Sol 1 (c)

we know,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad \text{And} \quad \bar{\hat{y}} = \frac{1}{n}\sum_{i=1}^{n} \hat{y}_i$$

$$\hat{y} = \beta_0 + \beta_1 x \quad \text{—— (i)}$$

And

$$\bar{y} = \beta_0 + \beta_1 \bar{x} \quad \text{—— (ii)}$$

where,
β_0 and β_1 are two coefficient.

Now,

$$\bar{\hat{y}} = \frac{1}{n}\sum_{i=1}^{n} \hat{y}_i$$

$$= \frac{1}{n}\sum_{i=1}^{n} (\beta_0 + \beta_1 x) \quad [\text{from eqn (i)}]$$

$$= \beta_0 + \beta_1 \frac{1}{n}\sum_{i=1}^{n} x$$

$$= \beta_0 + \beta_1 \bar{x}$$

$$= \bar{y} \quad [\text{from eqn (ii)}]$$

$$\boxed{\text{So, } \bar{\hat{y}} = \bar{y}}$$

Sol ③ ⓓ

we know, $P = X(X^TX)^{-1} \cdot X^T$

$\therefore P^2 = X \cdot (X^TX)^{-1} \cdot \underbrace{X^T \cdot X}\, (X^T \cdot X)^{-1} \cdot X^T$

$= X \cdot (X^TX)^{-1} (X^T \cdot X) \quad (X^T \cdot X)^{-1} X^T$

$= X \cdot I \cdot (X^TX^{-1}) \cdot X^T \qquad [\because I = M^{-1} \cdot M]$

$= X \cdot (X^TX^{-1}) X^T$

$= P$

Again. $(1-P)^2 = (1-P) \cdot (1-P)$

$= I(1-P) - P(1-P)$

$= I^2 - IP - PI + P^2 \qquad [A \cdot I = A]$

$= I - P - P + P^2$

$= I - P - P + P \qquad$ [we prove $P^2 = P$]

$= I - P$