Question 1 (7 points)

You are given a map of Egypt, represented as a weighted, undirected graph. Nodes represent cities, arcs represent two-way roads between cities, and arc weights represent distances between cities. (That is, if $\{a, b\}$ is an arc connecting nodes a and b, then the weight $w(\{a, b\})$ is the distance between city a and city b.) Two friends live in different cities on the map. The two friends want to meet (at some city) as quickly as possible. They can communicate using cell phones to synchronize their moves. At every step, each friend chooses a city adjacent (given the graph) to their current city and travels to it. The amount of time needed to move from city a to adjacent city b is taken to be $w(\{a, b\})$, but at every step the friend that arrives first must wait until the other one arrives (remember, they can communicate through cell phones). We would like to formulate this as a search problem. (Do not consider waiting and using the phone as operators, they are only there to justify the assumption of synchronicity.)

1. What is the state space?

   $\{\langle i, j\rangle \mid i$ and j are nodes in the graph$\}$.

2. What is the successor function?

   $Successor(\langle i, j\rangle) = \{\langle k, l\rangle \mid k$ is adjacent to i and l is adjacent to $j\}$.

3. What is the goal test?

   $GoalTest(\langle i, j\rangle)$ succeeds iff $i = j$.

4. What is an admissible heuristic for this problem?

   $h(\langle i, j\rangle) = \delta(i, j)/2$, where $\delta(i, j)$ is the shortest distance between i and j.

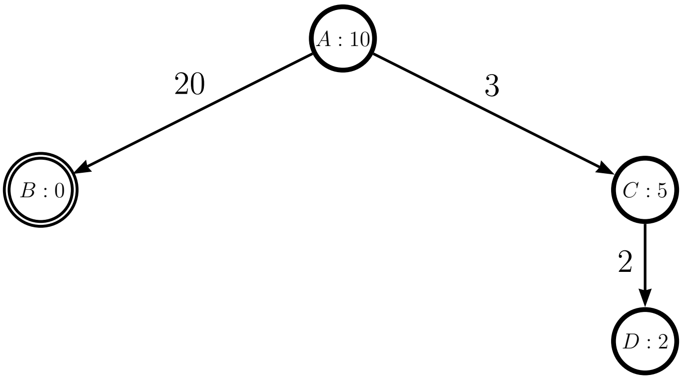
5. Are there completely connected maps for which no solution exists? Why?

   Yes. Consider a graph with only two nodes where the two friends start is different cities. Since both friends have to move at every step, they will never be in the simultaneously in the same city.

Question 2 (4 points)

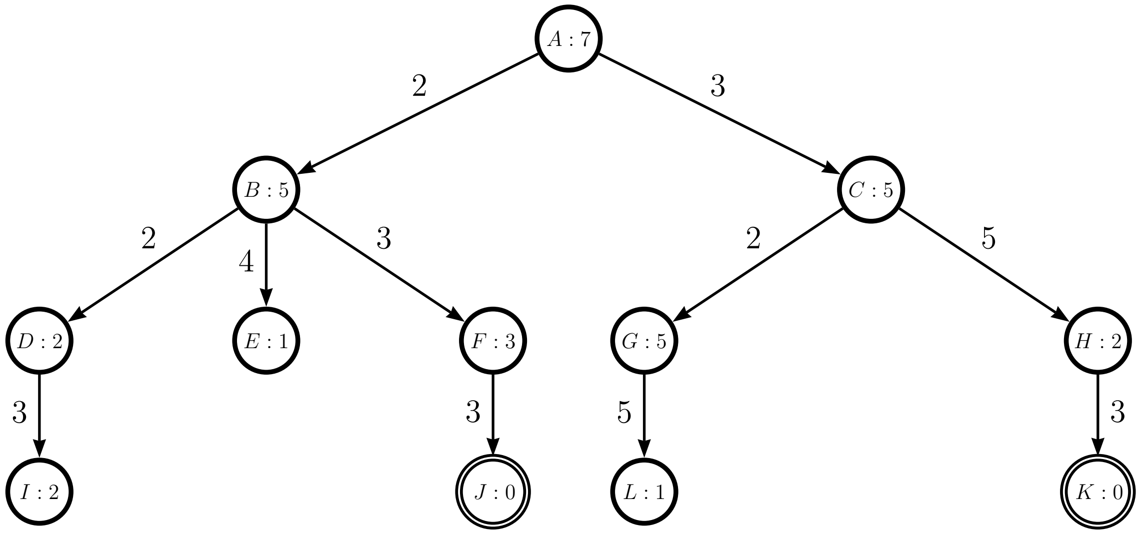
According to the generic search algorithm discussed in class, is it true that a breadth-first strategy always expands at least as many nodes as an A^* strategy with an admissible heuristic? Explain your answer.

No. In the following example, A^* will choose A, C, D, B for expansion, whereas breadth-first will only choose A, B or A, C, B.



Question 3 ($(3+2) \times 2 = 10$ points)

The following tree is a full search tree for some state space. Arc labels denote branch costs, double circles indicate goal nodes. The numbers following the ":" indicate the value of the heuristic function h for the corresponding node.



For each of the following search strategies, indicate the order in which nodes will be chosen for expansion (resolve ties using the lexicographic order of node names) and the chosen path to a goal:

(a) Iterative deepening search.

Expansion sequence: $AABCABDEFCGHABDIEFJ$

Path to goal: $ABFJ$

(b) A* search.

Expansion sequence: $ABDECFJ$

Path to goal: $ABDJ$

Question 4 (4 points)

Which of the following are semantically and syntactically correct translations of "No dog bites a child of its owner"? Circle all and only the correct translations. If you think a translation is incorrect, carefully explain why.

1. $\forall x[Dog(x) \Rightarrow \neg Bites(x, Child(Owner(x)))]$

   Incorrect. *Child*'s being a function symbol assumes that everyone has exactly one child (no less, no more) which by no means is implicated by the English sentence.

2. $\neg\exists x, y[Dog(x) \wedge Child(y, Owner(x)) \wedge Bites(x, y)]$

   Correct.

3. $\forall x[Dog(x) \Rightarrow (\forall y[Child(y, Owner(x)) \Rightarrow \neg Bites(x, y)])]$

   Correct.

4. $\neg\exists x[Dog(x) \Rightarrow (\exists y[Child(y, Owner(x)) \wedge Bites(x, y)])]$

   Incorrect. The above sentence is true iff all individuals in the domain are dogs, which is not the case for the English sentence.

Question 5 (9 points)

Which of the following are logically implied by $(P \vee Q) \wedge (\neg R \vee \neg S \vee T)$? Explain.

1. $(P \vee Q)$.

   Logically implied: Given the semantics of $\wedge$ (in terms of its truth table), if $(P \vee Q) \wedge (\neg R \vee \neg S \vee T)$ is true, $(P \vee Q)$ must also be true.

2. $(P \vee Q \vee R) \wedge (Q \wedge R \wedge S \Rightarrow T)$.

   Logically implied: Again, for $(P \vee Q) \wedge (\neg R \vee \neg S \vee T)$ to be true, both $(P \vee Q)$ and $(\neg R \vee \neg S \vee T)$ must be true. If $(P \vee Q)$ is true then, given the semantics of $\vee$, $(P \vee Q \vee R)$ is also true. Similarly, if $(\neg R \vee \neg S \vee T)$ is true then so is $(\neg Q \vee \neg R \vee \neg S \vee T)$, where the latter is logically equivalent to $(Q \wedge R \wedge S \Rightarrow T)$. Hence, if $(P \vee Q) \wedge (\neg R \vee \neg S \vee T)$ is true, $(P \vee Q \vee R) \wedge (Q \wedge R \wedge S \Rightarrow T)$ is also true.

3. $(P \vee Q) \wedge (\neg S \vee T)$.

   Not logically implied: For P and S true and R and T false, $(P \vee Q) \wedge (\neg R \vee \neg S \vee T)$ is true but $(P \vee Q) \wedge (\neg S \vee T)$ is false.

Question 6 (9 points)

In the mine-sweeper world, an object is broken if it is fragile and gets dropped, or if some object explodes next to it. A broken object can always be repaired. The following language, based on the situation calculus, will be used to reason about this world.

Predicates. There are three predicate symbols:

1. *Fragile*, where $\mathit{Fragile}(x)$ means that object x is fragile.
2. *Broken*, where $\mathit{Broken}(x, s)$ means that object x is broken in situation s.
3. *NextTo*, where $\mathit{NextTo}(y, x, s)$ means that object y is next to object x in situation s.

Actions. There are also three of these:

1. $\mathit{Drop}(r, x)$ denotes agent r's dropping of object x.
2. $\mathit{Repair}(r, x)$ denotes agent r's repairing of object x.
3. $\mathit{Explode}(x)$ denotes the explosion of object x.

Write a successor-state axiom for the predicate *Broken*.

$$
\begin{aligned}
&\forall x, a, s[\mathit{Broken}(x, \mathit{Result}(a, s)) \Leftrightarrow \\
&\qquad\qquad (\mathit{Fragile}(x) \wedge \exists r[a = \mathit{Drop}(r, x)]) \\
&\qquad \vee\ \exists y[\mathit{NextTo}(y, x, s) \wedge a = \mathit{Explode}(y)] \\
&\qquad \vee\ (\mathit{Broken}(x, s) \wedge \neg\exists r[a = \mathit{Repair}(r, x)]) \\
&]
\end{aligned}
$$