

CSEN 1003 Compiler, Spring Term 2019
Practice Assignment 4

Discussion: 19.02.19 - 24.02.19

Exercise 4-1

Eliminating Left Recursion

Eliminate left-recursion from each of the following grammars:

a) $S \rightarrow Sa \mid b$

Solution:

$$\begin{aligned} S &\rightarrow bS' \\ S' &\rightarrow aS' \mid \varepsilon \end{aligned}$$

b) $S \rightarrow Sab \mid cd$

Solution:

$$\begin{aligned} S &\rightarrow cdS' \\ S' &\rightarrow abS' \mid \varepsilon \end{aligned}$$

c) $S \rightarrow S \cup S \mid S S \mid S* \mid (S) \mid a$

Solution:

$$\begin{aligned} S &\rightarrow (S)S' \mid aS' \\ S' &\rightarrow \cup SS' \mid SS' \mid *S' \mid \varepsilon \end{aligned}$$

d)
$$\begin{aligned} rexpr &\rightarrow rexpr \cup rterm \mid rterm \\ rterm &\rightarrow rterm rfactor \mid rfactor \\ rfactor &\rightarrow rfactor* \mid rprimary \\ rprimary &\rightarrow a \mid b \end{aligned}$$

Solution:

$$\begin{aligned} rexpr &\rightarrow rterm rexpr' \\ rexpr' &\rightarrow \cup rterm rexpr' \mid \varepsilon \\ rterm &\rightarrow rfactor rterm' \\ rterm' &\rightarrow rfactor rterm' \mid \varepsilon \\ rfactor &\rightarrow rprimary rfactor' \\ rfactor' &\rightarrow *rfactor' \mid \varepsilon \\ rprimary &\rightarrow a \mid b \end{aligned}$$

Alternative solution:

⁰Exercises are due to Dr. Carmen Gervet

$$\begin{aligned}
rexpr &\rightarrow rterm \cup rexpr \mid rterm \\
rterm &\rightarrow rfactor \ rterm \mid rfactor \\
rfactor &\rightarrow rprimary \ rfactor' \\
rfactor' &\rightarrow *rfactor' \mid \varepsilon \\
rprimary &\rightarrow \mathbf{a} \mid \mathbf{b}
\end{aligned}$$

e)
$$\begin{aligned}
A &\rightarrow 0 \mid T1 \\
T &\rightarrow 1 \mid A0
\end{aligned}$$

Solution:

$$\begin{aligned}
A &\rightarrow 0 \mid T1 \\
T &\rightarrow 1T' \mid 00T' \\
T' &\rightarrow 10T' \mid \varepsilon
\end{aligned}$$

f)
$$\begin{aligned}
A &\rightarrow BC \\
B &\rightarrow Bb \mid \varepsilon \\
C &\rightarrow AC \mid \mathbf{a}
\end{aligned}$$

Solution:

$$\begin{aligned}
A &\rightarrow BC \mid C \\
B &\rightarrow \mathbf{b}B' \\
B' &\rightarrow \mathbf{b}B' \mid \varepsilon \\
C &\rightarrow \mathbf{b}B'CCC' \mid \mathbf{a}C' \\
C' &\rightarrow CC' \mid \varepsilon
\end{aligned}$$

Exercise 4-2

Left Factoring

Left-factor each of the following grammars:

a)
$$S \rightarrow 0S1 \mid 01$$

Solution:

$$\begin{aligned}
S &\rightarrow 0S' \\
S' &\rightarrow S1 \mid 1
\end{aligned}$$

b)
$$S \rightarrow \mathbf{a}bx \mid \mathbf{a}by \mid \mathbf{a}cx \mid \mathbf{a}cy$$

Solution:

$$\begin{aligned}
S &\rightarrow \mathbf{a}S' \\
S' &\rightarrow \mathbf{b}S'' \mid \mathbf{c}S'' \\
S'' &\rightarrow \mathbf{x} \mid \mathbf{y}
\end{aligned}$$

Exercise 4-3

Consider the following CFG:

$$\begin{aligned} S &\rightarrow 0T1S \mid \varepsilon \\ T &\rightarrow 0T1 \mid \varepsilon \end{aligned}$$

- a) Compute the FIRST and FOLLOW sets.

Solution:

$$\text{FIRST}(S) = \{0, \varepsilon\}$$

$$\text{FIRST}(T) = \{0, \varepsilon\}$$

$$\text{FOLLOW}(S) = \{\$ \}$$

$$\text{FOLLOW}(T) = \{1\}$$

- b) What language does this grammar recognize?

Solution:

$$\{0^n 1^n \mid n \geq 1\}^*$$

Exercise 4-4

Consider the following CFG:

$$\begin{aligned} S &\rightarrow SAB \mid SBC \mid \varepsilon \\ A &\rightarrow aAa \mid \varepsilon \\ B &\rightarrow bB \mid \varepsilon \\ C &\rightarrow cC \mid \varepsilon \end{aligned}$$

- a) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

Non-terminal	FIRST	FOLLOW
S	$\{a, b, c, \varepsilon\}$	$\{\$, a, b, c\}$
A	$\{a, \varepsilon\}$	$\{\$, a, b, c\}$
B	$\{b, \varepsilon\}$	$\{\$, a, b, c\}$
C	$\{c, \varepsilon\}$	$\{\$, a, b, c\}$

- b) Build the parsing table.

Solution:

Non-terminal	Input Symbol			
	a	b	c	\$
S	SAB SBC ε	SAB SBC ε	SAB SBC ε	SAB SBC ε
A	aAa ε	ε	ε	ε
B	ε	bB ε	ε	ε
C	ε	ε	cC ε	ε

- c) From the parsing table, show why the grammar is not LL(1).

Solution:

The grammar is not LL(1) because some entries of the parsing table contain more than one rule.

Exercise 4-5

Consider the following CFG:

$$\begin{aligned} S &\rightarrow (L) \mid \mathbf{a} \\ L &\rightarrow L, S \mid S \end{aligned}$$

- a) Eliminate left recursion.

Solution:

$$\begin{aligned} S &\rightarrow (L) \mid \mathbf{a} \\ L &\rightarrow (L)L' \mid \mathbf{a}L' \\ L' &\rightarrow ,SL' \mid \varepsilon \end{aligned}$$

- b) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

$$\begin{aligned} \text{FIRST}(S) &= \{ (, \mathbf{a} \} \\ \text{FIRST}(L) &= \{ (, \mathbf{a} \} \\ \text{FIRST}(L') &= \{ , , \varepsilon \} \\ \text{FOLLOW}(S) &= \{), ,, \$ \} \\ \text{FOLLOW}(L) &= \{) \} \\ \text{FOLLOW}(L') &= \{) \} \end{aligned}$$

Exercise 4-6

Consider the following CFG:

$$S \rightarrow SS+ \mid SS* \mid \mathbf{a}$$

- a) Left factor the grammar and eliminate left recursion.

Solution:

$$\begin{aligned} S &\rightarrow \mathbf{a}S' \\ S' &\rightarrow SXS' \mid \varepsilon \\ X &\rightarrow + \mid * \end{aligned}$$

- b) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

$$\begin{aligned} \text{FIRST}(S) &= \{ \mathbf{a} \} \\ \text{FIRST}(S') &= \text{FIRST}(S) \cup \{ \varepsilon \} = \{ \mathbf{a}, \varepsilon \} \end{aligned}$$

$$\text{FIRST}(X) = \{+, *\}$$

$$\text{FOLLOW}(S) = \text{FIRST}(X) \cup \{\$\} = \{+, *, \$\}$$

$$\text{FOLLOW}(S') = \text{FOLLOW}(S) = \{+, *, \$\}$$

$$\text{FOLLOW}(X) = \{\mathbf{a}, +, *, \$\}$$

c) Build the parsing table.

Solution:

Non-terminal	Input Symbol			
	a	+	*	\$
S	$\mathbf{a}S$			
S'	SXS'	ε	ε	ε
X		$+$	$*$	