Sheet 4

Exercis I Further problems on differentiating inverse trigonometric functions

In Problems 1 to 6, differentiate with respect to the variable.

1. (a) $\sin^{-1} 4x$ (b) $\sin^{-1} \frac{x}{2}$

$$\left[\text{(a) } \frac{4}{\sqrt{1-16x^2}} \text{ (b) } \frac{1}{\sqrt{4-x^2}}\right]$$

2. (a) $\cos^{-1} 3x$ (b) $\frac{2}{3}\cos^{-1}\frac{x}{3}$

$$\left[\text{(a) } \frac{-3}{\sqrt{1-9x^2}} \text{ (b) } \frac{-2}{3\sqrt{9-x^2}}\right]$$

3. (a) $3\tan^{-1} 2x$ (b) $\frac{1}{2}\tan^{-1}\sqrt{x}$

$$\left[\text{(a) } \frac{6}{1+4x^2} \text{ (b) } \frac{1}{4\sqrt{x}(1+x)}\right]$$

4. (a) $2\sec^{-1} 2t$ (b) $\sec^{-1}\frac{3}{4}x$

$$\left[\text{(a) } \frac{2}{t\sqrt{4t^2-1}} \text{ (b) } \frac{4}{x\sqrt{9x^2-16}}\right]$$

5. (a) $\frac{5}{2}\operatorname{cosec}^{-1}\frac{\theta}{2}$ (b) $\operatorname{cosec}^{-1} x^2$

$$\left[\text{(a) } \frac{-5}{\theta\sqrt{\theta^2-4}} \text{ (b) } \frac{-2}{x\sqrt{x^4-1}}\right]$$

6. (a) $3\cot^{-1} 2t$ (b) $\cot^{-1}\sqrt{\theta^2-1}$

$$\left[\text{(a) } \frac{-6}{1+4t^2} \text{ (b) } \frac{-1}{\theta\sqrt{\theta^2-1}}\right]$$

7. Show that the differential coefficient of $\tan^{-1}\frac{x}{1-x^2}$ is $\frac{1+x^2}{1-x^2+x^4}$

In Problems 8 to 11 differentiate with respect to the variable.

8. (a) $2x\sin^{-1} 3x$ (b) $t^2\sec^{-1} 2t$

$$\left[\begin{array}{l}\text{(a) } \dfrac{6x}{\sqrt{1-9x^2}} + 2\sin^{-1} 3x \\ \text{(b) } \dfrac{t}{\sqrt{4t^2-1}} + 2t\sec^{-1} 2t\end{array}\right]$$

9. (a) $\theta^2\cos^{-1}(\theta^2-1)$ (b) $(1-x^2)\tan^{-1} x$

$$\left[\begin{array}{l}\text{(a) } 2\theta\cos^{-1}(\theta^2-1) - \dfrac{2\theta^2}{\sqrt{2-\theta^2}} \\ \text{(b) } \left(\dfrac{1-x^2}{1+x^2}\right) - 2x\tan^{-1} x\end{array}\right]$$

10. (a) $2\sqrt{t}\cot^{-1} t$ (b) $x\operatorname{cosec}^{-1}\sqrt{x}$

$$\left[\begin{array}{l}\text{(a) } \dfrac{-2\sqrt{t}}{1+t^2} + \dfrac{1}{\sqrt{t}}\cot^{-1} t \\ \text{(b) } \operatorname{cosec}^{-1}\sqrt{x} - \dfrac{1}{2\sqrt{(x-1)}}\end{array}\right]$$

11. (a) $\frac{\sin^{-1} 3x}{x^2}$ (b) $\frac{\cos^{-1} x}{\sqrt{1-x^2}}$

$$\left[\begin{array}{l}\text{(a) } \dfrac{1}{x^3}\left\{\dfrac{3x}{\sqrt{1-9x^2}} - 2\sin^{-1} 3x\right\} \\ \text{(b) } \dfrac{-1+\dfrac{x}{\sqrt{1-x^2}}\cos^{-1} x}{(1-x^2)}\end{array}\right]$$

Exercise 2 Further problems on differentiation of inverse hyperbolic functions

In Problems 1 to 11, differentiate with respect to the variable.

1. (a) $\sinh^{-1}\frac{x}{3}$ (b) $\sinh^{-1} 4x$

$$\left[(a)\ \frac{1}{\sqrt{(x^2+9)}}\quad (b)\ \frac{4}{\sqrt{(16x^2+1)}}\right]$$

2. (a) $2\cosh^{-1}\frac{t}{3}$ (b) $\frac{1}{2}\cosh^{-1} 2\theta$

$$\left[(a)\ \frac{2}{\sqrt{(t^2-9)}}\quad (b)\ \frac{1}{\sqrt{(4\theta^2-1)}}\right]$$

3. (a) $\tanh^{-1}\frac{2x}{5}$ (b) $3\tanh^{-1} 3x$

$$\left[(a)\ \frac{10}{25-4x^2}\quad (b)\ \frac{9}{(1-9x^2)}\right]$$

4. (a) $\text{sech}^{-1}\frac{3x}{4}$ (b) $-\frac{1}{2}\text{sech}^{-1} 2x$

$$\left[(a)\ \frac{-4}{x\sqrt{(16-9x^2)}}\quad (b)\ \frac{1}{2x\sqrt{(1-4x^2)}}\right]$$

5. (a) $\text{cosech}^{-1}\frac{x}{4}$ (b) $\frac{1}{2}\text{cosech}^{-1} 4x$

$$\left[(a)\ \frac{-4}{x\sqrt{(x^2+16)}}\quad (b)\ \frac{-1}{2x\sqrt{(16x^2+1)}}\right]$$

6. (a) $\coth^{-1}\frac{2x}{7}$ (b) $\frac{1}{4}\coth^{-1} 3t$

$$\left[(a)\ \frac{14}{49-4x^2}\quad (b)\ \frac{3}{4(1-9t^2)}\right]$$

7. (a) $2\sinh^{-1}\sqrt{(x^2-1)}$

(b) $\frac{1}{2}\cosh^{-1}\sqrt{(x^2+1)}$

$$\left[(a)\ \frac{2}{\sqrt{(x^2-1)}}\quad (b)\ \frac{1}{2\sqrt{(x^2+1)}}\right]$$

8. (a) $\text{sech}^{-1}(x-1)$ (b) $\tanh^{-1}(\tanh x)$

$$\left[(a)\ \frac{-1}{(x-1)\sqrt{[x(2-x)]}}(b)\ 1\right]$$

9. (a) $\cosh^{-1}\left(\frac{t}{t-1}\right)$ (b) $\coth^{-1}(\cos x)$

$$\left[(a)\ \frac{-1}{(t-1)\sqrt{(2t-1)}}\ (b)\ -\text{cosec}\,x\right]$$

10. (a) $\theta\ \sinh^{-1}\theta$ (b) $\sqrt{x}\ \cosh^{-1} x$

$$\left[\begin{array}{l}(a)\ \dfrac{\theta}{\sqrt{(\theta^2+1)}}+\sinh^{-1}\theta\\ (b)\ \dfrac{\sqrt{x}}{\sqrt{(x^2-1)}}+\dfrac{\cosh^{-1} x}{2\sqrt{x}}\end{array}\right]$$

11. (a) $\dfrac{2\,\text{sech}^{-1}\sqrt{t}}{t^2}$ (b) $\dfrac{\tanh^{-1} x}{(1-x^2)}$

$$\left[\begin{array}{l}(a)\ \dfrac{-1}{t^3}\left\{\dfrac{1}{\sqrt{(1-t)}}+4\,\text{sech}^{-1}\sqrt{t}\right\}\\ (b)\ \dfrac{1+2x\tanh^{-1} x}{(1-x^2)^2}\end{array}\right]$$

12. Show that $\frac{d}{dx}[x\cosh^{-1}(\cosh x)]=2x$