d) $f(N) = N\log^2 N + N\log\log N$

$$\lim_{N\to\infty} \frac{f(N)}{N\log^2 N} = 1$$

$\rightarrow f(N) = O(N\log^2 N)$

e) $f(N) = N^2(N+2N) + (N^3 \times N^3)$

$$\lim_{N\to\infty} \frac{f(N)}{N^6} = 1$$

$\rightarrow f(N) = O(N^6)$

f) $f(N) =$

$$\lim_{N\to\infty} \frac{f(N)}{N^{1/4}} = 1$$

$\rightarrow f(N) = O(N^{1/4})$

Exercice 2 :

(A) - The outer loop runs n^2 times

- The first inner loop runs n times
- The last inner loop runs 5000 time so $O(1)$

- The second inner loop runs i times so

$$n^2 + n^2 - 1 + \ldots + 1 = \frac{(n^2+1)(n^2)}{2} = O(n^4)$$

so $T(N) = O(n^2 \times n) + O(n^4)$.

$T(N) = O(n^4)$

(B) we for K such that

$$\frac{n}{2^K} < 100$$

$$K < \log \frac{n}{100}$$

if $(n < 10)$ $\rightarrow O(1)$.

else if $(n < 100)$ return fct2$(n-2, m)$
$\rightarrow O(1)$.

else
return $(n/2, m)$ $O(\log n)$.

$T(n) = O(\log n)$.

(C) The outer loop n times

- The first inner loop $O(n^2)$.
- The second inner loop.

$$O(1+2+\dots+n-1)=O\left(\frac{n(n-1)}{2}\right)=O(n^2)$$

- The last is $O(1)$.

So $T(n)=O(n^2)$.

Exercice 3:

The first version:

$$n+n-1+\dots+1=\frac{(n+1)n}{2}=\frac{n^2+n}{2}$$
$$=O(n^2)$$

because it traverse n times then n-1 until 1.

- The second version :

  - it traverse firstly to reach the end of the linked list then it traverse to the first node and print each node

  so $O(n)$.

The second version is faster.