Syntax Analysis: LR(1) and LALR Parsing

Lecture 7

Objectives

By the end of this lecture you should be able to:

1. Identify LR(1) items.
2. Construct an LR(1) automaton for a CFG.
3. Construct the LR(1) parsing table for a CFG.
4. Trace the operation of an LR(1) parser.
5. Construct an LALR automaton for a CFG.
6. Construct the LALR parsing table for a CFG.
7. Trace the operation of an LALR parser.

Outline

1. Canonical LR(1) Parsing
2. LALR Parsing

Outline

Grammars Not SLR

Example

Consider the following grammar G_7:

$$\begin{array}{rcl} S & \longrightarrow & L{=}R \mid R \\ L & \longrightarrow & *R \mid \mathbf{id} \\ R & \longrightarrow & L \end{array}$$

Grammars Not SLR: States

Example

I_0: $S' \rightarrow \cdot S$
$S \rightarrow \cdot L = R$
$S \rightarrow \cdot R$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$
$R \rightarrow \cdot L$

I_1: $S' \rightarrow S \cdot$

I_2: $S \rightarrow L \cdot = R$
$R \rightarrow L \cdot$

I_3: $S \rightarrow R \cdot$

I_4: $L \rightarrow * \cdot R$
$R \rightarrow \cdot L$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$

I_5: $L \rightarrow \mathbf{id} \cdot$

I_6: $S \rightarrow L = \cdot R$
$R \rightarrow \cdot L$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$

I_7: $L \rightarrow * R \cdot$

I_8: $R \rightarrow L \cdot$

I_9: $S \rightarrow L = R \cdot$

Problems with SLR Parsing

One problem with SLR parsers is that it is always possible to reduce $A \longrightarrow \alpha$ if the next input is in $Follow(A)$.

Example

- With G_7 and input **id** $=$ **id**, an SLR parser may shift **id** and then reduce $L \longrightarrow$ **id**.
- Since $= \in Follow(R)$, the parser may decide to reduce $R \longrightarrow L$.
- Clearly, this is not correct; a sentential form starting with $R=$ can never reduce to S.

States should carry more information about when reduction is appropriate.

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LR(0) Items: Reprise

Definition

An LR(0) item of CFG $G = \langle V, \Sigma, R, S \rangle$ is a pair $\langle A \rightarrow \alpha, i \rangle$, where $(A \rightarrow \alpha) \in R$ and $0 \leq i \leq |\alpha|$.

- Intuitively, an LR(0) item is a rule and a position in the right side of the rule.
  - Thus, $\langle A \rightarrow \mathtt{a}B\mathtt{b}, 2 \rangle \equiv A \rightarrow \mathtt{a}B.\mathtt{b}$
- An LR(0) item represents a state of the parser: *We have already found the prefix of α before the dot and, if we find the suffix following the dot, we may reduce α to A.*

LR(1) Items

Definition

An LR(1) item of CFG $G = \langle V, \Sigma, R, S \rangle$ is a pair $\langle c, a \rangle$, where c is an LR(0) item and $a \in \Sigma \cup \{\$\}$.

- c is called the core of the item.
- a is the lookahead of the item.
  - Note that a is a single symbol, hence the "1" in "LR(1) item."
- An LR(1) item $[A \to \alpha.\beta, a]$ represents a state of the parser: *We have already found α in the input and, if we find β, we may reduce $\alpha\beta$ to A if the next symbol is a.*

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The Set of LR(1) Items

The smallest set $\mathcal{I}(G)$ satisfying the following

- $[S' \longrightarrow .S, \$] \in \mathcal{I}$.
- $[A \longrightarrow \alpha X.\beta, a] \in \mathcal{I}$ if $[A \longrightarrow \alpha.X\beta, a] \in \mathcal{I}$, for $X \in \Sigma \cup V$.
- $[B \longrightarrow .\gamma, b] \in \mathcal{I}$ if $[A \longrightarrow \alpha.B\beta, a] \in \mathcal{I}$ where $(B \longrightarrow \gamma) \in R$ and $b \in First(\beta a)$.

The LR(1) NFA

Definition

For the CFG $G = \langle V, \Sigma, R, S \rangle$, the LR(1) NFA is an NFA $N_G^1 = \langle \mathcal{I}(G), V \cup \Sigma, \delta, [S' \rightarrow .S, \$], \mathcal{I}(G) \rangle$, where

- $S' \notin V \cup \Sigma$;
- $\delta([A \rightarrow \alpha.X\beta, a], X) = \{[A \rightarrow \alpha X.\beta, a]\}$;
- $\delta([A \rightarrow \alpha.B\beta, a], \varepsilon) = \{[B \rightarrow .\gamma, b] \mid (B \rightarrow \gamma) \in R$ and $b \in First(\beta a)\}$.

The LR(1) Automaton

Definition

The LR(1) automaton for a CFG G is the DFA M_G^1 which is equivalent to N_G^1 and constructed using the standard subset construction.

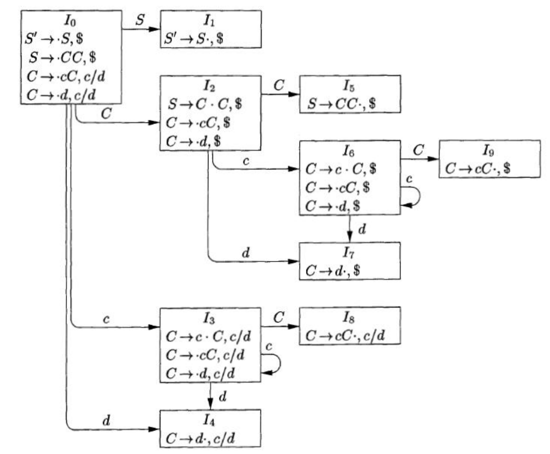
Grammar G_8

Example (The grammar)

$$
\begin{array}{llll}
1. & S & \longrightarrow & CC \\
2. & C & \longrightarrow & \mathtt{c}C \\
3. & C & \longrightarrow & \mathtt{d}
\end{array}
$$

Grammar G_8

Example (The automaton)



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The Canonical LR(1) Parsing Table

We are given a CFG $G = \langle V, \Sigma, R, S \rangle$.

1. Construct M_G^1.
2. For all states q of M_G^1
   1. GOTO$(q, A) = \delta(q, A)$, for every $A \in V$.
   2. If $a \in \Sigma$ and $[A \rightarrow \alpha.a\beta, b] \in q$, then ACTION$(q, a) =$ "shift $\delta(q, a)$".
   3. Else if $A \neq S'$ and $[A \rightarrow \alpha., a] \in q$, then ACTION$(q, a) =$ "reduce $A \rightarrow \alpha$".
   4. Else if $[S' \rightarrow S., \$] \in q$, then ACTION$(q, \$)$ = "accept".
   5. Else ACTION(q, a) = "error".

If any conflicting actions result from the above construction, we say that G is not LR(1).

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Example (Table ▸ Automaton)

STATE	ACTION			GOTO	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Grammar G_8

Example (Trace ▸ Automaton)

Stack	Input
0	ccdd$
03	cdd$
033	dd$
0334	d$
0338	d$
038	d$
02	d$
027	$
025	$
01	$

Canonical LR(1) and SLR Parsing

- Note that a canonical LR(1) table can include conflicts only if the corresponding SLR table does.
- Thus, every SLR grammar is a canonical LR(1) grammar, but not vice versa.
- The price, however, is the increased table size due to the increased number of automaton states.

Outline

1. Canonical LR(1) Parsing
2. LALR Parsing

Lookahead LR Parsing

- Lookahead LR parsers (LALR parsers) are often used in practice.
- Most common syntactic constructs in programming languages are representable by LALR grammars.
- LALR tables are considerably smaller than canonical LR(1) tables.
  - SLR and LALR tables always have the same number of states.
  - Such number is typically several hundred states.
  - A canonical LR(1) table typically has several thousand states!

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The Core

Definition

The core of an LR(1) state q is the set

$$core(q) = \{c | [c, l] \in q\}$$

Two LR(1) states q_1 and q_2 are core-equivalent if $core(q_1) = core(q_2)$.

Example (Automaton of G_8)

The following states are core-equivalent.

- I_4 and I_7.
- I_8 and I_9.
- I_3 and I_6.

How are they different?

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Core-Equivalence and the Automaton

Observation

Let M_G^1 be the LR(1) DFA for a CFG G, and let q_1, q_2 be states of M_G^1 and $X \in V \cup \Sigma$. If q_1 and q_2 are core-equivalent, then $\delta(q_1, X)$ and $\delta(q_2, X)$ are core-equivalent.

- The number of states of the automaton may be reduced considerably if we use core-equivalence-classes as states.
- In particular, if $\{q_1, \ldots, q_n\}$ is an equivalence class of core-equivalence, then we may replace the states $q_1, \ldots, q_n$ by the single state $Q = q_1 \cup \cdots \cup q_n$.
- Transitions are adjusted appropriately.

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The LALR Automaton

Definition

Let $\langle \mathcal{Q}(G), V \cup \Sigma, \delta, q_0, F \rangle$ be an LR(1) automaton. The LALR automaton $M_G^A = \langle \mathcal{Q}'(G), V \cup \Sigma, \delta', q_0', F' \rangle$ is such that

- $\mathcal{Q}'(G) = \{ \bigcup_{q_i \in [q]} q_i | [q]$ is an equivalence class of core-equivalence $\}$
- q_0 and q_0' are core-equivalent.
- $q' \in F'$ if and only if there is some $q \in F$ such that q and q' are core-equivalent.
- $\delta'(q', X)$ is core-equivalent to $\delta(q, X)$, where q and q' are core-equivalent.

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Grammar G_8

Example (LALR automaton)

Draw the state diagram of the LALR automaton for G_8.

The LALR Parsing Table

- The LALR parsing table is constructed in exactly the same way the canonical LR(1) table is constructed, but with M_G^A rather than M_G^1.
- The size of the LALR table is the same as the size of the SLR table.
- If the table has no conflicts, the grammar is said to be an LALR grammar.

Grammar G_8

Example (LALR parsing table)

STATE	ACTION			GOTO	
	c	d	$\$$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

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Grammar G_8

Example (Trace)

Trace the operation of both the canonical LR(1) and the LALR parsers on input `ccd`. What do you notice?

LALR Conflicts (I)

Observation

If an LR(1) table does not have shift/reduce conflicts, then the corresponding LALR table does not have shift/reduce conflicts.

Proof. A shift/reduce conflict occurs if an LALR state has an item $[A \rightarrow \alpha., a]$, calling for reducing by $A \rightarrow \alpha$, and an item $[B \rightarrow \beta.a\gamma, b]$ calling for a shift. But, then, there must be some LR(1) state with items $[A \rightarrow \alpha., a]$ and $[B \rightarrow \beta.a\gamma, c]$. This state would also have a shift/reduce conflict, contradicting our assumption. $\square$

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LALR Conflicts (II)

Observation

If an LR(1) table does not have reduce/reduce conflicts, the corresponding LALR table may have reduce/reduce conflicts.

Proof. Consider the grammar

$$\begin{array}{lcl} S & \longrightarrow & \mathtt{a}A\mathtt{d} \mid \mathtt{a}B\mathtt{e} \mid \mathtt{b}B\mathtt{d} \mid \mathtt{b}A\mathtt{e} \\ A & \longrightarrow & \mathtt{c} \\ B & \longrightarrow & \mathtt{c} \end{array}$$

- This grammar generates the strings $\mathtt{acd}, \mathtt{ace}, \mathtt{bcd}, \mathtt{bce}$.
- The LR(1) state $\{[A \rightarrow c., d], [B \rightarrow c., e]\}$ is reachable from the start state after reading $\mathtt{ac}$.
- Similarly, the LR(1) state $\{[A \rightarrow c., e], [B \rightarrow c., d]\}$ is reachable from the start state after reading $\mathtt{bc}$.
- The LALR automaton will include the state $\{[A \rightarrow c., d/e], [B \rightarrow c., d/e]\}$, which results in a reduce/reduce conflict. $\square$

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