Cairo University

Faculty of Engineering

`Computer Engineering Dept

Digital Communication

Assignment 1

Submitted to:

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By:

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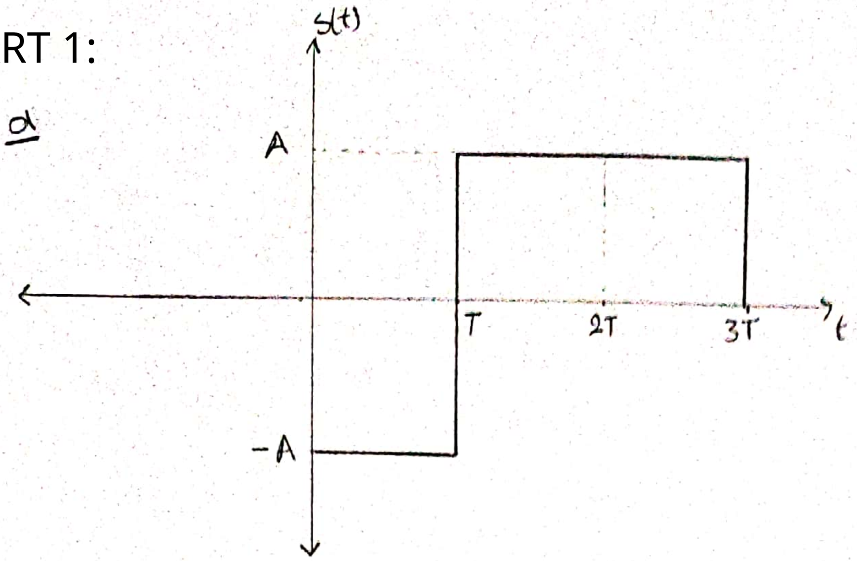
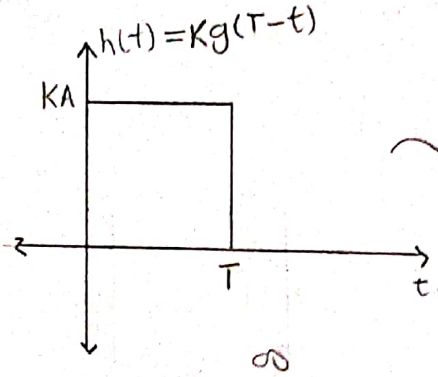
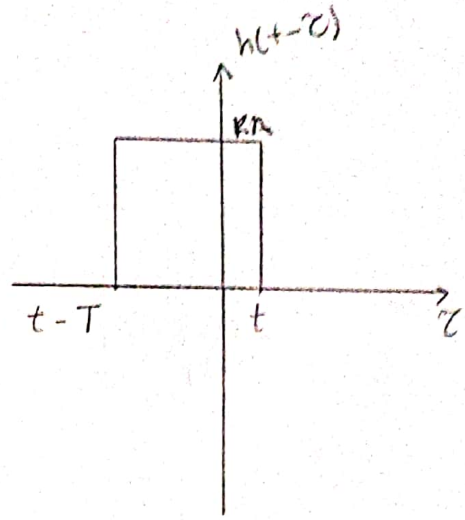
PART 1:

<u>a</u>







$$y(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau)h(t-\tau)d\tau$$

<u>$0 < t < T$:</u>

$$\int_{0}^{t} -KA^2 \, d\tau \boxed{= -KA^2 t}$$

<u>$T < t < 2T$:</u>

$$\int_{t-T}^{T} -KA^2 \, d\tau + \int_{T}^{t} KA^2 \, d\tau = KA^2(t-T-T+t-T)$$

$$\boxed{= KA^2(2t-3T)}$$

<u>$2T < t < 3T$:</u>

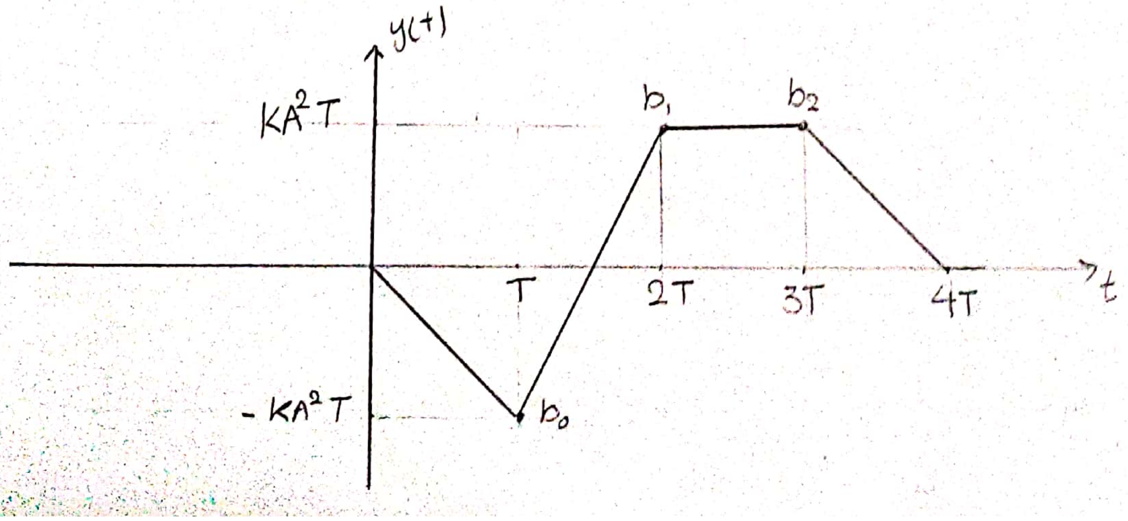
$$\int_{t-T}^{t} KA^2 \, d\tau = KA^2(\cancel{t}-\cancel{t}+T) \boxed{= KA^2 T}$$

<u>$3T < t < 4T$:</u>

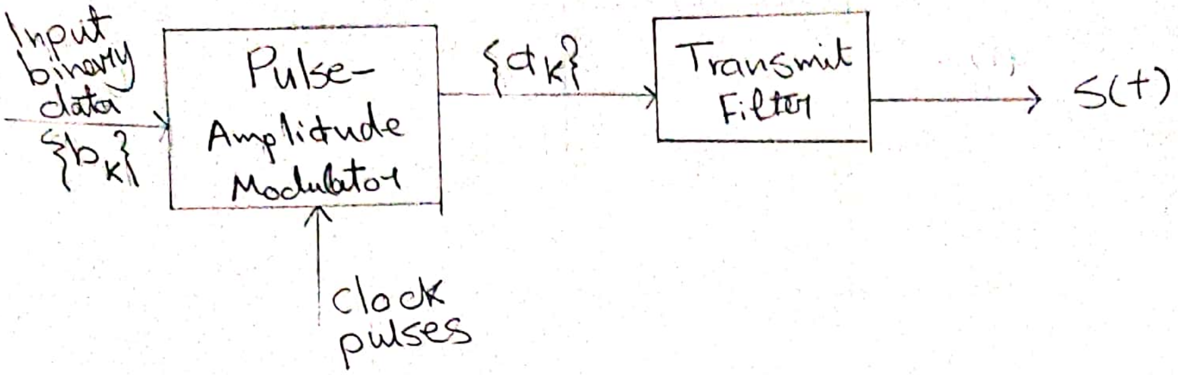
$$\int_{t-T}^{3T} KA^2 \, d\tau = KA^2(3T-t-T)$$

$$\boxed{= KA^2(4T-t)}$$

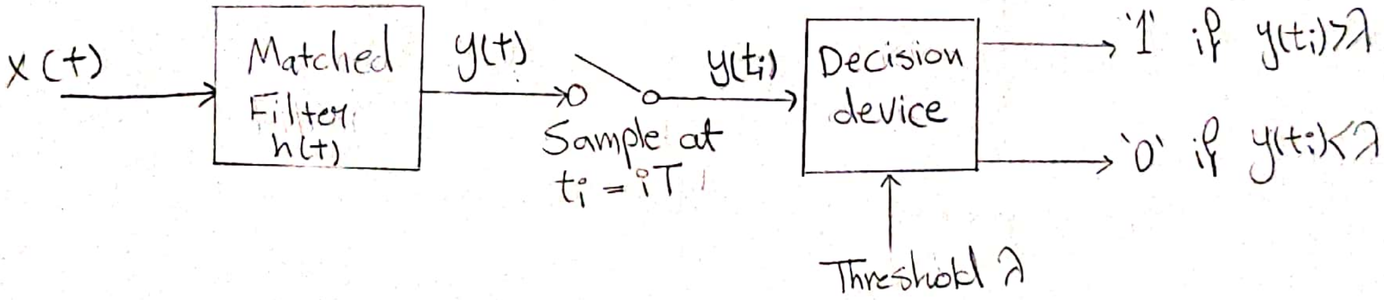
<u>b, c</u>



d) The Transmitter :



e) The Receiver:



PART 2:

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
import random

A = 1
T = 1
sample_rate = 100
num_random_processes = 1000
E_N0 = np.arange(-10, 20, 1)
```

In [2]:

```
def receiver_filter(rt, which):
    if which == 1:
        gt = np.fromfunction(lambda i: ((0*sample_rate < i) & (i <= T*sample_rate))*A,
(sample_rate+10,)).astype(np.float)
        return signal.convolve(rt,gt)/sample_rate
    elif which == 2:
        return rt
    else:
        ht = np.fromfunction(lambda i: ((0*sample_rate < i) & (i <= T*sample_rate))*((i
/sample_rate)*np.sqrt(3))/T, (sample_rate+10,)).astype(np.float)
        return signal.convolve(rt, ht)/sample_rate
```

In [3]:

```
def random_pulse():
    return A if random.getrandbits(1) == 1 else -A
```

In [4]:

```
def calc_N0(avg_powerDB):
    return ((A**2)*(T*sample_rate))/(10**(avg_powerDB/10))
```

In [5]:

```
def add_AWGN(gt, avg_powerDB):
    N0 = calc_N0(avg_powerDB)
    AWGN = np.random.normal(0, N0/2, sample_rate+10)
    return gt+AWGN
```

In [6]:

```
def sample_at_T(yt):
    return yt[T*sample_rate]
```

In [7]:

```
def mak_decision(y):
    return A if y >= 0 else -A
```

In [8]:

```
def calc_prob_error(num_random_processes, avg_powerDB, which_filter):
    num_wrong_decisions = 0
    for _ in range(num_random_processes):
        magnitude = random_pulse()
        gt = np.fromfunction(lambda i: ((0*sample_rate < i) & (i <= T*sample_rate))*mag
nitude, (sample_rate+10,)).astype(np.float)
        rt = add_AWGN(gt, avg_powerDB)
        yt = receiver_filter(rt, which_filter)
        yT = sample_at_T(yt)
        out = mak_decision(yT)
        if (out == A and magnitude == -A) or (out == -A and magnitude == A):
            num_wrong_decisions += 1
    return num_wrong_decisions/num_random_processes
```

In [9]:

```
def plot_out(avg_powerDB):
    gt = np.fromfunction(lambda i: ((0*sample_rate < i) & (i <= T*sample_rate))*A,
                         (sample_rate+10,)).astype(np.float)
    rt = add_AWGN(gt, avg_powerDB)
    yt1 = receiver_filter(rt, 1)
    yt2 = receiver_filter(rt, 2)
    yt3 = receiver_filter(rt, 3)

    fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(17,5))
    fig.suptitle("output of receive filters with AWGN with (E/N0 = {})".format(avg_powe
rDB))

    ax1.plot(np.stack((yt1,), axis=1))
    ax1.set_title("filter 1")
    ax1.set(xlabel='time', ylabel='y(t)')

    ax2.plot(np.stack((yt2,), axis=1))
    ax2.set_title("filter 2")
    ax2.set(xlabel='time', ylabel='y(t)')

    ax3.plot(np.stack((yt3,), axis=1))
    ax3.set_title("filter 3")
    ax3.set(xlabel='time', ylabel='y(t)')

    plt.show()
```

In [10]:

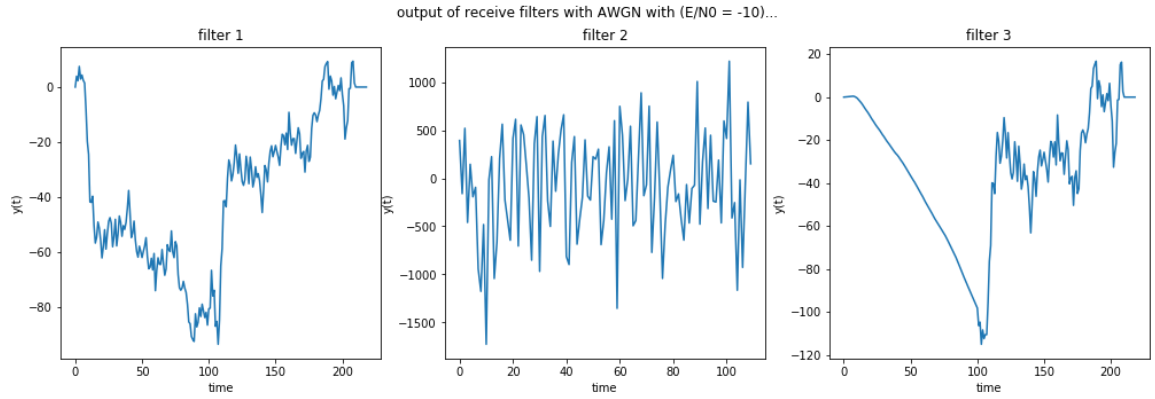
```
def calc_prob_all(avg_powerDB):
    prob1 = calc_prob_error(num_random_processes, avg_powerDB, 1)
    prob2 = calc_prob_error(num_random_processes, avg_powerDB, 2)
    prob3 = calc_prob_error(num_random_processes, avg_powerDB, 3)
    print("probability of error of receive filter1 with avg_power = {} db is {}".format
(avg_powerDB, prob1))
    print("probability of error of receive filter2 with avg_power = {} db is {}".format
(avg_powerDB, prob2))
    print("probability of error of receive filter3 with avg_power = {} db is {}".format
(avg_powerDB, prob3))
```

In [11]:

```
def prob_error_list(E_N0, which_filter):
    prob_error = []
    for each in E_N0:
        prob_error.append(calc_prob_error(num_random_processes, each, which_filter))
    return prob_error
```

In [12]:

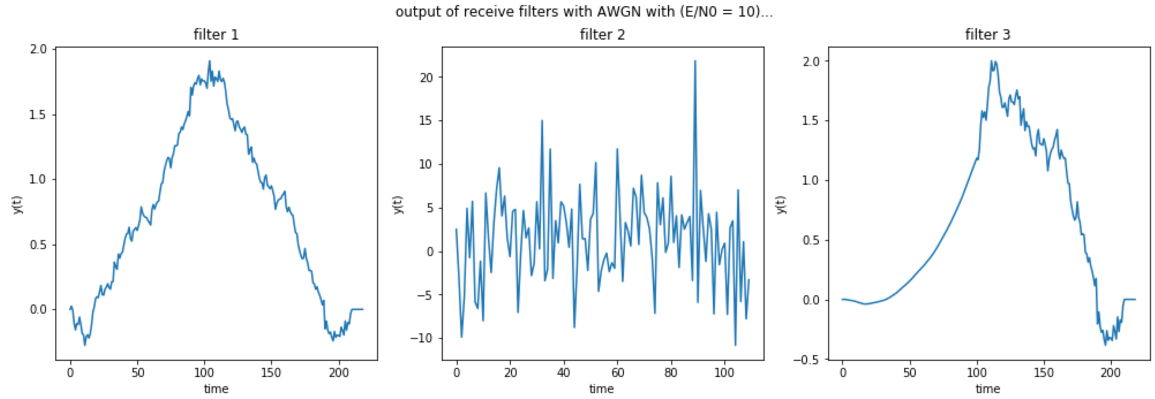
```
plot_out(avg_powerDB= -10)
calc_prob_all(avg_powerDB= -10)
```



```
probability of error of receive filter1 with avg_power = -10 db is 0.508
probability of error of receive filter2 with avg_power = -10 db is 0.537
probability of error of receive filter3 with avg_power = -10 db is 0.479
```

In [13]:

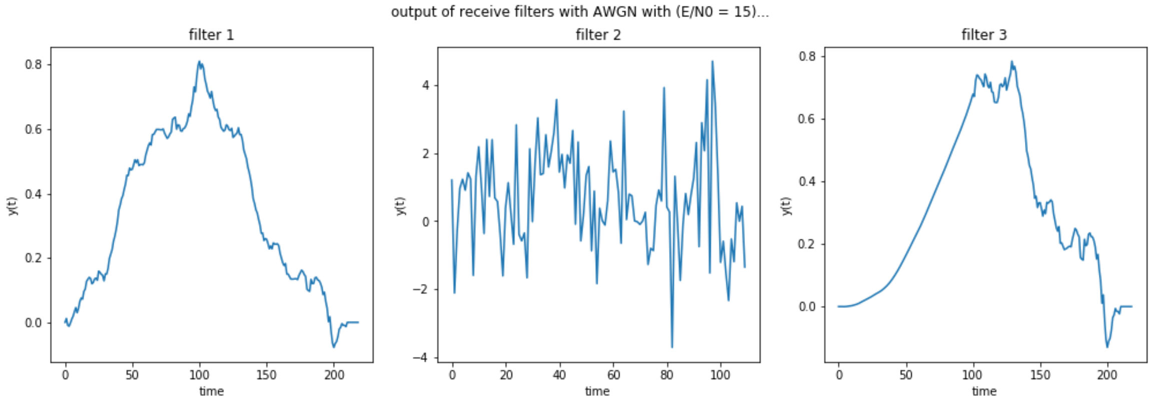
```
plot_out(avg_powerDB= 10)
calc_prob_all(avg_powerDB= 10)
```



```
probability of error of receive filter1 with avg_power = 10 db is 0.031
probability of error of receive filter2 with avg_power = 10 db is 0.418
probability of error of receive filter3 with avg_power = 10 db is 0.037
```

In [14]:

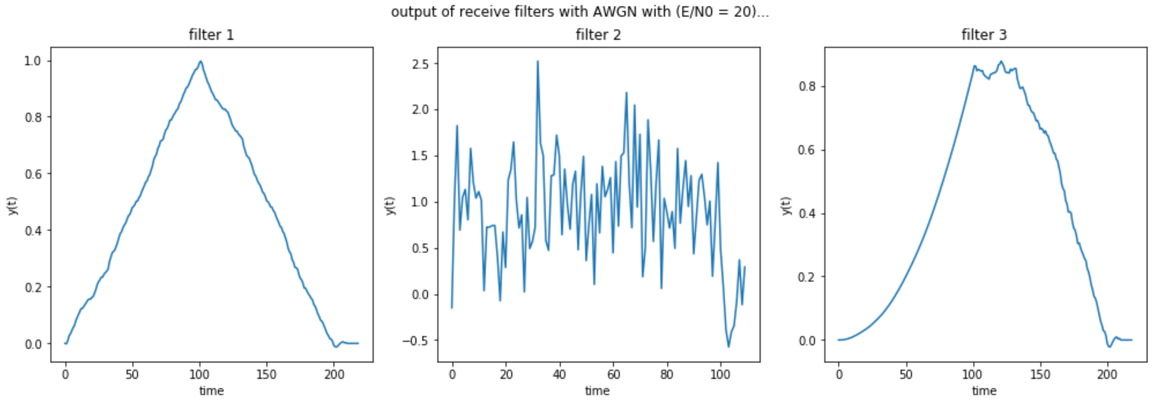
```
plot_out(avg_powerDB= 15)
calc_prob_all(avg_powerDB= 15)
```



```
probability of error of receive filter1 with avg_power = 15 db is 0.0
probability of error of receive filter2 with avg_power = 15 db is 0.249
probability of error of receive filter3 with avg_power = 15 db is 0.0
```

In [15]:

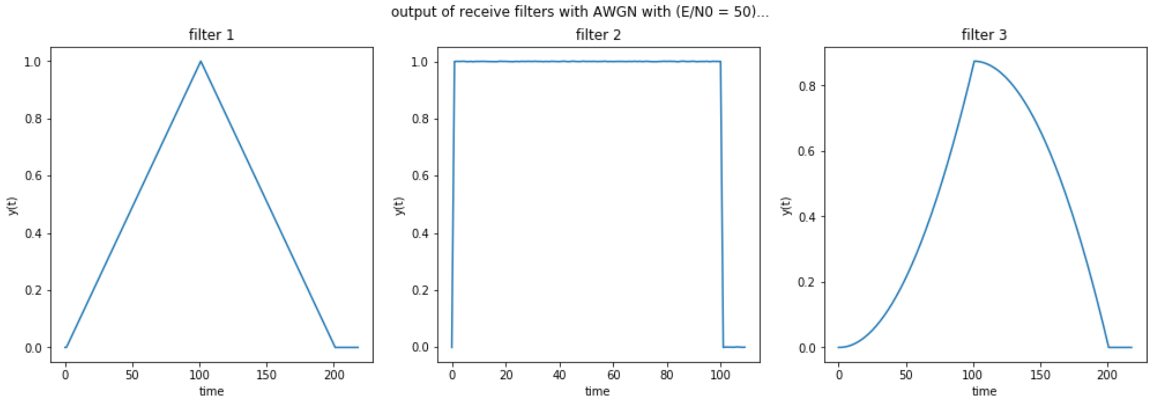
```
plot_out(avg_powerDB= 20)
calc_prob_all(avg_powerDB= 20)
```



```
probability of error of receive filter1 with avg_power = 20 db is 0.0
probability of error of receive filter2 with avg_power = 20 db is 0.016
probability of error of receive filter3 with avg_power = 20 db is 0.0
```

In [16]:

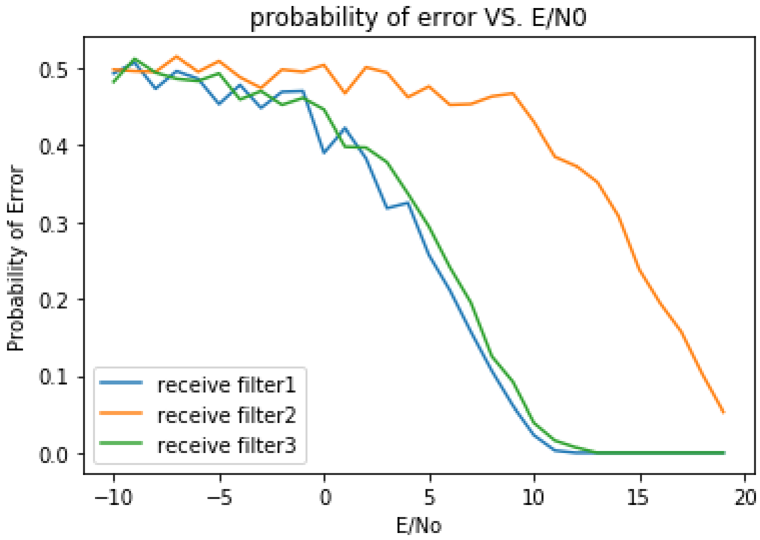
```
plot_out(avg_powerDB= 50)
calc_prob_all(avg_powerDB= 50)
```



```
probability of error of receive filter1 with avg_power = 50 db is 0.0
probability of error of receive filter2 with avg_power = 50 db is 0.0
probability of error of receive filter3 with avg_power = 50 db is 0.0
```

In [17]:

```
prob_error1 = prob_error_list(E_N0, 1)
prob_error2 = prob_error_list(E_N0, 2)
prob_error3 = prob_error_list(E_N0, 3)
plt.plot(E_N0, prob_error1, label='receive filter1')
plt.plot(E_N0, prob_error2, label='receive filter2')
plt.plot(E_N0, prob_error3, label='receive filter3')
plt.legend()
plt.title('probability of error VS. E/N0')
plt.xlabel('E/No')
plt.ylabel('Probability of Error')
plt.show()
```



Question 1: Is BER increasing or decreasing with E/N0, and why ?

Our answer:

As it is shown in the output, BER decreases while E/N0 is increasing. this beacause of, as long as E/N0 increases, N0 decreases and that means that the power of the noise becomes much smaller than the power of the pulse such that it cannot highly affect the pulse signal

Question 2: which has the lowest BER, and why ?

Our answer:

As it is shown in the output, filter one has the lowset BER as it is a matched filter = g(T-t), proved in the lec that h(t)optimal = g(T-t) maximizes the SNR and minimizes BER.. simply why?! as the output of the convolution at T is the energy of the pulse so the BER is minimum.