HW 12 Scalability

Case 1:

Computation Cost:

→ 1 iteration, work per processor:

$$T_{comp} = \gamma \cdot \frac{n^2}{p}$$

→ $n = 10^6$, $p = 1000$, $\gamma = 10^{-9}$

$$T_{comp} = 10^{-9} \times \frac{(10^6)^2}{1000} = 10^{-9} \times \frac{10^{12}}{10^3} = 10^{-9} \times 10^9 = \underline{1}\ s$$

Communication Cost:

→ each process must exchange boundary data with its top and bottom neighbors. Size of each boundary is n data points,

→ Single message:

$$T_{msg} = \alpha + \beta n = 10^{-6} + 10^{-9} \times 10^6 = 10^{-6} \times 10^{-3} = 0.001001\ s$$

→ both top and bottom:

$$T_{comm} \approx 2 \times T_{msg} \approx 0.002\ s$$

Weak Scaling Consideration

→ $N_{local} = \frac{n^2}{p} = \text{constant}$, therefore $n \propto \sqrt{p}$, thus,

$$T_{comm} \approx \alpha + \beta n \propto \alpha + \sqrt{p}$$

∴ this grows a $\sqrt{p}$, and in turn does not scale weakly

Case 2:

→ each processor subdomain size:

$$\frac{n}{\sqrt{p}} \times \frac{n}{\sqrt{p}}, \quad N_{local} = \frac{n^2}{p}$$

Computation Cost:

$$T_{comp} = \gamma \times \frac{n^2}{p} = 1s \quad \text{(same as case 1)}$$

Communication Cost:

→ each process exchanges boundary data across 4 edges: Each edge length is $\frac{n}{\sqrt{p}}$, thus:

$$T_{edge} = a + \beta \frac{n}{\sqrt{p}}$$

→ $n = 10^6$, $\sqrt{p} = \sqrt{1000} \approx 31.62$

$$T_{edge} = 10^{-6} + 10^{-9} \times \frac{10^6}{31.62} = 3.26 \times 10^{-5} s$$

→ assuming 4 neighbors:

$$T_{comm} = 4 \times T_{edge} = 1.3 \times 10^{-4} s$$

Weak Scaling Consideration:

→ $N_{local} = \frac{n^2}{p}$ = constant, therefore $n \propto \sqrt{p}$ so that $\frac{n}{\sqrt{p}}$ = constant

→ Thus communication cost remains constant as p increases and does scale weakly