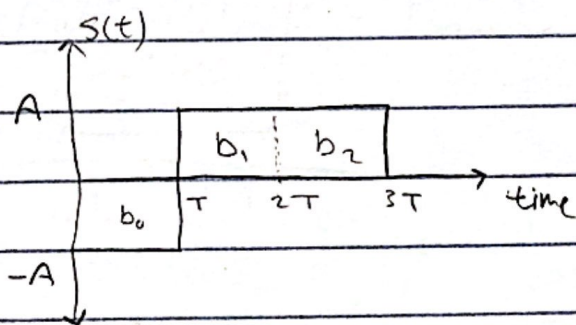# Assignment 2

* Part 1

a) baseband plot for "011" sequence
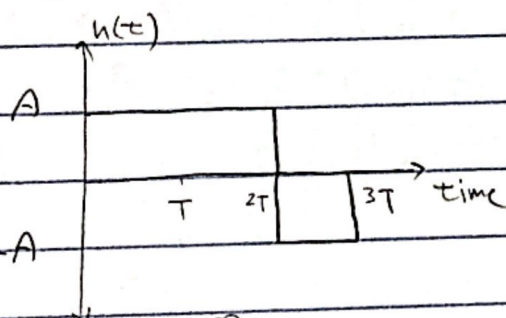

b,c) Matched filter output plot due to signal only

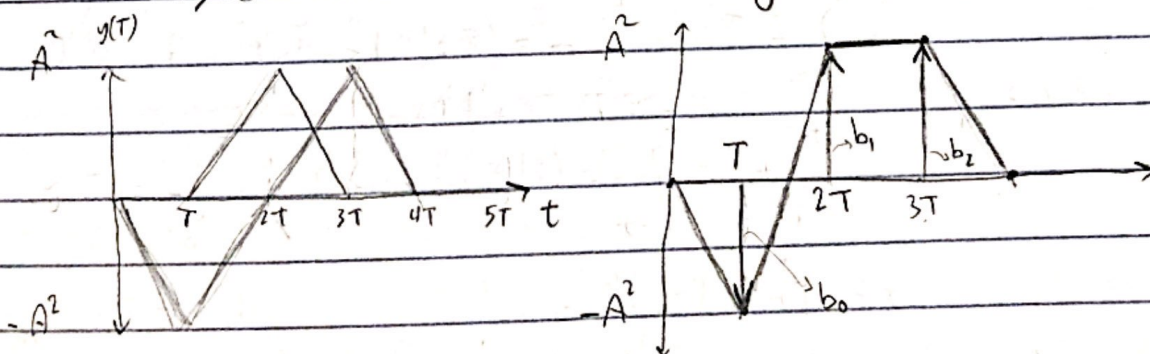$$y(t) = S(t) * h(t) \qquad \boxed{h(t) = S(T-t)}$$

flip around line of symmetry

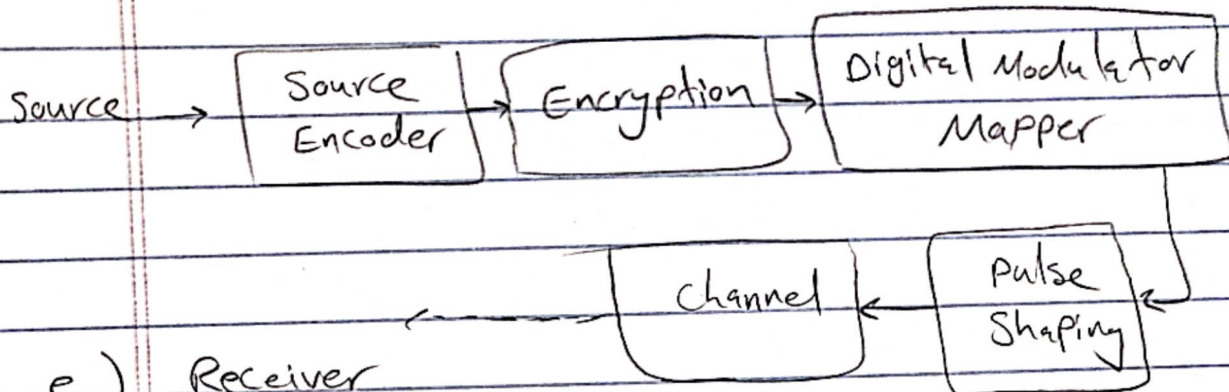$$h(T-t) = S(T-(T-t))$$
$$= S(t)$$

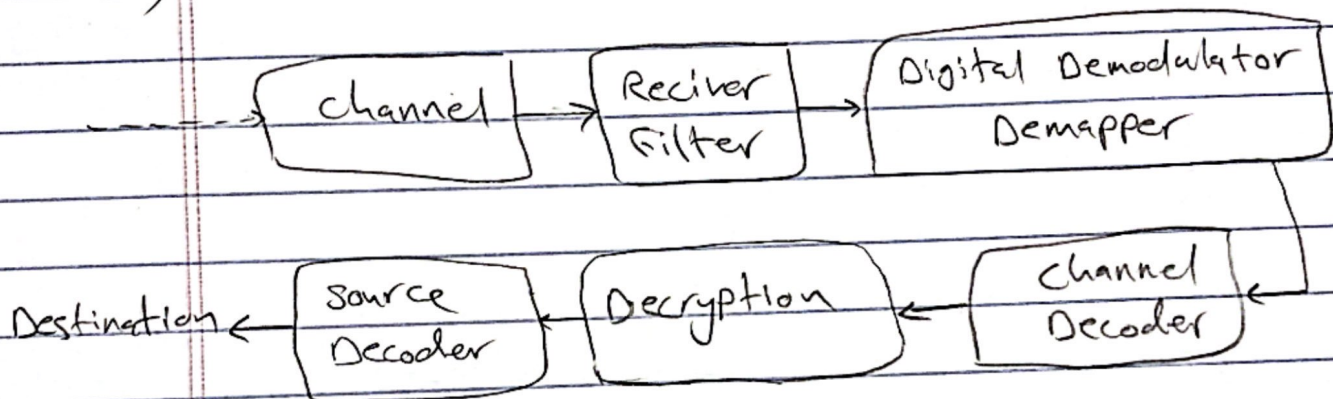$$y(t) = \int_{-\infty}^{\infty} S(\tau) h(t-\tau) \, d\tau \quad, \quad y(T) = \int_{-\infty}^{\infty} S(\tau)^2 \, d\tau$$

→ to make calculations easy we will compute this for every bit

d) Transmitter

```
Source ──→ ┌──────────┐   ┌──────────┐   ┌────────────────────┐
           │  Source  │──→│Encryption│──→│ Digital Modulator  │
           │ Encoder  │   │          │   │      Mapper        │
           └──────────┘   └──────────┘   └────────────────────┘
                                                    │
           ┌──────────┐   ┌──────────┐              ↓
       ←── │ Channel  │←──│  Pulse   │←─────────────┘
           │          │   │ Shaping  │
           └──────────┘   └──────────┘
```

e) Receiver

```
   ┌──────────┐   ┌──────────┐   ┌────────────────────┐
──→│ Channel  │──→│  Reciver │──→│ Digital Demodulator│
   │          │   │  Filter  │   │     Demapper       │
   └──────────┘   └──────────┘   └────────────────────┘
                                             │
   ┌──────────┐   ┌──────────┐   ┌──────────┐│
   │  Source  │←──│Decryption│←──│ Channel  │←┘
   │ Decoder  │   │          │   │ Decoder  │
   └──────────┘   └──────────┘   └──────────┘
        │
Destination ←
```

Part 2: Simulation

a) $y(t) = r(t) * h(t)$

$$g(t) \begin{cases} \rightarrow -A & \text{'0'} \\ \rightarrow A & \text{'1'} \end{cases}$$

$$\boxed{\begin{array}{l} A=1 \\ T=1 \end{array}}$$

$$y(T) = \begin{cases} \rightarrow -A^2 T + n(T) & \text{'0'} = n(T) - 1 \\ \rightarrow A^2 T + n(T) & \text{'1'} = n(T) + 1 \end{cases}$$

$n(T) = w(t) * h(t)$

$M_y = E[y(T)] = E[g_o(T)] + E[n(T)]$

$$E[g_o(t)] = \begin{cases} \rightarrow -1 & \text{'0'} \\ \rightarrow 1 & \text{'1'} \end{cases}$$

$E[n(T)] = E[\int w(\tau) h(T-\tau) d\tau]$
$\qquad = E[\int w(\tau) g(\tau) d\tau] = E[\int_0^T A w(\tau) d\tau]$
$\qquad = E[\int_0^1 w(\tau) d\tau] = 0 \qquad E[w(\tau)] = 0$

$$M_y = \begin{cases} \rightarrow -1 & \text{'0'} \\ \rightarrow 1 & \text{'1'} \end{cases}$$

$\sigma_y^2 = var[y(T)] = E[(g_o(T) + n(T) - M_y)^2]$
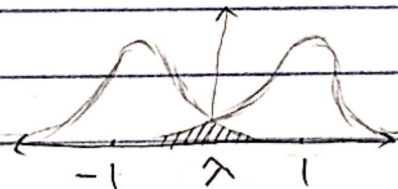$\qquad = E[n(T)^2] = P_n$ "Power of noise"

$\sigma_y^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2} \int_0^T |h(t)|^2 dt$

$\boxed{\sigma_y^2 = \frac{N_0}{2} A^2 T = \frac{N_0}{2}}$

$P(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-M_y)^2}{2\sigma^2}\right)$

$P(y|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y+1)^2}{N_0}\right)$

$P(y|1) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y-1)^2}{N_0}\right)$

$\lambda = \frac{1-1}{2} = 0$

$$P(e|0) = \int_\lambda^\infty P(y|0)\,dy = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y+1)^2}{N_0}\right) dy$$

let $z = \dfrac{y+1}{\sqrt{N_0}}$ $\qquad dz = \dfrac{1}{\sqrt{N_0}}\,dy$ $\qquad dy = \sqrt{N_0}\,dz$

$$P(e|0) = \frac{1}{\sqrt{\pi N_0}} \int_{\frac{1}{\sqrt{N_0}}}^\infty \exp(-z^2)\,\sqrt{N_0}\,dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{1}{\sqrt{N_0}}}^\infty \exp(-z^2)\,dz \qquad \frac{1}{2}$$

$$P(e) = P(0)\,P(e|0) + P(1)\,P(e|1) \qquad\qquad P(0) = P(1) = \frac{1}{2}$$

$$= \frac{1}{2}\left[P(e|0) + P(e|1)\right] \qquad\qquad P(e|0) = P(e|1)$$

$$= \frac{1}{2} \times 2\,P(e|0) = P(e|0)$$

$$\boxed{= \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{\sqrt{N_0}}\right)} \qquad \#$$

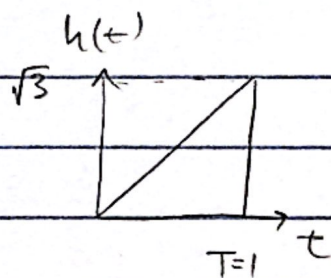b) $h(t) = \delta(t)$ $\qquad y(t) = r(t) * \delta(t)$ $\qquad n(T) = 1$

$$y(T) = \begin{cases} n(T) = 1 \\ \\ n(T) + 1 \end{cases}$$

It is the same as (a)

$$\boxed{P(e) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{\sqrt{N_0}}\right)} \quad \#$$

c) $y(t) = r(t) * h(t)$

$$g_0(T) = \int_0^T h(\tau) g(T-\tau) d\tau$$

$$= \int_0^T \frac{\sqrt{3}}{2} A\tau \, d\tau = \frac{\sqrt{3}}{2} AT = \frac{\sqrt{3}}{2}$$

$$y(T) = \begin{cases} -\frac{\sqrt{3}}{2} + n(T) & (0) \\ \frac{\sqrt{3}}{2} + n(T) & (1) \end{cases} \qquad \mu_y = \begin{cases} -\frac{\sqrt{3}}{2} & (0) \\ \frac{\sqrt{3}}{2} & (1) \end{cases}$$

$$\sigma_y^2 = \frac{N_0}{2} \int_0^T |h(t)|^2 dt = \frac{N_0}{2} \int_0^T (\sqrt{3}\, t)^2 dt = \frac{N_0}{2} \int_0^T 3t^2 dt$$

$$= \frac{3}{2} N_0 T^2 = \frac{3}{2} N_0$$

$$P(e) = P(e|0) \qquad P(y|0) = \frac{1}{\sqrt{3\pi N_0}} \exp\left(\frac{(y + \sqrt{3}/2)^2}{3N_0} \right)$$

$$P(e|0) = \int_0^\infty P(y|0) dy \qquad z = \frac{y + \frac{\sqrt{3}}{2}}{\sqrt{3N_0}} \qquad dz = \frac{dy}{\sqrt{3N_0}} \qquad dy = \sqrt{3N_0}\, dz$$

$$P(e) = \frac{1}{\sqrt{3\pi N_0}} \int_{\frac{1}{2\sqrt{N}}}^\infty \exp(-z^2) \sqrt{3N_0}\, dz = \frac{1}{\sqrt{\pi}} \int_{\frac{1}{2\sqrt{N}}}^\infty \exp(-z^2) dz$$

$$\boxed{P(e) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2\sqrt{N_0}} \right)} \qquad \#$$