Notes on Exercises 5

Exercise 1

The cdf of the standard logistic distribution is given by $F(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}$.

(a) What are the properties of a cdf? Explain why F fulfills these.

(b) Calculate the pdf $f(x)$.

(c) Use the R function `rlogis` to generate pseudo-random numbers for the logistic distribution. (The standard logistic has `location=0` and `scale=1`.) Simulate samples from the standard logistic distribution and illustrate that its expectation is 0 and the variance equals $\frac{\pi^2}{3}$.

(a) cdf - properties:
- has values in $[0, 1]$
- monotonously increasing (strictly if distribution is continuous)
- domain of the cdf: $x \in \mathbb{R} = (-\infty, \infty)$

(b) cdf: $F(x)$, pdf: $f(x)$ } $f(x) = F'(x)$

(F is an antiderivative of f)

$\Rightarrow f(x) = \frac{e^x}{(1+e^x)^2}$

symmetric around 0

(kind of similar to the Gaussian/normal pdf)

Exercise 5/2

See R code: cdfs of logistic and normal distributions are very close if the distributions are rescaled accordingly

⇒ you cannot compare coefficients of logit and probit models directly but you would also to compare them in rescaled way

but in general: results of logit and probit models are similar

pros & cons:

logit: $F(x) = \frac{1}{1+e^{-x}}$ is more simple to calculate than Φ

(but are smooth and differentiable)

probit: has the advantage that it could be easier generalized to multi-dimensional dependent variables / systems of regression equations

⇒ when using logit or probit take care to use the relevant inverse link function F or Φ when doing a prediction