

# Syntax-Directed Translation

## Lecture 8

# Objectives

By the end of this lecture you should be able to:

- 1 Identify the difference between SDDs and SDTs.
- 2 Find the side effects of parsing with an SDD or an SDT.
- 3 Determine whether an attribute of an SDD is synthesized or inherited.
- 4 Construct SDDs and SDTs.
- 5 Eliminate left-recursion from an SDD with synthesized attributes.
- 6 Construct an SDT which is equivalent to a given SDD.

# Outline

- 1 Semantic Analysis as Translation
- 2 Syntax-Directed Definitions
- 3 Digression: Eliminating Left Recursion
- 4 SDD Order of Evaluation
- 5 Syntax-Directed Translation Schemes

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# Semantic Analysis

- The **semantic analysis** of a source program amounts to *understanding* what the program means.
- Understanding what a program means is best demonstrated by translating the program to an intermediate representation which can be pretty directly mapped to equivalent machine language code.
- All aspects of a source program which are not accounted for by parsing are considered to fall under the rubric of semantic analysis; for example, type checking.
- We consider different translation schemes, all taking the result of parsing as a starting point.

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# Syntax-Directed Translation

## Syntax-Directed Definitions (SDDs):

- Symbols of the grammar have associated **attributes**.
- Values of attributes are computed by augmenting grammar rules with actions.
- Grammar symbols have distinguished attributes which carry the “meaning” (translation) of instances of the symbol.
- Better suited for specification.

## Syntax-Directed Translation Schemes (SDTs):

- Pieces of code to perform translation actions are added at various positions in the right-side of a production.
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## A Note on Notation

We will often need to distinguish different occurrences of the same grammar symbol in grammar productions. Subscripts will be used to do so. Thus, in

$$E \longrightarrow E_1 + T$$

$E_1$  is not a symbol different from  $E$ ; it is a different occurrence of  $E$ .

# Infix-to-Prefix

## Example (SDD)

$$\begin{array}{lll} E & \longrightarrow & E_1 + T \quad E.code = '+' \circ E_1.code \circ T.code \\ E & \longrightarrow & T \quad E.code = T.code \\ T & \longrightarrow & T_1 * F \quad T.code = '*' \circ T_1.code \circ F.code \\ T & \longrightarrow & F \quad T.code = F.code \\ F & \longrightarrow & (E) \quad F.code = E.code \\ F & \longrightarrow & \mathbf{id} \quad F.code = \mathbf{id.lexval} \end{array}$$

# Infix-to-Prefix

## Example (SDT)

$$E \longrightarrow \{\text{print}('+\')\}E_1 + T$$
$$E \longrightarrow T$$
$$T \longrightarrow \{\text{print}('*')\}T_1 * F$$
$$T \longrightarrow F$$
$$F \longrightarrow (E)$$
$$F \longrightarrow \mathbf{id} \{\text{print}(\mathbf{id.lexval})\}$$

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- 1 Semantic Analysis as Translation
- 2 **Syntax-Directed Definitions**
- 3 Digression: Eliminating Left Recursion
- 4 SDD Order of Evaluation
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# Synthesized Attributes

## Definition

- 1 An attribute  $a$  of an occurrence  $A_i$  of a variable  $A$  is a **synthesized attribute** if,  $A_i \rightarrow \alpha$  is a rule where the value of  $a$  is determined only by the values of other attributes of  $A_i$  and the values of attributes of symbols in  $\alpha$ .
- 2 All attributes of terminals are synthesized attributes; their values are provided by the lexical analyzer.

## Example ( ▶ Infix-to-Prefix SDD )

The attribute *code* is a synthesized attribute of  $E$ ,  $T$ , and  $F$  in the infix-to-prefix SDD. The attribute *lexval* is a synthesized attribute of *id*.

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# Inherited Attributes

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An attribute  $a$  of an occurrence  $A_i$  of a variable  $A$  is an **inherited attribute** if,  $B \rightarrow \alpha A_i \beta$  is a rule where the value of  $a$  is determined by the values of attributes of symbols occurring in the rule.

## Example

$$B \longrightarrow B_1 A C \quad A.inh = B.inh \circ B_1.syn \circ A.syn \circ C.syn$$



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## SDDs and Parse Trees

- It is convenient to suppose that parsing constructs a parse tree.
- SDD rules are applied to compute the values of attributes at various nodes of the tree.
- In general, all attribute values needed for a computation must be computed prior to this computation; this constrains the order in which we traverse the tree.
- If all attributes are synthesized, any bottom-up order will do—one resulting from a postorder traversal, for example.
- The parse tree with the values of attributes indicated is called an **annotated parse tree**.

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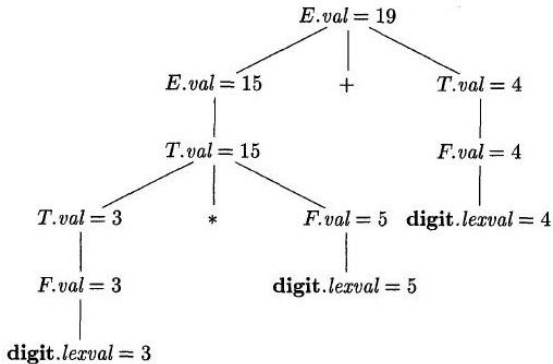
# Expression SDD

## Example (SDD)

$E$	$\longrightarrow$	$E_1 + T$	$E.val = E_1.val + T.val$
$E$	$\longrightarrow$	$T$	$E.val = T.val$
$T$	$\longrightarrow$	$T_1 * F$	$T.val = T_1.val \times F.val$
$T$	$\longrightarrow$	$F$	$T.val = F.val$
$F$	$\longrightarrow$	$(E)$	$F.val = E.val$
$F$	$\longrightarrow$	<b>digit</b>	$F.val = \mathbf{digit.lexval}$

# Expression SDD

## Example (Annotated Parse Tree)



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# Why Inherited Attributes?

- Why would we ever need inherited attributes?
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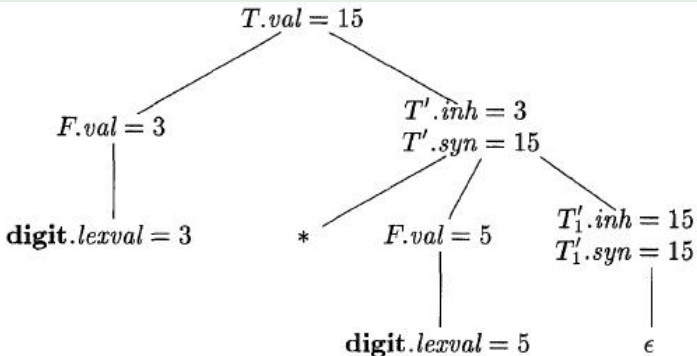
# Expression SDD Without Left Recursion

## Example (SDD)

$$\begin{array}{lll}
 T & \longrightarrow & FT' \quad T'.inh = F.val \\
 & & T.val = T'.syn \\
 T' & \longrightarrow & *FT'_1 \quad T'_1.inh = T'.inh \times F.val \\
 & & T'.syn = T'_1.syn \\
 T' & \longrightarrow & \varepsilon \quad T'.syn = T'.inh \\
 F & \longrightarrow & \mathbf{digit} \quad F.val = \mathbf{digit.lexval}
 \end{array}$$

# Expression SDD Without Left Recursion

## Example (Tree)



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# Left Recursion in SDDs

- Suppose we are given an SDD with a left recursive grammar having only synthesized attributes.
- We have seen how to construct an equivalent grammar which is not left-recursive.
- But how do we get an equivalent SDD?
- We shall see how to do so if we only have immediate left recursion.
- Can you extend it to the general case?

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# Eliminating Left Recursion

We are given

$$\begin{aligned} A &\longrightarrow A_1 Y & A.a &= g(A_1.a, Y.y) \\ A &\longrightarrow X & A.a &= f(X.x) \end{aligned}$$

We get

$$\begin{aligned} A &\longrightarrow XR & R.i &= f(X.x) \\ & & A.a &= R.s \\ R &\longrightarrow YR_1 & R_1.i &= g(R.i, Y.y) \\ & & R.s &= R_1.s \\ R &\longrightarrow \varepsilon & R.s &= R.i \end{aligned}$$

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# The Problem

- We are given an SDD and a parse tree for a given string.
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## Definition

Let  $T = (N_T, E_T)$  be a parse tree and let  $D$  be the associated SDD. The **dependency graph** of  $T$  and  $D$  is a graph  $G_D(T) = (N_G, E_G)$  where

- $N_G = \{n.a \mid n \in N_T \text{ and } a \text{ is an attribute of } n\}$
- $(u.a, v.b) \in E_G$  if and only if
  - 1  $b$  is a synthesized attribute of  $v$ ,  $u$  is a child of  $v$  in  $T$ , and the corresponding production has a rule which defines  $v.b$  in terms of  $u.a$ .
  - 2  $b$  is an inherited attribute of  $b$ ,  $u = v$  or  $u$  is a sibling or parent of  $v$  in  $T$ , and the corresponding production has a rule which defines  $v.b$  in terms of  $u.a$ .

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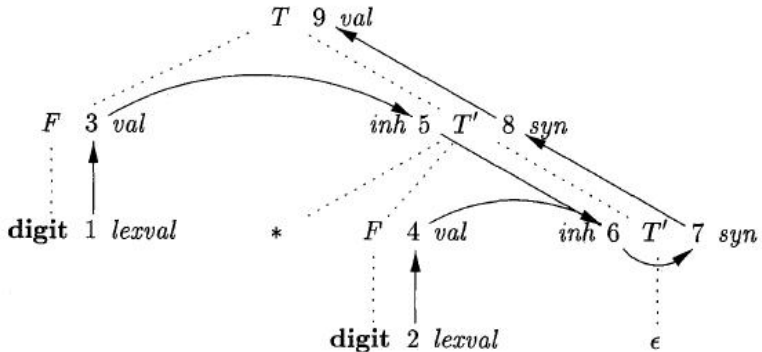
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# Expression SDD Without Left Recursion

## Example (Dependency Graph)



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# Order of Evaluation

- If the dependency graph is a **directed acyclic graph** (DAG), a **topological sort** of its nodes yields a valid order of evaluation:
  - 1 Initialize an empty queue.
  - 2 While there are nodes in the graph.
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# Cycles in the Dependency Graph

## Example

$$\begin{array}{l} A \longrightarrow B \quad A.s = B.i \\ \quad \quad \quad \quad \quad B.i = A.s + 1 \end{array}$$



# S-Attributed SDDs

## Definition

An SDD is **S-attributed** if every attribute is synthesized.

## Example

The left recursive expression grammar is an *S*-attributed grammar. ▶

## Observation

If  $D$  is an *S*-attributed SDD, then, for every parse tree  $T$ ,  $G_D(T)$  is a DAG.

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If  $D$  is an *S*-attributed SDD, then, for every parse tree  $T$ ,  $G_D(T)$  is a DAG.

# S-Attributed SDDs

## Definition

An SDD is **S-attributed** if every attribute is synthesized.

## Example

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# *L*-Attributed SDDs

## Definition

An SDD is ***L*-attributed** if for every rule  $B \rightarrow \alpha A \beta$

- 1 every inherited attribute  $a$  of  $A$  depends only on inherited attributes of  $B$  and attributes of symbols occurring in  $\alpha$ ; and
- 2 every synthesized attribute of  $B$  depends only on attributes of symbols in  $\alpha A \beta$  and on inherited attributes of  $B$ .

## Example

The non-left-recursive expression grammar is *L*-attributed. ▶

## Observation

If  $D$  is an *L*-attributed SDD, then, for every parse tree  $T$ ,  $G_D(T)$  is a DAG.


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# Outline

- 1 Semantic Analysis as Translation
- 2 Syntax-Directed Definitions
- 3 Digression: Eliminating Left Recursion
- 4 SDD Order of Evaluation
- 5 Syntax-Directed Translation Schemes

## From SDDs to SDTs

### Given an $L$ -attributed SDD

- 1 Insert the action that computes an inherited attribute of a non-terminal immediately before the occurrence of the non-terminal on the right-side of a rule.
  - If several inherited attributes are computed, make sure the actions are ordered in a way that is consistent with the dependency graph.
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## Expression SDT Without Left Recursion

### Example

Following is an SDT corresponding to the earlier non-left-recursive SDD. ▶

$$\begin{aligned}T &\longrightarrow F \{T'.inh = F.val\} T' \{T.val = T'.syn\} \\T' &\longrightarrow *F \{T'_1.inh = T'.inh \times F.val\} T'_1 \{T'.syn = T'_1.syn\} \\T' &\longrightarrow \varepsilon \{T'.syn = T'.inh\} \\F &\longrightarrow \mathbf{digit} \{F.val = \mathbf{digit.lexval}\}\end{aligned}$$