Algorithms Homework 5

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1 Homework 5

1.1 Question 1

We wish to show that EYR $\in NP$. This should not be too difficult to verify and one way we could do it is by the following algorithm where $G = (V, E)$ and D is the found subset of the vertices. So the verifier will return True if is satisfies the EYR problem and return False if it doesn't satisfy the conditions.

Algorithm 1 Verify(G, D)

```
for each vertex v in V \ D do
    We set connectivity to False
    for each vertex u in D do
        if (u, v) ∈ D then
            We set connectivity to True
    if not connectivity then
        return False
return True
```

The proposed algorithm runs in polynomial time and more concretely $\mathcal{O}(|V \setminus D| \cdot D)$.

1.2 Question 2

a) The first idea that I have to translate it would be to first go through each vertex, then for each subset make the vertices included in the subset into a complete graph resembling that if we choose that subset then we cover all of those vertices. Running EYR on this the problem then becomes after filling the vertices to find the subsets that correspond to it. Then it comes to a point where we need to find complete subgraphs to be able to pinpoint which subsets to take, which might not be a good approach since we know this is already a difficult problem (Clique Problem). I'm not sure if one is setting up some type of circular argument in this type of logic or if it is a valid reduction (I feel it should be correct (?)). Either way I have an alternative approach that does not include finding these complete subgraphs to identify the subsets chosen, but just present this idea that might be a good choice but feedback is appreciated.

My approach is the following: first start by drawing each vertex in V. Then draw each subset as vertex nodes and connect it to all vertices included in that subset. Then connect all subset vertices to each other forming a complete graph (since we do not want the requirement that we need to pick all subsets). This resulting graph which we call $G' = (V', E')$ can then be used to run EYR which we now move on to in order to show it is equivalent to the set cover problem for this specific graph instance. To me this is a problem that also illustrates that the EYR is at least as difficult as the set covering problem the EYR problem since the constructed graph is highly specific and is only part of the large collection of graphs that EYR could be solved on. Hence since it is clearly even more difficult to solve for arbitrary graphs and the idea behind relative difficulty in NP completeness becomes clear since it is then (very) much so at least as difficult as the set covering problem. This reduction is polynomial since we are only going through each vertex in the vertex set once, we are drawing the connections between the vertices and each subset, then lastly we make the subset vertices into a complete graph. We can provide a pessimistic upperbound but still showcasing that it is polynomial by having n vertices in $|V|$ and m vertices for the number of subsets then making this into a complete graph: $nm(nm + 1)/2 = \mathcal{O}(n^2m^2)$. The number of edges we will draw is upperbounded by this but still would be polynomial.

b) The proposition is that if we run EYR on G' and find a solution this will give us a solution to the set covering problem.

(Proof) Given a solution for EYR on the graph G' then if a vertex has been chosen then simply connect it to any subset that is connected to it. By the construction of the subsets vertices in the graph they correspond to the original subsets in the set covering and the marked subset vertices are then the solution to the set cover problem. Each vertex in $|V|$ needs to adjacent to a subset node in this solution of EYR and hence will satisfy that these subsets include all elements in the set covering.

c) The proposition is that if we a have a solution to set cover then we have a solution to EYR.

(Proof) Given a solution to the set covering problem then simply fill those subset vertices in G' and by the construction these will satisfy the EYR conditions. Specifically the union of all subsets needs to reach all elements in the set, in the graph all vertices will be adjacent to some subset vertex.

What we (hopefully) have proved or convinced is that by the translation/reduction of set cover to G' the set covering problem is equivalent to the EYR problem. Hence going back to the point that since this is a specific instance of EYR it needs to be at least as hard as the set covering problem and we conclude that EYR is NP-complete.

1.3 Question 3

This is an optional question so I havn't thought about it much but I feel the EYR problem is very closely related to the vertex covering problem. If I was asked to show this problem was EYR without the restriction to set covering I would probably start with the vertex covering problem and try to find a reduction there. Intuitively the difference is that in the vertex covering the edges should be incident to a vertex in the subset but here it is the vertices that should be incident instead. My thinking is there should be a way to translate these two in a way that should be quite straightforward. Feedback if my way of thinking here is on the right path is appreciated!