Question 1 (12 points)

We would like to model the following problem as a search problem: Given a set S of positive integers and a positive integer t, find subset T of S such that

$$\sum_{i \in T} i = t$$

The optimality criterion is that T should be as small as possible (with respect to cardinality).

1. How would you represent a state in this problem?

   $\langle C, R, t \rangle$, where $\{C, R\}$ is a partition of S.

2. What is the initial state?

   $\langle \varnothing, S, t \rangle$.

3. What is the goal test?

   $\sum_{i \in C} i = t$

4. What are the operators of the problem?

   In state $\langle C, R, t \rangle$, there is an operator Op_i, for each $i \in R$, where $Op_i(\langle C, R, t \rangle) = \langle C \cup \{i\}, R - \{i\}, t \rangle$

5. What is a suitable path cost function?

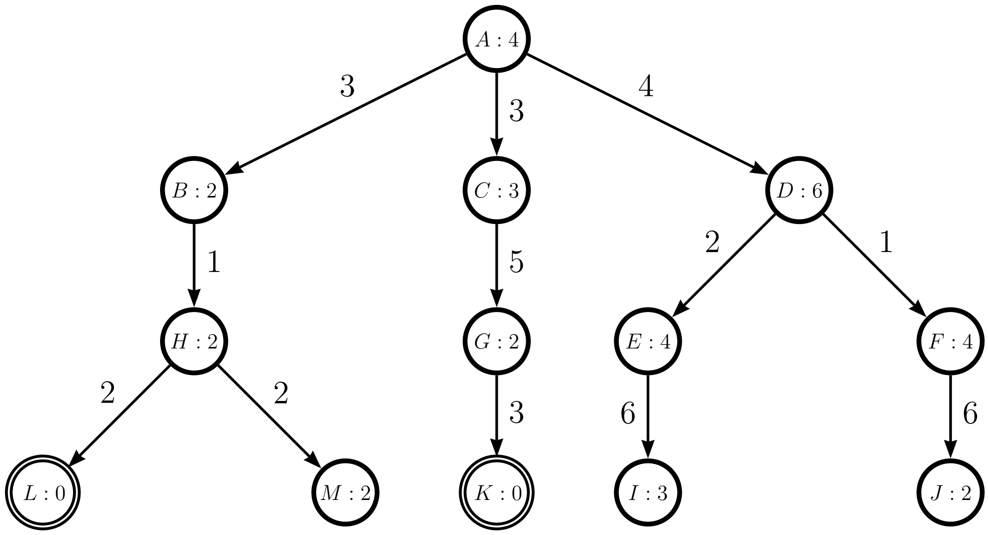
   For a sequence of operators s, $cost(s) = |s|$.

6. What is a suitable admissible heuristic function?

   $h(\langle C, R, t \rangle) = \lfloor \frac{t - \sum_{i \in C} i}{j} \rfloor$, where j is the largest member of R with $j \leq t - \sum_{i \in C} i$ if one exists, else $j = 1$.

Question 2 $((\mathbf{3}+\mathbf{2})\times\mathbf{2}=\mathbf{10}$ points)

The following tree is a full search tree for some state space. Arc labels denote branch costs, double circles indicate goal nodes. The numbers following the ":" indicate the value of the heuristic function h for the corresponding node.



For each of the following search strategies, indicate the order in which nodes will be chosen for expansion (resolve ties using the lexicographic order of node names) and the chosen path to a goal:

(a) Iterative deepening search.

Expansion sequence:

A A B C D A B H C G D E F A B H L

Path to goal:

ABHL

(b) A* search.

Expansion sequence:

A B C H L

Path to goal:

ABHL

Question 3 (8 points)

Which of the following are possible (semantically and syntactically correct) first-order-logic translations of "Every voter votes for a candidate which some voter does not vote for"? (Predicate symbols have the obvious semantics.) Circle all and only the correct translations. If you think a translation is incorrect, carefully explain why.

1. $\exists x[Cand(x) \land \forall y[(Voter(y) \land Votes(y,x)) \Rightarrow \exists z[Voter(z) \land \neg Votes(z,x)]]]$

   This would be false if there are no candidates and no voters, while the English statement would be true.

2. $\forall x[(Voter(x) \land \exists y[Cand(y) \land Votes(x,y)]) \Rightarrow \exists z[Voter(z) \land \neg Votes(z,y)]]$

   This would be true if some voters do not vote, while the English would be false.

3. $\forall x \forall y[(Voter(x) \land Cand(y) \land Votes(x,y)) \Rightarrow \exists z[Voter(z) \land \neg Votes(z,y)]]$

   Same as above.

4. $\neg \forall x \forall y \forall z[(Voter(x) \land Voter(y) \land Cand(z) \land Votes(x,z)) \Rightarrow Votes(y,z)]$

   This would be true if only one candidate votes; the English statement requires all candidates to vote.

Question 4 (6 points)

Answer the following questions.

(a) Under what conditions is depth-limited search complete? Under what conditions is it optimal?

Depth-limited search with limit l is complete if whenever there is goal state, then there is a goal state that may be reached in at most l steps. It is optimal if an optimal goal state is reached in at most l steps and that, within depth l, costs can only increase as we go deeper and to the right in the search tree.

(b) Give a set I_1 of propositional logic inference rules which is sound. Give a set I_2 which is complete. Explain your reasoning.

$I_1 = \dfrac{p}{p}$; $I_2 = \dfrac{}{p}$

Question 5 (16 points)

In the "Towers of Hanoi" problem, we have three pegs and five discs each of a distinct radius. Discs have center holes that allow them to be slid down pegs. At any time, discs are stacked on pegs in order of their sizes (largest disk at the bottom of the stack). We can move a disk off the top of the stack at one peg onto the top of the stack of another (or possibly the same) peg provided that we never place a large disk above a smaller disk.

We use the following predicate symbols to reason about the towers of Hanoi in the situation calculus:

1. $Smaller(x, y)$, which means that disk x is smaller than disc y.
2. $On(x, y, s)$, which means that disk x is directly on top of disk y in situation s.
3. $At(x, p, s)$, which means that disk x is on (but not necessarily on top of) the stack at peg p in situation s.
4. $2Stack(p, s)$, which means that the stack at peg p in situation s has exactly two disks on it.

In addition, there are functional terms of the form $Move(p, q)$ denoting the action of moving the disk (if there is one) on top of the stack at peg p onto the top of the stack at peg q.

Write down a successor-state axiom for the predicate $2Stack$.

Hint: You might find it useful to define some shorthand-predicates in terms of the given predicate and function symbols.

$Top(x,p,s) \equiv At(x,p,s) \wedge \neg\exists y On(y,x,s)$
$1Stack(x,p,s) \equiv Top(x,p,s) \wedge \neg\exists y On(x,y,s)$
$3Stack(x,p,s) \equiv Top(x,p,s) \wedge \exists y \exists z [On(x,y,s) \wedge On(y,z,s) \wedge \neg\exists w On(z,w,s)]$

$$
\begin{aligned}
&\forall p,s,a[2Stack(p,Result(a,s)) \Leftrightarrow \\
&\quad \exists x,y,q[1Stack(x,p,s) \wedge q \neq p \wedge Top(y,q,s) \wedge Smaller(y,x) \wedge a = Move(q,p)] \\
&\bigvee \\
&\quad \exists x,q[3Stack(x,p,s) \wedge q \neq p \wedge a = Move(p,q) \wedge \forall y[Top(y,q,s) \Rightarrow Smaller(x,y)] \\
&\bigvee \\
&\quad 2Stack(p,s) \wedge \\
&\quad\; \forall q[a = Move(p,q) \Rightarrow [q = p \vee \exists x,y[Top(x,p,s) \wedge Top(y,q,s) \wedge \neg Smaller(x,y)]]] \\
&\quad \wedge \\
&\quad\; \forall q[a = Move(q,p) \Rightarrow [q = p \vee \forall x,y[Top(x,p,s) \wedge Top(y,q,s) \Rightarrow \neg Smaller(y,x)]]] \\
&]
\end{aligned}
$$