Hierarchical Agglomeretive_Class 19-Completed

April 18, 2021

1 0. Together

Hierarchical clustering (as the name suggests) assumes that the clusters in the data have a hierarchical relationship. For example, in a McDonald's food dataset we could have clusters like: Dessert, Drinks, Sandwhiches, Other. Within Sandwhiches we could have chicken sandwhiches, Burgers, Vegan Sandwhiches...within Burgers we could have smaller, lower calorie options, and bigger, more substantial burgers...etc.

Blood cells is another great example of a hierarchical relationship:

Hierarchical Agglomeretive Clustering (which we perform here), goes bottom up: every datapoint starts as it's own singleton cluster, and at each step, we merge the two closest clusters together until all data points are in one big cluster. In order to decide which clusters are closest, we need to choose two things:

- **distance metric**: this is a measure that helps us define how close together two *data points* are. Euclidean distance is a common distance metric that you may be familiar with, but there is also cosine distance, manhattan distance, hamming distance, and even custom distance functions (like levenshtein distance between two strings!)
- **linkage criteria**: this is a measure of how close two *clusters* are. Because (most) clusters have more than one point, we need to define what it means for two clusters to be close.

Single Linkage: the distance between two clusters (A and B) as the minimum distance between any point in A and any point in B

Average Linkage: the distance between two clusters (A and B) as the average distance between points in A and points in B.

Complete Linkage: the distance between two clusters (A and B) as the maximum distance between any point in A and any point in B.

Centroid Method: the distance between two clusters (A and B) as the distance between their respective mean vectors (centroids).

Ward's Method (default): the distance between two clusters (A and B) as the Sum of Squared Errors when combining the two clusters together.

and MORE! You could technically define this anyway you wanted.

2 1. Linkage Critera

Let's build a few functions that return the distance between two clusters. Each of these functions take in two dataframes, A and B that contain the datapoints for the two respective clusters (number of features can vary).

The functions take in two arguments:

- A: an N1 x P dataframe containing the data points in cluster A. (N1 is the number of data points in cluster A; P is the number of features used)
- B: an N2 x P dataframe containing the data points in cluster B. (N2 is the number of data points in cluster B; P is the number of features used)

The function should calculate and return the distance between the clusters (assume you're using euclidean distance for all of these) according to their respective linkage criterion (single, average, and complete).

2.0.1 *Question

To calculate the distance between two clusters A (N1 x P) and B (N2 x P), how many distances (between 2 data points) would you have to calculate?

```
[57]: def single(A,B):
    distances = []

    for A_index in range(0, A.shape[0]):
        for B_index in range(0, B.shape[0]):
            d = np.linalg.norm(A.iloc[A_index]-B.iloc[B_index])
```

```
distances.append(d)
          return(min(distances))
      def average(A,B):
          distances = []
          for A_index in range(0, A.shape[0]):
              for B_index in range(0, B.shape[0]):
                  d = np.linalg.norm(A.iloc[A index]-B.iloc[B index])
                  distances.append(d)
          return(np.mean(distances))
      def complete(A,B):
          distances = []
          for A_index in range(0, A.shape[0]):
              for B_index in range(0, B.shape[0]):
                  d = np.linalg.norm(A.iloc[A_index]-B.iloc[B_index])
                  distances.append(d)
          return(max(distances))
[58]: # check if your functions are working
      df = pd.read_csv("https://raw.githubusercontent.com/cmparlettpelleriti/
       →CPSC392ParlettPelleriti/master/Data/HAC1.csv")
      dA = df.loc[df.cluster == "A"] # cluster A
      dB = df.loc[df.cluster == "B"] # cluster B
      dC = df.loc[df.cluster == "C"] # cluster C
[59]: # print(complete(dA[["x", "y"]], dB[["x", "y"]]))
      # print(average(dA[["x","y"]], dB[["x","y"]]))
      # print(single(dA[["x", "y"]], dB[["x", "y"]]))
      # if complete() is correct, this will print true
      completePASS = abs(complete(dA[["x","y"]], dB[["x","y"]]) - 4.718047025872837)_{L}
      = 0.01
      print("Complete:", completePASS)
      # if average() is correct, this will print true
      averagePASS = abs(average(dA[["x","y"]], dB[["x","y"]]) - 2.734811240314461) <= 
      \hookrightarrow 0.01
      print("Average:", averagePASS)
```

Complete: True Average: True Single: True

Using the dataset df below,

- 1. plot the clusters using ggplot, color by cluster
- 2. use your functions single(), average(), and complete() to calculate the distances between each pair of clusters.

2.0.2 Question

- 3. Look at which clusters are considered "close" and "far" in different methods. Are there differences between which are furthest/closest between methods? What are they?
- 4. Describe why you think you see these differences.

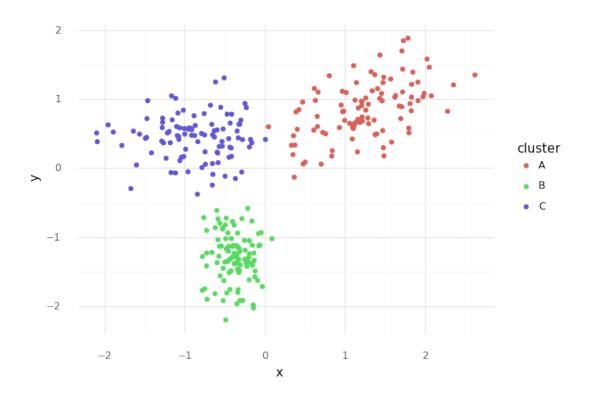
2.0.3 Answer

(example) The red cluster (A) is pretty spread out, so when you use methods like single linkage, Cluster C (blue) is closest to one end of cluster A. But the other end of cluster A is far away, so with complete linkage, Clusters A and C are furthest away from each other.

Average linkage is inbetween the two because while some points in cluster A are close to some points in cluster C, on average they're only moderately close.

The differences appear because the different linkage critera change WHICH points in each of the two clusters are influencing the measure of "closeness"

```
[67]: # plot
ggplot(df, aes(x = "x", y = "y", color = "cluster")) + geom_point() +
→theme_minimal()
```



[67]: <ggplot: (8795207390139)>

```
[61]: # calculate distances (this will likely take a few min to run)
      #---single----
      s_AB = single(dA[["x","y"]],dB[["x","y"]])
      s_AC = single(dA[["x","y"]],dC[["x","y"]])
      s_BC = single(dC[["x","y"]],dB[["x","y"]])
      print("AB:", s_AB)
      print("AC:", s_AC)
     print("BC:", s_BC)
      print("\n")
      #---average----
      a_AB = average(dA[["x","y"]],dB[["x","y"]])
      a_AC = average(dA[["x","y"]],dC[["x","y"]])
      a_BC = average(dC[["x","y"]],dB[["x","y"]])
      print("AB:", a_AB)
      print("AC:", a_AC)
     print("BC:", a_BC)
      print("\n")
      #---complete--
```

```
c_AB = complete(dA[["x","y"]],dB[["x","y"]])
c_AC = complete(dA[["x","y"]],dC[["x","y"]])
c_BC = complete(dC[["x","y"]],dB[["x","y"]])

print("AB:", c_AB)
print("AC:", c_AC)
print("BC:", c_BC)
print("\n")
```

AB: 0.7361237342164363 AC: 0.18809268564971213 BC: 0.33781439410788666

AB: 2.734811240314461 AC: 2.22451006115341 BC: 1.8723507863969513

AB: 4.718047025872837 AC: 4.81659588972283 BC: 3.5029348343614792

3 2. Exploring Linkage Critera

Using the predictors listed in predictors, perform HAC on the burger king dataset using sklearn and the following linkage critera:

- single linkage
- average linkage
- complete linkage
- · ward's method

3.0.1 Question

Plot and compare the different clusters/dendrograms that you get. What do you notice is similar/different?

Think hard: what do the dendrograms tell you about the data?

(NOTE: see documentation if you need a refresher on how to set linkage criteria in sklearn, and here for how to set it for the dendrogram)

3.0.2 Answer

(example)

The dendrograms are incredibly different. Using single linkage, a lot of the clusters tend to be closer together (shown by the high density of connections in the lower part of the dedrogram). Complete linkage on the other hand gives you a dendrogram with a lot of density in the middle of the dendrogram (which suggests not so cohesive clusters). Average linkage (as the name suggests) Gives us a combination of Single and Complete linkages. There's still a lot of density towards the bottom, suggesting more cohesion, but you don't have as much separability (shown by longer distances in the top of the dendrogram).

Ward's method has the nices looking dendrogram with high separability (long distances in the top of the dendrogram) and relatively good cohesion (short distances in the lower part of the dendrogram). While the orange cluster has a lot of variability, the Red and Green Clusters have pretty good cohesion.

The fact that single linkage gives you a lot of shorter distances compared to complete/average linkage suggests that edges of clusters might be close together while the rest of the cluster is quite spread out.

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[62]:				Item S	ervi	ing.size	Calor	ies F	at.Cal	Pro	tein(g)	F	at(g)	\	
	0	Hamburger Cheeseburger Double_Hamburger Double_Cheeseburger Buck_Double				109.0	-0.835	829	90.0	-0	.370060	-0.9	19796		
	1					121.0	-0.661780 -0.400706		130.0	-0	.145073	70844			
	2					146.0			160.0		0.304900 -0.4218				
	3					171.0	-0.009	095	230.0	0	.604882	0.0	76012		
	4					158.0	-0.183	144	200.0	0	.454891	-0.1	72940		
		Sat.Fat(g	;) Tra	ans.fat(g)	Chol(mg)) Sodi	um(mg)	Carb	s(g)	Fiber(g	g) \			
	0	4.0 6.0 8.0 12.0 10.0		0	.0	38	-0.801337		-0.580	0013	1	.0			
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	2			0			0.077055		69 -0.580013 65 -0.528996 07 -0.580013		1.0				
	3			1											
	4			0							1	.0			
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