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## Data structures

### Fenwick tree

```
template<typename T> struct fenwick_tree {
    /* can convert it to map, build what you need only
     * will be: memory  $O(q \cdot \log n)$  ,time  $O(\log n \cdot \log n)$  */
    vector<T> BIT;
    int n;
    fenwick_tree(int n) : n(n), BIT(n + 1) {}
    T getAccum(int idx) {
        T sum = 0;
        while (idx) {
            sum += BIT[idx];
            idx -= (idx & -idx);
        }
        return sum;
    }
    void add(int idx, T val) {
        assert(idx != 0);
        while (idx <= n) {
            BIT[idx] += val;
            idx += (idx & -idx);
        }
    }
    T getValue(int idx) {
        return getAccum(idx) - getAccum(idx - 1);
    }
    // ordered statistics tree // get index that has value >= accum
    // values must be positive
    int getIdx(T accum) {
        int start = 1, end = n, rt = -1;
        while (start <= end) {
            int mid = start + end >> 1;
            T val = getAccum(mid);
            if (val >= accum)
                rt = mid, end = mid - 1;
            else start = mid + 1;
        }
        return rt;
    }
    //not tested (from topcoder)
    //first index less than or equal accum  $O(\log n)$  (same as getIdx)
    int find(T accum) {
        int i = 1, idx = 0;
        while ((1 << i) <= n) i <<= 1;
        for (; i > 0; i >>= 1) {
            int tidx = idx + i;
            if (tidx > n) continue;
            if (accum >= BIT[tidx]) {
                idx = tidx; accum -= BIT[tidx];
            }
        }
        return idx;
    }
};
```

## Fenwick tree 2d

```
template<typename T>
struct fenwick_tree_2d {
#define Lbit(x) (x&-x)
    int n, m;
    vector<vector<T>> BIT;
    fenwick_tree_2d(int n, int m) :
        n(n), m(m), BIT(n + 1, vector<T>(m + 1)) {
    }
    T getAccum(int i, int j) {
        T sum = 0;
        for (; i; i -= Lbit(i))
            for (int idx = j; idx > 0; idx -= Lbit(idx))
                sum += BIT[i][idx];
        return sum;
    }
    void add(int i, int j, int val) {
        assert(i != 0 && j != 0);
        for (; i <= n; i += Lbit(i))
            for (int idx = j; idx <= m; idx += Lbit(idx))
                BIT[i][idx] += val;
    }
    T getRectangleSum(int x1, int y1, int x2, int y2) {
        if (y1 > y2)
            swap(y1, y2);
        if (x1 > x2)
            swap(x1, x2);
        return getAccum(x2, y2) - getAccum(x1 - 1, y2) - getAccum(x2, y1 - 1)
            + getAccum(x1 - 1, y1 - 1);
    }
};
```

## Fenwick tree update range

```
/*
x[i] = a[i] - a[i-1] //a is original array
y[i] = x[i]*(i-1)

sum(1,3) = a[1] + a[2] + a[3] = (x[1]) + (x[2] + x[1]) + (x[3] + x[2] + x[1])

= 3*(x[1] + x[2] + x[3]) - 0*x[1] - 1*x[2] - 2*x[3] //same equation but more complex
= sumX(1,3) * 3 - sumY(1,3)
so sum(1,n) = sumX(1,n)*n - sumY(1,n)

update:
x[l] += val, x[r+1] -= val
y[l] += val *(l-1), y[r+1] -= r*val
*/

template<typename T>
class fenwick_tree {
    int n;
    vector<T> x, y;
    T getAccum(vector<T>& BIT, int idx) {
        T sum = 0;
        while (idx) {
            sum += BIT[idx];
            idx -= (idx & -idx);
        }
        return sum;
    }
    void add(vector<T>& BIT, int idx, T val) {
        assert(idx != 0);
        while (idx <= n) {
            BIT[idx] += val;
            idx += (idx & -idx);
        }
    }
    T prefix_sum(int idx) {
        return getAccum(x, idx) * idx - getAccum(y, idx);
    }
public:
    fenwick_tree(int n) :
        n(n), x(n + 1), y(n + 1) {
    }
    void update_range(int l, int r, T val) {
        add(x, l, val);
        add(x, r + 1, -val);
        add(y, l, val * (l - 1));
        add(y, r + 1, -val * r);
    }
    T range_sum(int l, int r) {
        return prefix_sum(r) - prefix_sum(l - 1);
    }
};
```

## Segment tree

```
/*
for efficient memory (2*n)
#define LEFT (idx+1)
#define MID ((start+end)>>1)
#define RIGHT (idx+((MID-start+1)<<1))
*/
template<typename T>
class segment_tree { //1-based
#define LEFT (idx<<1)
#define RIGHT (idx<<1|1)
#define MID ((start+end)>>1)
    int n;
    vector<T> tree, lazy;
    T merge(const T& left, const T& right) {}
    inline void pushdown(int idx, int start, int end) {
        if (lazy[idx] == 0) return;
        //update tree[idx] with lazy[idx]
        tree[idx] += lazy[idx];
        if (start != end) {
            lazy[LEFT] += lazy[idx];
            lazy[RIGHT] += lazy[idx];
        }
        lazy[idx] = 0; //clear lazy
    }
    inline void pushup(int idx) {
        tree[idx] = merge(tree[LEFT], tree[RIGHT]);
    }
    void build(int idx, int start, int end) {
        if (start == end) return;
        build(LEFT, start, MID);
        build(RIGHT, MID + 1, end);
        pushup(idx);
    }
    void build(int idx, int start, int end, const vector<T>& arr) {
        if (start == end) {
            tree[idx] = arr[start];
            return;
        }
        build(LEFT, start, MID, arr);
        build(RIGHT, MID + 1, end, arr);
        pushup(idx);
    }
    T query(int idx, int start, int end, int from, int to) {
        pushdown(idx, start, end);
        if (from <= start && end <= to)
            return tree[idx];
        if (to <= MID)
            return query(LEFT, start, MID, from, to);
        if (MID < from)
            return query(RIGHT, MID + 1, end, from, to);
        return merge(query(LEFT, start, MID, from, to),
            query(RIGHT, MID + 1, end, from, to));
    }
}
```



```

void update(int idx, int start, int end, int lq, int rq, const T& val) {
    pushdown(idx, start, end);
    if (rq < start || end < lq)
        return;
    if (lq <= start && end <= rq) {
        lazy[idx] += val; //update lazy
        pushdown(idx, start, end);
        return;
    }
    update(LEFT, start, MID, lq, rq, val);
    update(RIGHT, MID + 1, end, lq, rq, val);
    pushup(idx);
}

public:
    segment_tree(int n) :n(n), tree(n << 2), lazy(n << 2) {}
    segment_tree(const vector<T>& v) {
        n = v.size() - 1;
        tree = vector<T>(n << 2);
        lazy = vector<T>(n << 2);
        build(1, 1, n, v);
    }
    T query(int l, int r) {
        return query(1, 1, n, l, r);
    }
    void update(int l, int r, const T& val) {
        update(1, 1, n, l, r, val);
    }
};

#ifdef LEFT
#ifdef RIGHT
#ifdef MID
};

```

### Max sum range node

```

struct MSR_Node {
    ll left, right, mid, sum;
    MSR_Node(const ll& val) { //be careful from the empty subarray
        left = right = mid = sum = val;
    }
    MSR_Node(const MSR_Node& a, const MSR_Node& b) {
        left = max(a.left, a.sum + b.left);
        right = max(b.right, b.sum + a.right);
        mid = max({ a.mid, b.mid, a.right + b.left });
        sum = a.sum + b.sum;
    }
    ll getMax() {
        return max({ left, right, mid, sum });
    }
};

```

## Extended Segment tree

```
struct segtree {
    segtree *left = nullptr, *right = nullptr;
    int mx = 0;
    segtree(int val = 0) :
        mx(val) {
    }
    void extend() {
        if (left == nullptr) {
            left = new segtree();
            right = new segtree();
        }
    }
    void pushup() {
        mx = max(left->mx, right->mx);
    }
    ~segment_tree() {
        if (left == nullptr) return;
        delete left;
        delete right;
    }
};

class extened_segment_tree {
#define MID ((start+end)>>1)
    void update(segtree *root, int start, int end, int pos, int val) {
        if (pos < start || end < pos)
            return;
        if (start == end) {
            root->mx = max(root->mx, val);
            return;
        }
        root->extend();
        update(root->left, start, MID, pos, val);
        update(root->right, MID + 1, end, pos, val);
        root->pushup();
    }
    int query(segtree *root, int start, int end, int l, int r) {
        if (root == nullptr || r < start || end < l)
            return 0;
        if (l <= start && end <= r)
            return root->mx;
        return max(query(root->left, start, MID, l, r),
                    query(root->right, MID + 1, end, l, r));
    }
public:
    int start, end;
    segtree *root;
    extened_segment_tree() {
    }
    ~extened_segment_tree() {
        delete root;
    }
    extened_segment_tree(int start, int end) : start(start), end(end) {
        root = new segtree();
    }
}
```

```

    void update(int pos, int val) {
        update(root, start, end, pos, val);
    }
    int query(int l, int r) {
        return query(root, start, end, l, r);
    }
#undef MID
};

```

## Persistent segment tree

```

struct segtree {
    static segtree *sentinel;
    segtree *left, *right;
    bool dirty = false;
    ll sum = 0, lazy = 0;
    segtree(ll val = 0) : sum(val) {
        left = right = this;
    }
    segtree(segtree *left, segtree *right) : left(left), right(right) {
        sum = left->sum + right->sum;
    }
};

segtree *segtree::sentinel = new segtree();
class persistent_segment_tree {
#define MID ((start+end)>>1)
    segtree* apply(segtree *root, int start, int end, ll val) {
        segtree *rt = new segtree(*root);
        rt->dirty = true;
        rt->sum += (end - start + 1) * val;
        rt->lazy += val;
        return rt;
    }
    void pushdown(segtree *root, int start, int end) {
        if (root->dirty == false || start == end)
            return;
        root->left = apply(root->left, start, MID, root->lazy);
        root->right = apply(root->right, MID + 1, end, root->lazy);
        root->lazy = 0;
        root->dirty = 0;
    }
    segtree* build(int start, int end, const vector<int> &v) {
        if (start == end)
            return new segtree(v[start]);
        return new segtree(build(start, MID, v), build(MID + 1, end, v));
    }
    segtree* Set(segtree *root, int start, int end, int pos, ll new_val) {
        pushdown(root, start, end);
        if (pos < start || end < pos)
            return root;
        if (pos <= start && end <= pos)
            return new segtree(new_val);
        return new segtree(Set(root->left, start, MID, pos, new_val),
            Set(root->right, MID + 1, end, pos, new_val));
    }
}

```

```

segtree* update(segtree *root, int start, int end, int l, int r, ll val) {
    pushdown(root, start, end);
    if (r < start || end < l)
        return root;
    if (l <= start && end <= r)
        return apply(root, start, end, val);
    return new segtree(update(root->left, start, MID, l, r, val),
        update(root->right, MID + 1, end, l, r, val));
}
ll query(segtree *root, int start, int end, int l, int r) {
    pushdown(root, start, end);
    if (r < start || end < l)
        return 0;
    if (l <= start && end <= r)
        return root->sum;
    return query(root->left, start, MID, l, r)
        + query(root->right, MID + 1, end, l, r);
}
public:
    int start, end;
    vector<segtree*> versions;
    persistent_segment_tree(int start, int end) :
        start(start), end(end) {
        versions.push_back(segtree::sentinel);
    }
    persistent_segment_tree(const vector<int> &v) :
        start(0), end(v.size() - 1) {
        versions.push_back(build(start, end, v));
    }
    void update(int l, int r, ll val) {
        versions.push_back(update(versions.back(), start, end, l, r, val));
    }
    ll query(int time, int l, int r) {
        return query(versions[time], start, end, l, r);
    }
};
#endif
};

```

## Ordered set

```

#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template<typename key>
using ordered_set = tree<key, null_type, less<key>, rb_tree_tag,
tree_order_statistics_node_update>;
/*
    find_by_order(k) :
    It returns to an iterator to the k-th element (counting from zero) in the set in O(logn)
    time.
    To find the first element k must be zero.
    order_of_key(k) :
    It returns to the number of items that are strictly smaller than our item k in O(logn)
    time. */

```

## Sparse table

```
template<typename T>
struct sparse_table {
    vector<vector<T>> sparseTable;
    using F = function<T(T, T)>;
    F merge;
    static int LOG2(int x) { //floor(log2(x))
        return 31 - __builtin_clz(x);
    }
    sparse_table(vector<T>& v, F merge) :
        merge(merge) {
        int n = v.size();
        int logN = LOG2(n);
        sparseTable = vector<vector<T>>(logN + 1);
        sparseTable[0] = v;
        for (int k = 1, len = 1; k <= logN; k++, len <= 1) {
            sparseTable[k].resize(n);
            for (int i = 0; i + len < n; i++)
                sparseTable[k][i] = merge(sparseTable[k - 1][i],
                    sparseTable[k - 1][i + len]);
        }
    }
    T query(int l, int r) {
        int k = LOG2(r - l + 1); // max k ==> 2^k <= length of range
        //check first 2^k from left and last 2^k from right //overlap
        return merge(sparseTable[k][l], sparseTable[k][r - (1 << k) + 1]);
    }
    T query_shifting(int l, int r) {
        T res;
        bool first = true;
        for (int i = (int)sparseTable.size() - 1; i >= 0; i--)
            if (l + (1 << i) - 1 <= r) {
                if (first)
                    res = sparseTable[i][l];
                else
                    res = merge(res, sparseTable[i][l]);
                first = false;
                l += (1 << i);
            }
        return res;
    }
};
```

## SQRT Decomposition

```
//zero based SQRT_Decomposition with lazy propagation
template<typename update_type, typename query_type>
class SQRT_Decomposition {
    struct Bucket {
        int l, r;
        update_type lazy;
        Bucket(int l, int r) : l(l), r(r) {
            //set default value to lazy
            //build bucket for the first time
        }
        void build() {
            //update all bucket with lazy if have
            //rebuild the bucket
            //clear lazy
        }
        //update all bucket
        //just update lazy
        void update(const update_type& val) {}

        //update range in bucket
        void update(int start, int end, const update_type& val) {
            if (start == l && end == r) {
                update(val);
                return;
            }
            //update bucket
            //rebuild the bucket if need
        }
        //query about all bucket
        //calc with lazy
        query_type query() {}
        //query about range in bucket
        query_type query(int start, int end) {
            if (start == l && end == r)
                return query();
            //push lazy if have
            //calc
        }
    };

};

int n, sqrtN;
vector<Bucket> bucket;
int begin(int idx) {
    return idx * sqrtN;
}
int end(int idx) {
    return min(sqrtN * (idx + 1), n) - 1;
}
int which_block(int idx) {
    return idx / sqrtN;
}
```

```

public:
    Sqrt_Decomposition(int n) {
        this->n = n;
        sqrtN = sqrt(n);
        for (int i = 0; i < n; i += sqrtN)
            bucket.push_back(Bucket(i, min(i + sqrtN, n) - 1));
    }
    void update(int left, int right, update_type val) {
        int st = which_block(left), ed = which_block(right);
        bucket[st].update(left, min(bucket[st].r, right), val);
        for (int i = st + 1; i < ed; i++)
            bucket[i].update(val);
        if (st != ed) bucket[ed].update(bucket[ed].l, right, val);
    }
    query_type query(int left, int right) {
        int st = which_block(left), ed = which_block(right);
        query_type rt = bucket[st].query(left, min(bucket[st].r, right));
        for (int i = st + 1; i < ed; i++)
            rt += bucket[i].query();
        if (st != ed) rt += bucket[ed].query(bucket[ed].l, right);
        return rt;
    }
};

```

## Treap

### Implicit Treap

```

enum DIR { L, R };
template<typename T> struct cartesian_tree {
    static cartesian_tree<T>* sentinel;
    T key = T();
    int priority = 0, size = 0, reverse = false;
    cartesian_tree* child[2];
    cartesian_tree() {
        child[L] = child[R] = this;
    }
    cartesian_tree(const T& x, int y) : key(x), priority(y) {
        size = 1;
        child[L] = child[R] = sentinel;
    }
    void push_down() {
        if (!reverse) return;
        reverse = 0;
        child[L]->doReverse();
        child[R]->doReverse();
    }
    void doReverse() {
        reverse ^= 1;
        swap(child[L], child[R]);
    }
    void push_up() {
        size = child[L]->size + child[R]->size + 1;
    }
};

```

```

template<typename T>
cartesian_tree<T>* cartesian_tree<T>::sentinel = new cartesian_tree<T>();

template<typename T, template<typename > class cartesian_tree>
class implicit_treap { //1 based
    typedef cartesian_tree<T> node;
    typedef cartesian_tree<T>* nodeptr;
#define emptyNode cartesian_tree<T>::sentinel
    nodeptr root;
    void split(nodeptr root, nodeptr& l, nodeptr& r, int firstXElment) {
        if (root == emptyNode) {
            l = r = emptyNode;
            return;
        }
        root->push_down();
        if (firstXElment <= root->child[L]->size) {
            split(root->child[L], l, root->child[L], firstXElment);
            r = root;
        }
        else {
            split(root->child[R], root->child[R], r,
                firstXElment - root->child[L]->size - 1);
            l = root;
        }
        root->push_up();
    }
    nodeptr merge(nodeptr l, nodeptr r) {
        l->push_down();
        r->push_down();
        if (l == emptyNode || r == emptyNode)
            return (l == emptyNode ? r : l);
        if (l->priority > r->priority) {
            l->child[R] = merge(l->child[R], r);
            l->push_up();
            return l;
        }
        r->child[L] = merge(l, r->child[L]);
        r->push_up();
        return r;
    }
    vector<nodeptr> split_range(int s, int e) { // [x<s,s<=x<=e,e<x]
        nodeptr l, m, r, tmp;
        split(root, l, tmp, s - 1);
        split(tmp, m, r, e - s + 1);
        return { l,m,r };
    }
public:
    implicit_treap() : root(emptyNode) {}
    int size() {
        return root->size;
    }
}

```



```

void insert(int pos, const T& key) {
    nodeptr tmp = new node(key, rand());
    nodeptr l, r;
    split(root, l, r, pos - 1);
    root = merge(merge(l, tmp), r);
}
void push_back(const T& value) {
    root = merge(root, new node(value, rand()));
}
T getByIndex(int pos) {
    vector<nodeptr> tmp = split_range(pos, pos);
    nodeptr l = tmp[0], m = tmp[1], r = tmp[2];
    T rt = m->key;
    root = merge(merge(l, m), r);
    return rt;
}
void erase(int pos) {
    vector<nodeptr> tmp = split_range(pos, pos);
    nodeptr l = tmp[0], m = tmp[1], r = tmp[2];
    delete m;
    root = merge(l, r);
}
void cyclic_shift(int s, int e) { //to the right
    vector<nodeptr> tmp = split_range(s, e);
    nodeptr l = tmp[0], m = tmp[1], r = tmp[2];
    nodeptr first, second;
    split(m, first, second, e - s);
    root = merge(merge(merge(l, second), first), r);
}
void reverse_range(int s, int e) {
    vector<nodeptr> tmp = split_range(s, e);
    nodeptr l = tmp[0], m = tmp[1], r = tmp[2];
    m->reverse ^= 1;
    root = merge(merge(l, m), r);
}
node range_query(int s, int e) {
    vector<nodeptr> tmp = split_range(s, e);
    nodeptr l = tmp[0], m = tmp[1], r = tmp[2];
    node rt = *m;
    root = merge(merge(l, m), r);
    return rt;
}
};

```

### Ordered multiset

```

enum DIR { L, R };
template<typename T>
struct cartesian_tree {
    static cartesian_tree<T>* sentinel;
    T key = T();
    int priority = 0, frequency = 0, size = 0;
    cartesian_tree* child[2];
    cartesian_tree() {
        child[L] = child[R] = this;
    }
}

```

```

cartesian_tree(const T& x, int y) : key(x), priority(y) {
    size = frequency = 1;
    child[L] = child[R] = sentinel;
}
void push_down() {
}

void push_up() {
    size = child[L]->size + child[R]->size + frequency;
}
};
template<typename T> //be
cartesian_tree<T>* cartesian_tree<T>::sentinel = new cartesian_tree<T>();
template<typename T>
void split(cartesian_tree<T>* root, T key, cartesian_tree<T>*& l,
    cartesian_tree<T>*& r) {
    if (root == cartesian_tree<T>::sentinel) {
        l = r = cartesian_tree<T>::sentinel;
        return;
    }
    root->push_down();
    if (root->key <= key) {
        split(root->child[R], key, root->child[R], r);
        l = root;
    }
    else {
        split(root->child[L], key, l, root->child[L]);
        r = root;
    }
    root->push_up();
}

template<typename T>
cartesian_tree<T>* merge(cartesian_tree<T>* l, cartesian_tree<T>* r) {
    l->push_down();
    r->push_down();
    if (l == cartesian_tree<T>::sentinel || r == cartesian_tree<T>::sentinel)
        return (l == cartesian_tree<T>::sentinel ? r : l);
    if (l->priority > r->priority) {
        l->child[R] = merge(l->child[R], r);
        l->push_up();
        return l;
    }
    r->child[L] = merge(l, r->child[L]);
    r->push_up();
    return r;
}

template<typename T, template<typename > class cartesian_tree>
class treap {
    typedef cartesian_tree<T> node;
    typedef node* nodeptr;
#define emptyNode node::sentinel
    nodeptr root;

```

```

void insert(nodeptr& root, nodeptr it) {
    if (root == emptyNode) {
        root = it;
    }
    else if (it->priority > root->priority) {
        split(root, it->key, it->child[L], it->child[R]);
        root = it;
    }
    else
        insert(root->child[root->key < it->key], it);
    root->push_up();
}

bool increment(nodeptr root, const T& key) {
    if (root == emptyNode)
        return 0;
    if (root->key == key) {
        root->frequency++;
        root->push_up();
        return root;
    }
    bool rt = increment(root->child[root->key < key], key);
    root->push_up();
    return rt;
}

nodeptr find(nodeptr root, const T& key) {
    if (root == emptyNode || root->key == key)
        return root;
    return find(root->child[root->key < key], key);
}

void erase(nodeptr& root, const T& key) {
    if (root == emptyNode)
        return;
    if (root->key == key) {
        if (--(root->frequency) == 0)
            root = merge(root->child[L], root->child[R]);
    }
    else
        erase(root->child[root->key < key], key);
    root->push_up();
}

T kth(nodeptr root, int k) {
    if (root->child[L]->size >= k)
        return kth(root->child[L], k);
    k -= root->child[L]->size;
    if (k <= root->frequency)
        return root->key;
    return kth(root->child[R], k - root->frequency);
}

```

```

int order_of_key(nodeptr root, const T& key) {
    if (root == emptyNode)
        return 0;
    if (key < root->key)
        return order_of_key(root->child[L], key);
    if (key == root->key)
        return root->child[L]->size;
    return root->child[L]->size + root->frequency
        + order_of_key(root->child[R], key);
}

public:
    treap() : root(emptyNode) {}
    void insert(const T& x) {
        if (increment(root, x)) //change it to find(x) to make it as a set
            return;
        insert(root, new node(x, rand()));
    }
    void erase(const T& x) {
        erase(root, x);
    }
    bool find(const T& x) {
        return (find(root, x) != emptyNode);
    }
    int get_kth_number(int k) {
        assert(1 <= k && k <= size());
        return kth(root, k);
    }
    int order_of_key(const T& x) {
        return order_of_key(root, x);
    }
    int size() {
        return root->size;
    }
};

```

## Big Integer

```

class BigInt {
private:
#define CUR (*this)
    const int BASE = 1000000000; //1e9
    vector<int> v;
public:
    BigInt(const long long &val = 0) {
        CUR = val;
    }
    BigInt(const string &val) {
        CUR = val;
    }
    int size() const {
        return v.size();
    }
    bool zero() const {
        return v.empty();
    }
}

```

```

BigInt& operator =(long long val) {
    v.clear();
    while (val) {
        v.push_back(val % BASE);
        val /= BASE;
    }
    return CUR;
}
BigInt& operator =(const BigInt &a) {
    v = a.v;
    return CUR;
}
BigInt& operator=(const string &s) {
    CUR = 0;
    for (const char &ch : s)
        CUR = CUR * 10 + (ch - '0');
    return CUR;
}
/*****compare*****/
bool operator <(const BigInt &a) const {
    if (a.size() != size())
        return size() < a.size();
    for (int i = size() - 1; i >= 0; i--) {
        if (v[i] != a.v[i])
            return v[i] < a.v[i];
    }
    return false;
}
/*****add*****/
BigInt operator +(const BigInt &a) const {
    BigInt res = CUR;
    int idx = 0, carry = 0;
    while (idx < a.size() || carry) {
        if (idx < a.size())
            carry += a.v[idx];
        if (idx == res.size())
            res.v.push_back(0);
        res.v[idx] += carry;
        carry = res.v[idx] / BASE;
        res.v[idx] %= BASE;
        idx++;
    }
    return res;
}
BigInt& operator +=(const BigInt &a) {
    CUR = CUR + a;
    return CUR;
}

```

```

/*****multiply*****/
BigInt operator *(const BigInt &a) const {
    BigInt res;
    if (CUR.zero() || a.zero())
        return res;
    res.v.resize(size() + a.size());
    for (int i = 0; i < size(); i++) {
        if (v[i] == 0)
            continue;
        long long carry = 0;
        for (int j = 0; carry || j < a.size(); j++) {
            carry += 1LL * v[i] * (j < a.size() ? a.v[j] : 0);
            while (i + j >= res.size())
                res.v.push_back(0);
            carry += res.v[i + j];
            res.v[i + j] = carry % BASE;
            carry /= BASE;
        }
    }
    while (!res.v.empty() && res.v.back() == 0)
        res.v.pop_back();
    return res;
}

BigInt& operator *=(const BigInt &a) {
    CUR = CUR * a;
    return CUR;
}

/*****Division*****/
BigInt& operator /=(const int &a) {
    ll carry = 0;
    for (int i = (int) v.size() - 1; i >= 0; i--) {
        ll cur = v[i] + carry * BASE;
        v[i] = cur / a;
        carry = cur % a;
    }
    while (!v.empty() && v.back() == 0)
        v.pop_back();
    return CUR;
}

/*****print*****/
friend ostream& operator<<(ostream &out, const BigInt &a) {
    out << (a.zero() ? 0 : a.v.back());
    for (int i = (int) a.v.size() - 2; i >= 0; i--)
        out << setfill('0') << setw(9) << a.v[i];
    return out;
}

#undef CUR
};

```

## Mod int

```
struct modint {
#define CUR (*this)
    static const int MOD = 1e9 + 7;
    int val;
    modint(const long long& a = 0) {
        val = a % MOD;
        if (val < 0) val += MOD;
    }
    modint& operator+=(const modint& a) {
        if ((val += a.val) >= MOD) val -= MOD;
        return CUR;
    }
    modint operator+(const modint& a) const {
        modint c = CUR;
        c += a;
        return c;
    }
    modint& operator--=(const modint& a) {
        if ((val -= a.val) < 0) val += MOD;
        return CUR;
    }
    modint operator-(const modint& a) const {
        modint c = CUR;
        c -= a;
        return c;
    }
    modint operator*(const modint& a) const {
        return modint((1LL * this->val * a.val) % MOD);
    }
    modint& operator*=(const modint& a) {
        CUR = CUR * a;
        return CUR;
    }
    modint operator/(const modint& a) {
        return CUR * power(a, MOD - 2);
    }
    modint& operator/=(const modint& a) {
        CUR = CUR / a;
        return CUR;
    }
    static modint power(modint x, long long y) {
        modint res = 1;
        while (y > 0) {
            if (y & 1) res *= x;
            x *= x; y >>= 1;
        }
        return res;
    }
    friend ostream& operator<<(ostream& out, const modint& a) {
        out << a.val;
        return out;
    }
}
#undef CUR
};
```

# Graph

## Graph Data structures

### DSU

```
struct DSU {
    vector<int> rank, parent, size;
    vector<vector<int>> component;
    int forsets;
    DSU(int n) {
        size = rank = parent = vector<int>(n + 1, 1);
        component = vector<vector<int>>(n + 1);
        forsets = n;
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
            component[i].push_back(i);
        }
    }
    int find_set(int v) {
        if (v == parent[v])
            return v;
        return parent[v] = find_set(parent[v]);
    }
    void link(int par, int node) {
        parent[node] = par;
        size[par] += size[node];
        for (const int& it : component[node])
            component[par].push_back(it);
        component[node].clear();
        if (rank[par] == rank[node])
            rank[par]++;
        forsets--;
    }
    bool union_sets(int v, int u) {
        v = find_set(v), u = find_set(u);
        if (v != u) {
            if (rank[v] < rank[u])
                swap(v, u);
            link(v, u);
        }
        return v != u;
    }
    bool same_set(int v, int u) {
        return find_set(v) == find_set(u);
    }
    int size_set(int v) {
        return size[find_set(v)];
    }
};
```

### DSU bipartiteness

```
struct DSU_bipartiteness {
    vector<int> bipartite, rank;
    vector<pair<int, int>> parent;
```



```

DSU_bipartiteness(int n) {
    bipartite = rank = vector<int>(n + 1, 1);
    parent = vector<pair<int, int>>(n + 1);
    for (int i = 0; i <= n; i++)
        parent[i] = { i, 0 };
}
pair<int, int> find_set(int x) {
    if (x == parent[x].first)
        return parent[x];
    int parity = parent[x].second;
    parent[x] = find_set(parent[x].first);
    parent[x].second ^= parity;
    return parent[x];
}
void union_sets(int x, int y) {
    pair<int, int> p = find_set(x);
    x = p.first;
    int paX = p.second;
    p = find_set(y);
    y = p.first;
    int paY = p.second;
    if (x == y) {
        if (paX == paY)
            bipartite[x] = false;
    }
    else {
        if (rank[x] < rank[y])
            swap(x, y);
        parent[y] = { x, paX ^ paY ^ 1 };
        bipartite[x] &= bipartite[y];
        if (rank[x] == rank[y])
            rank[x]++;
    }
}
bool is_bipartite(int x) {
    return bipartite[find_set(x).first];
}
};

```

## LCA

```

class LCA {
    int n, logN, root = 1;
    vector<int> depth;
    vector<vector<int>> adj, lca;
    void dfs(int node, int parent) {
        lca[node][0] = parent;
        depth[node] = (~parent ? depth[parent] + 1 : 0);
        for (int k = 1; k <= logN; k++) {
            int up_parent = lca[node][k - 1];
            if (~up_parent)
                lca[node][k] = lca[up_parent][k - 1];
        }
    }
}

```

```

        for (int child : adj[node])
            if (child != parent)
                dfs(child, node);
    }
public:
    LCA(const vector<vector<int>> &adj, int root = 1) : root(root), adj(_adj) {
        adj = _adj;
        n = adj.size() - 1;
        logN = log2(n);
        lca = vector<vector<int>>(n + 1, vector<int>(logN + 1, -1));
        depth = vector<int>(n + 1);
        dfs(root, -1);
    }
    int get_LCA(int x, int y) {
        if (depth[x] < depth[y])
            swap(x, y);
        for (int k = logN; k >= 0; k--)
            if (depth[x] - (1 << k) >= depth[y])
                x = lca[x][k];
        if (x == y)
            return x;
        for (int k = logN; k >= 0; k--) {
            if (lca[x][k] != lca[y][k]) {
                x = lca[x][k], y = lca[y][k];
            }
        }
        return lca[x][0];
    }
    int get_distance(int u, int v) {
        return depth[u] + depth[v] - 2 * depth[get_LCA(u, v)];
    }
    int shifting(int node, int dist) {
        for (int i = logN; i >= 0 && ~node; i--)
            if (dist & (1 << i))
                node = lca[node][i];
        return node;
    }
    edge get_path(int u, int LCA) {
        edge rt;
        for (int k = logN; k >= 0; k--)
            if (depth[u] - (1 << k) >= depth[LCA]) {
                rt = merge(rt, lca[u][k]);
                u = lca[u][k].to;
            }
        return rt;
    }
};

```

### Heavy light decomposition

//1-based,if value in node,just update it after build chains

//don't forget to call build\_chains after add edges.

```

class heavy_light_decomposition {
    int n, is_value_in_edge;
    vector<int> parent, depth, heavy, root, pos_in_array, pos_to_node, size;

```

```

const static int merge(int a, int b); //implement it
struct array_ds { //implement it
    int n;
    array_ds(int n) : n(n) {}
} seg;

struct TREE {
    int cnt_edges = 1;
    vector<vector<int>> adj;
    //need for value in edges
    vector<vector<int>> edge_idx;
    //edge_to need for undirected tree //end of edge in directed tree
    vector<int> edge_to, edge_cost;
    TREE(int n) : adj(n + 1), edge_idx(n + 1), edge_to(n + 1), edge_cost(n + 1) {
    }
    void add_edge(int u, int v, int c) {
        adj[u].push_back(v);
        adj[v].push_back(u);
        edge_idx[u].push_back(cnt_edges);
        edge_idx[v].push_back(cnt_edges);
        edge_cost[cnt_edges] = c;
        cnt_edges++;
    }
} tree;
int dfs_hld(int node) {
    int size = 1, max_sub_tree = 0;
    for (int i = 0; i < (int) tree.adj[node].size(); i++) {
        int ch = tree.adj[node][i], edge_idx = tree.edge_idx[node][i];
        if (ch != parent[node]) {
            tree.edge_to[edge_idx] = ch;
            parent[ch] = node;
            depth[ch] = depth[node] + 1;
            int child_size = dfs_hld(ch);
            if (child_size > max_sub_tree)
                heavy[node] = ch, max_sub_tree = child_size;
            size += child_size;
        }
    }
    return size;
}

vector<tuple<int, int, bool>> get_path(int u, int v) { //l,r,must_reverse?
    vector<pair<int, int>> tmp[2];
    bool idx = 1;
    while (root[u] != root[v]) {
        if (depth[root[u]] > depth[root[v]]) {
            swap(u, v);
            idx = !idx;
        }
        //if value in edges ,you need value of root[v] also (connector edge)
        tmp[idx].push_back( { pos_in_array[root[v]], pos_in_array[v] });
        v = parent[root[v]];
    }
}

```

```

    if (depth[u] > depth[v]) {
        swap(u, v);
        idx = !idx;
    }
    if (!is_value_in_edge || u != v)
        tmp[idx].push_back( { pos_in_array[u] + is_value_in_edge, pos_in_array[v] });
    reverse(all(tmp[1]));
    vector<tuple<int, int, bool>> rt;

    for (int i = 0; i < 2; i++)
        for (auto &it : tmp[i])
            rt.emplace_back(it.first, it.second, i == 0);
    return rt; //u is LCA
}

public:
heavy_light_decomposition(int n, bool is_value_in_edge) :
    n(n), is_value_in_edge(is_value_in_edge), seg(n + 1), tree(n + 1) {
    heavy = vector<int>(n + 1, -1);
    parent = depth = root = pos_in_array = pos_to_node = size = vector<int>(n + 1);
}

void add_edge(int u, int v, int c = 0) {
    tree.add_edge(u, v, c);
}

void build_chains(int src = 1) {
    parent[src] = -1;
    dfs_hld(src);
    for (int chain_root = 1, pos = 1; chain_root <= n; chain_root++) {
        if (parent[chain_root] == -1 || heavy[parent[chain_root]] != chain_root)
            for (int j = chain_root; j != -1; j = heavy[j]) {
                root[j] = chain_root;
                pos_in_array[j] = pos++;
                pos_to_node[pos_in_array[j]] = j;
            }
    }
    if (is_value_in_edge)
        for (int i = 1; i < n; i++)
            update_edge(i, tree.edge_cost[i]);
}

void update_node(int node, int value) {
    seg.update(pos_in_array[node], value);
}

void update_edge(int edge_idx, int value) {
    update_node(tree.edge_to[edge_idx], value);
}

void update_path(int u, int v, ll c) {
    vector<tuple<int, int, bool>> intervals = get_path(u, v);
    for (auto &it : intervals)
        seg.update(get<0>(it), get<1>(it), c);
}

node query_in_path(int u, int v) {
    vector<tuple<int, int, bool>> intervals = get_path(u, v);
    //initial value, check if handling u == v
    node query_res = 0;

```

```

    for (auto &it : intervals) {
        int l, r;
        bool rev;
        tie(l, r, rev) = it;
        node cur = seg.query(l, r);
        if (rev) cur.reverse();
        query_res = node(query_res, cur);
    }
    return query_res;
}
};

```

## Centroid decomposition

```

class centroid_decomposition {
    vector<bool> centroidMarked;
    vector<int> size;
    void dfsSize(int node, int par) {
        size[node] = 1;
        for (int ch : adj[node])
            if (ch != par && !centroidMarked[ch]) {
                dfsSize(ch, node);
                size[node] += size[ch];
            }
    }
    int getCenter(int node, int par, int size_of_tree) {
        for (int ch : adj[node]) {
            if (ch == par || centroidMarked[ch])
                continue;
            if (size[ch] * 2 > size_of_tree)
                return getCenter(ch, node, size_of_tree);
        }
        return node;
    }
    int getCentroid(int src) {
        dfsSize(src, -1);
        int centroid = getCenter(src, -1, size[src]);
        centroidMarked[centroid] = true;
        return centroid;
    }
    int decomposeTree(int root) {
        root = getCentroid(root);
        solve(root);
        for (int ch : adj[root]) {
            if (centroidMarked[ch])
                continue;
            int centroid_of_subtree = decomposeTree(ch);
            //note: root and centroid_of_subtree probably not have a direct edge in adj
            centroidTree[root].push_back(centroid_of_subtree);
            centroidParent[centroid_of_subtree] = root;
        }
        return root;
    }
}

```

```

void calc(int node, int par) {
    //TO-DO
    for (int ch : adj[node]) if (ch != par && !centroidMarked[ch])
        calc(ch, node);
}
void add(int node, int par) {
    //TO-DO
    for (int ch : adj[node]) if (ch != par && !centroidMarked[ch])
        add(ch, node);
}
void remove(int node, int par) {
    //TO-DO
    for (int ch : adj[node]) if (ch != par && !centroidMarked[ch])
        remove(ch, node);
}
void solve(int root) {
    //add root
    for (int ch : adj[root])
        if (!centroidMarked[ch]) {
            calc(ch, root);
            add(ch, root);
        }
    //TO-DO
    //remove root
    for (int ch : adj[root])
        if (!centroidMarked[ch])
            remove(ch, root);
}
public:
    int n, root;
    vector<vector<int>> adj, centroidTree;
    vector<int> centroidParent;
    centroid_decomposition(vector<vector<int>> &adj) : adj(adj) {
        n = (int) adj.size() - 1;
        size = vector<int>(n + 1);
        centroidTree = vector<vector<int>>(n + 1);
        centroidParent = vector<int>(n + 1, -1);
        centroidMarked = vector<bool>(n + 1);
        root = decomposeTree(1);
    }
};

```

## Shortest path algorithms

### Dijkstra

```

struct edge {
    int from, to, weight;
    edge() { from = to = weight = 0;}
    edge(int from, int to, int weight) :
        from(from), to(to), weight(weight) {
    }
    bool operator <(const edge& other) const {
        return weight > other.weight;
    }
};

```

```

vector<vector<edge>> adj;

//O(E*log(v))
void dijkstra(int src, int dest = -1) {
    priority_queue<edge> q;
    vector<int> dis(adj.size(), INT_MAX), prev(adj.size(), -1);
    q.push(edge(-1, src, 0));
    dis[src] = 0;
    while (!q.empty()) {
        edge e = q.top();
        q.pop();
        if (e.weight > dis[e.to])
            continue;
        prev[e.to] = e.from;
        if (e.to == dest)
            break;
        for (edge ne : adj[e.to])
            if (dis[ne.to] > dis[e.to] + ne.weight) {
                ne.weight = dis[ne.to] = dis[e.to] + ne.weight;
                q.push(ne);
            }
    }
    vector<int> path;
    while (dest != -1) {
        path.push_back(dest);
        dest = prev[dest];
    }
    reverse(path.begin(), path.end());
}

```

### Bellmanford

```

vector<edge> edgeList;
//O(V*E)
void bellmanford(int n, int src, int dest = -1) {
    vector<int> dis(n + 1, oo), prev(n + 1, -1);
    dis[src] = 0;
    bool negativeCycle = false;
    int last = -1, tmp = n;
    while (tmp--) {
        last = -1;
        for (edge e : edgeList)
            if (dis[e.to] > dis[e.from] + e.weight) {
                dis[e.to] = dis[e.from] + e.weight;
                prev[e.to] = e.from;
                last = e.to;
            }
        if (last == -1)
            break;
        if (tmp == 0)
            negativeCycle = true;
    }
    if (last != -1) {
        for (int i = 0; i < n; i++)
            last = prev[last];
        vector<int> cycle;
    }
}

```

```

        for (int cur = last; cur != last || cycle.size() > 1; cur = prev[cur])
            cycle.push_back(cur);
        reverse(cycle.begin(), cycle.end());
    }
    vector<int> path;
    while (dest != -1) {
        path.push_back(dest);
        dest = prev[dest];
    }
    reverse(path.begin(), path.end());
}

```

### *Difference constraints*

```

void difference_constraints() {
    int m;
    cin >> m;
    int cnt = 1;
    while (m--) {
        string x1, x2;
        int w; // x1 - x2 <= w
        cin >> x1 >> x2 >> w;
        map<string, int> id;
        if (id.find(x1) == id.end())
            id[x1] = cnt++;
        if (id.find(x2) == id.end())
            id[x2] = cnt++;
        edgeList.emplace_back(id[x2], id[x1], w);
    }
    for (int i = 1; i < cnt; i++)
        edgeList.emplace_back(cnt, i, 0);
    bellmanford(cnt, cnt);
}

```

### **Floyed**

```

vector<vector<int>>> adj, par;
// adj[i][j] = oo , adj[i][i] = 0 , par[i][j] = i
void init(int n) {
    par = adj = vector<vector<int>>>(n + 1, vector<int>(n + 1, oo));
    for (int i = 1; i <= n; i++)
        adj[i][i] = 0;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++)
            par[i][j] = i;
}
void floyd() {
    for (int k = 1; k < adj.size(); k++)
        for (int i = 1; i < adj.size(); i++)
            for (int j = 1; j < adj.size(); j++)
                if (adj[i][j] > adj[i][k] + adj[k][j]) {
                    adj[i][j] = adj[i][k] + adj[k][j];
                    par[i][j] = par[k][j];
                }
}

```



```

void buildPath(int src, int dest) {
    vector<int> path;
    while (src != dest) {
        path.push_back(dest);
        dest = par[src][dest];
    }
    path.push_back(src);
    reverse(path.begin(), path.end());
}

```

## SPFA

```

vector<vector<edge>> adj;
enum visit {finished, in_queue, not_visited };

void spfa(int src) {
    int n = adj.size();
    vector<int> dis(n, INF), prev(n, -1), state(n, not_visited);
    dis[src] = 0;
    deque<int> q;
    q.push_back(src);
    while (!q.empty()) {
        int u = q.front();
        q.pop_front();
        state[u] = finished;
        for (auto &e : adj[u]) {
            if (dis[e.to] > dis[e.from] + e.cost) {
                dis[e.to] = dis[e.from] + e.cost;
                prev[e.to] = e.from;
                if (state[e.to] == not_visited) {
                    q.push_back(e.to);
                } else if (state[e.to] == finished) {
                    q.push_front(e.to);
                }
                state[e.to] = in_queue;
            }
        }
    }
}

```

## MST

### Kruskal

```

struct edge {
    int from, to;
    ll weight;
    edge() {
        from = to = weight = 0;
    }
    edge(int from, int to, ll weight) :
        from(from), to(to), weight(weight) {
    }
    bool operator <(const edge& other) const {
        return weight < other.weight;
    }
};

```

```

vector<edge> edgeList;
pair<int, vector<edge>> MST_Kruskal(int n) { //O(edges*log(edges))
    DSU uf(n);
    vector < edge > edges;
    int mstCost = 0;
    sort(edgeList.begin(), edgeList.end());
    for (auto e : edgeList)
        if (uf.union_sets(e.from, e.to)) {
            mstCost += e.weight;
            edges.push_back(e);
        }
    if (edges.size() != n - 1)
        return { INT_MAX, vector<edge>() };
    return { mstCost, edges };
}

int miniMax(int src, int dest, int n) {
    int max = INT_MIN;
    DSU uf(n);
    sort(edgeList.begin(), edgeList.end());
    for (auto e : edgeList) {
        if (uf.same_set(src, dest))
            return max;
        uf.union_sets(e.from, e.to);
        max = e.weight;
    }
    return max;
}

//O(edges*log(edges) + nodes*nodes)
pair<int, vector<edge>> SMST_Kruskal(int n) {
    DSU uf(n);
    sort(edgeList.begin(), edgeList.end());
    vector<edge> take, leave;
    int mstCost = 0;
    for (auto e : edgeList)
        if (uf.union_sets(e.from, e.to)) {
            mstCost += e.weight;
            take.push_back(e);
        }
        else leave.push_back(e);
    pair<int, vector<edge>> ret = { INT_MAX, vector<edge>() };
    for (int i = 0; i < take.size(); i++) {
        uf = DSU(n);
        vector <edge> edges;
        mstCost = 0;
        for (int j = 0; j < take.size(); j++) {
            if (i == j)
                continue;
            uf.union_sets(take[j].from, take[j].to);
            mstCost += take[j].weight;
            edges.push_back(take[j]);
        }
    }
}

```

```

        for (edge e : leave) {
            if (uf.union_sets(e.from, e.to)) {
                mstCost += e.weight;
                edges.push_back(e);
                break;
            }
        }
        if (edges.size() == n - 1 && ret.first < mstCost)
            ret = { mstCost, edges };
    }
    return ret;
}

```

### Prim

```

struct edge {
    int from, to, weight;
    edge() {
        from = to = weight = 0;
    }
    edge(int from, int to, int weight) : from(from), to(to), weight(weight) {}
    bool operator <(const edge& other) const {
        return weight > other.weight;
    }
};

vector<vector<edge>> adj;
vector<edge> prim(int node) {
    vector<bool> vis(adj.size());
    priority_queue<edge> q;
    vector<edge> edges;
    q.push(edge(-1, node, 0));
    while (!q.empty()) {
        edge e = q.top();
        q.pop();
        if (vis[e.to])
            continue;
        vis[e.to] = true;
        if (e.from != -1)
            edges.push_back(e);
        for (edge ch : adj[e.to])
            if (!vis[ch.to])
                q.push(ch);
    }
    return edges; // check it connected or not
}

```

### SMST $O(n * \log(n))$

```

struct edge {
    int from, to;
    ll weight;
    edge() {
        from = to = weight = 0;
    }
    edge(int from, int to, ll weight) :
        from(from), to(to), weight(weight) {
    }
}

```

```

    bool operator <(const edge& other) const {
        return weight < other.weight;
    }
};

int MST_Kruskal(int n, vector<edge> edgeList, vector<edge>& take,
vector<edge>& leave) {
    DSU uf(n);
    vector<edge> edges;
    sort(edgeList.begin(), edgeList.end());
    int mst_cost = 0;
    for (auto e : edgeList)
        if (uf.union_sets(e.from, e.to)) {
            take.push_back(e);
            mst_cost += e.weight;
        }
        else leave.push_back(e);
    return mst_cost;
}

struct LCA {
#define INIT { -1, -2 }
    struct data {
        int lca = -1;
        pair<int, int> max_edges = INIT; //first max,second max (distinct)
    };
    pair<int, int> merge(pair<int, int> a, pair<int, int> b) {
        if (a.first < b.first)
            swap(a, b);
        if (b.first == a.first)
            a.second = max(a.second, b.second);
        else if (b.first > a.second)
            a.second = b.first;
        return a;
    }
    int logN;
    vector<vector<data>> lca;
    vector<vector<edge>> adj;
    vector<int> depth;
    void dfs(int node, int par) {
        for (edge e : adj[node])
            if (e.to != par) {
                depth[e.to] = depth[node] + 1;
                lca[e.to][0].max_edges.first = e.weight;
                lca[e.to][0].lca = node;
                dfs(e.to, node);
            }
    }
    LCA(int n, vector<edge>& edges) :
        adj(n + 1) {
            for (auto& e : edges) {
                adj[e.from].push_back(e);
                adj[e.to].push_back(edge(e.to, e.from, e.weight));
            }
            logN = log2(n);
            depth = vector<int>(n + 1);

```

```

lca = vector<vector<data>>(n + 1, vector<data>(logN + 1));
dfs(1, -1);
for (int k = 1; k <= logN; k++)
    for (int node = 1; node <= n; node++) {
        int par = lca[node][k - 1].lca;
        if (~par) {
            lca[node][k].lca = lca[par][k - 1].lca;
            lca[node][k].max_edges = merge(lca[node][k - 1].max_edges,
            lca[par][k - 1].max_edges);
        }
    }
}

pair<int, int> max_two_edges(int u, int v) {
    pair<int, int> ans = INIT;
    if (depth[u] < depth[v]) swap(u, v);
    for (int i = logN; i >= 0; i--)
        if (depth[u] - (1 << i) >= depth[v]) {
            ans = merge(ans, lca[u][i].max_edges);
            u = lca[u][i].lca;
        }
    if (u == v) return ans;
    for (int i = logN; i >= 0; i--)
        if (lca[u][i].lca != lca[v][i].lca) {
            ans = merge(ans, lca[u][i].max_edges);
            ans = merge(ans, lca[v][i].max_edges);
            u = lca[u][i].lca;
            v = lca[v][i].lca;
        }
    ans = merge(ans, lca[u][0].max_edges);
    ans = merge(ans, lca[v][0].max_edges);
    return ans;
}

};
int main() {
    run();
    int t;
    cin >> t;
    for (int I = 1; I <= t; I++) {
        int n, e;
        cin >> n >> e;
        vector<edge> edgeList(e);
        for (auto& it : edgeList)
            cin >> it.from >> it.to >> it.weight;
        vector<edge> take, leave;
        int mst_cost = MST_Kruskal(n, edgeList, take, leave);
        if (take.size() != n - 1) {
            cout << "No way\n";
            continue;
        }
        LCA tree(n, take);
        ll rt = INF;
        for (edge e : leave) {
            pair<int, int> p = tree.max_two_edges(e.from, e.to);
            rt = min(rt, mst_cost - p.first + e.weight);
        }
    }
}

```

```

        if (rt == INF)
            cout << "No second way\n";
        else
            cout << rt << endl;
    }
}

```

## Tarjan

### SCC

```

vector<vector<int>> adj, scc;
vector<set<int>> dag;
vector<int> dfs_num, dfs_low, compId;
vector<bool> inStack;
stack<int> stk;
int timer;
void dfs(int node) {
    dfs_num[node] = dfs_low[node] = ++timer;
    stk.push(node);
    inStack[node] = 1;
    for (int child : adj[node])
        if (!dfs_num[child]) {
            dfs(child);
            dfs_low[node] = min(dfs_low[node], dfs_low[child]);
        }
        else if (inStack[child])
            dfs_low[node] = min(dfs_low[node], dfs_num[child]);
    //can be dfs_low[node] = min(dfs_low[node], dfs_low[child]);
    if (dfs_low[node] == dfs_num[node]) {
        scc.push_back(vector<int>());
        int v = -1;
        while (v != node) {
            v = stk.top();
            stk.pop();
            inStack[v] = 0;
            scc.back().push_back(v);
            compId[v] = scc.size() - 1;
        }
    }
}
void SCC() {
    timer = 0;
    dfs_num = dfs_low = compId = vector<int>(adj.size());
    inStack = vector<bool>(adj.size());
    scc = vector<vector<int>>();
    for (int i = 1; i < adj.size(); i++)
        if (!dfs_num[i]) dfs(i);
}
void DAG() {
    dag = vector<set<int>>(scc.size());
    for (int i = 1; i < adj.size(); i++)
        for (int j : adj[i])
            if (compId[i] != compId[j])
                dag[compId[i]].insert(compId[j]);
}

```

## Articulation points and bridges

```
vector<vector<int>> adj;
vector<int> dfs_num, dfs_low;
vector<bool> articulation_point;
vector<pair<int, int>> bridge;
stack<pair<int, int>> edges;
vector<vector<pair<int, int>>> BCC; //biconnected components
int timer, cntChild;

void dfs(int node, int par) {
    dfs_num[node] = dfs_low[node] = ++timer;
    for (int child : adj[node]) {
        if (par != child && dfs_num[child] < dfs_num[node])
            edges.push({ node, child });

        if (!dfs_num[child]) {
            if (par == -1)
                cntChild++;
            dfs(child, node);
            if (dfs_low[child] >= dfs_num[node]) {
                articulation_point[node] = 1;
                //get biconnected component
                BCC.push_back(vector<pair<int, int>>());
                pair<int, int> edge;
                do {
                    edge = edges.top();
                    BCC.back().push_back(edge);
                    edges.pop();
                } while (edge.first != node || edge.second != child);
            }
            //can be (dfs_low[child] == dfs_num[child])
            if (dfs_low[child] > dfs_num[node])
                bridge.push_back({ node, child });
            dfs_low[node] = min(dfs_low[node], dfs_low[child]);
        }
        else if (child != par)
            dfs_low[node] = min(dfs_low[node], dfs_num[child]);
    }
}

void articulation_points_and_bridges() {
    timer = 0;
    dfs_num = dfs_low = vector<int>(adj.size());
    articulation_point = vector<bool>(adj.size());
    bridge = vector<pair<int, int>>();
    for (int i = 1; i < adj.size(); i++)
        if (!dfs_num[i]) {
            cntChild = 0;
            dfs(i, -1);
            articulation_point[i] = cntChild > 1;
        }
}
```

## Edge classification

```
vector<vector<int>> adj;
vector<int> start, finish;
int timer;
void dfsEdgeClassification(int node) {
    start[node] = timer++;
    for (int child : adj[node]) {
        if (start[child] == -1)
            dfsEdgeClassification(child);
        else {
            if (finish[child] == -1)
                ; // Back Edge
            else if (start[node] < start[child])
                ; // Forward Edge
            else; // Cross Edge
        }
    }
    finish[node] = timer++;
}
```

## 2-SAT

```
int n;
int Not(int x) {
    return (x > n ? x - n : x + n);
}

void addEdge(int a, int b) {
    adj[Not(a)].push_back(b);
    adj[Not(b)].push_back(a);
}

void add_xor_edge(int a, int b) {
    addEdge(Not(a), Not(b));
    addEdge(a, b);
}

bool _2SAT(vector<int>& value) {
    SCC();
    for (int i = 1; i <= n; i++)
        if (compId[i] == compId[Not(i)])
            return false;
    vector<int> assign(scc.size(), -1);
    for (int i = 0; i < scc.size(); i++)
        if (assign[i] == -1) {
            assign[i] = true;
            assign[compId[Not(scc[i].back())]] = false;
        }
    for (int i = 1; i <= n; i++)
        value[i] = assign[compId[i]];
    return true;
}
```



## Flows

### Maximum bipartite matching

```
vector<vector<int>> adj;
vector<int> rowAssign, colAssign, vis; //make vis array instance of vector
int test_id;
bool canMatch(int i) {
    if (vis[i] == test_id) return false;
    vis[i] = test_id;
    for (int j : adj[i])
        if (colAssign[j] == -1) {
            colAssign[j] = i;
            rowAssign[i] = j;
            return true;
        }
    for (int j : adj[i])
        if (canMatch(colAssign[j])) {
            colAssign[j] = i;
            rowAssign[i] = j;
            return true;
        }
    return false;
}
// O(rows * edges) //number of operation could be strictly less than order (1e5*1e5->AC)
int maximum_bipartite_matching(int rows, int cols) {
    int maxFlow = 0;
    rowAssign = vector<int>(rows, -1);
    colAssign = vector<int>(cols, -1);
    vis = vector<int>(rows);
    for (int i = 0; i < rows; i++) {
        test_id++;
        if (canMatch(i)) maxFlow++;
    }
    vector<pair<int, int>> matches;
    for (int j = 0; j < cols; j++)
        if (~colAssign[j]) matches.push_back( { colAssign[j], j } );
    return maxFlow;
}
```

### Hopcroft Karp for bipartite matching

```
//O(sqrt(V) * E)
struct Hopcroft_Karp { //1-based
#define NIL 0
#define INF INT_MAX
    int n, m;
    vector<vector<int>> adj;
    vector<int> rowAssign, colAssign, dist;
    bool bfs() {
        queue<int> q;
        dist = vector<int>(adj.size(), INF);
        for (int i = 1; i <= n; i++)
            if (rowAssign[i] == NIL) {
                dist[i] = 0;
                q.push(i);
            }
    }
}
```

```

        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            if (dist[cur] >= dist[NIL]) break;
            for (auto& nxt : adj[cur]) {
                if (dist[colAssign[nxt]] == INF) {
                    dist[colAssign[nxt]] = dist[cur] + 1;
                    q.push(colAssign[nxt]);
                }
            }
        }
        return dist[NIL] != INF;
    }
    bool dfs(int i) {
        if (i == NIL)
            return true;
        for (int j : adj[i]) {
            if (dist[colAssign[j]] == dist[i] + 1 && dfs(colAssign[j])) {
                colAssign[j] = i;
                rowAssign[i] = j;
                return true;
            }
        }
        dist[i] = INF;
        return false;
    }
    Hopcroft_Karp(int n, int m)
        : n(n), m(m), adj(n + 1), rowAssign(n + 1), colAssign(m + 1) {}
    void addEdge(int u, int v) {
        adj[u].push_back(v);
    }
    int maximum_bipartite_matching() {
        int rt = 0;
        while (bfs()) {
            for (int i = 1; i <= n; i++)
                if (rowAssign[i] == NIL && dfs(i))
                    rt++;
        }
        return rt;
    }
};

```

### Edmonds Karp

```

//O( V * E * E)
#define INF 0x3f3f3f3f3f3f3f3fLL
int n;
int capacity[101][101];
int getPath(int src, int dest, vector<int> &parent) {
    parent = vector<int>(n + 1, -1);
    queue<pair<int, int>> q;
    q.push( { src, INF } );
    while (q.size()) {
        int cur = q.front().first, flow = q.front().second;
        q.pop();
    }
}

```

```

        if (cur == dest) return flow;
        for (int i = 1; i <= n; i++)
            if (parent[i] == -1 && capacity[cur][i]) {
                parent[i] = cur;
                q.push( { i, min(flow, capacity[cur][i]) } );
                if (i == dest) return q.back().second;
            }
    }
    return 0;
}

int Edmonds_Karp(int source, int sink) {
    int max_flow = 0;
    int new_flow = 0;
    vector<int> parent(n + 1, -1);
    while (new_flow = getPath(source, sink, parent)) {
        max_flow += new_flow;
        int cur = sink;
        while (cur != source) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return max_flow;
}

```

## Dinic

//O(V\*V\*E) more faster

```

struct Dinic { //O-based
    struct flowEdge {
        int from, to;
        ll cap, flow = 0;
        flowEdge(int from, int to, ll cap) :
            from(from), to(to), cap(cap) {
        }
    };
    vector<flowEdge> edges;
    int n, m = 0, source, sink;
    vector<vector<int>> adj;
    vector<int> level, ptr;
    Dinic(int n, int source, int sink) :
        n(n), source(source), sink(sink), adj(n), level(n), ptr(n) {
    }
    void addEdge(int u, int v, ll cap) {
        edges.emplace_back(u, v, cap);
        edges.emplace_back(v, u, 0);
        adj[u].push_back(m);
        adj[v].push_back(m + 1);
        m += 2;
    }
}

```

```

bool bfs() {
    queue<int> q;
    level = vector<int>(n, -1);
    level[source] = 0;
    q.push(source);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (auto &id : adj[cur]) {
            if (edges[id].cap - edges[id].flow <= 0)
                continue;
            int nxt = edges[id].to;
            if (level[nxt] != -1)
                continue;
            level[nxt] = level[cur] + 1;
            q.push(nxt);
        }
    }
    return level[sink] != -1;
}

ll dfs(int node, ll cur_flow) {
    if (cur_flow == 0 || node == sink)
        return cur_flow;
    for (int &cid = ptr[node]; cid < adj[node].size(); cid++) {
        int id = adj[node][cid];
        int nxt = edges[id].to;
        if (level[node] + 1 != level[nxt] || edges[id].cap - edges[id].flow <= 0)
            continue;
        ll tmp = dfs(nxt, min(cur_flow, edges[id].cap - edges[id].flow));
        if (tmp == 0)
            continue;
        edges[id].flow += tmp;
        edges[id ^ 1].flow -= tmp;
        return tmp;
    }
    return 0;
}

ll flow() {
    ll max_flow = 0;
    while (bfs()) {
        fill(ptr.begin(), ptr.end(), 0);
        while (ll pushed = dfs(source, INF))
            max_flow += pushed;
    }
    return max_flow;
}
};

```

### Min cost Max flow

```

struct MCMF { //0-based
    struct edge {
        int from, to, cost, cap, flow, backEdge;
        edge() {
            from = to = cost = cap = flow = backEdge = 0;
        }
    };
};

```

```

    edge(int from, int to, int cost, int cap, int flow, int backEdge) :
        from(from), to(to), cost(cost), cap(cap), flow(flow), backEdge(
            backEdge) {
    }
    bool operator <(const edge &other) const {
        return cost < other.cost;
    }
};
int n, src, dest;
vector<vector<edge>> adj;
const int OO = 1e9;
MCMF(int n, int src, int dest) : n(n), src(src), dest(dest), adj(n) {}

void addEdge(int u, int v, int cost, int cap) {
    edge e1 = edge(u, v, cost, cap, 0, adj[v].size());
    edge e2 = edge(v, u, -cost, 0, 0, adj[u].size());
    adj[u].push_back(e1);
    adj[v].push_back(e2);
}
pair<int, int> minCostMaxFlow() {
    int maxFlow = 0, cost = 0;
    while (true) {
        vector<pair<int, int>> path = spfa();
        if (path.empty())
            break;
        int new_flow = 0;
        for (auto &it : path) {
            edge &e = adj[it.first][it.second];
            new_flow = min(new_flow, e.cap - e.flow);
        }
        for (auto &it : path) {
            edge &e = adj[it.first][it.second];
            e.flow += new_flow;
            cost += new_flow * e.cost;
            adj[e.to][e.backEdge].flow -= new_flow;
        }
        maxFlow += new_flow;
    }
    return {maxFlow, cost};
}
enum visit { finished, in_queue, not_visited };
vector<pair<int, int>> spfa() {
    vector<int> dis(n, OO), prev(n, -1), from_edge(n), state(n,
        not_visited);
    deque<int> q;
    dis[src] = 0;
    q.push_back(src);
    while (!q.empty()) {
        int u = q.front();
        q.pop_front();
        state[u] = finished;
        for (int i = 0; i < adj[u].size(); i++) {
            edge e = adj[u][i];
            if (e.flow >= e.cap || dis[e.to] <= dis[u] + e.cost)
                continue;

```

```

        dis[e.to] = dis[u] + e.cost;
        prev[e.to] = u;
        from_edge[e.to] = i;
        if (state[e.to] == in_queue) continue;
        if (state[e.to] == finished
            || (!q.empty() && dis[q.front()] > dis[e.to]))
            q.push_front(e.to);
        else
            q.push_back(e.to);
        state[e.to] = in_queue;
    }
}
if (dis[dest] == 00)
    return {};
vector<pair<int, int>> path;
int cur = dest;
while (cur != src) {
    path.push_back( { prev[cur], from_edge[cur] } );
    cur = prev[cur];
}
reverse(path.begin(), path.end());
return path;
}
};

```

## Hungarian

```

// nodes are 0-based
/* There are n workers and n tasks.
You know exactly how much you need to pay each worker to perform one or another task.
You also know that every worker can only perform one task.
Your goal is to assign each worker some a task,
while minimizing your expenses.
*/
// fill vector a with costs
// if you want maximize final cost then you will multiply edges cost with -1
// this algorithm works only on bipartite graph
// the maximum matching must equal to n
template<typename T>
class hungarian {
public:
    int n;
    int m;
    vector< vector<T> > a;
    vector<T> u;
    vector<T> v;
    vector<int> pa;
    vector<int> pb;
    vector<int> way;
    vector<T> minv;
    vector<bool> used;
    T inf;
    hungarian(int _n, int _m) : n(_n), m(_m) {
        assert(n <= m);
        a = vector< vector<T> >(n, vector<T>(m));
        u = vector<T>(n + 1);

```

```

v = vector<T>(m + 1);
pa = vector<int>(n + 1, -1);
pb = vector<int>(m + 1, -1);
way = vector<int>(m, -1);
minv = vector<T>(m);
used = vector<bool>(m + 1);
inf = numeric_limits<T>::max();
}
inline void add_row(int i) {
    fill(minv.begin(), minv.end(), inf);
    fill(used.begin(), used.end(), false);
    pb[m] = i;
    pa[i] = m;
    int j0 = m;
    do {
        used[j0] = true;
        int i0 = pb[j0];
        T delta = inf;
        int j1 = -1;
        for (int j = 0; j < m; j++) {
            if (!used[j]) {
                T cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j]) {
                    minv[j] = cur;
                    way[j] = j0;
                }
                if (minv[j] < delta) {
                    delta = minv[j];
                    j1 = j;
                }
            }
        }
        for (int j = 0; j <= m; j++) {
            if (used[j]) {
                u[pb[j]] += delta;
                v[j] -= delta;
            }
            else {
                minv[j] -= delta;
            }
        }
        j0 = j1;
    } while (pb[j0] != -1);
    do {
        int j1 = way[j0];
        pb[j0] = pb[j1];
        pa[pb[j0]] = j0;
        j0 = j1;
    } while (j0 != m);
}
inline T current_score() {
    return -v[m];
}

```

```

inline T solve() {
    for (int i = 0; i < n; i++) {
        add_row(i);
    }
    return current_score();
}
};

```

## String

### Hashing

```

struct hashing {
    int MOD, BASE;
    vector<int> Hash, modInv;
    hashing(string s, int MOD, int BASE, char first_char = 'a') :
        MOD(MOD), BASE(BASE), Hash(sz(s) + 1), modInv(sz(s) + 1) {
        modInv[0] = 1;
        ll base = 1;
        for (int i = 1; i <= sz(s); i++) {
            Hash[i] = (Hash[i - 1] + (s[i - 1] - first_char + 1) * base) % MOD;
            modInv[i] = power(base, MOD - 2, MOD);
            base = (base * BASE) % MOD;
        }
    }
    int getHash(int l, int r) { //1-based
        return (1LL * (Hash[r] - Hash[l - 1] + MOD) % MOD * modInv[l]) % MOD;
    }
};
//MOD = 1e9 + 9 ,BASE = 31
//MOD = 2000000011 ,BASE = 53 ->careful of overflow
//*****
//MOD = 998634293,BASE = 953
//MOD = 986464091,BASE = 1013

```

### KMP

```

struct KMP {
    string pattern;
    vector<int> longestPrefix;
    KMP(string& str) :pattern(str) {
        failure_function();
    }
    int fail(int k, char nxt) {
        while (k > 0 && pattern[k] != nxt)
            k = longestPrefix[k - 1];
        if (nxt == pattern[k]) k++;
        return k;
    }
    void failure_function() {
        int n = pattern.size();
        longestPrefix = vector<int>(n);
        for (int i = 1, k = 0; i < n; i++)
            longestPrefix[i] = k = fail(k, pattern[i]);
    }
}

```



```

void match(const string& str) {
    int n = str.size();
    int m = pattern.size();
    for (int i = 0, k = 0; i < n; i++) {
        k = fail(k, str[i]);
        if (k == m) {
            cout << i - m + 1 << endl; //0-based
            k = longestPrefix[k - 1]; // if you want next match
        }
    }
}

};
vector<bool> suffix_pal(string s) { //[i..n-1] pal?
    string r = s;
    reverse(all(r));
    vector<bool> v(s.size());
    v[0] = (s == r);
    string pattern = r + "#" + s;
    int n = pattern.size();
    vector<int> longestPrefix(n);
    int k = 0;
    for (int i = 1; i < n; i++) {
        while (k > 0 && pattern[k] != pattern[i])
            k = longestPrefix[k - 1];
        if (pattern[i] == pattern[k]) k++;
        longestPrefix[i] = k;
    }
    while (k > 0) {
        v[s.size() - k] = true;
        k = longestPrefix[k - 1];
    }
    return v;
}
vector<bool> prefix_pal(string s) { // [0..i] pal?
    string r = s;
    reverse(all(r));
    vector<bool> v(s.size());
    v.back() = (s == r);
    string pattern = s + "#" + r;
    int n = pattern.size();
    vector<int> longestPrefix(n);
    int k = 0;
    for (int i = 1; i < n; i++) {
        while (k > 0 && pattern[k] != pattern[i])
            k = longestPrefix[k - 1];
        if (pattern[i] == pattern[k])
            k++;
        longestPrefix[i] = k;
    }
    while (k > 0) {
        v[k - 1] = true;
        k = longestPrefix[k - 1];
    }
    return v;
}

```

```

//frq[i] = number of occur s[0..i] in s
vector<int> build_fre_prefix(const string& s) {
    KMP kmp(s);
    kmp.failure_function();
    vector<int> f = kmp.longestPrefix;
    int n = sz(s);
    vector<int> frq(n);
    for (int i = n - 1; i >= 0; i--)
        if (f[i]) frq[f[i] - 1] += frq[i] + 1;
    for (auto& it : frq) it++;
    return frq;
}

```

## Trie

```

class trie {
    struct trie_node {
        bool is_leaf = false;
        map<char, int> next;
        bool have_next(char ch) {
            return next.find(ch) != next.end();
        }
        int& operator[](char ch) {
            return next[ch];
        }
    };
    vector<trie_node> t;
public:
    trie() {
        t.push_back(trie_node());
    }
    void insert(const string &s) {
        int root = 0;
        for (const char &ch : s) {
            if (!t[root].have_next(ch)) {
                t.push_back(trie_node());
                t[root][ch] = t.size() - 1;
            }
            root = t[root][ch];
        }
        t[root].is_leaf = true;
    }
    bool find(const string &s) {
        int root = 0;
        for (const char &ch : s) {
            if (!t[root].have_next(ch))
                return false;
            root = t[root][ch];
        }
        return t[root].is_leaf;
    }
};

```

## Aho Corasick

```
struct aho_corasick {
    struct trie_node {
        vector<int> pIdxs; //probably take memory limit
        map<char, int> next;
        int fail;
        trie_node() : fail(0) {}
        bool have_next(char ch) {
            return next.find(ch) != next.end();
        }
        int& operator[](char ch) {
            return next[ch];
        }
    };
    vector<trie_node> t;
    vector<string> patterns;
    vector<int> end_of_pattern;
    vector<vector<int>> adj;
    int insert(const string &s, int patternIdx) {
        int root = 0;
        for (const char &ch : s) {
            if (!t[root].have_next(ch)) {
                t.push_back(trie_node());
                t[root][ch] = t.size() - 1;
            }
            root = t[root][ch];
        }
        t[root].pIdxs.push_back(patternIdx);
        return root;
    }
    int next_state(int cur, char ch) {
        while (cur > 0 && !t[cur].have_next(ch))
            cur = t[cur].fail;
        if (t[cur].have_next(ch))
            return t[cur][ch];
        return 0;
    }
    void buildAhoTree() {
        queue<int> q;
        for (auto &child : t[0].next)
            q.push(child.second);

        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (auto &child : t[cur].next) {
                int k = next_state(t[cur].fail, child.first);
                t[child.second].fail = k;
                vector<int> &idxs = t[child.second].pIdxs;
                //dp[child.second] = max(dp[child.second], dp[k]);
                idxs.insert(idxs.end(), all(t[k].pIdxs));
                q.push(child.second);
            }
        }
    }
}
```

```

void buildFailureTree() {
    adj = vector<vector<int>>(t.size());
    for (int i = 1; i < t.size(); i++)
        adj[t[i].fail].push_back(i);
}
aho_corasick(const vector<string> &_patterns) {
    t.push_back(trie_node());
    patterns = _patterns;
    end_of_pattern = vector<int>(patterns.size());
    for (int i = 0; i < patterns.size(); i++)
        end_of_pattern[i] = insert(patterns[i], i);
    buildAhoTree();
    //buildFailureTree();
}
vector<vector<int>> match(const string &str) {
    int k = 0;
    vector<vector<int>> rt(patterns.size());
    for (int i = 0; i < str.size(); i++) {
        k = next_state(k, str[i]);
        for (auto &it : t[k].pIdxs)
            rt[it].push_back(i);
    }
    return rt;
}
};

```

## Suffix Automaton

```
struct suffix_automaton {
    struct state {
        int len, link = 0, cnt = 0;
        bool terminal = false, is_clone = false;
        map<char, int> next;
        state(int len = 0) : len(len) {}
        bool have_next(char ch) {
            return next.find(ch) != next.end();
        }
        void clone(const state &other, int nlen) {
            len = nlen;
            next = other.next;
            link = other.link;
            is_clone = true;
        }
    };
    vector<state> st;
    int last = 0;
    suffix_automaton() {
        st.push_back(state());
        st[0].link = -1;
    }
    suffix_automaton(const string &s) : suffix_automaton() {
        for (char ch : s) extend(ch);
        for (int cur = last; cur > 0; cur = st[cur].link)
            st[cur].terminal = true;
    }

    void extend(char c) {
        int cur = st.size();
        st.push_back(state(st[last].len + 1));
        st[cur].cnt = 1;
        int p = last;
        last = cur;
        while (p != -1 && !st[p].have_next(c)) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if (p == -1) return;
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len) {
            st[cur].link = q;
            return;
        }
        int clone = st.size();
        st.push_back(state());
        st[clone].clone(st[q], st[p].len + 1);
        while (p != -1 && st[p].next[c] == q) {
            st[p].next[c] = clone;
            p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
    }
}
```

```

void calc_number_of_occurrences() {
    vector<vector<int>> lvl(st[last].len + 1);
    for (int i = 1; i < st.size(); i++)
        lvl[st[i].len].push_back(i);
    for (int i = st[last].len; i >= 0; i--)
        for (auto cur : lvl[i])
            st[st[cur].link].cnt += st[cur].cnt;
}
vector<ll> dp;
ll Count(int cur) { //count number of paths
    ll &rt = dp[cur];
    if (rt) return rt;
    rt = 1;
    for (auto ch : st[cur].next)
        rt += Count(ch.second);
    return rt;
}
string kth_substring(ll k) { //1-based,different substring,0 = ""
    assert(k <= Count(0));
    string rt;
    int cur = 0;
    while (k > 0) {
        for (auto ch : st[cur].next) {
            if (Count(ch.second) < k)
                k -= Count(ch.second);
            else {
                rt += ch.first;
                cur = ch.second;
                k--;
                break;
            }
        }
    }
    return rt;
}
string longest_common_substring(const string &t) {
    int cur = 0, l = 0, mx = 0, idx = 0;
    for (int i = 0; i < t.size(); i++) {
        while (cur > 0 && !st[cur].have_next(t[i])) {
            cur = st[cur].link;
            l = st[cur].len;
        }
        if (st[cur].have_next(t[i])) {
            cur = st[cur].next[t[i]];
            l++;
        }
        if (l > mx) {
            mx = l;
            idx = i;
        }
    }
    return t.substr(idx - mx + 1, mx);
}
};

```

## Suffix array

```
class suffix_array {
    int getOrder(int a) const {
        return (a < (int) order.size() ? order[a] : 0);
    }
    void radix_sort(int k) {
        vector<int> frq(n), tmp(n);
        for (auto &it : suf)
            frq[getOrder(it + k)]++;
        for (int i = 1; i < n; i++)
            frq[i] += frq[i - 1];
        for (int i = n - 1; i >= 0; i--)
            tmp[--frq[getOrder(suf[i] + k)]] = suf[i];
        suf = tmp;
    }
public:
    int n;
    string s;
    vector<int> suf, lcp, order; // order store position of suffix i in suf array
    suffix_array(const string &s) :
        n(s.size() + 1), s(s) {
        suf = order = vector<int>(n);
        vector<int> newOrder(n);
        for (int i = 0; i < n; i++)
            suf[i] = i;
        { //sort according to first character
            vector<int> tmp(n);
            for (int i = 0; i < n; i++)
                tmp[i] = s[i];
            sort(all(tmp));
            for (int i = 0; i < n; i++)
                order[i] = (lower_bound(all(tmp), s[i]) - tmp.begin());
        }
        for (int len = 1; newOrder.back() != n - 1; len <= 1) {
            auto cmp = [&](const int &a, const int &b) {
                if (order[a] != order[b])
                    return order[a] < order[b];
                return getOrder(a + len) < getOrder(b + len);
            };
            //sort(all(suf), cmp); //run in 576ms (n<=4e5)
            radix_sort(len); //sort second part
            radix_sort(0); //sort first part
            newOrder[0] = 0;
            for (int i = 1; i < n; i++)
                newOrder[i] = newOrder[i - 1] + cmp(suf[i - 1], suf[i]);
            for (int i = 0; i < n; i++)
                order[suf[i]] = newOrder[i];
        }
        buildLCP();
    }
}
```

```

/*
 * longest common prefix
 * O(n)
 * lcp[i] = lcp(suf[i],suf[i-1])
 */
void buildLCP() {
    lcp = vector<int>(n);
    int k = 0;
    for (int i = 0; i < n - 1; i++) {
        int pos = order[i];
        int j = suf[pos - 1];
        while (s[i + k] == s[j + k])
            k++;
        lcp[pos] = k;
        if (k)
            k--;
    }
}

int LCP_by_order(int a, int b) {
    if (a > b) swap(a, b);
    int mn = n - suf[a] - 1;
    for (int k = a + 1; k <= b; k++)
        mn = min(mn, lcp[k]);
    return mn; }

//LCP(i,j) : longest common prefix between suffix i and suffix j
int LCP(int i, int j) {
    //return LCP_by_order(order[i],order[j]);
    if (order[j] < order[i])
        swap(i, j);
    int mn = n - i - 1;
    for (int k = order[i] + 1; k <= order[j]; k++)
        mn = min(mn, lcp[k]);
    return mn;
}

//compare s[a.first..a.second] with s[b.first..b.second]
//-1:a<b ,0:a==b,1:a>b
int compare_substrings(pair<int, int> a, pair<int, int> b) {
    int lcp = min({ LCP(a.first, b.first), a.second - a.first + 1,
                    b.second - b.first + 1 });
    a.first += lcp;
    b.first += lcp;
    if (a.first <= a.second) {
        if (b.first <= b.second) {
            if (s[a.first] == s[b.first])
                return 0;
            return (s[a.first] < s[b.first] ? -1 : 1);
        }
        return 1;
    }
    return (b.first <= b.second ? -1 : 0);
}

```



```

pair<int, int> find_string(const string &x) {
    int st = 0, ed = n;
    for (int i = 0; i < sz(x) && st < ed; i++) {
        auto cmp = [&](int a, int b) {
            if (a == -1)
                return x[i] < s[b + i];
            return s[a + i] < x[i];
        };
        st = lower_bound(suf.begin() + st, suf.begin() + ed, -1, cmp)
            - suf.begin();
        ed = upper_bound(suf.begin() + st, suf.begin() + ed, -1, cmp)
            - suf.begin();
    }
    return {st, ed-1};
}
};
11 number_of_different_substrings(string s) {
    int n = s.size();
    suffix_array sa(s);
    ll cnt = 0;
    for (int i = 0; i <= n; i++)
        cnt += n - sa.suf[i] - sa.lcp[i];
    return cnt;
}
string longest_common_substring(const string &s1, const string &s2) {
    suffix_array sa(s1 + "#" + s2);
    vector<int> suf = sa.suf, lcp = sa.lcp;
    auto type = [&](int idx) {
        return idx <= s1.size();
    };
    int mx = 0, idx = 0;
    int len = s1.size() + 1 + s2.size();
    for (int i = 1; i <= len; i++)
        if (type(suf[i - 1]) != type(suf[i]) && lcp[i] > mx) {
            mx = lcp[i];
            idx = min(suf[i - 1], suf[i]);
        }
    return s1.substr(idx, mx);
}
int longest_common_substring(const vector<string> &v) {
    int n = v.size();
    int len = n - 1;
    for (auto &it : v)
        len += it.size();
    string s(len, '.');
    vector<int> type(len + 1, n), frq(n + 1);
    for (int i = 0, j = 0; i < v.size(); i++) {
        if (i) s[j] = 'z' + i; //review this
        for (char ch : v[i]) {
            s[j] = ch;
            type[j] = i;
            j++;
        }
    }
    suffix_array sa(s);
}

```

```

vector<int> suf = sa.suf, lcp = sa.lcp;
monoqueue q;
int st = 0, ed = 0, cnt = 0, mx = 0;
while (st <= s.size()) {
    while (ed <= s.size() && cnt < v.size()) {
        q.push(lcp[ed], ed);
        if (++frq[type[suf[ed]]] == 1)
            cnt++;
        ed++;
    }
    q.pop(st);
    if (cnt == v.size()) mx = max(mx, q.getMin()); //st+1,ed
    if (--frq[type[suf[st]]] == 0) cnt--;
    st++;
}
return mx;
}

string kth_substring(string s, int k) { //1-based,repated
    int n = s.size();
    suffix_array sa(s);
    vector<int> suf = sa.suf, lcp = sa.lcp;
    for (int i = 1; i <= n; i++) {
        int len = n - suf[i];
        int cnt = 0;
        for (int l = 1; l <= len; l++) {
            cnt++;
            int st = i + 1, ed = n, ans = i;
            while (st <= ed) {
                int md = st + ed >> 1;
                if (sa.LCP_by_order(i, md) >= l)
                    st = md + 1, ans = md;
                else
                    ed = md - 1;
            }
            cnt += ans - i;
            if (cnt >= k) return s.substr(suf[i], l);
        }
        k -= len;
    }
    assert(0);
}

```

### Suffix array Faster

```

class suffix_array {
    const static int alpha = 128;
    int getOrder(int a) const {
        return (a < (int) order.size() ? order[a] : 0);
    }
public:
    int n;
    string s;
    vector<int> suf, order, lcp; // order store position of suffix i in suf array
    suffix_array(const string &s) : n(s.size() + 1), s(s) {
        suf = order = lcp = vector<int>(n);
        vector<int> bucket_idx(n), newOrder(n), newsuff(n);
        vector<int> prev(n), head(alpha, -1);
    }

```

```

    for (int i = 0; i < n; i++) {
        prev[i] = head[s[i]];
        head[s[i]] = i;
    }
    int buc = -1, idx = 0;
    for (int i = 0; i < alpha; i++) {
        if (head[i] == -1) continue;
        bucket_idx[++buc] = idx;
        for (int j = head[i]; ~j; j = prev[j])
            suf[idx++] = j, order[j] = buc;
    }
    int len = 1;
    do {
        auto cmp = [&](int a, int b) {
            if (order[a] != order[b])
                return order[a] < order[b];
            return getOrder(a + len) < getOrder(b + len);
        };
        for (int i = 0; i < n; i++) {
            int j = suf[i] - len;
            if (j < 0)
                continue;
            newsuff[bucket_idx[order[j]]++] = j;
        }
        for (int i = 1; i < n; i++) {
            suf[i] = newsuff[i];
            bool cmpres = cmp(suf[i - 1], suf[i]);
            newOrder[suf[i]] = newOrder[suf[i - 1]] + cmpres;
            if (cmpres)
                bucket_idx[newOrder[suf[i]]] = i;
        }
        order = newOrder;
        len <= 1;
    } while (order[suf[n - 1]] != n - 1);
}
};

```

## Z algorithm

/\* z[i] equal the length of the longest substring starting from s[i]  
which is also a prefix of s \*/

```

vector<int> z_algo(string s) {
    int n = s.size();
    vector<int> z(n);
    z[0] = n;
    for (int i = 1, L = 1, R = 1; i < n; i++) {
        int k = i - L;
        if (z[k] + i >= R) {
            L = i;
            R = max(R, i);
            while (R < n && s[R - L] == s[R]) R++;
            z[i] = R - L;
        } else z[i] = z[k];
    }
    return z;
}

```

# Math

## Primes

### Sieve

```
const int N = 1e8;
bool isPrime[N + 1];
vector<int> prime;
void sieve() {
    memset(isPrime, true, sizeof(isPrime));
    isPrime[0] = isPrime[1] = false;
    for (int i = 4; i <= N; i += 2)
        isPrime[i] = false;
    for (int i = 3; i * i <= N; i += 2)
        if (isPrime[i])
            for (int j = i * i; j <= N; j += i + i)
                isPrime[j] = false;
    for (int i = 1; i <= N; i++)
        if (isPrime[i])
            prime.push_back(i);
}
```

### Linear Sieve

```
const int N = 1e7;
int lpf[N + 1];
vector<int> prime;
void sieve() {
    for (int i = 2; i <= N; i++) {
        if (lpf[i] == 0) {
            lpf[i] = i;
            prime.push_back(i);
        }
        for (int j : prime) {
            if (j > lpf[i] || 1LL * i * j > N) break;
            lpf[i * j] = j;
        }
    }
}
```

### Prime Factors

```
// return number of Divisors(n) using prime factorization
ll numOfDivisors(primeFactors mp) {
    ll cnt = 1;
    for (auto it : mp) cnt *= (it.second + 1);
    return cnt;
}
// return sum of Divisors(n) using prime factorization
ll sumOfDivisors(primeFactors mp) {
    ll sum = 1;
    for (auto it : mp) sum *= sumPower(it.first, it.second);
    return sum;
}
```

## Phi function

```
ll phi_function(ll n) {
    ll result = n;
    primeFactors pf = prime_factors(n);
    for (auto &it : pf) {
        ll p = it.first;
        result -= (result / p);
    }
    return result;
}

void phi_1_to_n(int n) {
    for (int i = 0; i <= n; i++)
        phi[i] = i;
    for (int i = 2; i <= n; i++)
        if (phi[i] == i)
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
}
```

## Moebius function

```
char mob[N];
bool prime[N];
void moebius() {
    memset(mob, 1, sizeof mob);
    memset(prime + 2, 1, sizeof(prime) - 2);
    mob[0] = 0;
    mob[2] = -1;
    for (int i = 4; i < N; i += 2) {
        mob[i] *= (i & 3) ? -1 : 0;
        prime[i] = 0;
    }
    for (int i = 3; i < N; i += 2)
        if (prime[i]) {
            mob[i] = -1;
            for (int j = 2 * i; j < N; j += i) {
                mob[j] *= j % (1LL * i * i) ? -1 : 0;
                prime[j] = 0;
            }
        }
}
```

## Extended Euclidean

```
ll egcd(ll a, ll b, ll& x, ll& y) {
    if (a < 0) {
        auto g = egcd(-a, b, x, y);
        x *= -1;
        return g;
    }
    if (b < 0) {
        auto g = egcd(a, -b, x, y);
        y *= -1;
        return g;
    }
}
```

```

    if (!b) {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll g = egcd(b, a % b, x1, y1);
    x = y1, y = x1 - y1 * (a / b);
    return g;
}

```

## Linear Diophantine Equation

```

// return false if there is no solution
// return true if there exist a solution
// x, y are the solutions and g is the gcd between a and b
bool Diophantine_Solution(ll a, ll b, ll c, ll& x, ll& y, ll& g) {
    if (!a && !b) {
        if (c) return false;
        x = y = g = 0;
        return true;
    }
    g = egcd(a, b, x, y);
    if (c % g) return false;
    x *= c / g;
    y *= c / g;
    return true;
}

void shift_solution(ll& x, ll& y, ll a, ll b, ll cnt) {
    x += b * cnt;
    y -= a * cnt;
}

// find all number of solutions of ax + by = c
// x in range {minx, maxx}
// y in range {miny, maxy}
ll Diophantine_Solutions(ll a, ll b, ll c, ll minx, ll maxx, ll miny, ll maxy) {
    if (minx > maxx || miny > maxy) return 0;
    if (!a && !b && !c)
        return (maxx - minx + 1) * (maxy - miny + 1);
    if (!a && !b) return 0;
    if (!a) {
        if (c % b) return 0;
        ll num = c / b;
        return (num >= miny && num <= maxy) * (maxx - minx + 1);
    }
    if (!b) {
        if (c % a) return 0;
        ll num = c / a;
        return (num >= minx && num <= maxx) * (maxy - miny + 1);
    }
    ll x, y, g;
    if (!Diophantine_Solution(a, b, c, x, y, g))
        return 0;
}

```

```

11 lx1, lx2, rx1, rx2;
// a * x + b * y = c
// (a / g) * x + (b / g) * y = c / g
a /= g, b /= g, c /= g;
g = 1;
int sign_a = (a > 0 ? 1 : -1);
int sign_b = (b > 0 ? 1 : -1);
// x + k * b >= minx
// k * b >= minx - x
// k >= (minx - x) / b
// k >= ceil((minx - x) / b)
shift_solution(x, y, a, b, (minx - x) / b);
// if x is less than minx so we need to increase it by one step only
// from the upove equation x + k * b >= minx
// if b is positive so choose k equal to 1 to increase x one
// if b is negattive so choose k equal to -1 because -1 * -1 = 1 ans also increase x
if (x < minx)
    shift_solution(x, y, a, b, sign_b);
if (x > maxx) return 0;
lx1 = x;

// x + k * b <= maxx
// k * b <= maxx - x
// k <= (maxx - x) / b
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
rx1 = x;
// y - k * a >= miny
// y - miny >= k * a
// k * a <= y - miny
// k <= (y - miny) / a
shift_solution(x, y, a, b, (y - miny) / a);
if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
if (y > maxy) return 0;
lx2 = x;
// y - k * a <= maxy
// y - maxy <= k * a
// k * a >= y - maxy
// k >= (y - maxy) / a
shift_solution(x, y, a, b, (y - maxy) / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
rx2 = x;
if (lx2 > rx2) swap(lx2, rx2);
// becuase we calculate the equations lx2, rx2 from shifting y
// not from shifting x directly
11 lx = max(lx1, lx2);
11 rx = min(rx1, rx2);
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}

```

## Extended Euclidean for n variables

```
//O(n * log(m)) Memory & Time; coefficients.size() <= n, coefficients[i] <= m
// 0-based implementation
template<typename T>
T extended_euclidean(const deque<T>& cof, deque<T>& var) {
    int n = cof.size();
    if (!cof.back()) {
        int cnt = 0, id = 0;
        for (int i = 0; i < n; i++)
            if (!cof[i]) {
                cnt++;
                var[i] = 0;
            }
        else id = i;
        if (cnt >= n - 1) {
            var[id] = 1;
            return cof[id];
        }
        deque<T> new_cof, new_var;
        for (int i = 0; i < n; i++)
            if (cof[i]) {
                new_cof.push_back(cof[i]);
                new_var.push_back(var[i]);
            }
        T g = extended_euclidean(new_cof, new_var);
        for (int i = 0; !new_var.empty(); i++)
            if (cof[i]) {
                var[i] = new_var.front();
                new_var.pop_front();
            }
        return g;
    }
    deque<T> new_cof = cof;
    for (int i = 0; i < n - 1; i++)
        new_cof[i] %= new_cof.back();
    new_cof.push_front(new_cof.back());
    new_cof.pop_back();
    var.push_front(var.back());
    var.pop_back();
    T g = extended_euclidean(new_cof, var);
    var.push_back(var.front());
    var.pop_front();
    for (int i = 0; i < n - 1; i++)
        var.back() -= cof[i] / cof.back() * var[i];
    return g;
}

template<typename T>
vector<T> find_any_solution(const vector<T>& cof, T rhs) {
    int n = cof.size();
    if (!n)
        return vector<T>();
    deque<T> deque_cof(cof.begin(), cof.end()), deque_var(n);
    T g = extended_euclidean(deque_cof, deque_var);
    if (g && rhs % g)
        return vector<T>();
}
```



```

vector<T> var(deque_var.begin(), deque_var.end());
if (g) {
    rhs /= g;
    for (auto& it : var)
        it *= rhs;
}
return var;
}

```

## Sum Sequence

```

//return sum of sequence a, a+x , a+2x .... b
ll sumSequence(ll a, ll b, ll x) {
    a = ((a + x - 1) / x) * x;
    b = (b / x) * x;
    return (b + a) * (b - a + x) / (2 * x);
}

```

## Sum Range Divisors

```

// return sum of divisors for all number from 1 to n //O(n)
ll sumRangeDivisors(int n) {
    ll ans = 0;
    for (int x = 1; x <= n; x++)
        ans += (n / x) * x;
    return ans;
}

// calc 1e9 in 42ms, can calc more but need big integer
ll sumRangeDivisors(ll x) {
    ll ans = 0, left = 1, right;
    for (; left <= x; left = right + 1) {
        right = x / (x / left);
        ans += (x / left) * (left + right) * (right - left + 1) / 2;
    }
    return ans;
}

```

## Combinatorics

```

/*
 * nCr = n!/((n-r)! * r!)
 * nCr(n,r) = nCr(n,n-r)
 * nPr = n!/(n-r)!
 * nPr(circle) = nPr/r
 * nCr(n,r) = pascal[n][r]
 * catalan[n] = nCr(2n,n)/(n+1)
 */
ull nCr(int n, int r) {
    if (r > n)
        return 0;
    r = max(r, n - r);
    ull ans = 1, div = 1, i = r + 1;
    while (i <= n) {
        ans *= i++;
        ans /= div++;
    }
    return ans;
}

```

```

ull nPr(int n, int r) {
    if (r > n)
        return 0;
    ull p = 1, i = n - r + 1;
    while (i <= n)
        p *= i++;
    return p;
}
// return catalan number n-th using dp O(n^2)//max = 35 then overflow
vector<ull> catalanNumber(int n) {
    vector<ull> catalan(n + 1);
    catalan[0] = catalan[1] = 1;
    for (int i = 2; i <= n; i++) {
        ull& rt = catalan[i];
        for (int j = 0; j < n; j++)
            rt += catalan[j] * catalan[n - j - 1];
    }
    return catalan;
}

// count number of paths in matrix n*m
// go to right or down only
ull countNumberOfPaths(int n, int m) {
    return nCr(n + m - 2, n - 1);
}

```

### nCr pre calculation

```

const int N = 1e5 + 100;
const int mod = 1e9 + 7;
ll fact[N];
ll inv[N]; //mod inverse for i
ll invfact[N]; //mod inverse for i!
void factInverse() {
    fact[0] = inv[1] = fact[1] = invfact[0] = invfact[1] = 1;
    for (long long i = 2; i < N; i++) {
        fact[i] = (fact[i - 1] * i) % mod;
        inv[i] = mod - (inv[mod % i] * (mod / i) % mod);
        invfact[i] = (inv[i] * invfact[i - 1]) % mod;
    }
}
ll nCr(int n, int r) {
    if (r > n) return 0;
    return (((fact[n] * invfact[r]) % mod) * invfact[n - r]) % mod;
}

```

## Matrices

```
typedef vector<int> row;
typedef vector<row> matrix;

matrix initial(int n, int m, int val = 0) {
    return matrix(n, row(m, val));
}

matrix identity(int n) {
    matrix rt = initial(n, n);
    for (int i = 0; i < n; i++)rt[i][i] = 1;
    return rt;
}

matrix addIdentity(const matrix& a) {
    matrix rt = a;
    for (int i = 0; i < sz(a); i++)rt[i][i] += 1;
    return rt;
}

matrix add(const matrix& a, const matrix& b) {
    matrix rt = initial(sz(a), sz(a[0]));
    for (int i = 0; i < sz(a); i++)for (int j = 0; j < sz(a[0]); j++)
        rt[i][j] = a[i][j] + b[i][j];
    return rt;
}

matrix multiply(const matrix& a, const matrix& b) {
    matrix rt = initial(sz(a), sz(b[0]));
    for (int i = 0; i < sz(a); i++) for (int k = 0; k < sz(a[0]); k++) {
        if (a[i][k] == 0)continue;
        for (int j = 0; j < sz(b[0]); j++)
            rt[i][j] += a[i][k] * b[k][j];
    }
    return rt;
}

matrix power(const matrix& a, ll k) {
    if (k == 0)return identity(sz(a));
    if (k & 1)return multiply(a, power(a, k - 1));
    return power(multiply(a, a), k >> 1);
}

matrix power_itr(matrix a, ll k) {
    matrix rt = identity(sz(a));
    while (k) {
        if (k & 1)rt = multiply(rt, a);
        a = multiply(a, a); k >>= 1;
    }
    return rt;
}

matrix sumPower(const matrix& a, ll k) {
    if (k == 0)return initial(sz(a), sz(a));
    if (k & 1)return multiply(a, addIdentity(sumPower(a, k - 1)));
    return multiply(sumPower(a, k >> 1), addIdentity(power(a, k >> 1)));
}
```

```

/* return matrix contains
   a^k      0
a^1+a^2.. a^k  I
*/
matrix sumPowerV2(const matrix& a, ll k) {
    int n = sz(a);
    matrix rt = initial(2 * n, 2 * n);
    for (int i = 0; i < 2 * n; i++)
        for (int j = 0; j < n; j++)
            rt[i][j] = a[i % n][j];
    for (int i = n; i < 2 * n; i++) rt[i][i] = 1;
    return power(rt, k);
}

```

## Matrix class

```

struct matrix {
    using T = int;
    using row = vector<T>;
    vector<vector<T>> v;
    matrix() {}
    matrix(int n, int m, T val = 0) : v(n, row(m, val)) {}
    int size() const {
        return v.size();
    }
    int cols() const {
        return v[0].size();
    }
    matrix operator*(T a) const {
        matrix rt = *this;
        REP(i, rt.size())
            REP(j, rt.cols())
                rt.v[i][j] *= a;
        return rt;
    }
    friend matrix operator*(T a, const matrix& b) {
        return (b * a);
    }
    friend matrix operator+(const matrix& a, const matrix& b) {
        assert(a.size() == b.size() && a.cols() == b.cols());
        matrix rt(a.size(), a.cols());
        REP(i, rt.size()) REP(j, rt.cols())
            rt.v[i][j] = a.v[i][j] + b.v[i][j];
        return rt;
    }
    friend matrix operator*(const matrix& a, const matrix& b) {
        assert(a.cols() == b.size());
        matrix rt(a.size(), b.cols());
        REP(i, rt.size()) REP(k, a.cols()) {
            if (a.v[i][k] == 0) continue;
            REP(j, rt.cols()) rt.v[i][j] += a.v[i][k] * b.v[k][j];
        }
        return rt;
    }
};

```

```

matrix identity(int n) {
    matrix r(n, n);
    for (int i = 0; i < n; i++)
        r.v[i][i] = 1;
    return r;
}
matrix addIdentity(const matrix& a) {
    matrix rt = a;
    REP(i, a.size()) rt.v[i][i]++;
    return rt;
}
matrix power(matrix a, long long y) {
    assert(y >= 0 && a.size() == a.cols());
    matrix rt = identity(a.size());
    while (y > 0) {
        if (y & 1)
            rt = rt * a;
        a = a * a;
        y >>= 1;
    }
    return rt;
}
matrix sumPower(const matrix& a, ll k) {
    if (k == 0)
        return matrix(sz(a), sz(a));
    if (k & 1)
        return a * addIdentity(sumPower(a, k - 1));
    return (sumPower(a, k >> 1) * addIdentity(power(a, k >> 1)));
}
/* return matrix contains
    a^k      0
a^1+a^2.. a^k  I
*/
matrix sumPowerV2(const matrix& a, ll k) {
    int n = sz(a);
    matrix rt(2 * n, 2 * n);
    REP(i, n) REP(j, n) {
        rt.v[i][j] = a.v[i][j];
        rt.v[i + n][j] = a.v[i][j];
    }
    for (int i = n; i < 2 * n; i++)
        rt.v[i][i] = 1;
    return power(rt, k);
}

```

## Mod

### Fast power

```

ll power(ll x, ll y, int mod) {
    if (y == 0) return 1;
    if (y == 1) return x % mod;
    ll r = power(x, y >> 1, mod);
    return (((r * r) % mod) * power(x, y & 1, mod)) % mod;
}

```

## Sum of powers

```
// return  $a^1 + a^2 + a^3 + \dots + a^k$ 
ll sumPower(ll a, ll k, int mod) {
    if (k == 1) return a % mod;
    ll half = sumPower(a, k / 2, mod);
    ll p = half * power(a, k / 2, mod) % mod;
    p = (p + half) % mod;
    if (k & 1) p = (p + power(a, k, mod)) % mod;
    return p;
}
```

## Mod Inverse

```
ll modInverse(ll b, ll mod) { // if mod is Prime
    return power(b, mod - 2, mod);
}
ll modInverse(ll b, ll mod) { // if mod is not Prime, gcd(a,b) must be equal 1
    return power(b, phi_function(mod) - 1, mod);
}
```

## $(a^n) \% p = \text{result}$ , return n

```
//  $(a^n) \% p = \text{result}$ , return minimum n
int getPower(int a, int result, int mod) {
    int sq = sqrt(mod);
    map<int, int> mp;
    ll r = 1;
    for (int i = 0; i < sq; i++) {
        if (mp.find(r) == mp.end())
            mp[r] = i;
        r = (r * a) % mod;
    }
    ll tmp = modInverse(r, mod);
    ll cur = result;
    for (int i = 0; i <= mod; i += sq) {
        if (mp.find(cur) != mp.end())
            return i + mp[cur];
        cur = (cur * tmp) % mod; // val / (a^sq)
    }
    return INF;
}
// Returns minimum x for which  $a^x \% m = b \% m$ .
// a, m not coprime
int getPower(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 111 * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
```

```

for (int i = 0; i < n; ++i)
    an = (an * 111 * a) % m;

unordered_map<int, int> vals;
for (int q = 0, cur = b; q <= n; ++q) {
    vals[cur] = q;
    cur = (cur * 111 * a) % m;
}

for (int p = 1, cur = k; p <= n; ++p) {
    cur = (cur * 111 * an) % m;
    if (vals.count(cur)) {
        int ans = n * p - vals[cur] + add;
        return ans;
    }
}
return -1;
}

```

## CRT

```

ll CRT(vector<ll>& a, vector<ll>& m){
    ll lcm = m[0], rem = a[0];
    int n = a.size();
    for(int i = 1; i < n; i++){
        ll x, y;
        ll gcd = extended_euclidean(lcm, m[i], x, y);
        if((a[i] - rem) % gcd)
            return -1;
        x *= (a[i] - rem) / gcd;
        rem += lcm * x;
        lcm = lcm * m[i] / gcd;
        rem = (rem % lcm + lcm) % lcm;
    }
    return rem;
}

```

## FFT

```

typedef valarray<complex<double>> polynomial;
vector<complex<double>> CM1[3][LGN + 1];
const double PI = acos(-1);
void prepare() {
    for (int sign = -1; sign <= 1; sign += 2) {
        for (int i = 0; i <= LGN; i++) {
            int N = 1 << i;
            double theta = sign * 2 * PI / N;
            complex<double> cm1 = 1;
            complex<double> cm2(cos(theta), sin(theta));
            for (int j = 0; j < N / 2; j++) {
                CM1[sign + 1][i].push_back(cm1);
                cm1 *= cm2;
            }
        }
    }
}

```

```

void fft(polynomial &a, int sign = -1) {
    int N = a.size();
    int lgn = log2(N);
    for (int m = N; m >= 2; m >>= 1, lgn--) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            const complex<double> &w = CM1[sign + 1][lgn][i];
            for (int j = i; j < N; j += m) {
                int k = j + mh;
                complex<double> x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
    }
    int i = 0;
    for (int j = 1; j < N - 1; j++) {
        for (int k = N >> 1; k > (i ^ k); k >>= 1)
            ;
        if (j < i)
            swap(a[i], a[j]);
    }
}

void inv_fft(polynomial &a) {
    complex<double> N = a.size();
    fft(a, 1);
    a /= N;
}

valarray<int> mul(const valarray<int> &a, const valarray<int> &b) {
    int addeg = (int) a.size() - 1, bdeg = (int) b.size() - 1;
    int N = 1;
    while (N <= addeg + bdeg)
        N <<= 1;
    polynomial A(N), B(N);
    for (int i = 0; i < a.size(); i++)
        A[i] = a[i];
    for (int i = 0; i < b.size(); i++)
        B[i] = b[i];
    fft(A);
    fft(B);
    polynomial m = A * B;
    inv_fft(m);
    valarray<int> rt(N);
    for (int i = 0; i < N; i++)
        rt[i] = round(m[i].real());
    return rt;
}

```



## NNT

```
typedef valarray<modint> polynomial;
vector<modint> CM1[2][LGN + 1];
bool validRoot(modint root) {
    modint rootinv = modint(1) / root;
    for (int invert = 0; invert <= 1; invert++) {
        for (int i = 1; i <= LGN; i++) {
            int N = 1 << i;
            assert((MOD - 1) % N == 0);
            int C = (MOD - 1) / N;
            modint cm2 = modint::power(invert ? root : rootinv, C);
            if (cm2.val <= 1) return false;
        }
    }
    return true;
}
void prepare() {
    modint root = 2;
    while (!validRoot(root)) root += 1;
    modint rootinv = modint(1) / root;
    for (int invert = 0; invert <= 1; invert++) {
        for (int i = 0; i <= LGN; i++) {
            int N = 1 << i;
            int C = (MOD - 1) / N;
            modint cm2 = modint::power(invert ? root : rootinv, C);
            modint cm1 = 1;
            set<int> st;
            for (int j = 0; j < N / 2; j++) {
                CM1[invert][i].push_back(cm1);
                cm1 *= cm2;
            }
        }
    }
}
void fft(polynomial& a, bool invert = 0) {
    int N = a.size();
    int lgn = log2(N);
    for (int m = N; m >= 2; m >>= 1, lgn--) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            const modint& w = CM1[invert][lgn][i];
            for (int j = i; j < N; j += m) {
                int k = j + mh;
                modint x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
    }
    int i = 0;
    for (int j = 1; j < N - 1; j++) {
        for (int k = N >> 1; k > (i ^ k); k >>= 1) continue;
        if (j < i) swap(a[i], a[j]);
    }
}
```

```

void inv_fft(polynomial& a) {
    int N = a.size();
    fft(a, 1);
    a /= N;
}
valarray<modint> mul(const polynomial& a, const polynomial& b) {
    int addeg = (int)a.size() - 1, bdeg = (int)b.size() - 1;
    int N = 1;
    while (N <= addeg + bdeg)
        N <<= 1;
    polynomial A(N), B(N);
    for (int i = 0; i < a.size(); i++)
        A[i] = a[i];
    for (int i = 0; i < b.size(); i++)
        B[i] = b[i];
    fft(A);
    fft(B);
    polynomial rt = A * B;
    inv_fft(rt);
    return rt;
}

```

## Misc

### Bitmask

```

bool getBit(ll num, int ind) {
    return ((num >> ind) & 1);
}
ll setBit(ll num, int ind, bool val) {
    return val ? (num | (1LL << ind)) : (num & ~(1LL << ind));
}
ll flipBit(ll num, int ind) {
    return (num ^ (1LL << ind));
}
ll leastBit(ll num) {
    return (num & -num);
}
template<class Int>
Int turnOnLastZero(Int num) {
    return num | num + 1;
}
template<class Int>
Int turnOnLastConsecutiveZeroes(Int num) {
    return num | num - 1;
}
template<class Int>
Int turnOffLastBit(Int num) {
    return num & num - 1;
}
template<class Int>
Int turnOffLastConsecutiveBits(Int num) {
    return num & num + 1;
}

```

```

//num%mod, mod is a power of 2
11 Mod(11 num, 11 mod) {
    return (num & mod - 1);
}
bool isPowerOfTwo(11 num) {
    return (num & num - 1) == 0;
}
void genAllSubmask(int mask) {
    for (int subMask = mask;; subMask = (subMask - 1) & mask) {
        //code
        if (subMask == 0)
            break;
    }
}
/*
 * __builtin functions:
 * __builtin_popcount -> used to count the number of one's
 * __builtin_clz -> used to count the leading zeros of the integer
 * __builtin_ctz -> used to count the trailing zeros of the integer
 *
int LOG2(int x) { //floor(log2(x))
    return 31 - __builtin_clz(x);
}
int LOG2(long long x) { //floor(log2(x))
    return 63 - __builtin_clzll(x);
}

```

## Coordinate Compress

```

void coordinateCompress(vector<int>& axes, vector<int>& iTov,
    map<int, int> vToI, int start = 2, int step = 2) {
    for (auto& it : axes) vToI[it] = 0;
    iTov.resize(start + step * vToI.size());
    int idx = 0;
    for (auto& it : vToI) {
        it.second = start + step * idx;
        iTov[it.second] = it.first;
        idx++;
    }
}

```

## Random numbers

```

#include <chrono>
#include <random>
//write this line once in top
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count()* ((uint64_t) new char
| 1));
// use this instead of rand()
template<typename T> T Rand(T low, T high) {
    return uniform_int_distribution<T>(low, high)(rng);
}

```

## Custom hash

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(pair<uint64_t, uint64_t> x) const {
        static const uint64_t FIXED_RANDOM =
chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x.first + FIXED_RANDOM) ^ (splitmix64(x.second +
FIXED_RANDOM) >> 1);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

## Max histogram area

```
int maxHistogramArea(vector<int> v) {
    stack<int> st;
    int maxArea = 0, area = 0, i = 0;
    while (i < sz(v)) {
        if (st.empty() || v[st.top()] <= v[i])
            st.push(i++);
        else {
            int top = st.top(); st.pop();
            if (st.empty()) area = v[top] * i;
            else area = v[top] * (i - st.top() - 1);
            maxArea = max(maxArea, area);
        }
    }
    while (!st.empty()) {
        int top = st.top(); st.pop();
        if (st.empty())
            area = v[top] * i;
        else area = v[top] * (i - st.top() - 1);
        maxArea = max(maxArea, area);
    }
    return maxArea;
}
```

## Sorting

```
long long cnt = 0;
vector<int> v, temp;

// e the first index not have in range array
// like end()
template<class T = less<int>>
void merge_sort(int s, int e, T cmp = less<int>()) {
    if (s + 1 >= e) return;
    int m = s + (e - s >> 1);
    merge_sort(s, m, cmp);
    merge_sort(m, e, cmp);
    for (int i = s; i < e; i++)
        temp[i] = v[i];
    int i = s, j = m, k = s;
    while (i < m && j < e)
        if (cmp(temp[i], temp[j]))
            v[k++] = temp[i++];
        else
            v[k++] = temp[j++], cnt += j - k;
    while (i < m)
        v[k++] = temp[i++];
    while (j < e)
        v[k++] = temp[j++];
}

// O(n*log(n)/log(base))
// O(n + base) memory
void radix_sort(vector<int>& v, int base) {
    vector<int> tmp(v.size());
    for (int it = 0, p = 1; it < 10; it++, p *= base) {
        vector<int> frq(base);
        for (auto& it : v)
            frq[(it / p) % base]++;
        for (int i = 1; i < base; i++)
            frq[i] += frq[i - 1];
        for (int i = v.size() - 1; i >= 0; i--)
            tmp[--frq[(v[i] / p) % base]] = v[i];
        v = tmp;
    }
}

void quick_sort(int s, int e) {
    if (s >= e) return;
    int j = rand() % (e - s + 1) + s;
    swap(v[s], v[j]);
    j = s;
    int pivot = v[s];
    for (int i = s + 1; i <= e; i++)
        if (v[i] <= pivot)
            swap(v[i], v[++j]);
    swap(v[s], v[j]);
    quick_sort(s, j - 1);
    quick_sort(j + 1, e);
}
```

## LIS binary Search

```
/* without build
 * make upper_bound if can take equal elements */
int LIS(const vector<int>& v) {
    vector<int> lis(v.size()); //put value less than zero if needed
    int l = 0;
    for (int i = 0; i < sz(v); i++) {
        int idx = lower_bound(lis.begin(), lis.begin() + l, v[i]) - lis.begin();
        if (idx == l)
            l++;
        lis[idx] = v[i];
    }
    return l;
}

void LIS_binarySearch(vector<int> v) {
    int n = v.size();
    vector<int> last(n), prev(n, -1);
    int length = 0;
    auto BS = [&](int val) {
        int st = 1, ed = length, md, rt = length;
        while (st <= ed) {
            md = st + ed >> 1;
            if (v[last[md]] >= val)
                ed = md - 1, rt = md;
            else st = md + 1;
        }
        return rt;
    };
    for (int i = 1; i < n; i++) {
        if (v[i] < v[last[0]])
            last[0] = i;
        else if (v[i] > v[last[length]]) {
            prev[i] = last[length];
            last[++length] = i;
        }
        else {
            int index = BS(v[i]);
            prev[i] = last[index - 1];
            last[index] = i;
        }
    }
    cout << length + 1 << "\n";
    vector<int> out;
    for (int i = last[length]; i >= 0; i = prev[i])
        out.push_back(v[i]);
    reverse(out.begin(), out.end());
    for (auto it : out)
        cout << it << endl;
}
```

## Mo algorithm

```
int sqrtN; //use a constant value
struct query {
    int l, r, qIdx, block;
    query(int l, int r, int qIdx) : l(l), r(r), qIdx(qIdx), block(1 / sqrtN) {}
    bool operator <(const query& o) const {
        if (block != o.block) return block < o.block;
        return (block % 2 == 0 ? r < o.r : r > o.r);
    }
};

int curL, curR, ans;
vector<query> q;
void add(int index);
void remove(int index);
void solve(int l, int r) {
    while (curL > l) add(--curL);
    while (curR < r) add(++curR);
    while (curL < l) remove(curL++);
    while (curR > r) remove(curR--);
}

vector<int> MO(int n) {
    vector<int> rt(q.size());
    ans = curL = curR = 0;
    add(0);
    sort(q.begin(), q.end());
    for (auto it : q) {
        solve(it.l, it.r);
        rt[it.qIdx] = ans;
    }
    return rt;
}
```

## floyd cycle detection algorithm

```
template<class IntFunction>
pair<int, int> find_cycle_floyd(IntFunction f, int x0) {
    int tortoise = f(x0), hare = f(f(x0));
    while (tortoise != hare) {
        tortoise = f(tortoise);
        hare = f(f(hare));
    }
    int start = 0;
    tortoise = x0;
    while (tortoise != hare) {
        tortoise = f(tortoise);
        hare = f(hare);
        start++;
    }
    int length = 1;
    hare = f(tortoise);
    while (tortoise != hare) {
        hare = f(hare);
        length++;
    }
    return make_pair(start, length);
}
```

## Convex Hull Trick (Line Container)

```
struct Line {
    ll m, c;
    mutable ll p; //p is intersection between cur and next
    bool operator<(const Line &o) const {
        //change to (m>o.m,c < o.c) to get min
        if (m != o.m)
            return m < o.m;
        return c < o.c;
    }
    bool operator<(ll x) const {
        return p < x;
    }
};

struct LineContainer: multiset<Line, less<>> {
    // (for doubles, use inf = INFINITY, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end())
            return x->p = inf, 0;
        if (x->m == y->m)
            x->p = inf;
        else
            x->p = div(y->c - x->c, x->m - y->m);
        return x->p >= y->p;
    }
    void add(ll m, ll c) {
        auto z = insert( { m, c, 0 } ), y = z++, x = y;
        while (isect(y, z))
            z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.m * x + l.c;
    }
};
```

## Java Scanner

```
static class MScanner {
    StringTokenizer st;
    BufferedReader br;
    public MScanner(InputStream system) {
        br = new BufferedReader(new InputStreamReader(system));
    }
}
```



```

public MScanner(String file) throws Exception {
    br = new BufferedReader(new FileReader(file));
}
public String next() throws IOException {
    while (st == null && !st.hasMoreTokens())
        st = new StringTokenizer(br.readLine());
    return st.nextToken();
}
public String nextLine() throws IOException {
    return br.readLine();
}
public int nextInt() throws IOException {
    return Integer.parseInt(next());
}
public double nextDouble() throws IOException {
    return Double.parseDouble(next());
}
public char nextChar() throws IOException {
    return next().charAt(0);
}
public long nextLong() throws IOException {
    return Long.parseLong(next());
}
public int[] intArr(int n) throws IOException {
    int[] in = new int[n]; for (int i = 0; i < n; i++) in[i] = nextInt();
    return in;
}
public long[] longArr(int n) throws IOException {
    long[] in = new long[n]; for (int i = 0; i < n; i++) in[i] = nextLong();
    return in;
}
public int[] intSortedArr(int n) throws IOException {
    int[] in = new int[n]; for (int i = 0; i < n; i++) in[i] = nextInt();
    shuffle(in);
    Arrays.sort(in);
    return in;
}
public long[] longSortedArr(int n) throws IOException {
    long[] in = new long[n]; for (int i = 0; i < n; i++) in[i] = nextLong();
    shuffle(in);
    Arrays.sort(in);
    return in;
}
static void shuffle(int[] in) {
    for (int i = 0; i < in.length; i++) {
        int idx = (int)(Math.random() * in.length);
        int tmp = in[i];
        in[i] = in[idx];
        in[idx] = tmp;
    }
}
static void shuffle(long[] in) {
    for (int i = 0; i < in.length; i++) {
        int idx = (int)(Math.random() * in.length);
        long tmp = in[i];

```

```

        in[i] = in[idx];
        in[idx] = tmp;
    }
}
public Integer[] IntegerArr(int n) throws IOException {
    Integer[] in = new Integer[n]; for (int i = 0; in; i++)in[i] = nextInt();
    return in;
}
public Long[] LongArr(int n) throws IOException {
    Long[] in = new Long[n]; for (int i = 0; in; i++)in[i] = nextLong();
    return in;
}

public boolean ready() throws IOException {
    return br.ready();
}

public void waitForInput() throws InterruptedException {
    Thread.sleep(3000);
}
}

```

# Geometry

## point

```
#define ll long long
typedef long double ld;
typedef complex<double> point; // it can be long long not double
template<class T>
istream& operator>>(istream& is, complex<T>& p) {
    T value;
    is >> value;
    p.real(value);
    is >> value;
    p.imag(value);
    return is;
}
#define PI acos(-1.0)
#define EPS 1e-8
#define X real()
#define Y imag()
#define angle(a) (atan2((a).imag(), (a).real())) // angle with original
#define length(a) (hypot((a).imag(), (a).real()))
#define vec(a,b) ((b)-(a))
#define dp(a,b) ( (conj(a)*(b)).real() )
#define cp(a,b) ( (conj(a)*(b)).imag() ) // product = area of parallelogram
#define normalize(a) (a)/length(a)
// norm(a) // return x^2 + y^2 //a is point //can use dp(a,a)
bool same(point p1, point p2) { // check to points same or not
    return dp(vec(p1, p2), vec(p1, p2)) < EPS;
}
```

## rotate

```
point rotate(point p, double angle, point around = point(0, 0)) {
    p -= around;
    return (p * exp(point(0, angle))) + around;
}
```

## reflect

```
// Reflect v around m and origin
point reflect0(point v, point m) {
    return conj(v / m) * m;
}
// Reflect point p around l1-l2
point reflect(point p, point l1, point l2) {
    point z = p - l1, w = l2 - l1;
    return conj(z / w) * w + l1;
}
```

## Triangles

```
/*  
sin(A)/a = sin(B)/b = sin(C)/c  
a^2 = b^2 + c^2 - 2b*c*cos(A)  
sin(A+B) = sin(A) * cos(B) + sin(B) * cos(A)  
sin(A-B) = sin(A) * cos(B) - sin(B) * cos(A)  
cos(A+B) = cos(A) * cos(B) - sin(A) * sin(B)  
cos(A-B) = cos(A) * cos(B) + sin(A) * sin(B)  
tan(A+B) = (tan(A) + tan(B))/(1 - tan(A) * tan(B))  
tan(A-B) = (tan(A) - tan(B))/(1 - tan(A) * tan(B))  
*/
```

## Get Angles/Sides

```
double fixAngle(double A) {  
    return A > 1 ? 1 : (A < -1 ? -1 : A);  
}  
// return min angle: aOb / bOa  
// dp(v1, v2) = |v1|*|v2|*cos(theta)  
double angle0(point a, point O, point b) {  
    point v1(a - O), v2(b - O);  
    return acos(fixAngle(dp(v1, v2) / dist(v1) / dist(v2)));  
}  
double getSide_a_bAB(double b, double A, double B) {  
    return (sin(A) * b) / sin(B);  
}  
double getAngle_A_abB(double a, double b, double B) {  
    return asin(fixAngle((a * sin(B)) / b));  
}  
// give me wrong answer in team formation :D  
double getAngle_A_abc(double a, double b, double c) {  
    return acos(fixAngle((b * b + c * c - a * a) / (2 * b * c)));  
}  
double triangleArea(double a, double b, double c) {  
    double s = (a + b + c) / 2.0;  
    return sqrt((s - a) * (s - b) * (s - c) * s);  
}  
double triangleArea(point p0, point p1, point p2) {  
    double a = length(vec(p1, p0)), b = length(vec(p2, p0)), c = length(  
        vec(p2, p1));  
    return triangleArea(a, b, c);  
}
```

## Point In Triangle

```
bool pointInTriangle(point a, point b, point c, point pt) {  
    ll s1 = fabs(cp(vec(a,b), vec(a,c)));  
    ll s2 = fabs(cp(vec(pt,a), vec(pt,b))) + fabs(cp(vec(pt, b), vec(pt, c)))  
        + fabs(cp(vec(pt, a), vec(pt, c)));  
    return s1 == s2;  
}
```

## Get largest Circle Inside the triangle

```
//A triangle with area A and semi-perimeter s has an inscribed circle (incircle) with
//radius r = A/s
bool circleInTriangle(point a, point b, point c, point& ctr, double& r) {
    double ab = length(a - b), bc = length(b - c),
           ca = length(c - a);
    double s = 0.5 * (ab + bc + ca);
    r = triangleArea(ab, bc, ca) / s;

    if (fabs(r) < EPS) return 0; // no inCircle center
    double ratio = length(a - b) / length(a - c);
    point p1 = b + (vec(b, c) * (ratio / (1 + ratio)));
    ratio = length(b - a) / length(b - c);
    point p2 = a + (vec(a, c) * (ratio / (1 + ratio)));
    return intersectSegments(a, p1, b, p2, ctr); // get their intersection point
}
```

## Lines

### IsCollinear

```
//o = anlge(a) - angle(b)
//if o = 0 || o = 180 isCollinear
//else not
bool isCollinear(point a, point b, point c) {
    return fabs(cp(vec(a, b), vec(a, c))) < EPS;
}
```

### isPointOnRay

```
//o = anlge(a) - angle(b)
//if o = 0 isPointOnRay a->b
//else not
bool isPointOnRay(point a, point b, point c) {
    if (!isCollinear(a, b, c))
        return false;
    return dcmp(dp(vec(a, b), vec(a, c)), 0) == 1;
}
bool isPointOnRay(point a, point b, point c) {
    if (length(vec(a, c)) < EPS) return true;
    return same(normalize(vec(a, b)), normalize(vec(a, c)));
}
```

### isPointOnSegment

```
bool isPointOnSegment(point a, point b, point c) {
    return isPointOnRay(a, b, c) && isPointOnRay(b, a, c);
}
bool isPointOnSegment(point a, point b, point c) {
    double acb = length(vec(b, a)), ac = length(vec(c, a)), cb = length(vec(c, b));
    return dcmp(acb - (ac + cb), 0) == 0;
}
```

### distToLine

```
// dist point p2 to line p0-p1
double distToLine(point p0, point p1, point p2) {
    return fabs(cp(vec(p0, p1), vec(p0, p2)) / length(vec(p1, p0)));
}
```

### distToSegment

```
//minimum distance from point p2 to segment p0-p1
double distToSegment(point p0, point p1, point p2) {
    double d1, d2;
    point v1 = p1 - p0, v2 = p2 - p0;
    if ((d1 = dp(v1, v2)) <= 0) return length(vec(p0, p2));
    if ((d2 = dp(v1, v1)) <= d1) return length(vec(p1, p2));
    double t = d1 / d2;
    return length(vec((p0 + v1 * t), p2));
}
```

### pointToSegment

```
// minimum point in segment po-p1 to point p2
point pointToSegment(point p0, point p1, point p2) {
    double d1, d2;
    point v1 = p1 - p0, v2 = p2 - p0;
    if ((d1 = dp(v1, v2)) <= 0) return p0;
    if ((d2 = dp(v1, v1)) <= d1) return p1;
    double t = d1 / d2;
    return (p0 + v1 * t);
}
```

### intersectSegments

```
// return point intersect in line a-b with c-d using parametric equations
bool intersectSegments(point a, point b, point c, point d, point& intersect) {
    double d1 = cp(vec(b, a), vec(c, d)), d2 = cp(vec(c, a), vec(c, d)), d3 = cp(vec(b,
a), vec(c, a));
    if (fabs(d1) < EPS)
        return false; // Parallel || identical
    double t1 = d2 / d1, t2 = d3 / d1;
    intersect = a + (b - a) * t1;
    if (t1 < -EPS || t2 < -EPS || t2 > 1 + EPS)
        return false; //e.g ab is ray, cd is segment ... change to whatever
    return true;
}
```

### CCW

```
// return 1 if point c is counter-clockwise about segment a-b
// -1 if point c is clockwise about segment a-b
// 0 if c is isCollinear about a-b
int ccw(point a, point b, point c) {
    point v1(b - a), v2(c - a);
    double t = cp(v1, v2);
    if (t > EPS)
        return 1;
    if (t < -EPS)
        return -1;
    if (v1.X * v2.X < -EPS || v1.Y * v2.Y < -EPS)
        return -1;
    if (norm(v1) < norm(v2) - EPS)
        return 1;
    return 0;
}
```

## Circles

### Find circle passes with 3 points

```
// 2 points has infinite circles
// Find circle passes with 3 points, some times, there is no circle! (in case colinear)
// Draw two perpendicular lines and intersect them
pair<double, point> findCircle(point a, point b, point c) {
    //create median, vector, its perpendicular
    point m1 = (b + a) * 0.5, v1 = b - a, pv1 = point(v1.Y, -v1.X);
    point m2 = (b + c) * 0.5, v2 = b - c, pv2 = point(v2.Y, -v2.X);
    point end1 = m1 + pv1, end2 = m2 + pv2, center;
    intersectSegments(m1, end1, m2, end2, center);
    return make_pair(length(vec(center, a)), center);
}
```

### intersectLineCircle

```
// If line intersect circle at point p, and  $p = p_0 + t(p_1 - p_0)$ 
// Then  $(p - C)(p - C) = r^2$  substitute p and rearrange
//  $(p_1 - p_0)(p_1 - p_0)t^2 + 2(p_1 - p_0)(p_0 - C)t + (p_0 - C)(p_0 - C) = r^2$ ; -> Quadratic
vector<point> intersectLineCircle(point p0, point p1, point C, double r) {
    double a = dp(vec(p0, p1), vec(p0, p1)), b = 2 * dp(vec(p0, p1), vec(C, p0)),
        c = dp(vec(C, p0), vec(C, p0)) - r * r;
    double f = b * b - 4 * a * c;
    vector<point> v;
    if (dcmp(f, 0) >= 0) {
        if (dcmp(f, 0) == 0) f = 0;
        double t1 = (-b + sqrt(f)) / (2 * a);
        double t2 = (-b - sqrt(f)) / (2 * a);
        v.push_back(p0 + t1 * (p1 - p0));
        if (dcmp(f, 0) != 0) v.push_back(p0 + t2 * (p1 - p0));
    }
    return v;
}
```

### Circle Circle Intersection

```
vector<point> intersectCircleCircle(point c1, double r1, point c2, double r2) {
    // Handle infinity case first: same center/radius and  $r > 0$ 
    if (same(c1, c2) && dcmp(r1, r2) == 0 && dcmp(r1, 0) > 0)
        return vector<point>(3, c1); // infinity 2 same circles (not points)

    // Compute 2 intersection case and handle 0, 1, 2 cases
    double ang1 = angle(vec(c1, c2)), ang2 = getAngle_A_abc(r2, r1, length(vec(c1, c2)));
    if (::isnan(ang2)) ang2 = 0; // if r1 or d = 0 => nan in getAngle_A_abc (/0)

    vector<point> v(1, polar(r1, ang1 + ang2) + c1);
    // if point NOT on the 2 circles = no intersection
    if (dcmp(dp(vec(c1, v[0]), vec(c1, v[0])), r1 * r1) != 0 ||
        dcmp(dp(vec(c2, v[0]), vec(c2, v[0])), r2 * r2) != 0)
        return vector<point>();
    v.push_back(polar(r1, ang1 - ang2) + c1);
    if (same(v[0], v[1])) // if same, then 1 intersection only
        v.pop_back();
    return v;
}
```

## Circle Circle Intersection Area

```
ld circleCircleIntersectionArea(point cen1, ld r1, point cen2, ld r2) {
    ld dis = hypot(cen1.X - cen2.X, cen1.Y - cen2.Y);
    if (dis > r1 + r2) return 0;
    if (dis <= fabs(r2 - r1) && r1 >= r2)
        return PI * r2 * r2;
    if (dis <= fabs(r2 - r1) && r1 < r2)
        return PI * r1 * r1;
    ld a = r1 * r1, b = r2 * r2;
    ld ang1 = acos((a + dis * dis - b) / (2 * r1 * dis)) * 2;
    ld ang2 = acos((b + dis * dis - a) / (2 * r2 * dis)) * 2;
    ld ret1 = .5 * b * (ang2 - sin(ang2));
    ld ret2 = .5 * a * (ang1 - sin(ang1));
    return ret1 + ret2;
}
```

## Check if polygon is convex

```
// CCW function must return 0 if the 3 points are collinear
bool isConvex(vector<point>& v) {
    int n = v.size(), m = n, sum = 0;
    v.push_back(v[0]);
    v.push_back(v[1]);
    char tmp;
    for (int i = 0; i < n; i++) {
        tmp = ccw(v[i], v[i + 1], v[i + 2]);
        if (tmp) sum += tmp;
        else m--;
    }
    v.pop_back();
    v.pop_back();
    return abs(sum) == m;
}
```

## Convex hull

```
bool cmp(point a, point b) {
    return a.X < b.X || (a.X == b.X && a.Y < b.Y);
}
bool cw(point a, point b, point c) {
    return cp(vec(a, b), vec(b, c)) < 0;
}
bool ccw(point a, point b, point c) {
    return cp(vec(a, b), vec(b, c)) > 0;
}
```



```

vector<point> convex_hull(vector<point>& p) {
    if (p.size() == 1) return p;
    sort(p.begin(), p.end(), &cmp);
    point p1 = p[0], p2 = p.back();
    vector<point> up, down;
    up.push_back(p1);
    down.push_back(p1);
    for (int i = 1; i < (int)p.size(); i++) {
        if (i == p.size() - 1 || cw(p1, p[i], p2)) {
            while (up.size() >= 2
                && !cw(up[up.size() - 2], up[up.size() - 1], p[i]))
                up.pop_back();
            up.push_back(p[i]);
        }
        if (i == p.size() - 1 || ccw(p1, p[i], p2)) {
            while (down.size() >= 2
                && !ccw(down[down.size() - 2], down[down.size() - 1], p[i]))
                down.pop_back();
            down.push_back(p[i]);
        }
    }
    vector<point> convex;
    for (int i = 0; i < (int)down.size(); i++)
        convex.push_back(down[i]);
    for (int i = up.size() - 2; i > 0; i--)
        convex.push_back(up[i]);
    return convex;
}

```

### Point in polygon $O(\log(n))$

```

void prepare(vector<point>& polygon) {
    int pos = 0;
    for (int i = 0; i < sz(polygon); i++) {
        if (make_pair(polygon[i].X, polygon[i].Y)
            < make_pair(polygon[pos].X, polygon[pos].Y))
            pos = i;
    }
    rotate(polygon.begin(), polygon.begin() + pos, polygon.end());
}

int dcmp(double x, double y) {
    if (fabs(x - y) <= EPS)
        return 0;
    return (x < y ? -1 : 1);
}

ll cross(point a, point b, point c) {
    return cp(vec(a, b), vec(a, c));
}

bool isPointOnSegment(point a, point b, point c) {
    double acb = length(a - b), ac = length(a - c), cb = length(b - c);
    return dcmp(acb - (ac + cb), 0) == 0;
}

```

```

//call prepare(polygon) before start
bool pointInConvexPolygon(const vector<point>& polygon, point pt) {
    if (isPointOnSegment(polygon[0], polygon.back(), pt))
        return true;
    if (cross(polygon[0], polygon.back(), pt) > 0)
        return false;
    if (cross(polygon[0], polygon[1], pt) < 0)
        return false;
    if (polygon.size() == 2)
        return false;
    int st = 2, ed = sz(polygon) - 2, ans = 1;
    while (st <= ed) {
        int md = st + ed >> 1;
        if (cross(polygon[0], polygon[md], pt) >= 0)
            st = md + 1, ans = md;
        else
            ed = md - 1;
    }
    return cross(polygon[ans], polygon[ans + 1], pt) >= 0;
}

```

### line sweep for lines intersections

```

struct segment {
    point p, q;
    int idx;
    segment() {
    }
    segment(point a, point b, int idx) :
        p(a), q(b), idx(idx) {
    }
    double CY(int x) const {
        if (p.X == q.X)
            return p.Y;
        double t = 1.0 * (x - p.X) / (q.X - p.X);
        return p.Y + (q.Y - p.Y) * t;
    }
    bool operator<(const segment& o) const {
        if (p == o.p && q == o.q)
            return idx < o.idx;
        int maxX = max(p.X, o.p.X);
        int res = dcmp(CY(maxX), o.CY(maxX));
        if (res == 0)
            return idx < o.idx;
        return res < 0;
    }
};

```

```

struct event {
    bool entry;
    point p;
    int idx;
    event(point p, bool entry, int idx) :
        p(p), entry(entry), idx(idx) {
    }
}

```

```

    bool operator<(const event& o) const {
        if (p.X != o.p.X)
            return p.X < o.p.X;
        if (entry != o.entry)
            return entry > o.entry;
        return idx < o.idx;
    }
};

int ccw(point a, point b, point c) {
    point v1(b - a), v2(c - a);
    double t = cp(v1, v2);
    if (t > EPS)
        return 1;
    if (t < -EPS)
        return -1;
    if (v1.X * v2.X < -EPS || v1.Y * v2.Y < -EPS)
        return -1;
    if (norm(v1) < norm(v2) - EPS)
        return 1;
    return 0;
}

bool intersect(point p1, point p2, point p3, point p4) {
    // special case handling if a segment is just a point
    bool x = (p1 == p2), y = (p3 == p4);
    if (x && y) return p1 == p3;
    if (x) return ccw(p3, p4, p1) == 0;
    if (y) return ccw(p1, p2, p3) == 0;
    return ccw(p1, p2, p3) * ccw(p1, p2, p4) <= 0
        && ccw(p3, p4, p1) * ccw(p3, p4, p2) <= 0;
}

pair<int, int> solve(vector<segment> v) {
    int n = v.size();
    vector<event> events;
    for (int i = 0; i < n; i++) {
        point& p = v[i].p;
        point& q = v[i].q;
        if (q.X < p.X || (q.X == p.X && q.Y < p.Y))
            swap(p, q);
        events.push_back(event(p, true, i));
        events.push_back(event(q, false, i));
    }
    sort(all(events));
    set<segment> st;
    auto before = [&](set<segment>::iterator it) {
        if (it == st.begin())
            return st.end();
        return --it;
    };
    auto check = [&](set<segment>::iterator x,
        set<segment>::iterator y) -> bool {
        if (x == st.end() || y == st.end())
            return false;
        return intersect(x->p, x->q, y->p, y->q);
    };
};

```

```

for (auto& cur : events) {
    if (cur.entry) {
        auto it = st.insert(v[cur.idx]).first;
        auto below = before(it);
        auto above = next(it);
        if (check(below, it)) return { below->idx, it->idx };
        if (check(above, it)) return { above->idx, it->idx };
    }
    else {
        auto it = st.find(v[cur.idx]);
        auto below = before(it);
        auto above = next(it);
        if (check(below, above)) return { below->idx, above->idx };
        st.erase(it);
    }
}
return { -1, -1 };
}

```

## Pyramid Volume

```

ld pyramidVolume(ld ab, ld ac, ld ad, ld bc, ld bd, ld cd) {
    ld w = ab, v = ac, u = ad, U = bc, V = bd, W = cd;
    ld A = (w - U + v) * (U + v + w);
    ld x = (U - v + w) * (v - w + U);
    ld B = (u - V + w) * (V + w + u);
    ld y = (V - w + u) * (w - u + V);
    ld Z = (v - W + u) * (W + u + v);
    ld z = (W - u + v) * (u - v + W);
    ld a = sqrt(x * B * Z);
    ld b = sqrt(A * y * Z);
    ld c = sqrt(A * B * z);
    ld d = sqrt(x * y * z);
    ld volume = -a + b + c + d;
    volume *= a - b + c + d;
    volume *= a + b - c + d;
    volume *= a + b + c - d;
    volume = sqrt(volume) / (192.0 * u * v * w);
    return volume;
}

```

## Math notes

- To get the summation of  $a^n$  sequence from  $x$  to  $y$ , transform  $a^n$  form to  $a^{\{n\}}$  form (factorial polynomial =  $a * (a - 1) * (a - 2) \dots (a - n + 1)$ ) then integrate the equation with limits from  $x$  to  $(y + 1)$
- To get the summation law from known values substitute and compute the equation:  $S(0 \rightarrow n) + x[n + 1] = x[0] + S(1 \rightarrow n + 1)$ , then take a factor from  $S(1 \rightarrow n + 1)$  to transform it to  $S(0 \rightarrow n)$ , so you can substitute and compute  $S(0 \rightarrow n)$  easily. You can use replacement in the original formula and the bounds of the summation (like replacing every  $(k - j)$  by  $j$ ). You can also multiply the summand by  $k$  - it might help. For example:
  - $S(0 \rightarrow n) = 2^0 + 2^1 + \dots + 2^n$
  - $S(0 \rightarrow n) + 2^{(n + 1)} = 2^0 + (2^1 + 2^2 + \dots + 2^{(n + 1)})$
  - $S(0 \rightarrow n) + 2^{(n + 1)} = 1 + 2 * S(0 \rightarrow n)$
  - $S(0 \rightarrow n) = 2^{(n + 1)} - 1$

$$\circ (nC_0)^2 + (nC_1)^2 + (nC_2)^2 + \dots + (nC_n)^2 = 2nC_n$$

## Congruence

- IF  $ax=by \pmod n$ , THEN  $x=y \pmod n$  IFF  $\gcd(a, n)=1$  (Division Rule)
- IF  $a=b \pmod m$ , THEN  $a^n=b^n \pmod m$  (Powers Rule)
- $x^n = x^{(\phi(m) + (n \% \phi(m))) \% m}$  (Euler Totient Function Application)
- ALL  $nCr$  for  $n=\text{prime}$ ,  $r!=0$ ,  $r!=n$  is divisible by  $n$  ( $nCr$  Rule)
- Distribution is ok IFF  $p$  is prime:  $(x+y)^p=x^p+y^p \pmod p$  (Powers Distribution Rule)
- IF  $a=b \pmod m$ , THEN  $a+c=b+c \pmod m$  (Adding Rule)
- IF  $a=b \pmod m$ , THEN  $ac=bc \pmod m$  (Multiplication Rule)
- IF  $a=b \pmod m$ , THEN  $a^{m^k}=b^{m^k} \pmod{m^{(k+1)}}$ ;  $k = 0,1,2,3,\dots$
- To find MOD INVERSE for an arbitrary number  $a$  and an arbitrary mod  $m$  provided that  $\gcd(a,m)=1$ , just use extended euclidean algorithm to find  $x$ :  $a*x-m*q=1$
- The sum of the Euler Totient function values over all the divisors of  $n$  is equal to  $n$
- $\phi(n!) = (n - \text{isPrime}(n)) * \phi((n - 1)!)$
- summation over totient of every divisor of  $n = n$

## Geometry

- Translation can be done by adding another dimension and translating by applying the translation to the new basis vector and holding the original basis vectors
- Rotation can be done by knowing the angle only and rotating the original basis vectors, and so does the reflection