Numerical study of compression response of open-celled Aluminum foam structure using 3-D Voronoi diagram model

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Abstract. The 3-D Voronoi tessellation technique is used for modeling the open-celled Aluminum foam structure. Random points were generated in 3-D space to form Voronoi seeds. The level of randomness of these seeds is controlled with a simple sequential inhibition (SSI) process. This level of seeds randomness identifies Voronoi cells regularity. The influence of the cell regularity on the compression response of open-celled Aluminum foam is studied in the elastic and plastic deformation regimes. Moreover, the effect of relative density is analysed for different regularities. The obtained results were compared with published experimental and mathematical modeling results and showed a good agreement with them. Simulations results reveal that irregular structures are stiffer and have larger plastic collapse strength compared with the honeycomb structure. For irregular structure, plastic collapse strength increases with the increase of degree regularity. Young's modulus decreases with increasing degree of regularity for relative density below 5% and increases for relative density above 5%.

1. Introduction

Open-celled Aluminum foams are man-made cellular solids. They have low relative density, high strength-to-weight ratios and energy absorption characteristics. These characteristics make open-celled Aluminum foams suitable for aerospace [1], automotive [2], military ([3], [4] and [5]) and other applications [6]. Due to a lot of foam applications, researchers studied their characteristics to give designers correct and reliable information. Simulations of virtual model methods are commonly used in early design stages as designers can do them repeatedly on different designs with low-cost and short time. Moreover, the closer the model to reality, the designers get better results. Therefore, in recent years, researchers have developed a number of virtual models to simulate foams. A virtual model with synthesis structure is constructed. Papka and Kyriakides [7] constructed their models using regular only periodic honeycombs structure. Silva and Gibson [8] generated 2D non-periodic Voronoi structure to capture the random morphology of real foam. Van Der Burg, et al [9] modeled foam structure by 2D random Voronoi structure with normal struts at the boundaries. Zhu, et al [10] constructed 3-D Voronoi model consist of 125 cells with different regularities, and applied finite element analysis to find the effect of irregularity on elastic properties. H. Zhu, et al [11] determined the effect of irregularity of 2D Voronoi cells on mechanical properties at high strain. They assumed that the solid material of Voronoi cells is elastic throughout the deformation process. Previous works citation in modeling and simulation of the foam structure shows a number of gaps. A small amount of Voronoi cells was used due to low computing power, which affects the accuracy of results. Using repeated regular unit did not represent the real randomness of foam. Only elastic properties of solid

martial were used to describe the model behavior. The most common models were in 2D, which are inaccurately representing the reality. The 3-D models were used only with low strain values. Strain rate was neglected in reviewing papers. These gaps have undesirable effects on the results accuracy and credibility.
The aim of the present work is developing a 3-D Voronoi model, which able to describe the real characteristics of Aluminum foam at different conditions. This developed model will be validated using results from earlier experiments and mathematical works. The model will be used to study the effect of cell relative density and regularity on the compression response of open-celled Aluminum foam.

2. Solid Modeling

The suggested model consists of three deformable parts: upper plate, lower plate, and Aluminum foam structure (see Figure 1). Voronoi diagram is used to model Aluminum foam structure. Voronoi diagram construction is based on Voronoi seeds distribution. This distribution defines Voronoi diagram regularity. The Number and size of cells in Voronoi diagram represent the pores of the Aluminum foam. The next sections describe in detail the steps of constructing the Voronoi model.

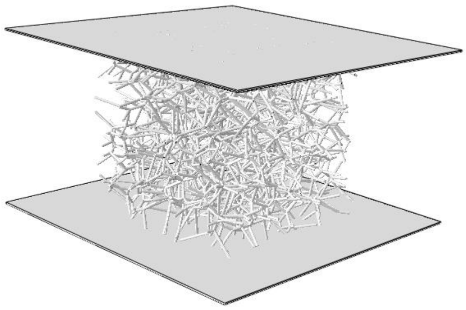
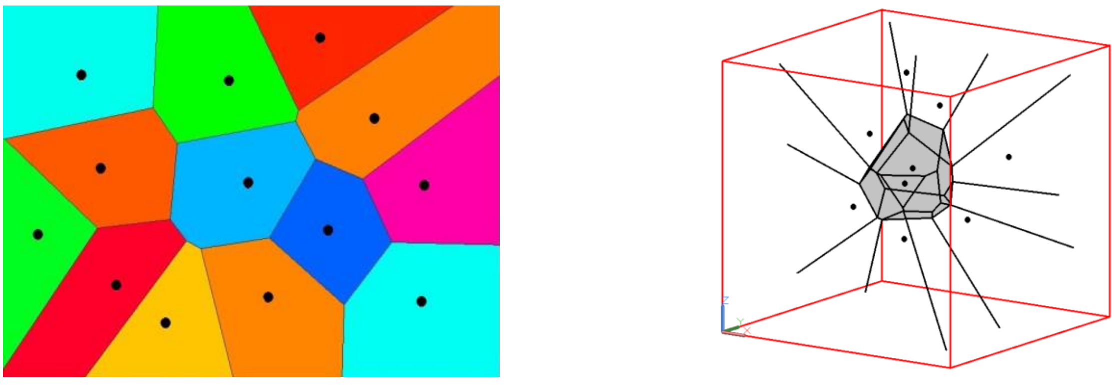


Figure 1. Parts of Aluminum foam model.

2.1. Voronoi diagram

Voronoi diagram is defined as partitioning of a plane into regions based on distance to points in a specific subset of the plane [12]. This set of points is called seeds. These seeds should be specified beforehand. For every seed, there is a corresponding region consisting of all points closer to this seed than to any other. These regions are called Voronoi cells (see Figure 2).



(a) (b)

Figure 2. Voronoi diagrams in different dimensions. (a) 2-D Voronoi diagram. (b) 3-D Voronoi diagram.

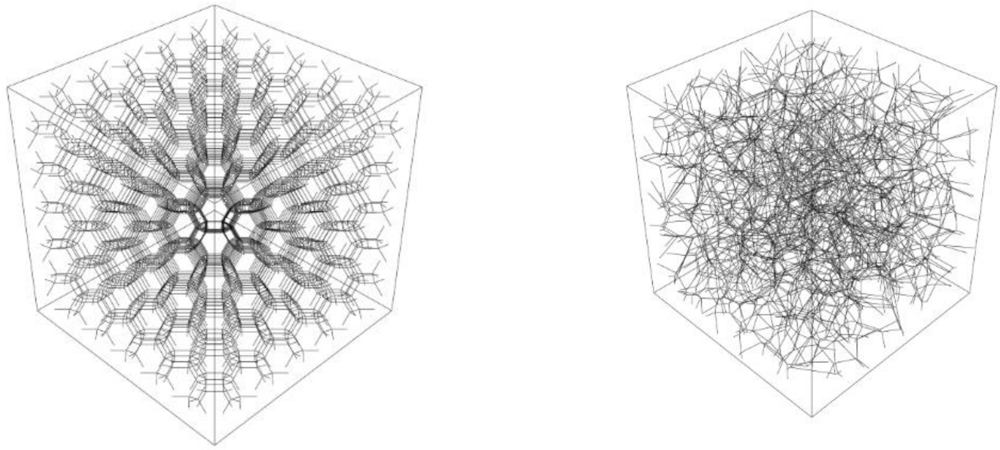
The Voronoi regularity (δ) is defined using Zhu, et al [13] regularity definition as:

$$\delta = \frac{s}{r} \tag{1}$$

Where s and r are the maximum distances between the neighbouring seeds in the cases of irregular and regular Voronoi diagram, respectively, with the same volume V and the number of seeds n. The distance r is given by:

$$r = \frac{\sqrt{6}}{2}\left(\sqrt[3]{\frac{V}{\sqrt{2}\,n}}\right) \tag{2}$$

Figure 3(a) and Figure 3(b) show 3-D of open-cell Aluminum foam for honeycomb structure and irregular Voronoi structure, respectively. A value of regularity $\delta = 0$ represents a complete spatial randomness of points. By increasing the value of regularity δ, model becomes more regular.



(a) (b)

Figure 3. 3-D Voronoi diagrams (a) Honeycomb. (b) Irregular Voronoi diagram.

Voronoi seeds must be defined first to generate the Voronoi diagram. Seeds were generated regularly to produce honeycombs structure. Simple sequential inhibition process (SSI) [14] was used to produce random points to get irregular Voronoi cells. In SSI process, points in space are generated based on a completely random process described by the Poisson distribution. After every point is generated, its distances with respect to the previous ones are evaluated. If none of these distances are greater than a threshold value s then the point is kept, otherwise, it is eliminated. The process continues until points are generated

2.2. Model size and number of cells in the model

Beam section with a constant circular profile is assigned for all the line features created using Voronoi diagram and it is called strut [15]. Zhu, et al [10] used formula to calculate the value of the strut diameter, which depends on the relative density $\bar{\rho}$ of the foam. This formula is used in present research and given by:

$$\bar{\rho} = \rho^*/\rho_s = \left(\frac{1}{V}\right)\sum_{i=1}^{n}\frac{\pi}{4}\,{d_i}^2 l_i \tag{3}$$

Where ρ^* and ρ_s are the density of foam and the density of solid material forming foam, respectively. d is the diameter of struts forming foam. l is the length of struts. n is number of struts. Andrews, et al [16] have experimentally studied the effect of the specimen size on the compression response of Aluminum foams. They suggested that a ratio of the specimen size to cell size should be not less than 6 to predict the elastic modulus and present a tolerable difference in the prediction of the plastic collapse strength of Aluminum foams. The creation of a single 3-D voronoi Diagram inside a unit cube requires nine nuclei in a as shown in Figure 4. By studying Figure 4, it can be concluded that the number of nuclei (n) required to generate (l) 3-D voronoi Diagram cells along the border of a unit cube can be calculated with the formula,

$$n = (l + 1)^3 + l^3 \tag{4}$$

In the current case l = 6, and hence the number of nuclei required is equal to n =559.

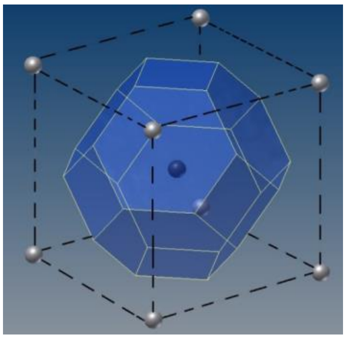


Figure 4. A single 3-D voronoi Diagram inside a unit cube.

2.3. Creation of the model

In the present work, Aluminum foam Voronoi models were generated using 559 seeds inside unite cube. In order to study the effects of cells regularity and relative density on the open-celled Aluminum foam compression response, honeycomb structure and four different irregular structures δ= {50%, 70%, 80%} and five different relative densities $\bar{\rho}$ = {0.01, 0.03, 0.05, 0.07, 0.09} were concerned. For every regularity and relative density, 20 models were generated by a different random distribution of seeds. These combinations generated 300 models for (δ<1). A C++ program was developed to generate these models automatically. This program encapsulates TetGen library [17] to get Voronoi cells. The developed model was enhanced by an increasing number of Voronoi cells, taking into account not only elastic properties of materials forming of Voronoi struts but also plastic properties. In addition, Johnson-Cook model was encapsulated in the model to define plastic properties. The advantages of using Johnson-Cook model are that the developed model can be simulated at different strain rates and temperature.

3. Finite-element modelling

The text of your paper should be formatted as follows: For the present case study, the finite-element package Abaqus/Standard was used. In the pre-processing stage, the Voronoi diagram geometry, in DXF format, is obtained by the developed model generation program. Then DXF format is transformed to IGS format using the Autodesk/Inventor package. Then, IGS file format is imported to Abaqus to form foam structure. In the presented research, Al-6061-T6 was used as a base material forming struts and face-sheets. It was modelled as an elastoplastic material. The elastic region was defined by isotropic hardening model [10] with an elastic modulus of 68.9 GPa and a Poisson's ratio of 0.33 [18]. The plastic region was defined by Johnson-Cook model [19]. Jacob, et al [20] produced these coefficients and validated them against the experimental data. Johnson Cook model was used as its analytical equation is a function of equivalent plastic strain, strain rate, and temperature. Consequently, the strain rate is included into the model for the present research. Therefore, using this enhanced model, parametric studies can be carried on the effect of strain rate and temperature on the response of Aluminum foam in future works. The strain rate in the present research was set to be 0.01 / S^{-1} for low strain below 0.01 and 0.15 /S^{-1} for high strain up to 0.15.

In the present model, boundary conditions were applied to the upper and lower plate. The upper plate and the lower plate were allowed to move only in one direction perpendicular to each other. Equal and opposite displacement were applied to plates in this direction. Voronoi ligaments in present research were discretized with spatial beam elements B32 in Abaqus. Quadratic interpolation was used. In order to avoid divergence of solution due to small ligaments, especially in high irregularities models, two elements per ligaments were used. The upper and lower plates were discretized with four-node Shell elements. They are generated to coincide with nodes in the beams. A total number of elements in

all models are varying from 11,000 to 12,000. The general static method was selected to capture changes in reaction forces and displacement of upper and lower plates. In general static method, non-linear effects were activated due to large deformations, which were expected during simulations.
In the post-processing stage, at every step of the solution, forces at every element of lower plate were added to form the total reaction force. Displacements of lower plate and upper plate were added to form total displacement. In order to draw stress-stain curve, total reaction force was divided by cross section area of Aluminum foam cube to form stress. Displacement was divided by the original length of Aluminum foam cube to form strain.

4. Results

Simulation of compression test was carried out on the developed model at different regularities and relative densities. The obtained Stress-strain curves were evaluated for every simulation case. Intensive studies were carried out on the effect of the regularity and relative density on the response of Aluminum foam under compression load.

4.1. Effect of regularity and relative density on stress-strain curve

Based on the simulations of the developed model, the stress-strain curves were established and evaluated for different structure regularities and relative densities. Figure 5 shows an example for these curves. Stresses (σ) were normalized by the yield stress of the solid material (σ_{ys}) and $\bar{\rho}^{1.5}$. Stresses were normalized by $\bar{\rho}^{\,1.5}$ due to foams strength scales. The value of $\bar{\rho}^{\,1.5}$ was obtained as discussed in section 4.2. Normalized stress was defined as:

$$\bar{\sigma} = \frac{\sigma}{\sigma_{ys} \cdot \bar{\rho}^{1.5}} \tag{5}$$

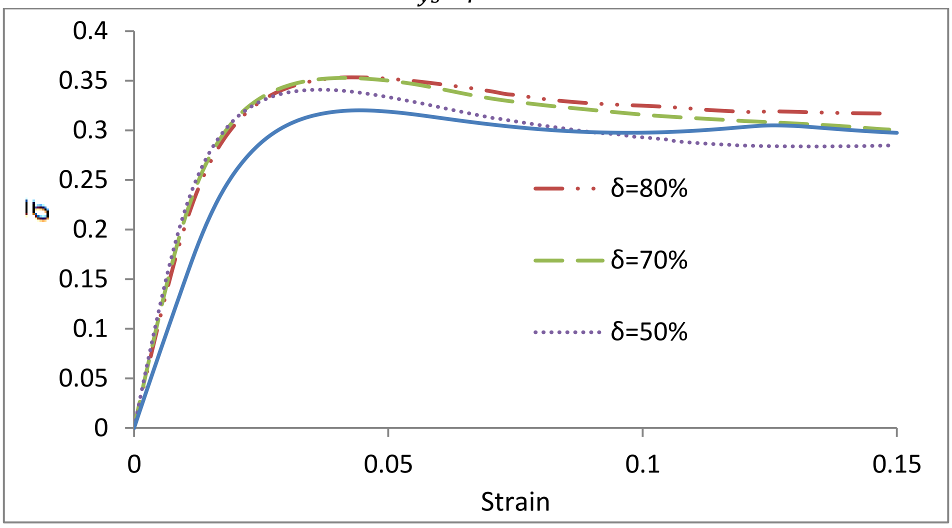


Figure 5. Stress-strain curve of 3-D Voronoi structure with relative density 1% at different regularities for strain up to 0.15.

As shown in Figure 5, honeycomb structure sustains a lower compressive stress compared with more irregular structures. For plastic region, the obtained results showed that high geometric irregularity sustains a lower compressive stress compared with the more regular structure. For elastic region, the obtained results of irregular structures showed that stresses decreasing with increasing of degree regularity for low relative density below 5% (see Figure 6). Stresses became equal for different regularities among relative density equal 5% (see Figure 7). Stresses increase with the increase of the degree of regularity in relative density above 5% (see Figure 8).

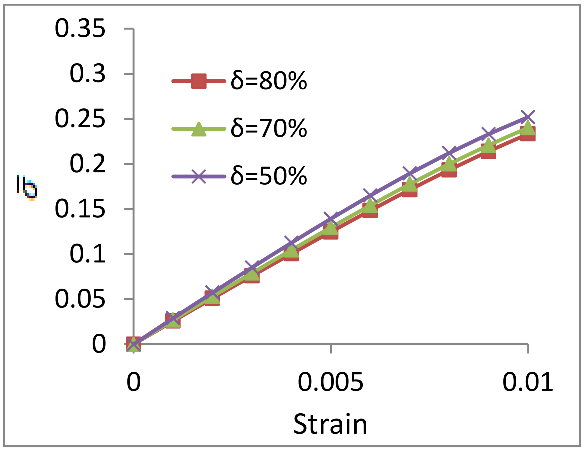


Figure 6. Stress-strain curve of 3-D Voronoi structure with relative density 1% at different regularities for strain up to 0.01.

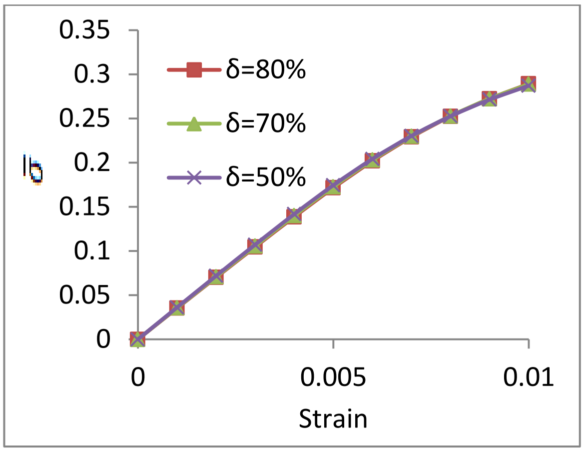


Figure 7. Stress-strain curve of 3-D Voronoi structure with relative density 5% at different regularities for strain up to 0.01.

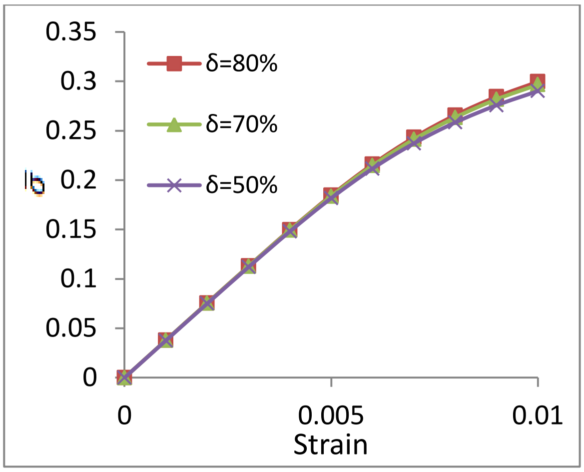


Figure 8. Stress-strain curve of 3-D Voronoi structure with relative density 9% at different regularities for strain up to 0.01.

4.2. Effect of relative density on modulus of elasticity and plastic collapse strength

The mean normalized modulus of elasticity (E^*/E_s) of different cell regularities and honeycomb structure, which obtained from numerical simulation (see Figure 9), can be represented by quadratic form as:

$$\frac{E^*}{E_s} = C2\,.\,\bar{\rho}^{\,2} + C3\,.\,\bar{\rho} \tag{6}$$

Where E^* and E_s are the modulus of elasticity of foam and base material forming foam, respectively. The proportionality coefficients C2 and C3 were obtained by fitting results computed from equation (6) and listed in Table 1. These results are compared with published results and discussed in section 5.

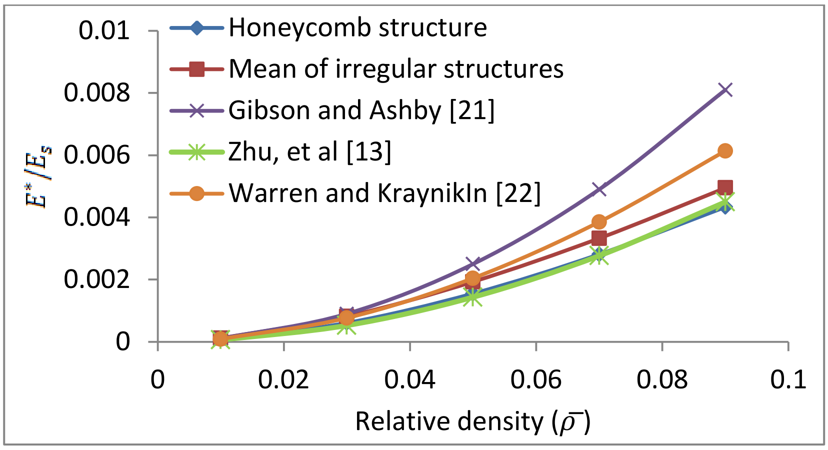


Figure 9. Variation of the normalized elastic modulus of open-celled Aluminum foam with relative density.

Table 1. Proportionality coefficients C2 and C3.

Regularity	C2	C3	Coefficient of determination
Irregular structure	0.3779	0.0231	0.9998
Honeycomb structure	0.5265	0.0029	1

Similarly, the mean normalized plastic collapse strength ($\sigma_{pl}{}^*/\sigma_{ys}$) of different cell regularities and honeycomb structure, which obtained from numerical simulation (see Figure 10), can be represented by an equation of the quadratic form as:

$$\frac{\sigma_{pl}{}^*}{\sigma_{ys}} = C4 \, . \, \bar{\rho}^{1.5} \tag{7}$$

Where $\sigma_{pl}{}^*$ and σ_{ys} are plastic collapse strength of foam and base material forming foam, respectively. The proportionality coefficient C4 was obtained by fitting results computed from equation (7) and listed in Table 2. These results are compared with published results and discussed in section 5.

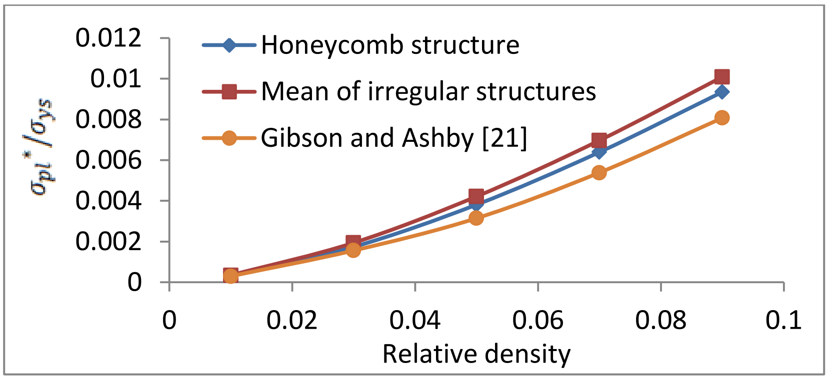


Figure 10 Variation of the normalized plastic collapse strength of open-celled Aluminum foam with relative density.

Table 2. Proportionality coefficient C4.

Regularity	C4	Coefficient of determination
Irregular structure	0.3677	1
Honeycomb structure	0.3799	1

4.3. Effect of structure regularity on modulus of elasticity and plastic collapse strength

The effect of structure regularity on the modulus of elasticity and plastic collapse strength was studied through simulating the developed model. The set of tests structure regularities is $\delta = \{50\%, 70\%, 80\%\}$. The obtained results show that young's modulus decreases with increasing degree of regularity up to relative density 5% (see Figure 11, Figure 12 and Figure 13). For relative density bigger than 5% up to 9% young's modulus increases with increasing degree of regularity (see Figure 14 and Figure 15).

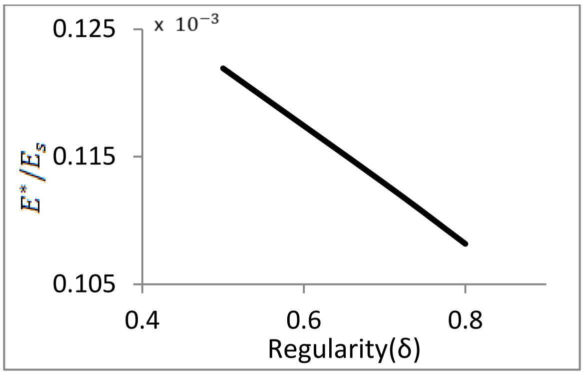


Figure 11. Effect of cell regularity on normalized elastic modulus of 3-D Voronoi structure for relative density equal 0.01

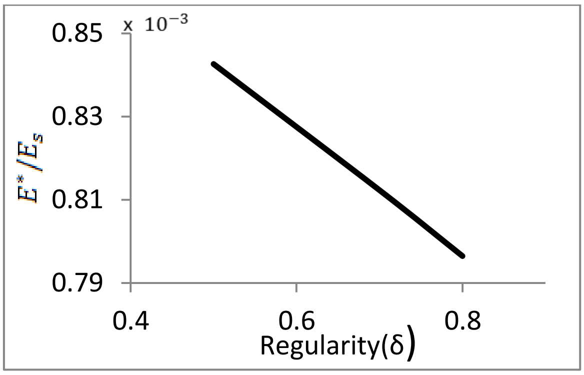


Figure 12. Effect of cell regularity on normalized elastic modulus of 3-D Voronoi structure for relative density equal 0.03

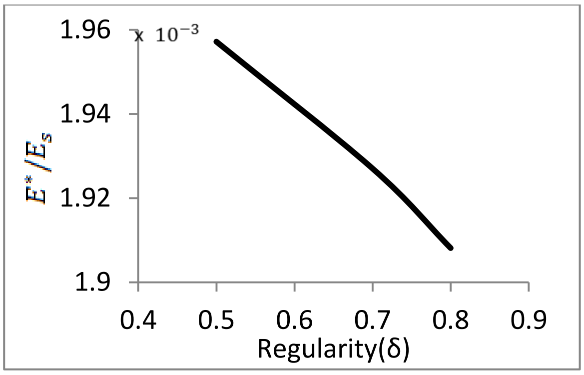


Figure 13. Effect of cell regularity on normalized elastic modulus of 3-D Voronoi structure for relative density equal 0.05

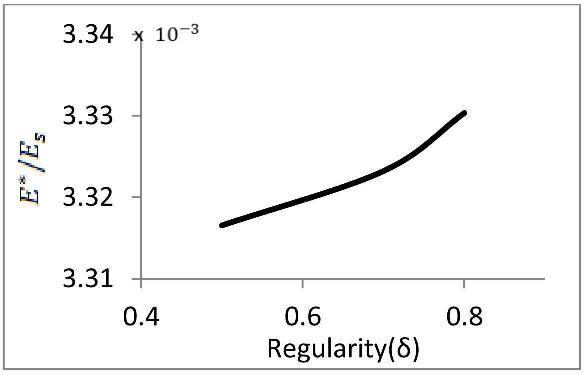


Figure 14. Effect of cell regularity on normalized elastic modulus of 3-D Voronoi structure for relative density equal 0.07

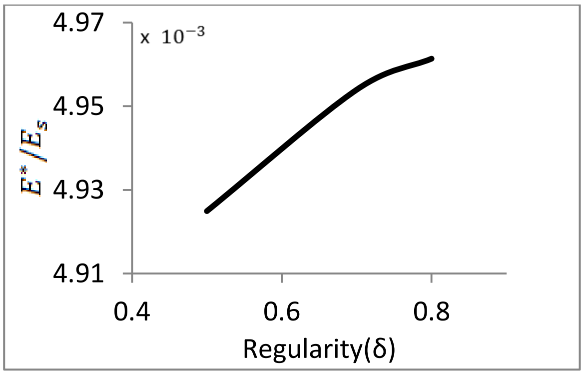


Figure 15. Effect of cell regularity on normalized elastic modulus of 3-D Voronoi structure for relative density equal 0.09

The plastic collapse strength increases with the increase of degree regularity. This increase became more noticeable by increase relative density (see Figure 16).

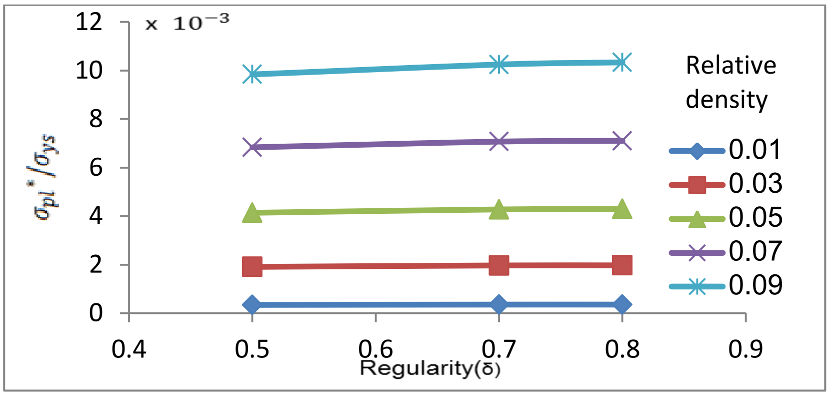


Figure 16. Effect of cell regularity on the normalized plastic collapse strength of 3-D Voronoi structure for different relative density

5. Discussions

The mean of normalized modulus of elasticity of different regularities was compared with those obtained from three different previous works ([21], [13] and [22]) at different relative density (see Figure 9). The obtained results of the honeycomb structure shows excellent compatibility with the previous work of Zhu, et al [13]. They used a regular honeycomb cell as a unit structure to estimate normalized modulus of elasticity of foam. By increasing the irregularity, the obtained results became more comparable with those obtained in references [21] and [22]. Similarly, the plastic collapse region was validated using normalized plastic collapse strength ($\sigma_{pl}^*/\sigma_{ys}$) of Voronoi structures. In Figure 10, the results were compared with those published in reference [21]. This showed an acceptable agreement. In this reference, Gibson and Ashby observed that the plastic-collapse strength depends on $\bar{\rho}^{1.5}$ as observed in present research (see Equation (7)). The change in relation between stresses and regularities for relative densities above 5% occurred because the difference between structures diameter is too small for low density approximately below 5% (see Figure 17).

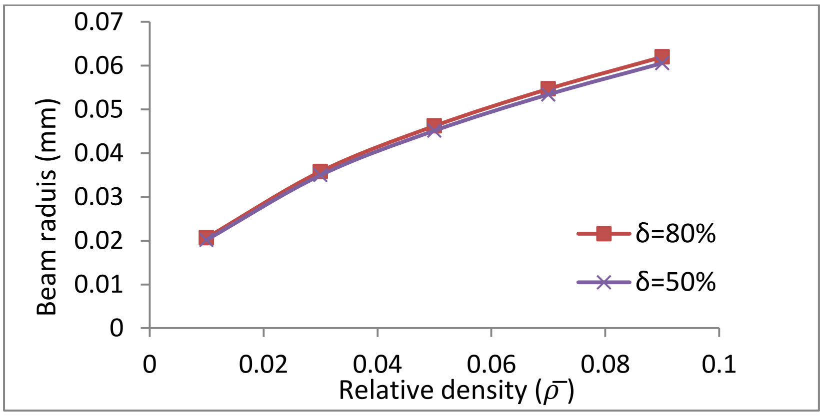


Figure 17. Effect of relative density on beam radius for 50% and 80% regularity.

Thus, for low relative density approximately below 5%, the main effect on stresses is the difference between structure regularities. However, this difference in beams diameter increases by increasing of relative density as the beam diameter is a function of square root of relative density (see Eq. (3)). This increase has a great effect on stresses as Timoshenko beam theory [23], which is used to calculate bending moment and shear force for the elastic zone. The bending moment M_{xx} and shear force Q_x is given by:

$$M_{xx} = -EI\frac{\partial\varphi}{\partial x} \quad (8) \qquad Q_x = kAG\left(-\varphi + \frac{\partial w}{\partial x}\right) \quad (9)$$

Where A is the cross-section area. E is the elastic modulus. I is the second moment of area. G is the shear modulus. k is called the Timoshenko shear coefficient depends on the geometry. φ is the angle of rotation of the normal to the mid-surface of the beam. Finally, w is the displacement of the mid-surface in Z direction. The bending moment is function of the second moment of area of beam cross-section $(\pi/4)r^4$. Shear force is function of cross-section area of beam. Thus, bending moment is function of beam radius to power four and shear force is function of beam diameter to power two. Consequently, any small change in beam diameter has a great effect on bending moment and shear force, which finally cause changing the relation between stresses and regularities for relative density above 5%

6. Conclusion

In this research, a large number of Voronoi cells was used to enhance the imitation of real Aluminum foam structure. Not only the elastic properties of the model are defined, but also plastic properties are considered. A finite element analysis was carried out using this developed model to study the effect of cell regularity and relative density on the compression response of open-celled Aluminum foam. The obtained results showed that irregular structures are stiffer and have larger plastic collapse strength compared with honeycomb structures. For irregular structure, young's modulus decreases with increasing degree of regularity up to relative density 5%. For relative density bigger than 5% up to 9% young's modulus increases with increasing degree of regularity. Plastic collapse strength increases with the increase of degree regularity. The obtained elastic modulus of irregular Voronoi and honeycomb structures are bounded by the results of references [21], [13] and [22]. Similarly, the variation of plastic collapse strength of 3-D Voronoi structures with relative density depends on $(\rho^*/\rho_s)^{1.5}$ as observed experimentally by reference [21]. The developed 3-D model was proven to have the ability to mimic the Aluminum foam structure and can be used for studying the mechanical behavior of the Aluminum foam and other foam materials.

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