

# Homework 1

January 2020

## 1 Combinatorics

1. How many different ways are there to order a list containing 100 distinct elements?
2. A *palindrome* is a word that can be read in both directions, such as “madam” or “noon”. How many 7-letter palindromes can be formed using the letters of the alphabet  $\mathcal{A}$ ? Start by choosing the alphabet and write down the number of letters, e.g. the English alphabet has 26 letters. (Please only choose alphabets that have  $> 10$  and  $< 50$  letters). Then calculate the number of palindromes.
3. A graph is *complete* if any pair of its vertices is connected by an edge. How many edges are there in a complete graph with 5 vertices? What about 25 vertices and  $n$  vertices?
4. How many different ways are there to order the letters contained in the word “engineering”?

## 2 Asymptotic order

1. Take the following list of functions and arrange them in ascending order of growth rate. That is, if function  $f(n)$  comes before function  $g(n)$  in your list, then it should be the case that  $f(n)$  is  $\mathcal{O}(g(n))$ .

$$\begin{aligned}f_1(n) &= 5^n \\f_2(n) &= n^{1.5} \\f_3(n) &= 2^{2^n} \\f_4(n) &= n^{300} \\f_5(n) &= 2^{n^2} \\f_6(n) &= n(\log n)^2 \\f_7(n) &= n \log \log n \\f_8(n) &= n^{\log n}\end{aligned}$$

2. For any two functions  $f(n)$  and  $g(n)$  that immediately follow each other in your list, prove that  $f(n)$  is  $\mathcal{O}(g(n))$ .