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Frequently Asked Questions

How many divisors does a number have?

Suppose you wish to find the number of divisors of 48. Starting with 1 we can work through the set of natural numbers and test divisibility in each case, noting that divisors can be listed in factor pairs.

$$48 = 1\times 48 = 2\times 24 = 3\times 16 = 4\times 12 = 6\times 8$$

Hence we can see that 48 has exactly ten divisors. It should also be clear that, using this method, we only ever need to work from 1 up to the square root of the number.

Although this method is quick and easy with small numbers, it is tedious and impractical for larger numbers. Fortunately there is a quick and accurate method using the divisor, or Tau, function.

Let d(n) be the number of divisors for the natural number, n.

We begin by writing the number as a product of prime factors: $n = p^a q^b r^c \ldots$
then the number of divisors, d(n) = $(a+1)(b+1)(c+1)\ldots$

To prove this, we first consider numbers of the form, $n = p^a$. The divisors are 1, p, p^2, ..., p^a; that is, d(p^a)=a+1.

Now consider $n = p^a q^b$. The divisors would be:

$$\begin{array}{ccccc}
1 & p & p^2 & \ldots & p^a \\
q & pq & p^2q & \ldots & p^aq \\
q^2 & pq^2 & p^2q^2 & \ldots & p^aq^2 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
q^b & pq^b & p^2q^b & \ldots & p^aq^b
\end{array}$$

Hence we prove that the function, d(n), is multiplicative, and in this particular case, d($p^a q^b$)=(a+1)(b+1). It should be clear how this can be extended for any natural number which is written as a product of prime factors.

The number of divisor function can be quickly demonstrated with the example we considered earlier: 48 = $2^4\times 3^1$, therefore d(48)=5×2=10.