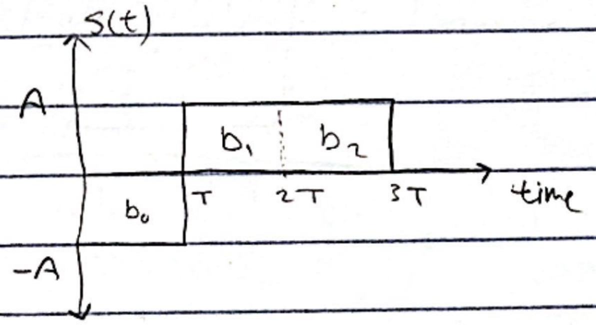
Assignment 2

* Part 1

a) baseband plot for "011" sequence



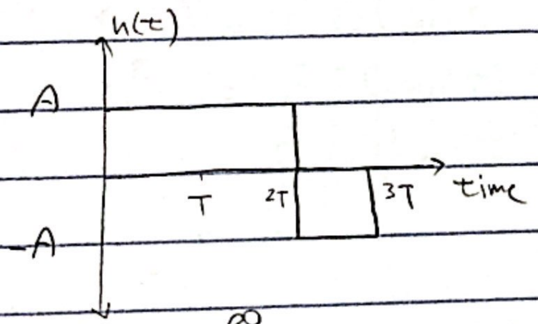
b,c) Matched filter output plot due to signal only

$y(t) = s(t) * h(t)$ $\qquad$ $\boxed{h(t) = s(T-t)}$

flip around line of symmetry

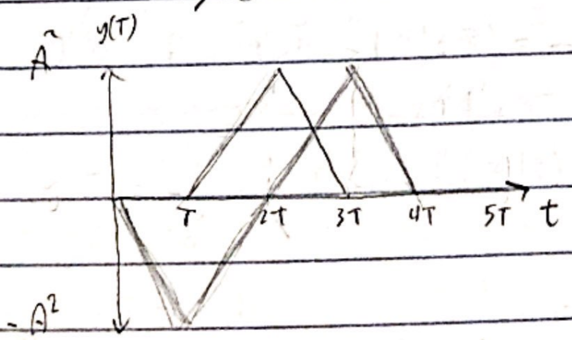
$h(T-t) = s(T-(T-t))$

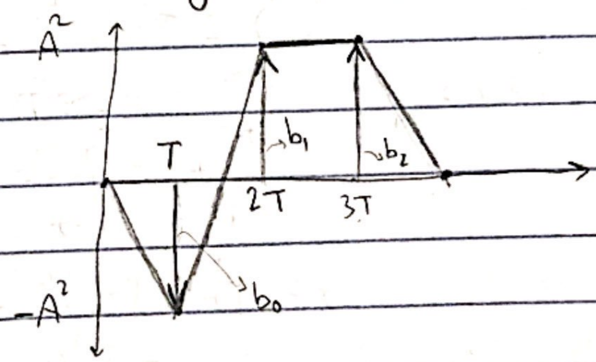
$= s(t)$



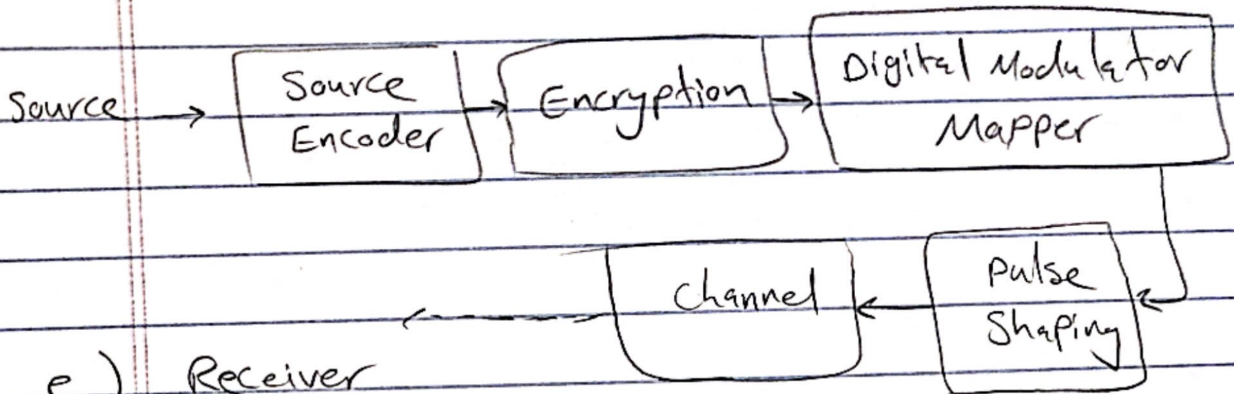
$$y(t) = \int_{-\infty}^{\infty} s(\tau)h(t-\tau)\,d\tau \,, \quad y(T) = \int_{-\infty}^{\infty} s(\tau)^2\,d\tau$$

→ to make calculations easy we will compute this for every bit

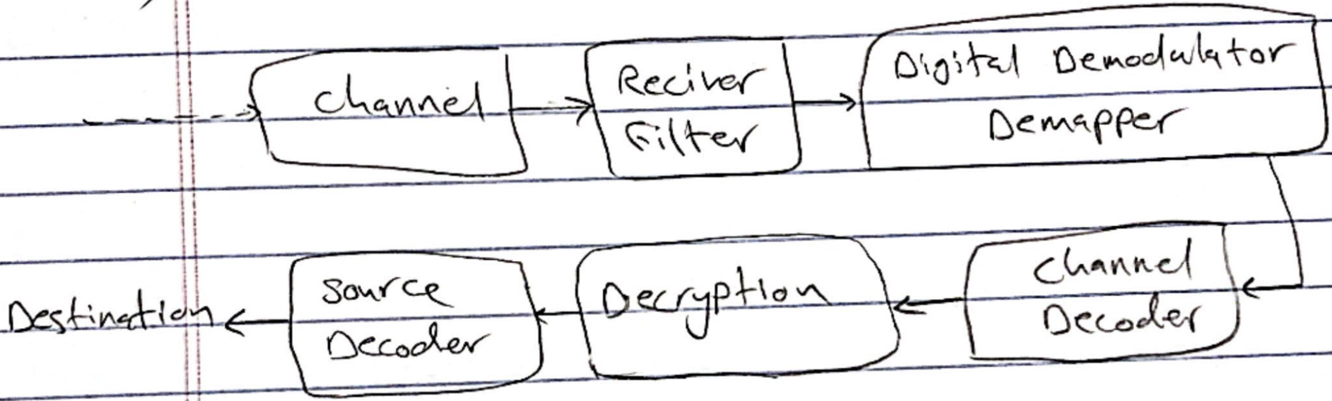




d) Transmitter



e) Receiver



* part 2: simulation

$$g(t)\begin{cases} -A & \text{'0'} \\ A & \text{'1'} \end{cases}$$

$$\boxed{\begin{matrix} A=1 \\ T=1 \end{matrix}}$$

a) $y(t) = r(t) * h(t)$

$$y(T) = \begin{cases} -A^2T + n(T) & \text{'0'} = n(T) - 1 \\ A^2T + n(T) & \text{'1'} = n(T) + 1 \end{cases}$$

$$n(T) = w(t) * h(t)$$

$$\mu_y = E[y(T)] = E[g_0(T)] + E(n(T))$$

$$E[g_0(t)] = \begin{cases} -1 & \text{'0'} \\ 1 & \text{'1'} \end{cases}$$

$$E[n(T)] = E\left[\int w(\tau) h(T-\tau)\, d\tau\right]$$
$$= E\left[\int w(\tau)\, g(\tau)\, d\tau\right] = E\left[\int_0^T A\, w(\tau)\, d\tau\right]$$
$$= E\left[\int_0^1 w(\tau)\, d\tau\right] = 0 \qquad E[w(\tau)] = 0$$

$$\boxed{\mu_y = \begin{cases} -1 & \text{'0'} \\ 1 & \text{'1'} \end{cases}}$$

$$\sigma_y^2 = \mathrm{var}[y(T)] = E[(\cancel{g_0(T)} + n(T) - \cancel{\mu_y})^2]$$
$$= E[n(T)^2] = P_n \quad \text{"Power of noise"}$$

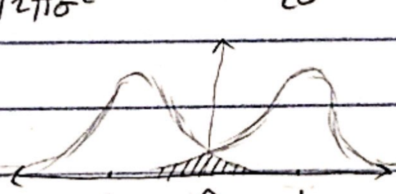
$$\sigma_y^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0}{2}\int_0^T |h(t)|^2\, dt$$

$$\boxed{\sigma_y^2 = \frac{N_0}{2} A^2 T = \frac{N_0}{2}}$$

$$p(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma^2}\right)$$

$$p(y|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y+1)^2}{N_0}\right)$$

$$p(y|1) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y-1)^2}{N_0}\right)$$



$$\lambda = \frac{1-1}{2} = 0$$

$$P(e|0) = \int_{\lambda}^{\infty} P(y|0)\,dy = \int_{0}^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y+1)^2}{N_0}\right) dy$$

let $z = \frac{y+1}{\sqrt{N_0}}$ $\quad dz = \frac{1}{\sqrt{N_0}}\,dy \quad dy = \sqrt{N_0}\,dz$

$$P(e|0) = \frac{1}{\sqrt{\pi N_0}} \int_{\frac{1}{\sqrt{N_0}}}^{\infty} \exp(-z^2)\sqrt{N_0}\,dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{1}{\sqrt{N_0}}}^{\infty} \exp(-z^2)\,dz$$

$$P(e) = P(0)P(e|0) + P(1)P(e|1) \qquad P(0) = P(1) = \frac{1}{2}$$

$$= \frac{1}{2}\left[P(e|0) + P(e|1)\right] \qquad P(e|0) = P(e|1)$$

$$= \frac{1}{2} * 2\,P(e|0) = P(e|0)$$

$$= \boxed{\frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{N_0}}\right)} \quad \#$$

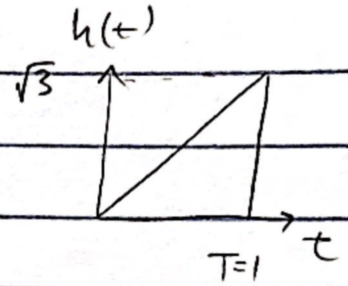
b) $h(t) = \delta(t)$ $\quad y(t) = r(t) * \delta(t)$

$$y(T) = \begin{cases} n(T) - 1 \\ n(T) + 1 \end{cases}$$

It is the same as (a)

$$\boxed{P(e) = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{N_0}}\right)} \quad \#$$

c) $y(t) = r(t) * h(t)$



$$g_0(T) = \int_0^T h(\tau)\, g(T-\tau)\, d\tau$$

$$= \int_0^T \frac{\sqrt{3}}{2} A \tau \, d\tau = \frac{\sqrt{3}}{2} A T = \frac{\sqrt{3}}{2}$$

$$y(T) = \begin{cases} -\frac{\sqrt{3}}{2} + n(T) & (0) \\ \frac{\sqrt{3}}{2} + n(T) & (1) \end{cases} \qquad \mu_y = \begin{cases} -\frac{\sqrt{3}}{2} & (0) \\ \frac{\sqrt{3}}{2} & (1) \end{cases}$$

$$\sigma_y^2 = \frac{N_0}{2} \int_0^T |h(t)|^2 dt = \frac{N_0}{2} \int_0^T (\sqrt{3}\, t)^2 dt = \frac{N_0}{2} \int_0^T 3t^2 \, dt$$

$$= \frac{N_0}{2} 3 \, \frac{1}{3} t^3 \Big|_0^T = \frac{N_0}{2} T^3 = \frac{N_0}{2}$$

$$p(e) = P(e|0) \qquad p(y|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left(\frac{(y+\sqrt{3}/2)^2}{N_0} \right)$$

$$P(e|0) = \int_0^\infty p(y|0)\, dy \qquad z = \frac{y + \frac{\sqrt{3}}{2}}{\sqrt{N_0}} \qquad dz = \frac{dy}{\sqrt{N_0}} \qquad dy = \sqrt{N_0}\, dz$$

$$P(e) = \frac{1}{\sqrt{\pi N_0}} \int_{\frac{\sqrt{3}}{2\sqrt{N_0}}}^{\infty} \exp(-z^2) \sqrt{N_0}\, dz = \frac{1}{\sqrt{\pi}} \int_{\frac{\sqrt{3}}{2\sqrt{N}}}^{\infty} \exp(-z^2)\, dz$$

$$\boxed{P(e) = \frac{1}{2} \operatorname{erfc}\left(\frac{\sqrt{3}}{2\sqrt{N_0}} \right)} \quad \#$$