Regression

Master Data Science
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Exercises 1

Exercise 1

(a) Two (continuous) variables have a correlation of -0.4 and a covariance of -1.84. One of the variables has a variance of 4. Calculate the variance of the other variable.

(b) The transformation between Celsius ($^\circ C$) and Fahrenheit ($^\circ F$) degrees for temperatures is given by:

$$X_{^\circ F} = X_{^\circ C} \cdot 1.8 + 32$$

Assume that the average temperature in summer in Berlin is $25^\circ C$ where we have a standard deviation of $3^\circ C$.

How could you transform these values into $^\circ F$? Do also calculate the variances in both degree measures.

Exercise 2

Consider the observations of two variables (quite artificial data! ☺):

x_i	1	1	2	2	3	3
y_i	1	3	2	4	3	5

The following tasks should be solved without R:

(a) Draw a scatterplot.
(b) Calculate the regression line.
(c) Check that the line goes through $(\overline{x}, \overline{y})$. (Is that always the case?)
(d) Calculate the correlation. How do you obtain the coefficient of determination?

Exercise 3

Assume we have observations $x_1, \ldots, x_n$ of a variable X. Determine the value a, which minimizes the following criterion:

$$Q(a) = \sum_{i=1}^{n} (x_i - a)^2$$

Exercise 4

Consider $X \sim N(2, 9)$. What does that mean?

Calculate the following probabilities (with and without R):

(a) $P(X \leq 0)$ (b) $P(X \leq -1)$
(c) $P(X \geq 5)$ (d) $P(-2 \leq X \leq 2)$

Exercise 5

Consider the following data for the speed versus braking distance (of a car):

Speed X (in km/h)	20	25	30	35	40	45	50	55	60	65	70
Braking distance Y (in m)	18	26	33	40	46	59	72	85	97	120	141

Here are some (possibly) useful values when calculating without R:

$$\overline{x} = 45, \quad \overline{y} = 67, \quad s_X^2 = 275, \quad s_Y^2 = 1600.6, \quad s_{XY} = 649.5$$

(a) Explain the meaning of: $\overline{x}$, $\overline{y}$, s_X^2, s_Y^2 and s_{XY} .

(b) What are the R functions to calculate them? (Check with R!)

(c) Display the data in R using a scatterplot.

(d) Calculate the correlation and check with R. What could we conclude from this value?

(e) Calculate the linear regression coefficients (first without R). Do the same using R and display the line in the scatterplot.

Exercise 6

On Moodle you find the file `MunichRent2003.csv`. This is a sample of 2053 appartments in Munich from 2003 (Munich was already an expensive city at this time ...). The variables are coded as follows:

Variable	Meaning	Values
`netrent`	net rent (per month)	in Euro
`netrent.sqm`	net rent per square metre	in Euro
`living.space`	living space	in square metres
`rooms`	no. of rooms	1,...,6
`year`	year of construction	year
`district`	Munich district	1,...,25
`location`	quality of location	best, good, simple
`warm.water`	warm water provided	yes, no
`central.heating`	central heating	yes, no
`bath.tiles`	bath room with tiles	yes, no
`bath.extras`	bath room with extras	yes, no
`upscale.kitchen`	kitchen with upscale equipment	yes, no

Original source: https://doi.org/10.5282/ubm/data.2

(a) For which pairs of variables would it be useful to estimate a simple linear regression model? (Choose at least two different examples. Explain which of the variables do you consider the dependent and the independent one.)

(b) Load the data into R. If you don't know what to do, check `?read.csv`.
(Extra task: Try also to load the original data into R!)

(c) Estimate the model that you have chosen in (a), i.e. calculate the coefficients, draw scatterplots and regression lines, determine R^2.

(d) Now consider `netrent` and `location`. Do you think it is useful to consider simple linear regression here? Do you know any other technique(s) to analyse the relationship between these two? (Maybe also a graphical technique?)

(e) Again consider two variables: `bath.tiles` and `upscale.kitchen`. Do you think there is a relationship between these two? How could you analyse it?