5DV005, Fall 2018, Lab session 6

Carl Christian Kjelgaard Mikkelsen

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Contents

1 The time and the place

The lab session will take place on

Wednesday, December 12th, 2018, (kl. 13.00-16.00), Room MA416-426.

2 The problems

Problem 1 Copy `rintmwe1` into `/lab6/work/l6p1.m` and adapt it to the problem of computing the integral

$$\int_0^\pi \exp(x)\sin(x)dx$$

numerically using the trapezoidal rule.

1. What evidence do you find to support support the conjecture that there exist an asymptotic error expansion of the form
$$T - A_h = \alpha h^p + \beta h^q + O(h^r), \quad 0 < p < q < r.$$
2. Based on the numerical evidence, what is a reasonable value of p?
3. Based on the numerical evidence, what is a reasonable value of q?
4. What is the smallest value of k for which the integral can be computed with a relative error less than $\tau = 10^{-6}$?

You must explain why your error estimate is reliable!

5. Compute the exact value of the integral and include this information in `l6p2`.
6. Is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?

Problem 2 Copy `rintmwe1.m` into `/work/l6p2.m` and adapt it to the problem of computing the integral

$$T = \int_{-1}^{1} \sqrt{1-x^2}dx$$

using the trapezoidal rule as your approximation A_h.

1. What evidence do you find to support support the conjecture that there exist an asymptotic error expansion of the form

$$T - A_h = \alpha h^p + \beta h^q + O(h^r), \quad 0 < p < q < r.$$

2. Based on the numerical evidence, what is a reasonable value of p?

3. Based on the numerical evidence, what is a reasonable value of q?

4. What is the smallest value of k for which the integral can be computed with a relative error less than $\tau = 10^{-6}$?

You must explain why your error estimate is reliable

5. Compute the exact value of the integral and include this information in `l6p1`. **Hint:** It is quite easy to compute the integral if you make a drawing of the graph first.

6. Is the behavior of Richardson's fraction related to the quality of Richardson's error estimate?

Problem 3 Copy `rintmwe1.m` into `/work/l6p3.m` and adapt it to the problem of computing the integral

$$T = \frac{1}{\sqrt{\pi}} \int_0^3 \exp(-x^2) dx$$

using the trapezoidal rule as your approximation A_h.

1. What evidence do you find to support support the conjecture that there exist an asymptotic error expansion of the form

$$T - A_h = \alpha h^p + \beta h^q + O(h^r), \quad 0 < p < q < r.$$

2. Based on the numerical evidence, what is a reasonable value of p?
3. Based on the numerical evidence, what is a reasonable value of q?
4. Why is Richardson's fraction not close to 2^p for small values of k?
5. Why is Richardson's fraction not close to 2^p for very large values of k?.
6. What is the smallest value of k for which the integral can be computed with a relative error less than $\tau = 10^{-6}$?

You must explain why your error estimate is reliable!

Problem 4 Copy `rdifmwe1` into `/work/16p4.m` and adapt it to the problem of computing the the target $T = f'(x)$, where f is you favorite differentiable function and x is your favorite real number using the mysterious rule

$$M_h = A_h + \frac{A_h - A_{2h}}{2^p - 1}$$

where A_h is your favorite rule for computing $f'(x)$ which obeys an asymptotic error expansion of the form

$$T - A_h = \alpha h^p + \beta h^q + O(h^r), \quad 0 < p < q < r.$$

1. What evidence can you uncover that suggests that M_h obeys an asymptotic error expansion of the form

$$T - M_h = \bar{\alpha} h^q + \bar{\beta} h^r + O(h^s), \quad 0 < q < r < s. \tag{1}$$

2. Based on your numerical evidence, what is a reasonable value of q?
3. Based on your numerical evidence, what is a reasonable value of r?
4. Include the exact value of the derivative of f in the script.
5. Examine the relationship between Richardson's fraction and the quality of the error estimate.

3 Concluding remarks

1. You will find that quality of the error estimate improves even after the computed value of Richardson's fractions have start to deviate from the expected pattern. This happens from time to time, but it is not something you can count on.

2. If you return an approximation without an error estimate or an error bound, then you work is incomplete.

3. If you return an error bound or an error estimate without explaining why it is reliable, then your work is incomplete.