5DV005, Fall 2018, Lab session 5

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Contents

1 The time and the place

The lab session will take place on

Wednesday, December 5th, 2018, (kl. 13.00-16.00), Room MA416-426.

2 Introduction

This week's problems do not require much in terms of programming. However, however careful inspection and analysis of the output is required. Use any excess time to complete problems from previous weeks.

3 The problems

Problem 1 Consider the problem of computing the derivative $f'(x)$ using the finite difference approximation

$$D_1(f, x, h) = \frac{f(x+h) - f(x)}{h}$$

Execute the script `rdifmwe1` and examine output in detail:

1. Determine the value of k where the computed value of Richardson's fraction has executed an illegal jump.
2. Determine the range of k values for which the computed value of Richardson's fraction convergences monotonically to 2^p for a suitable value of p.
3. Determine the range of k values for which the computed value of Richardson's fraction converges to 2^p at the correct rate.

4. Determine the range of k values where the error estimates become more and more accurate.

5. How is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?

Problem 2 Copy `rdifmwe1.m` into `/work/l5p2.m` and adapt it to the problem of computing $f'(2)$ where

$$f(x) = e^x \sin(x)$$

Do *not* include the derivative when you call `rdif`.

1. Verify that the computed value of Richardson's fraction appears to converge towards 2^p for a suitable value of p as h tends to zero.
2. Find the last value of k, where the computed value of Richardson's fraction behaved exactly as predicted for the exact value of Richardson's fraction.
3. Include the exact derivative when you call `rdif`. Find the value of k where the accuracy of Richardson's error estimate is maximal.
4. How is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?

Problem 3 Consider the problem of computing the derivative $f'(x)$ using the finite difference approximation

$$D_2(f, x, h) = \frac{f(x+h) - f(x-h)}{2h}$$

Execute the script `rdifmwe2` and examine output in detail:

1. Determine the value of k where the computed value of Richardson's fraction has executed an illegal jump.
2. Determine the range of k values for which the computed value of Richardson's fraction convergences monotonically to 2^p for a suitable value of p.
3. Determine the range of k values for which the computed value of RichRichardson's fraction converges to 2^p at the correct rate.
4. Determine the range of k values where the error estimates become more and more accurate.
5. Is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?

Problem 4 Copy `rdifmwe2.m` into `/work/l5p4.m` and adapt it to the problem of computing $f'(2)$ where

$$f(x) = e^x \sin(x).$$

Do *not* include the derivative when you call `rdif` initially.

1. Verify that the computed value of Richardson's fraction appears to converge towards 2^p for a suitable value of p as h tends to zero.
2. Find the last value of k, where the computed value of Richardson's fraction behaved exactly as predicted for the exact value of Richardson's fraction.
3. Include the exact derivative when you call `rdif`. Find the value of k where the accuracy of Richardson's error estimate is maximal.
4. Is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?