Fully Closed) and ELP (Fully Open)** configurations. **Velocity analysis** for **ERP** and **ELP** with constant **Angular Velocity = +3 RPM.**
- Force analysis.
- Calculations were cross-checked collaboratively by over five team members for higher accuracy.





First: Right Extreme Position



I-Position Analysis

$r_1 = 85$ mm, $r_2 = 120$ mm, $r_3 = 150$ mm, $r_4 = 100$ mm, $\theta_2 = 326.28°$

1. Coordinates of Point A:

$\because$ Point O is at the origin.

$\therefore \overrightarrow{oA} = \vec{r} \rightarrow [\, X_A + iY_A \,] = r_2 e^{i\theta_2}$

Taking the real part:

$X_a = r_2 \cos\theta_2 = 120 \cos(326.28°) = 99.81$ mm [p-1]

Taking the imaginary part:

$Y_a = r_2 \sin\theta_2 = 120 \sin(326.28°) = -66.62$ mm [p-2]

2. Vector $\vec{d}$:

Using vector loop OAQ

$$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$$

$$[x_Q + iy_Q] = [x_A + iy_A] + de^{i\theta_d}$$

$$de^{i\theta_d} = [x_Q - x_A] + i[y_Q - y_A]$$

By multiplying both sides by the conjugate:

$$de^{i\theta_d} * de^{-i\theta_d} = [\, [x_Q - x_A] + i[y_Q - y_A] * [x_Q - x_A] - i[y_Q - y_A] \,]$$

$$d^2 = (x_Q - x_A)^2 + (y_Q - y_A)^2$$

$$d = \sqrt{(x_Q - x_A)^2 + (y_Q - y_A)^2}$$

From 1:

taking the Real Parts $\Rightarrow d\cos(\theta_d) = (x_Q - x_A)$

taking the Imaginary parts $\Rightarrow d\sin(\theta_d) = (y_Q - y_A)$

$$d = \sqrt{(0-99.81)^2 + (85+66.62)^2} = 181.52 \text{ mm} \qquad [\text{ p-3 }]$$

$$\cos(\theta_d) = \frac{x(Q)-x(A)}{d} = \frac{0-99.81}{181.52} = -0.54 \qquad [\text{ p-4 }]$$

$$\sin(\theta_d) = \frac{Y(Q)-Y(A)}{d} = \frac{85+66.62}{181.52} = 0.84 \qquad [\text{ p-5 }]$$

from [p-4] and [p-5] $\therefore \theta_d = 123.36°$

3- θ_3 , β :

$$\overrightarrow{AB} = \overrightarrow{AQ} + \overrightarrow{QB}$$

$$\vec{r_3} = \vec{d} + \vec{r_4} \qquad \therefore \vec{r_4} = \vec{r_3} - \vec{d}$$

$$r_4\, e^{i\theta_4} = r_3\, e^{i\theta_3} - d\, e^{i\theta_d}$$

By multiplying by the conjugate: $r_4\, e^{i\theta_4} * r_4\, e^{-i\theta_4} = (r_3\, e^{i\theta_3} - d\, e^{i\theta_d}) * (r_3\, e^{-i\theta_3} - d\, e^{-i\theta_d})$

$$r_4^2 = r_3^2 + d^2 - r_3 d(e^{i\theta_\beta} + e^{-i\theta_\beta}) \quad for \quad \beta = \theta_3 - \theta_d$$

$$r_4^2 = r_3^2 + d^2 - 2r_3 d\cos(\theta_\beta) \qquad \therefore \quad \cos(\theta_\beta) = \frac{r_3^2 + d^2 - r_4^2}{2r_3 d}$$

$$\theta_3 = \theta_d - \beta$$

$$\beta = \cos^{-1}\left(\frac{150^2 + 181.5^2 - 100^2}{2*150*181.5}\right) = 33.43° \qquad [P-6]$$

$$\theta_3 = 123.36° - 33.43° = 89.93° \qquad [P-7]$$

θ_4 :

from 2 : <u>taking real parts</u> $\quad r_4 \cos(\theta_4) = r_3 \cos(\theta_3) - d\cos(\theta_d)$

$$\text{Cos}(\theta_4) = \frac{r_3 * \cos(\theta_3) - d*\cos(\theta_d)}{r_4} = \frac{150\cos(89.9) - 181.52\cos(123.36)}{100} = 0.9998 \qquad [P-8]$$

<u>Taking imaginary parts</u> $\quad r_4 \sin(\theta_4) = r_3 \sin(\theta_3) - d\sin(\theta_d)$

$$\sin(\theta_4) = \frac{r3*\sin(\theta_3) - d*\sin(\theta_d)}{r_4} = \frac{150\sin(89.9) - 181.52\sin(123.36)}{100} = 0.0166 \; [P-9]$$

from $[P-8]$ and $[P-9]$ $\qquad \theta_4 = 359.08°$

II- Velocity Analysis

$\boldsymbol{\omega_2}$ ***constant and equal 3 rpm***

$$\omega_2 = 3\,rpm = \frac{3*2\pi}{60} = 0.1\pi\,rad/sec$$

$$\overrightarrow{v_A} = ir_2\omega_2 e^{i\theta_2}$$

$$\overrightarrow{v_A} = v_A^x + iv_A^y$$

$$v_A^x = -\omega_2 * r_2 * \sin(\theta_2) = -0.1\pi * 120 * \sin(326.28) = 20.928\,mm/sec$$

$$v_A^y = \omega_2 * r_2 * \cos(\theta_2) = 0.1\pi * 120 * \cos(326.28) = 31.36\,mm/sec$$

$$v_B = v_A + v_{BA}$$

$$i\omega_4 * r_4 e^{i\theta_4} = \left(v_A^x + iv_A^y\right) + i\omega_3 * r_3 * e^{i\theta_3} \longrightarrow * e^{-i\theta_4}$$

$$i\omega_4 * r_4 = \left(v_A^x + iv_A^y\right)e^{-i\theta_4} + i\omega_3 * r_3 * e^{i(\theta_3-\theta_4)}$$

$$Real\ Part:\quad 0 = v_A^x * \cos(\theta_4) + v_A^y * \sin(\theta_4) - \omega_3 * r_3 * \sin(\theta_3 - \theta_4)$$

$$\therefore \omega_3 = \frac{v_A^x * \cos(\theta_4) + v_A^y * \sin(\theta_4)}{r_3 * \sin(\theta_3 - \theta_4)}$$

$$\omega_3 = \frac{20.928 * \cos(359.074) + 31.36\sin(359.074)}{150 * \sin(89.9 - 359.074)} = 0.136\,rad/sec$$

$$Img\ Part: \omega_4 * r_4 = -v_A^x * \sin(\theta_4) + v_A^y * \cos(\theta_4) + \omega_3 * r_3 * \cos(\theta_3 - \theta_4)$$

$$\therefore \omega_4 = \frac{= -v_A^x * \sin(\theta_4) + v_A^y * \cos(\theta_4) + \omega_3 * r_3 * \cos(\theta_3 - \theta_4)}{r_4}$$

$$= \frac{-20.928 * \sin(359.074) + 31.36\cos(359.074) + 0.136 * 150 * \cos(89.9 - 359.074)}{100}$$

$$\omega_4 = 0.314\,rad/sec$$

III- Acceleration Analysis

$\boldsymbol{\alpha_2 = 0}$

$\overrightarrow{a_A} = r_2\,(-\omega_2^2 + i\alpha_2)e^{i\theta_2}$

$\overrightarrow{a_A} = a_A^x + \, ia_A^y$

$a_A^x = -\omega_2^2 * r_{2*} cos\,(\theta_2) - \alpha_2 * r_2 * sin(\theta_2) = -(0.1\pi)^2 * 120 * cos(326.28) - 0$

$a_A^x = -9.85\, mm/sec^2$

$a_A^y = -\omega_2^2 * r_{2*}\, sin(\theta_2) + \alpha_2 * r_2 * cos(\theta_2) = -(0.1\pi)^2 * 120 * sin(326.28) + 0$

$a_A^y = 6.57 mm/sec^2$

$a_B = a_A + a_{BA}$

$(-\omega_4^2 * r_4 + i\alpha_4 * \therefore)\, e^{i\theta_4} = \left(a_A^x + ia_A^y\right) + (-\omega_3^2 * r_3 + i\alpha_3 * r_3)\, e^{i\theta_3} \qquad * \; e^{-i\theta_4}$

$(-\omega_4^2 * r_4 + i\alpha_4 * r_4) = \left(a_A^x + ia_A^y\right) e^{-i\theta_4} + (-\omega_3^2 * r_3 + i\alpha_3 * r_3)\, e^{i(\theta_3 - \theta_4)}$

$Real\ part := -\omega_4^2 * r_4 = a_A^x * cos(\theta_4) + a_A^y * sin(\theta_4) - \omega_3^2 * r_3 * cos(\theta_3 - \theta_4) - \alpha_3 * r_3 * sin(\theta_3 - \theta_4)$

$$\therefore \alpha_3 = \frac{\omega_4^2 * r_4 + a_A^x * cos(\theta_4) + a_A^y * sin(\theta_4) - \omega_3^2 * r_3 * cos(\theta_3 - \theta_4)}{r_3 * sin(\theta_3 - \theta_4)}$$

$$= \frac{0.314^2 * 100 - 9.85 * cos(359.074) + 6.57 * sin(359.074) - 0.136^2 * 150 * cos(89.9 - 359.074)}{150 * sin(89.9 - 359.074)}$$

$\alpha_3 = -3.69 * 10^{-4}\, rad/sec^2$

$Img\ part: \alpha_4 * r_4 = -a_A^x * sin\,(\theta_4) + a_A^y * cos(\theta_4) - \omega_3^2 * r_3 * sin(\theta_3 - \theta_4) + \alpha_3 * r_3 * cos(\theta_3 - \theta_4)$

$$\therefore \alpha 4 = \frac{-a_A^x * sin\,(\theta_4) + a_A^y * cos(\theta_4) - \omega_3^2 * r_3 * sin(\theta_3 - \theta_4) + \alpha_3 * r_3 * cos(\theta_3 - \theta_4))}{r_4}$$

$$\frac{9.85 * sin(359.074) + 6.57 * cos(359.074) - 0.136^2 * 150 * sin(89.9 - 359.074) + 3.69 * 10^{-4} * 150\, cos(89.9 - 359.074)}{100}$$

$\alpha 4 = 0.036\, rad/sec^2$

IV- Force &Torque Analysis

Inertia

For link (4) a B:

$$a_{G_4}^x = -\omega_4^2 g_4 \, cos(\theta_4) - \alpha_4 g_4 \, sin(\theta_4)$$

$$= -((0.3138)^2) \times 50 \times 10^{-3} \; cos(359.07) - 0.0364 \times 50 \times 10^{-3} \sin(359.07)$$
$$= -4.893462579 \times 10^{-3} \; m/s^2$$

$$a_{G_4}^y = -\omega_4^2 g_4 \, sin(\theta_4) + \alpha_4 g_4 \, cos(\theta_4)$$

$$= -((0.3138)^2) \times 50 \times 10^{-3} \; sin(359.07) - 0.0364 \times 50 \times 10^{-3} \cos(359.07)$$
$$= 1.899340114 \times 10^{-3} \; m/s^2$$

$$X_4 = -m_4 * \; a_{G_4}^x = -3.42 \times 10^{-3} \times (-4.893462579 \times 10^{-3}) = 1.673564202 \times 10^{-5} \, \mathrm{N}$$

$$Y_4 = -m_4 * \; a_{G_4}^y = -3.42 \times 10^{-3} \times (1.899340114 \times 10^{-3}) = -6.49574319 \times 10^{-6} \, \mathrm{N}$$

$$T_4 = -I_4 * \alpha_4 = -5089.54 \times 10^{-6} \times 0.0364 = -1.85259256 \times 10^{-4} \; N.m$$

For link (3) AB:

$$a_{G_3}^x = a_A^x - \omega_3^2 g_3 \, cos(\theta_3) - \alpha_3 g_3 \, sin(\theta_3)$$

$$= -9.85 - (0.1361)^2 \times 75 \times 10^{-3} \times \cos(89.93) + 0.0004 \times 75 \times 10^{-3} \times \sin(89.93)$$
$$= -9.8216 \times 10^{-3} \; m/s^2$$

$$a_{G_3}^y = a_A^y - \omega_3^2 g_3 \, sin(\theta_3) + \alpha_3 g_3 \, cos(\theta_3)$$

$$= 6.57 - (0.1361)^2 \times 75 \times 10^{-3} \times \sin(89.93) + 0.0004 \times 75 \times 10^{-3} \times \cos(89.93)$$
$$= 5.1507 \times 10^{-3} \; m/s^2$$

$$X_3 = -m_3 * \; a_{G_3}^x = -6.2 \times 10^{-3} \times (-9.84997) = 6.0884 \times 10^{-5} \, \mathrm{N}$$

$$Y_3 = -m_3 * \; a_{G_3}^y = -6.2 \times 10^{-3} \times (6.56861) = -3.198208 \times 10^{-4} \, \mathrm{N}$$

$$T_3 = -I_3 * \alpha_3 = -18438.11 \times 10^{-6} \times -0.0004 = 7.375244 \times 10^{-6} \; N.m$$

For link (2) OA :

$$a_{G_2}^{x} = -\omega_2^2 g_2 \, cos(\theta_2) - \alpha_2 g_2 \, sin(\theta_2)$$

$$= -(0.1\pi)^2 \times 60 \times 10^{-3} \times \cos(326.28) - 0 = -4.925487628 \times 10^{-3} \; m/s^2$$

$$a_{G_2}^{y} = -\omega_2^2 g_2 \, sin(\theta_2) + \alpha_2 g_2 \, cos(\theta_2)$$

$$= -(0.1\pi)^2 \times 60 \times 10^{-3} \times \sin(326.28) = 3.287376522 \times 10^{-3} \; m/s^2$$

$$X_2 = -m_2 * a_{G_2}^{x} = -4.13 \times 10^{-3} \times (-4.925487628 \times 10^{-3}) = 2.034226391 \times 10^{-6} \, \mathrm{N}$$

$$Y_2 = -m_2 * a_{G_2}^{y} = -4.13 \times 10^{-3} \times (3.287376522 \times 10^{-3}) = -1.357686503 \times 10^{-5} \, \mathrm{N}$$

$$T_2 = -I_2 * \alpha_2 = -8496.82 \times 10^{-6} \times 0 = 0$$

For link (4) :

$$\sum M_a = 0$$

$$M_{14} - x_4 g_4 \sin\theta_4 + y_4 \, g_4 \, \cos\theta_4 - x_{34} r_4 \sin\theta_4 + y_{34} r_4 \cos\theta_4 = 0$$

$$3 \times 10^{-5} - (1.674 \times 10^{-5} \times 50 \times 10^{-3} \sin(359.07)) - (6.4887 \times 10^{-6} \times 50 \times 10^{-3} \cos(359.07)) - x_{34} \times 0.1 \sin(359.07) + y_{34} \times 0.1 \cos(359.07) = 0$$

$$1.6162 \times 10^{-3} \, x_{34} + 0.09998 \, y_{34} = 2.9689 \times 10^{-5} \qquad (1)$$

For link (3) :

$$\sum M_A = 0$$

$$-x_3 g_3 \sin\theta_3 + y_3 \, g_3 \, \cos\theta_3 + x_{34} r_3 \sin\theta_3 - y_{34} r_3 \cos\theta_3 = 0$$

$$-(6.0886 \times 10^{-5} \times 76 \times 10^{-3} \times \sin(89.9)) - (3.214 \times 10^{-5} \times 75 \times 10^{-3} \times \cos(89.9)) + 0.15 \sin(89.9) \times x_{34} - 0.15 \cos(89.9) \times y_{34} = 0$$

$$0.149 \, x_{34} - 2.618 \times 10^{-4} \, y_{34} = 4.5692 \times 10^{-6} \qquad (2)$$

from (1), (2)

$$x_{34} = 3.1186 \times 10^{-5} \, N$$

$$y_{34} = 2.964 \times 10^{-4} \, N$$

For link (4) :

$$\sum F_X = 0$$

$$x_{14} = -x_4 - x_{34} = -(1.674 \times 10^{-5}) - (3.1186 \times 10^{-5}) = -4.7926 \times 10^{-5}\ N$$

$$\sum F_Y = 0$$

$$y_{14} = -y_4 - y_{34} = -(-6.488 \times 10^{-6}) - (2.964 \times 10^{-4}) = -2.89912 \times 10^{-4}\ N$$

For link (3) :

$$\sum F_X = 0$$

$$x_{23} = -(x_3 - x_{34}) = -((6.0886 \times 10^{-5}) - (3.1186 \times 10^{-5})) = -2.97 \times 10^{-5}\ N$$

$$\sum F_Y = 0$$

$$y_{23} = -(y_3 - y_{34}) = -((-3.2145 \times 10^{-5}) - (2.964 \times 10^{-4})) = 3.28545 \times 10^{-4}\ N$$

For link (2) :

$$\sum F_X = 0$$

$$x_{12} = -(x_2 - x_{23}) = -((2.0342 \times 10^{-5}) + (2.97 \times 10^{-5})) = -5.0042 \times 10^{-5}\ N$$

$$\sum F_Y = 0$$

$$y_{12} = -(y_2 - y_{23}) = -((-1.3576 \times 10^{-5}) - (3.28545 \times 10^{-4})) = 3.421 \times 10^{-4}\ N$$

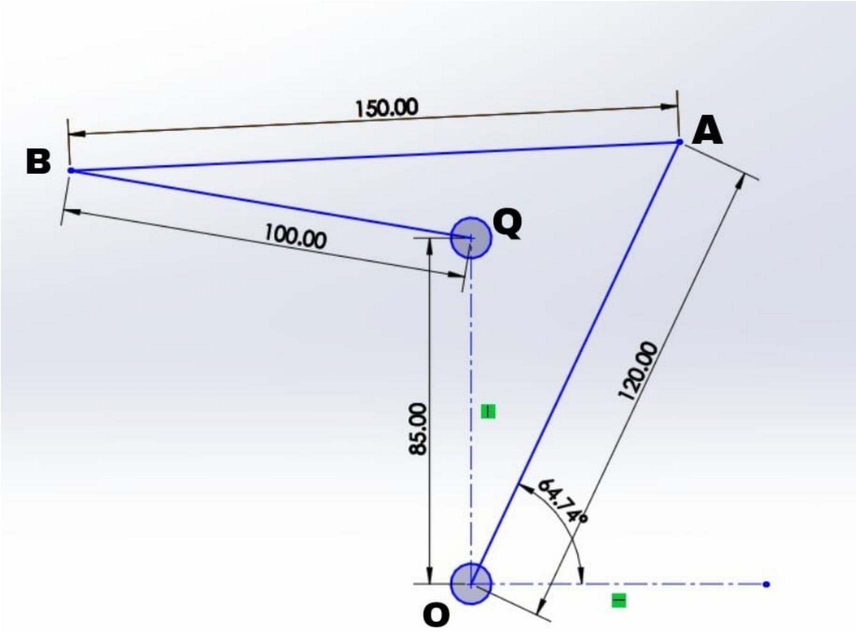
$$\sum M_O = 0$$

$$\begin{aligned} M_{12} = {} & (2.034 \times 10^{-5} \times 60 \times 10^{-3} \times \sin(326.28)) \\ & + (1.3576 \times 10^{-5} \times 60 \times 10^{-3} \times \cos(326.28)) \\ & + (2.97 \times 10^{-5} \times 0.12 \sin(326.28)) \\ & + (3.28545 \times 10^{-4} \times 0.12 \cos(326.28)) \end{aligned}$$

$$M_{12} - x_2 g_2 \sin\theta_2 + y_2 g_2 \cos\theta_2 + x_{23} r_2 \sin\theta_2 - y_{23} r_2 \cos\theta_2 = 0$$

$$M_{12} = 3.0814 \times 10^{-5}\ N.m$$

Second- left Extreme Position



I-Position Analysis

$$r_1 = 85 \text{ mm}, r_2 = 120 \text{ mm}, r_3 = 150 \text{ mm}, r_4 = 100 \text{ mm}$$

$$\theta_2 = 64.74°$$

1. Coordinates of Point A:

$\because$ Point O is at the origin.

$$\therefore \overrightarrow{oA} = \vec{r} \rightarrow [\, X_A + iY_A \,] = r_2 e^{i\theta_2}$$

Taking the real part: $X_A = r_2 \cos\theta_2 = 120 \cos(64.74°) = 51.21\text{mm}$ [p-1]

Taking the imaginary part: $Y_A = r_2 \sin\theta_2 = 120 \sin(64.74°) = 108.53\text{mm}$ [p-2]

2. Vector $\overrightarrow{d}$:

Using vector loop OAQ

$$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$$

$$[x_Q + iy_Q] = [x_A + iy_A] + de^{i\theta_d}$$

$$de^{i\theta_d} = [x_Q - x_A] + i[y_Q - y_A]$$

By multiplying both sides by the conjugate :

$$de^{i\theta_d} * de^{-i\theta_d} = [\,[x_Q - x_A] + i[y_Q - y_A] * [x_Q - x_A] - i[y_Q - y_A]\,]$$

$$d^2 = (x_Q - x_A)^2 + (y_Q - y_A)^2$$

$$\Rightarrow d = \sqrt{(x_Q - x_A)^2 + (y_Q - y_A)^2}$$

From 1:

taking the Real Parts $\Rightarrow$ $d\cos(\theta_d) = (x_Q - x_A)$

taking the Imaginary parts $\Rightarrow$ $d\sin(\theta_d) = (y_Q - y_A)$

$d = \sqrt{(0-51.21)^2 + (85-108.53)^2}$ =56.36mm [p-3]

$\cos(\theta_d) = \frac{x(Q)-x(A)}{d} = \frac{0-51.21}{56.36} = -0.91$ [p-4]

$\sin(\theta_d) = \frac{Y(Q)-Y(A)}{d} = \frac{85-108.532}{56.36} = -0.42$ [p-5]

from [p-4] and [p-5] $\therefore \theta_d = 204.83°$

3- θ_3 , β :

$\overrightarrow{AB} = \overrightarrow{AQ} + \overrightarrow{QB}$

$\overrightarrow{r_3} = \vec{d} + \overrightarrow{r_4}$ $\therefore$ $\overrightarrow{r_4} = \overrightarrow{r_3} - \vec{d}$

$r_4\, e^{i\theta_4} = r_3\, e^{i\theta_3} - d\, e^{i\theta_d}$

By multiplying by the conjugate:

$r_4\, e^{i\theta_4} * r_4\, e^{-i\theta_4} = (r_3\, e^{i\theta_3} - d\, e^{i\theta_d}) * (r_3\, e^{-i\theta_3} - d\, e^{-i\theta_d})$

$r_4^2 = r_3^2 + d^2 - r_3 d(e^{i\theta_\beta} + e^{-i\theta_\beta})$ for $\beta = \theta_3 - \theta_d$

$r_4^2 = r_3^2 + d^2 - 2r_3 d\cos(\theta_\beta)$ $\therefore$ $\cos(\theta_\beta) = \frac{r_3^2 + d^2 - r_4^2}{2r_3 d}$

$\theta_3 = \theta_d - \beta$

$\beta = \cos^{-1}(\frac{150^2 + 56.36^2 - 100^2}{2*150*56.36}) = 22°$ [P–6]

$\theta_3 = 204.83° - 22° = 182.83°$ [P–7]

θ_4 :

from 2 : <u>taking real parts</u> $r_4\cos(\theta_4) = r_3\cos(\theta_3) - d\cos(\theta_d)$

$\text{Cos}(\theta_4) = \frac{r_3 * \cos(\theta_3) - d * \cos(\theta_d)}{r_4} = \frac{150\cos(182.83) - 56.36\cos(204.83)}{100} = -0.987$ [P–8]

<u>Taking imaginary parts</u> $r_4\sin(\theta_4) = r_3\sin(\theta_3) - d\sin(\theta_d)$

$\sin(\theta_4) = \frac{r3 * \sin(\theta_3) - d * \sin(\theta_d)}{r_4} = \frac{150\sin(182.83) - 56.36\sin(204.83)}{100} = 0.163$ [P–9]

from [P–8] and [P–9] $\theta_4 = 170.62°$

II- Velocity Analysis

$$\boldsymbol{\omega_2}\ \textbf{\textit{constant and equal 3 rpm}}$$

$$\omega_2 = 3\ rpm = \frac{3 * 2\pi}{60} = 0.1\pi\ rad/sec$$

$$\overrightarrow{v_A} = ir_2\omega_2 e^{i\theta_2}$$

$$\overrightarrow{v_A} = v_A^x + iv_A^y$$

$$v_A^x = -\omega_2 * r_2 * sin(\theta_2) = -0.1\pi * 120 * sin(64.74) = -34.09\ mm/sec$$

$$v_A^y = \omega_2 * r_2 * cos\ (\theta_2) = 0.1\pi * 120 * cos(64.74) = 16.08\ mm/sec$$

$$v_B = v_A + v_{BA} \longrightarrow$$

$$i\omega_4 * r_4 e^{i\theta_4} = \left(v_A^x + iv_A^y\right) + i\omega_3 * r_3 * e^{i\theta_3} \qquad * e^{-i\theta_4}$$

$$i\omega_4 * r_4 = \left(v_A^x + iv_A^y\right)e^{-i\theta_4} + i\omega_3 * r_3 * e^{i(\theta_3-\theta_4)}$$

$$Real\ Part:\ \ 0 = v_A^x * cos(\theta_4) + v_A^y * sin(\theta_4) - \omega_3 * r_3 * sin(\theta_3 - \theta_4)$$

$$\therefore \omega_3 = \frac{v_A^x * cos(\theta_4) + v_A^y * sin(\theta_4)}{r_3 * sin(\theta_3 - \theta_4)}$$

$$\omega_3 = \frac{-34.09 * cos(170.62) + 16.08 sin\ (170.62)}{150 * sin\ (182.83 - 170.62)} = 1.1428\ rad/sec$$

$$Img\ Part: \omega_4 * r_4 = -v_A^x * sin(\theta_4) + v_A^y * cos(\theta_4) + \omega_3 * r_3 * cos(\theta_3 - \theta_4)$$

$$\therefore \omega_4 = \frac{= -v_A^x * sin(\theta_4) + v_A^y * cos(\theta_4) + \omega_3 * r_3 * cos(\theta_3 - \theta_4)}{r_4}$$

$$= \frac{- - 34.09 * sin(170.62) + 16.08\, cos(170.62) + 1.1428 * 150 * cos(182.83 - 170.62)}{100}$$

$$\omega_4 = 1.5723\ rad/sec$$

III- Acceleration Analysis

$$\boldsymbol{\alpha_2 = 0}$$

$$\overrightarrow{a_A} = r_2\,(-\omega_2^2 + i\alpha_2)e^{i\theta_2}$$

$$\overrightarrow{a_A} = a_A^x + \; ia_A^y$$

$$a_A^x = -\omega_2^2 * r_2 * cos\,(\theta_2) - \alpha_2 * r_2 * sin(\theta_2) = -(0.1\pi)^2 * 120 * cos(64.74) - 0$$

$$a_A^x = -5.05mm/sec^2$$

$$a_A^y = -\omega_2^2 * r_2 * sin(\theta_2) + \alpha_2 * r_2 * cos(\theta_2) = -(0.1\pi)^2 * 120 * sin(64.74) + 0$$

$$a_A^y = -10.71mm/sec^2$$

$$a_B = a_A + a_{BA}$$

$$(-\omega_4^2 * r_4 + i\alpha_4 * r_4)\,e^{i\theta_4} = \left(a_A^x + ia_A^y\right) + (-\omega_3^2 * r_3 + i\alpha_3 * r_3)\,e^{i\theta_3} \qquad * \; e^{-i\theta_4}$$

$$(-\omega_4^2 * r_4 + i\alpha_4 * r_4) = \left(a_A^x + ia_A^y\right)e^{-i\theta_4} + (-\omega_3^2 * r_3 + i\alpha_3 * r_3)\,e^{i(\theta_3-\theta_4)}$$

$$Real\,part := -\omega_4^2 * r_4 = a_A^x * cos(\theta_4) + a_A^y * sin(\theta_4) - \omega_3^2 * r_3 * cos(\theta_3 - \theta_4) - \alpha_3 * r_3 * sin(\theta_3 - \theta_4)$$

$$\therefore \alpha_3 = \frac{\omega_4^2 * r_4 + a_A^x * cos(\theta_4) + a_A^y * sin(\theta_4) - \omega_3^2 * r_3 * cos(\theta_3 - \theta_4)}{r_3 * sin(\theta_3 - \theta_4)}$$

$$= \frac{1.5723^2 * 100 - 5.05 * cos(170.62) \; - \; 10.71 * sin(170.62) - 1.1428^{\,2} * 150 * cos(182.83 - 170.62)}{150 * sin(182.83 - 170.62)}$$

$$\alpha_3 = 1.77rad/sec^2$$

$$Img\;part{:}\, \alpha_4 * r_4 = -a_A^x * sin\,(\theta_4) + a_A^y * cos(\theta_4) - \omega_3^2 * r_3 * sin(\theta_3 - \theta_4) + \alpha_3 * r_3 * cos(\theta_3 - \theta_4)$$

$$\therefore \alpha 4 = \frac{-a_A^x * sin\,(\theta_4) + a_A^y * cos(\theta_4) - \omega_3^2 * r_3 * sin(\theta_3 - \theta_4) + \alpha_3 * r_3 * cos(\theta_3 - \theta_4))}{r_4}$$

$$\frac{5.05 * sin(170.62) - 10.71 * cos(170.62) - 1.1428^2 * 150 * sin(182.83 - 170.62) + 1.77 * 150\, cos(182.83 - 170.62)}{100}$$

$$\alpha 4 = 2.29rad/sec^2$$

IV- Force &Torque Analysis

For link (2) point(G_2) :

$$\overrightarrow{a_{G_2}} = a_{G_2}^x + a_{G_2}^y = -(\omega_2^2 + i\alpha_2)\frac{r_2}{2}e^{i\theta_2}$$

$$a_{G_2}^x = -\omega_2^2\frac{r_2}{2}cos(\theta_2) - \alpha_2\frac{r_2}{2}sin(\theta_2)$$

$$= -\left(\left(\frac{\pi}{10}\right)^2\right) * \frac{120}{2}cos(64.74°) - 0 = -2.52 \times 10^3\ m/s^2$$

$$a_{G_2}^y = -\omega_2^2\frac{r_2}{2}sin(\theta_2) + \alpha_2\frac{r_2}{2}cos(\theta_2)$$

$$= -\left(\left(\frac{\pi}{10}\right)^2\right) * \frac{120}{2}sin(64.74°) = -5.35 \times 10^3\ m/s^2$$

For link (3) point(G_3) :

$$\overrightarrow{a_{G_3}} = \overrightarrow{a_A} + \overrightarrow{a_{G_3/A}}$$

$$a_{G_3}^x + ia_{G_3}^y = a_A^x + ia_A^y + (-\omega_3^2 + i\alpha_3)\frac{r_3}{2}e^{i\theta_3}$$

$$a_{G_3}^x = a_A^x + \frac{r_3}{2}(-\omega_3^2\,cos(\theta_3) - \alpha_3\,sin(\theta_3))$$

$$= -5.05 + \frac{150}{2}(-1{,}1428^2\,cos(182.83°) - 1.77\,sin(182.83°))$$

$$= 99.33 \times 10^{-3}\ m/s^2$$

$$a_{G_3}^y = a_A^y + \frac{r_3}{2}(-\omega_3^2\,sin(\theta_3) + \alpha_3\,cos(\theta_3))$$

$$= -10.71 + \frac{150}{2}(-1{,}1428^2\,sin(182.83°) - 1.77\,cos(182.83°))$$

$$= 126.71 \times 10^{-3}\ m/s^2$$

For link (4): point(G_4) :

$$\overrightarrow{a_{G_4}} = a_{G_4}^x + ia_{G_4}^y = (-\omega_4^2 + i\alpha_4)\frac{r_4}{2}e^{i\theta_4}$$

$$a_{G_4}^x = \frac{r_4}{2}(-\omega_4^2\cos(\theta_4) - \alpha_4\sin(\theta_4))$$

$$= \frac{100}{2}(-1.5723^2\cos(170.62°) - 2.29\sin(170.62°))$$

$$= 103.29 \times 10^{-3}\ \mathrm{m/s^2}$$

$$a_{G_4}^{y} = \frac{r_4}{2}(-\omega_4^2 \sin(\theta_4) + \alpha_4 \cos(\theta_4))$$

$$= \frac{100}{2}(-1.5723^2 \sin(170.62°) + 2.29 \cos(170.62°))$$

$$= -133.11 \times 10^{-3}\ \mathrm{m/s^2}$$

Inertia forces :

$$X_2 = -m_2 * a_{G_2}^{x} = -4.13 \times 10^{-3} - 2.52 \times 10^{-3} = 1.04 \times 10^{-5}\ \mathrm{N}$$

$$Y_2 = -m_2 * a_{G_2}^{y} = -4.13 \times 10^{-3} \times -5.35 \times 10^{-3} = 2.2 \times 10^{-5}\ \mathrm{N}$$

$$X_3 = -m_3 * a_{G_3}^{x} = -6.2 \times 10^{-3} \times 99.33 \times 10^{-3} = -61.5 \times 10^{-5}\ \mathrm{N}$$

$$Y_3 = -m_3 * a_{G_3}^{y} = -6.2 \times 10^{-3} \times 126.71 \times 10^{-3} = -78.5 \times 10^{-5}\ \mathrm{N}$$

$$X_4 = -m_4 * a_{G_4}^{x} = -3.42 \times 10^{-3} \times 103.29 \times 10^{-3} = -35.3 \times 10^{-5}\ \mathrm{N}$$

$$Y_4 = -m_4 * a_{G_4}^{y} = -3.42 \times 10^{-3} \times (-133.11 \times 10^{-3}) = 45.5 \times 10^{-5}\ \mathrm{N}$$

Link(3) :

$$\sum M_A = 0 \qquad \text{"}+\circlearrowleft\text{"}$$

$$-Y_3 * \frac{r_3}{2}\cos(\theta_3) - Y_{43} * r_3 \cos(\theta_3) + X_3 * \frac{r_3}{2}\sin(\theta_3) + X_{43} \cdot r_3 \sin(\theta_3) = 0$$

$$(-78.5 \times 10^{-5}) \times \frac{150}{2} \times \cos(182.83°) - 150 \times \cos(182.83°) \times Y_{43}$$
$$- 61.5 \times 10^{-5} \times \frac{150}{2} \times \sin(182.83°) + 150 \times \sin(182.83°) \times X_{43} = 0$$

$$-7.4 * X_{43} + 149.8 * Y_{43} = 0.0565 \qquad (1)$$

Link(4) :

$$\sum M_Q = 0$$

$$\theta_4' = 180° - \theta_4 = 9.38°$$

$$M_{14} - x_4 \frac{r_4}{2} \sin\theta_4' - y_4\left(\frac{r_4}{2}\cos\theta_4'\right) - x_{34} r_4 \sin\theta_4' - y_{34} r_4 \cos\theta_4' = 0$$

$$3 \times 10^{-5} - (-35.5 \times 10^{-5}) \frac{100}{2} \times \sin(9.38) - 45.5 \times 10^{-5} \times \frac{100}{2}$$
$$\times \cos(9.38) - 100 \times \sin(9.38) \times x_{34} - 100 \times \cos(9.38) \times y_{34} = 0$$
$$-16.3 \times x_{34} - 98.6 \times y_{34} = -0.0104$$
$$16.3 \times x_{43} + 98.6 \times y_{34} = -0.0104 \qquad (2)$$

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from (1), (2)

$$x_{43} = -2.24 \times 10^{-3}\ N$$

$$y_{43} = 2.66 \times 10^{-4}\ N$$

Link(3) **:**

$$\sum F_X = 0$$

$$x_{23} = -x_{43} - \ \ x_3 \ = -(-2.24 \ \times 10^{-3}) - (-61.5 \ \times 10^{-5}) = 2.865 \ \times 10^{-3} \ N$$

$$\sum F_Y = 0$$

$$y_{23} = -y_{43} - \ \ y_3 \ = -(2.66 \ \times 10^{-4}) - (-78.5 \ \times 10^{-5}) = 5.19 \ \times 10^{-4} \ N$$

Link(2) **:**

$$\sum M_O = 0 \qquad "+\circlearrowleft"$$

$$x_{32} = -x_{23} = (-2.865 \ \times 10^{-3}) \ N$$

$$y_{32} = -y_{23} = (-5.19 \ \times 10^{-4}) \ N$$

$$M_{12} - x_2 \frac{r_2}{2} \sin\theta_2 + y_2 \left(\frac{r_2}{2} \cos\theta_2\right) - x_{32} r_2 \sin\theta_2 + y_{32} r_2 \cos\theta_2 \ = \ 0$$

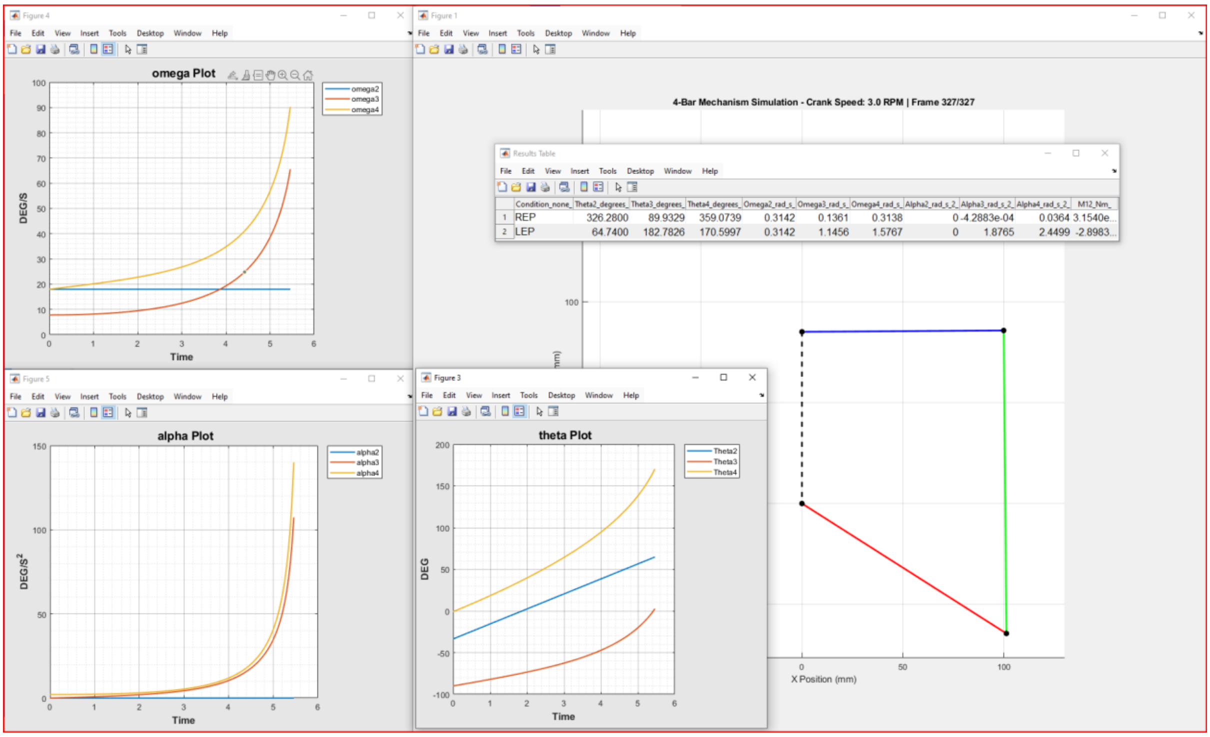
$$M_{12} - 1.04 \ \times 10^{-5} \times \frac{120}{2} \sin(64.74) + 2.2 \times 10^{-5} \left(\frac{120}{2} \cos(64.74)\right)$$

$$- (-2.865 \times 10^{-3}) \times 120 \sin(64.74)$$

$$+ (-5.19 \times 10^{-4}) \times 120 \cos(64.74) \ = \ 0$$

$$M_{12} = -0.283 \ N.mm = -2.83 \ \times 10^{-4} \ N.m$$

4. MATLAB Calculations



- Kinematic analysis over a full rotation
- Input angular velocity: +3 rad/s (constant)
- Output graphs for angular displacement, velocity, and acceleration of each link

5. Printing operation

- The parts were manufactured using a CNC laser cutting machine on 5 mm MDF sheets.

6. Assembly operation

- Assembly was performed using a tap-and-slot method with supplementary adhesive (super glue) where needed.
- The mechanism was mounted onto the box using 3 mm steel pins for joint articulation.

Results

- Smooth and realistic trunk motion achieved.
- MATLAB plots show continuous and stable output motion.
- SolidWorks simulation confirms the mechanical feasibility.
- Physical prototype replicates the designed motion effectively.

Discussion

The simulation and physical results closely match the theoretical predictions. The 4-bar mechanism provides an effective and simple solution for controlled trunk operation. Minor deviations in the prototype motion are due to material flexibility and joint clearances.

Conclusion

The project successfully demonstrates the application of a 4-bar mechanism in automotive trunk design. It integrates theoretical understanding, software modeling, and hands-on prototyping, reflecting a strong grasp of mechanical system analysis.

References

1. MATLAB Documentation – MathWorks
2. SolidWorks Help Center
3. Lecture Slides