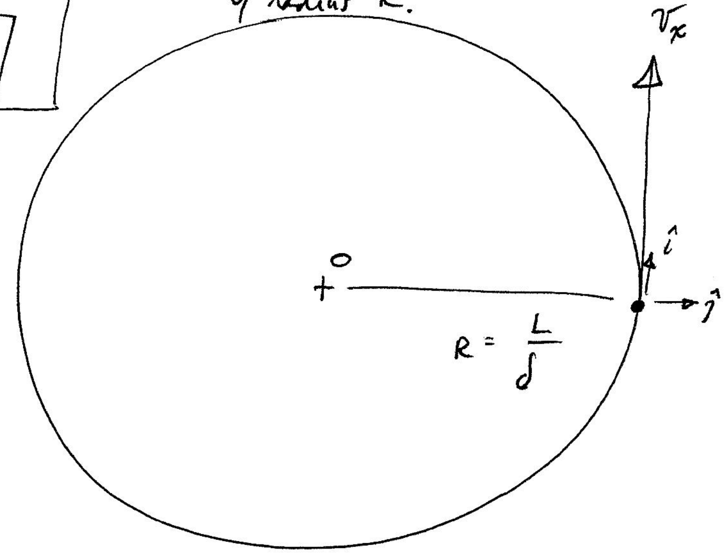
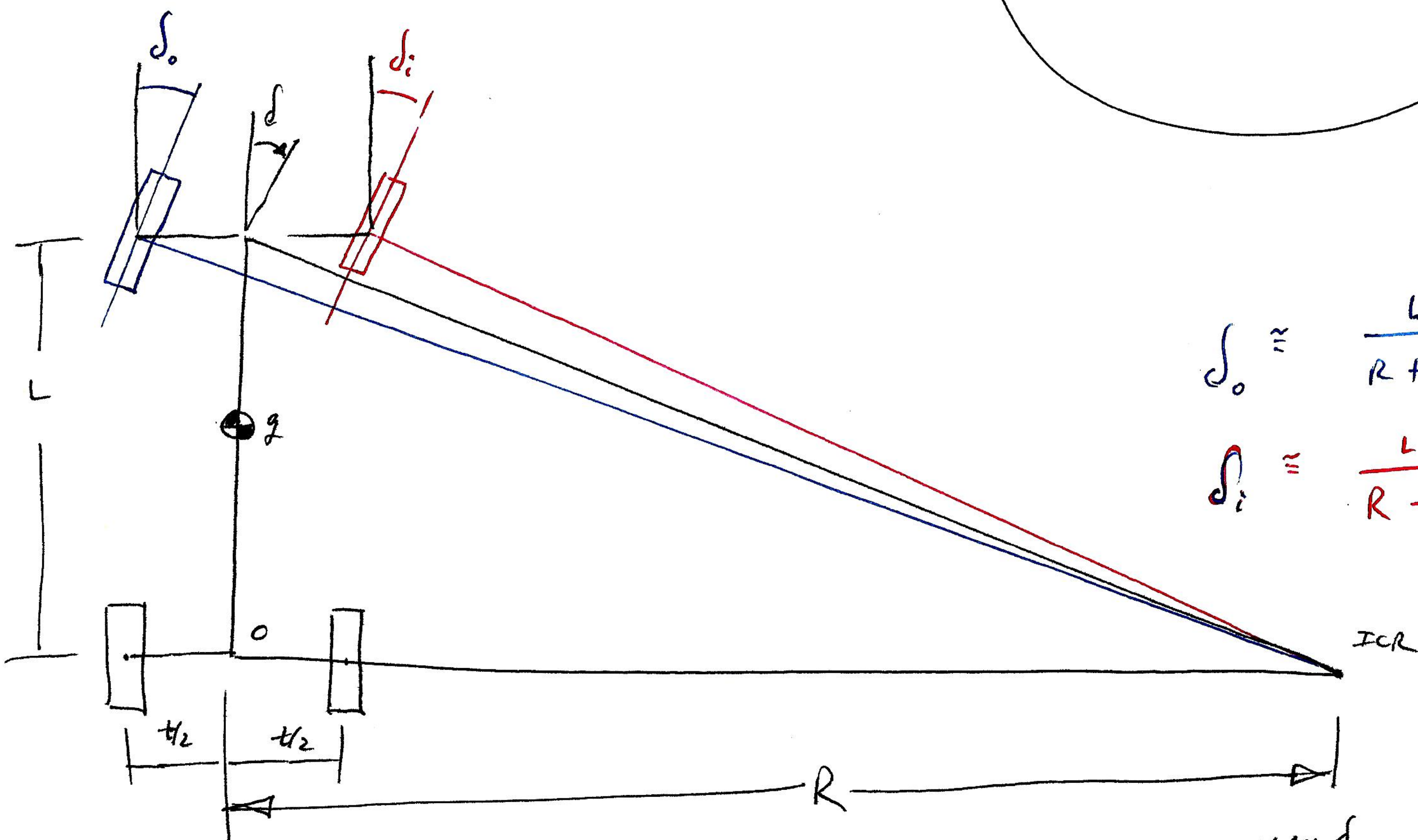
A KINEMATIC 2D STEERING MODEL WITH VEHICLE WHEELBASE LENGTH + TRACK WIDTH INCORPORATED WELL ENOUGH TO ANIMATE A 4-wheel VEHICLE

COMPLETE DEC 2015

if $\delta_{ack} = \frac{L}{R}$ + is the average of $\delta_o + \delta_i$ from Gillespie 1992, p. 196, then $\boxed{R = \frac{L}{\delta_{ack}}}$ defines the radius about which the vehicle steers at low speeds (no α_f or α_r).

the body-fixed velocity vector is $\vec{v}^{\,xy} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}$ while turning on circle of radius R.





$$\delta_o \cong \frac{L}{R + t/2}$$

$$\delta_i \cong \frac{L}{R - t/2}$$

$$\tan(\delta) = \frac{L}{R}$$

For small δ,

$$\tan(\delta) = \frac{s(\delta)}{c(\delta)} \approx \frac{\delta}{1} = \delta$$

THIS MEANS THE VEHICLE ROLLS "FORWARD" AS IT DRIVES IN A CIRCLE OF RADIUS, R.

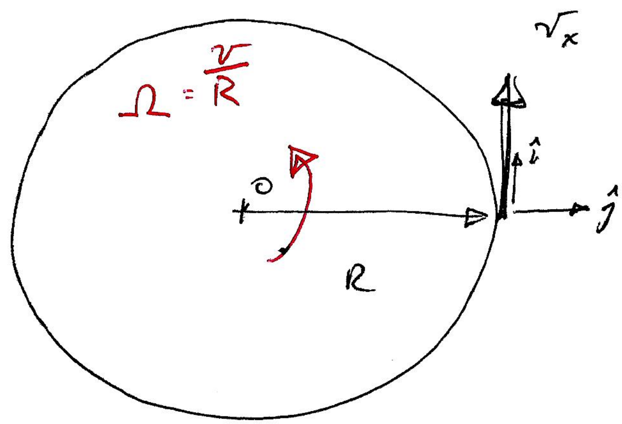
$\vec{v}^{\,xy}$) – velocity expressed in body-fixed coordinates.

Q: if $\vec{v}^{\,xy} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}$ REGARDLESS OF TURN RADIUS, THEN HOW DOES STEERING INPUT CHANGE A KINEMATIC MODEL'S YAW ANGLE?

ψ – VEHICLE HEADING (DIRECTION)

$\dot{\psi}$ – VEHICLE YAW RATE

A:



$R = \frac{L}{\tan(\delta)}$

$\dot{\psi} = \frac{v_x\, t(\delta)}{L}$

Recall..

$v = r \cdot \omega$

so:

$v_x = R \cdot \Omega$

$v_x = R \cdot \dot{\psi}$ if $R \to$ huge, $\dot{\psi} \to 0$

$\hookrightarrow \dot{\psi} = \frac{v_x}{R}$ if $R \to$ small, $\dot{\psi} \to$ large

RECALLING $R \approx \frac{L}{\delta_{ack}}$

$\dot{\psi} = \frac{v_x}{R} = v_x \cdot \left(\frac{1}{R}\right)$

$\dot{\psi} = v_x \cdot \left(\frac{\delta_{ack}}{L}\right)$

$\dot{\psi} = \left(\frac{v_x}{L}\right) \cdot \delta_{ack}$

↑ STEER ANGLE IN RADIANS SPECIFIED BY DRIVER.

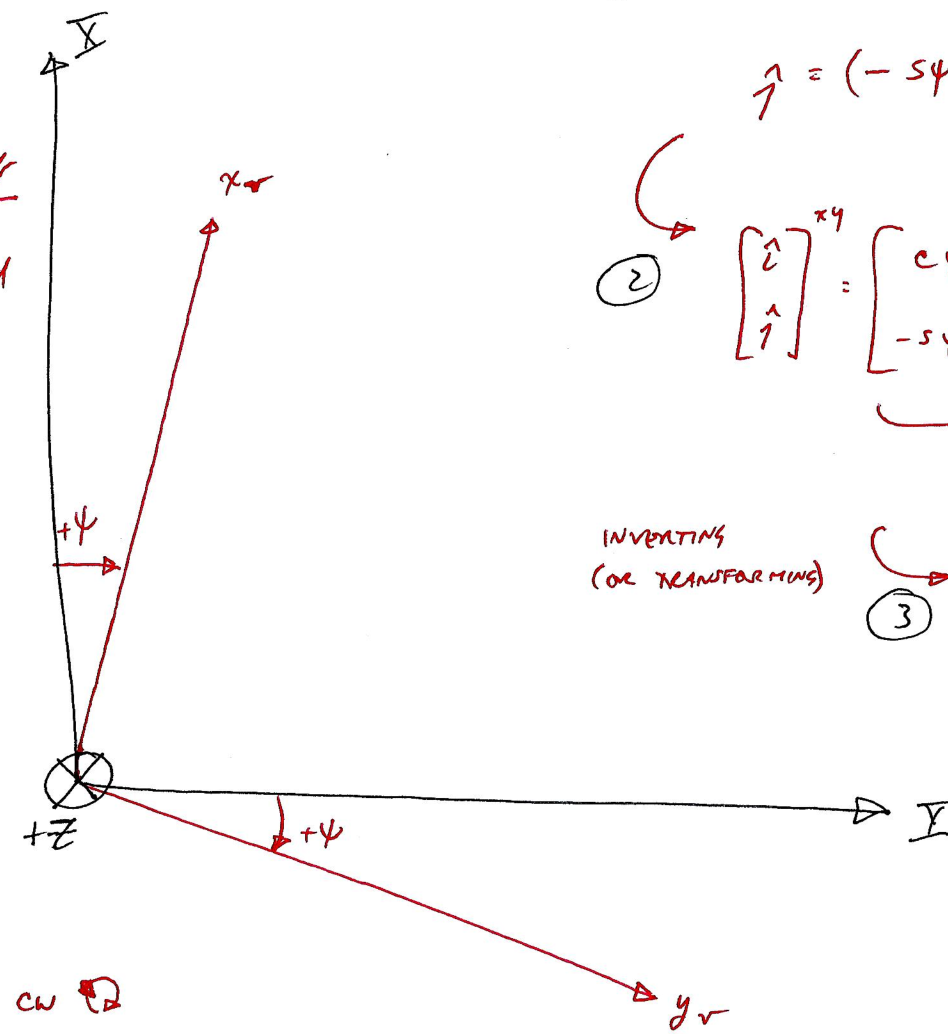
SAE COORDS
+Z "down"

SAE body-fixed coords

x_v - points out vehicle's front

y_v - points out passenger side window

ψ - vehicle heading + yaw angle w.r.t. terrain frame, XY



STARTING HERE

(1) $$\hat{i} = (c\psi)\hat{I} + (+s\psi)\hat{J}$$

$$\hat{j} = (-s\psi)\hat{I} + (c\psi)\hat{J}$$

(2) $$\begin{bmatrix}\hat{i}\\ \hat{j}\end{bmatrix}^{xy} = \underbrace{\begin{bmatrix}c\psi & s\psi\\ -s\psi & c\psi\end{bmatrix}}_{[T]^{xy}_{XY}} \cdot \begin{bmatrix}\hat{I}\\ \hat{J}\end{bmatrix}^{XY}$$

INVERTING (OR TRANSFORMING)

(3) $$\begin{bmatrix}\hat{I}\\ \hat{J}\end{bmatrix} = \underbrace{\begin{bmatrix}c\psi & -s\psi\\ s\psi & c\psi\end{bmatrix}}_{[T^T]^{XY}_{xy}} \cdot \begin{bmatrix}\hat{i}\\ \hat{j}\end{bmatrix}^{xy}$$

(4) $$\vec{v}_{veh}^{\,XY} = [T^T] \cdot \vec{v}_{veh}^{\,xy}$$

$$\begin{bmatrix}v_X\\ v_Y\end{bmatrix}^{XY} = \underbrace{\begin{bmatrix}c\psi & -s\psi\\ s\psi & c\psi\end{bmatrix}}_{T^T} \cdot \begin{bmatrix}v_x\\ 0\end{bmatrix}^{xy}$$

(5) $$v_{veh}^{X} = (c\psi \cdot v_x)\ \hat{I}$$

$$v_{veh}^{Y} = (s\psi \cdot v_x)\ \hat{J}$$

(6) $$X_{veh} = \int v_{veh}^{X} \cdot dt$$

$$Y_{veh} = \int v_{veh}^{Y} \cdot dt$$

(7) $$\psi = \int \dot{\psi} \cdot dt$$

$$+\quad \dot{\psi} = \left(\frac{v_x}{L}\right) \cdot \delta$$

$+\psi$ is CW

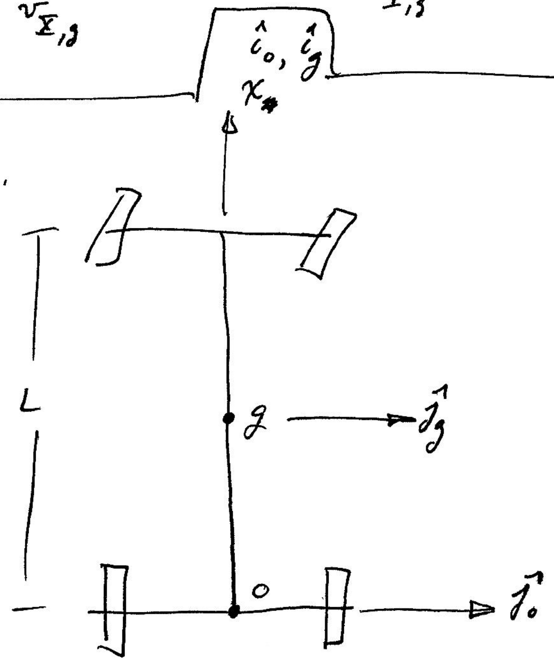
$$\begin{bmatrix} v_{X\,vel\,g} \\ v_{Y\,vel\,g} \end{bmatrix}^{XY} = \underbrace{\begin{bmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{bmatrix}}_{T} \cdot \begin{bmatrix} v_x \\ \frac{L}{2}\dot{\psi} \end{bmatrix}^{xy}$$

USE THIS TO PERFORM THE COORD TRANSFORM FROM xy TO XY @ pt g.

$$\begin{bmatrix} v_{X,vel,g} \\ v_{Y,vel,g} \end{bmatrix}^{XY} = \underbrace{\left(v_x \cdot c\psi - \left(\tfrac{L}{2}\dot{\psi}\right) s\psi \right)}_{v_{X,g}} \hat{I} + \left(\underbrace{v_x s\psi + \tfrac{L}{2}\dot{\psi} c\psi}_{v_{Y,g}} \right) \hat{J}$$

VELOCITY OF "g" IN TERRAIN FRAME.

INTEGRATE TO GET XY OF VEHICLE FOR ANIMATION



PROBLEM: CAR ANIMATES POORLY WHEN GRAPHICS ARE CENTERED @ g (or "o") AND MOTION IS SPECIFIED WITH $\vec{v}_o^{\,xy}$

SOLUTION: SPECIFY v_g^{xy} + TRANSFORM TO XY FRAME

- ACKERMANN STEERING IS BEST DEFINED AT "o". $\vec{v}_o^{\,xy}$ IS KNOWN.

$$\vec{v}_o^{\,xy} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}$$

- SPECIFYING $\vec{v}_g^{\,xy}$ AS A FUNCTION OF $\vec{v}_o^{\,xy}$ IS DONE BY TRANSLATING BY $\left(\frac{L}{2}\right)\hat{i}$ TO CG (or GEOMETRIC CENTER)

- $\cancel{\vec{v}_g^{\,xy} = \vec{v}_o^{\,xy} + \vec{\omega}^{\,xy} \times \vec{v}_o^{\,xy}}$

$$\boxed{\vec{v}_g^{\,xy} = \vec{v}_o^{\,xy} + \vec{\omega}^{\,xy} \times \vec{r}_{og}}$$

$$\vec{r}_{og} = \left(\frac{L}{2}\right)\hat{i} \qquad \vec{v}_o^{\,xy} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}^{xy}$$

$$\vec{v}_g^{\,xy} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \dot{\psi} \\ \left(\frac{L}{2}\right) & 0 & 0 \end{vmatrix} = \begin{matrix} (0-0)\,\hat{i} \\ -\left(0 - \left(\frac{L}{2}\right)\dot{\psi}\right)\hat{j} \\ +(0-0)\,\hat{k} \end{matrix} \;+ \vec{v}_o^{\,xy}$$

$$\vec{v}_g^{\,xy} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}^{xy} + \begin{bmatrix} 0 \\ \left(\frac{L}{2}\right)\dot{\psi} \end{bmatrix}^{xy} \sim \vec{v}_g = (v_x)\,\hat{i} + \left[\left(\frac{L}{2}\right)\dot{\psi}\right]\hat{j}$$

IN xy COORDS. THEN TRANSFORM TO XY + INTEGRATE.