The Situation Calculus

Lecture 4

Outline

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1. The Wumpus World
2. Logic
3. Reasoning Agent Meets the Wumpus

The Wumpus-less Wumpus World

- Version 1: Location of gold is known.

	G		
A			

- Possible strategy: Use greedy search with h as city block distance.

The Wumpus-less Wumpus World

- Version 2: Location of gold is unknown.
- But agent can sense a glitter when it is in the same cell as the gold.
- Will search help in this case?
- What should the agent do?
  - Perhaps, systematically visit all cells.

The Wumpus World(s)

S		B	P
$\mathfrak{W}$	S G B	P	B
S		B	
A	B	P	B

Goal

- The agent's goal is to find the gold and bring it back to the start, where it will climb out of the cave, as quickly as possible, and without getting killed.
- Performance measure:
  - $+1000$ for climbing out of the cave with the gold.
  - -1 for each action performed.
  - -10000 for getting killed.

Percepts

- In the cell containing the wumpus, and in the adjacent cells, the agent will perceive a stench.
- In the cells adjacent to a pit, the agent will perceive a breeze.
- In the cell containing the gold, the agent will perceive a glitter.
- If the agent walks into a wall, it perceives a bump.
- When the wumpus is killed, the agent perceives a scream.
- The agent cannot perceive its location.

Thus, a percept is a quintuple

$$\langle\ \mathit{Stench}, \mathit{Breeze}, \mathit{Glitter}, \mathit{Bump}, \mathit{Scream}\ \rangle$$

Actions

- Forward.
- Right.
- Left.
- Grab.
- Shoot.
- Climb.

Can search help the wumpus agent?

What is a Wumpus-World State?

- States encode deeper knowledge of the environment.
- For example, sensing no stench while in [1,1], the agent should *know* that the wumpus is neither in [1,2] nor in [2,1].
- Given the percept sequence, a state would include things like:
  - Cell [i,j] is safe.
  - There is a pit in cell [i,j].
  - The wumpus is in cell [i,j] or cell [k,l].

Logical Reasoning

- In order to update the state, given a new percept, the agent needs the following:
  1. Background knowledge linking percepts to possible contents of cells.
  2. Some way of representing this knowledge.
  3. Some way of mapping percepts to structures of the same representation.
  4. A method that manipulates these structures to update the agent's knowledge of the state of the environment.
- That is, the agent needs full-fledged logical reasoning.

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What is Logic?

- There are different uses of the word "logic".
- But think of "a logic" as a language used to represent knowledge.
  - For example, knowledge of the Wumpus-world agent.
- It is an artificial language, it should have precise syntax and semantics.
- In addition, it should have a proof theory.
- A set of sentences of a logic representing knowledge of an agent is referred to as a knowledge base (KB).

Logical Implication

- A sentence ϕ logically implies (or entails) a sentence ψ if every state of affairs in which ϕ is true is a state of affairs in which ψ is true.

Example

"All men are mortal and Socrates is a man" logically implies "Socrates is mortal"

- Logical implication does not depend at all on the proof theory, only on the semantics of the language.
- We write $KB \models \phi$ whenever the conjunction of all sentences in KB logically implies ϕ.

$\models$ and $\vdash$

- An agent that reasons logically needs to compute the logical implications of its KB.
  - That is, it should be able to generate new sentences that are necessarily true, assuming the truth of what is in the KB.
  - Note: the agent will then be rational, but not necessarily well-informed.
- However, computing using $\models$ requires access to the world; you cannot put the world inside the agent.
- Rather, all reasoning should be done on a representation of the world: the KB.
- Move from $\models$ to $\vdash$—derivation using inference rules.

Soundness and Completeness

- A logic is sound if $KB \vdash_I \phi$ implies $KB \models \phi$.
  - The rules of inference in I should be sensitive to the semantics; they should be truth preserving.
- A logic is complete if $KB \models \phi$ implies $KB \vdash_I \phi$.
- In practice, soundness is easier to achieve.

A Knowledge-Based Agent

function KB-AGENT(*percept*) **returns** action
    static: *KB*
        t //A counter, initially 0

    TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept, t*))
    action $\longleftarrow$ ASK(*KB*, MAKE-ACTION-QUERY(*t*))
    TELL(*KB*, MAKE-ACTION-SENTENCE(*action, t*))
    $t \longleftarrow t + 1$
    return *action*

Outline

A Propositional KB Agent

- $A_{i,j}$ means that the agent is in cell $[i,j]$.
- $B_{i,j}$ means that there is a breeze in cell $[i,j]$.
- $G_{i,j}$ means that there is a glitter in cell $[i,j]$.
- $P_{i,j}$ means that there is a pit in cell $[i,j]$.
- $S_{i,j}$ means that there is a stench in cell $[i,j]$.
- $W_{i,j}$ means that there is a wumpus in cell $[i,j]$.
- $\ldots$

Note

The above symbols have no internal structure.

Sample Situation

- The agent visited $[1,1]$, $[2,1]$, and $[1,2]$.
- The current KB contains the following percept statements.

$$\begin{array}{cc} \neg S_{1,1} & \neg B_{1,1} \\ \neg S_{2,1} & B_{2,1} \\ S_{1,2} & \neg B_{1,2} \end{array}$$

- The KB also contains background knowledge about the wumpus world.

Sample Background Knowledge

- Where is the wumpus?
  - $S_{1,1} \Leftrightarrow (W_{1,1} \vee W_{1,2} \vee W_{2,1})$
  - $S_{2,1} \Leftrightarrow (W_{1,1} \vee W_{2,1} \vee W_{2,2} \vee W_{3,1})$
  - $S_{1,2} \Leftrightarrow (W_{1,1} \vee W_{1,2} \vee W_{2,2} \vee W_{1,3})$
  - $\ldots$
- We should have similar rules for each cell.
- Similarly, for the whereabouts of pits and gold.

Sample Conclusions

- Assuming a reasonable proof theory, the agent can conclude the following.

$$\begin{array}{lll} \neg W_{1,1} & \neg W_{1,2} & \neg W_{2,1} \\ \neg W_{2,2} & & \\ W_{1,3} & & \end{array}$$

- In fact, a propositional KB agent can successfully perform in the wumpus world.

Problems with the Propositional KB Agent

- Cannot ask “What action should I take?”; can only check, for each action, if it is doable.
  - Sample action rule:
    $$A_{1,1} \wedge East \wedge W_{2,1} \Rightarrow \neg Forward$$
  - We can only ASK the KB if, for example, *Forward* is doable.
  - Thus, to decide what to do, we should find all doable actions and pick one.
- Proliferation of propositions. Gets worse if time is taken into account.

First-Order Agent with State

- What is a state?
- Well, it is a set of facts about the world at some particular time, t.
- These facts are derivable from percepts at, or before, t.
- For example, how would the agent know that it holds the gold?
- To represent states, we need to update an initial state description with percepts and action performances.
- We need to represent change.

The Situation Calculus

- One of the simplest and oldest AI schemes for representing change.
- Developed by John McCarthy.
- Ontological commitments: In addition to objects and agents, the world consists of
  1. actions and
  2. situations
- A situation is a snapshot of the world; a complete state if you will.
- No two situations are identical.
- Situations are not times; they are real or hypothetical.

Representing Change

- Every predicate, corresponding to a relation or property that changes over time, should have an extra situation argument.
- The initial situation is denoted by the constant S_0.
- Any other situation results from the initial situation by the performance of a sequence of actions.

Example

- $Result(Forward, S_0)$: The situation resulting from performing the action *Forward* in the initial situation.
- $Result(Turn(Right), S_0)$: The situation resulting from turning right in the initial situation.
- $Result(Turn(Right), Result(Forward, S_0))$: ?

Effect Axioms

- Actions are defined in terms of their effects.
- For every action, we should write axiom(s) specifying the properties of situations resulting from performing it.

Example

- $\forall x, s(\neg Holding(x, Result(Release, s)))$

Grabbing the Gold

- $Portable(Gold)$
- $\forall s(AtGold(s) \Rightarrow Present(Gold, s))$
- $\forall x, s(Present(x, s) \wedge Portable(x) \Rightarrow Holding(x, Result(Grab, s)))$

Frame Axioms

- Effect axioms allow the agent to deduce, for example, that it holds the gold right after grabbing it.
- They do not allow the agent to infer the persistence of any facts.
- Frame axioms do:
    - $\forall a, x, s(Holding(x, s) \wedge (a \neq Release) \Rightarrow Holding(x, Result(a, s)))$
    - $\forall a, x, s(\neg Holding(x, s) \wedge (a \neq Grab \vee \neg Portable(x) \vee \neg Present(x, s)) \Rightarrow \neg Holding(x, Result(a, s)))$
- Frame axioms are useful, but typically *false*.

Successor-State Axioms

- Ray Reiter introduced successor-state axioms.
- They combine both effect and frame axioms.
- By writing one for each predicate, we can always tell whether a predicate holds or not in any given situation.

Example

$$\forall a, x, s(Holding\ (x, Result(a, s)) \Leftrightarrow [(a = Grab \wedge Present(x, s) \wedge Portable(x)) \vee (Holding(x, s) \wedge a \neq Release)])$$

Location and Orientation

- Locations: functional terms of the form $[x, y]$.
- Orientations: angles in degrees; 0 for East and 90 for North.
- Location statements: $At(g, l, s)$.
  - For example, $At(Agent, [1, 1], S_0)$
- Orientation statements: $Orientation(g, s) = d$.
  - For example, $Orientation(Agent, S_0) = 0$

A Simple Map

- $\forall x, y(LocToward([x, y], 0) = [x + 1, y])$
- $\forall x, y(LocToward([x, y], 90) = [x, y + 1])$
- $\forall x, y(LocToward([x, y], 180) = [x - 1, y])$
- $\forall x, y(LocToward([x, y], 270) = [x, y - 1])$

We need a logical formalization of simple arithmetic.

Some Useful Definitions

- $\forall g, l, s(At(g, l, s) \Rightarrow LocAhead(g, s) = LocToward(l, Orientation(g, s)))$
- $\forall l1, l2(Adjacent(l1, l2) \Leftrightarrow \exists d(l1 = LocToward(l2, d)))$
- $\forall x, y(Wall([x, y]) \Leftrightarrow (x = 0 \lor x = 5 \lor y = 0 \lor y = 5))$

Successor-State Axiom for Location

$$\begin{aligned}
&\forall a, g, l, s(At(g, l, Result(a, s)) \Leftrightarrow \\
&\qquad [(a = Forward \wedge l = LocAhead(g, s) \wedge \neg Wall(l)) \\
&\qquad \vee\ (At(g, l, s) \wedge (a = Forward \Rightarrow Wall(LocAhead(g, s))))])
\end{aligned}$$