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CSEN 1003 Compiler, Spring Term 2019
Practice Assignment 4

Discussion: 19.02.19 - 24.02.19

Exercise 4-1

Eliminating Left Recursion

Eliminate left-recursion from each of the following grammars:

a) $S \rightarrow S\texttt{a} \mid \texttt{b}$

Solution:

$$\begin{array}{lcl} S & \rightarrow & \texttt{b}S' \\ S' & \rightarrow & \texttt{a}S' \mid \varepsilon \end{array}$$

b) $S \rightarrow S\texttt{ab} \mid \texttt{cd}$

Solution:

$$\begin{array}{lcl} S & \rightarrow & \texttt{cd}S' \\ S' & \rightarrow & \texttt{ab}S' \mid \varepsilon \end{array}$$

c) $S \rightarrow S \cup S \mid S\ S \mid S* \mid (S) \mid \texttt{a}$

Solution:

$$\begin{array}{lcl} S & \rightarrow & \texttt{(}S\texttt{)}S' \mid \texttt{a}S' \\ S' & \rightarrow & \cup SS' \mid SS' \mid *S' \mid \varepsilon \end{array}$$

d)
$$\begin{array}{lcl} rexpr & \rightarrow & rexpr \cup rterm \mid rterm \\ rterm & \rightarrow & rterm\ rfactor \mid rfactor \\ rfactor & \rightarrow & rfactor* \mid rprimary \\ rprimary & \rightarrow & \mathbf{a} \mid \mathbf{b} \end{array}$$

Solution:

$$\begin{array}{lcl} rexpr & \rightarrow & rterm\ rexpr' \\ rexpr' & \rightarrow & \cup rterm\ rexpr' \mid \varepsilon \\ rterm & \rightarrow & rfactor\ rterm' \\ rterm' & \rightarrow & rfactor\ rterm' \mid \varepsilon \\ rfactor & \rightarrow & rprimary\ rfactor' \\ rfactor' & \rightarrow & *rfactor' \mid \varepsilon \\ rprimary & \rightarrow & \mathbf{a} \mid \mathbf{b} \end{array}$$

Alternative solution:

[0] Exercises are due to Dr. Carmen Gervet

$$
\begin{array}{lcl}
rexpr & \rightarrow & rterm \cup rexpr \mid rterm \\
rterm & \rightarrow & rfactor\ rterm \mid rfactor \\
rfactor & \rightarrow & rprimary\ rfactor' \\
rfactor' & \rightarrow & *rfactor' \mid \varepsilon \\
rprimary & \rightarrow & \mathbf{a} \mid \mathbf{b}
\end{array}
$$

e)
$$
\begin{array}{lcl}
A & \rightarrow & \mathtt{0} \mid T\mathtt{1} \\
T & \rightarrow & \mathtt{1} \mid A\mathtt{0}
\end{array}
$$

Solution:

$$
\begin{array}{lcl}
A & \rightarrow & \mathtt{0} \mid T\mathtt{1} \\
T & \rightarrow & \mathtt{1}T' \mid \mathtt{00}T' \\
T' & \rightarrow & \mathtt{10}T' \mid \varepsilon
\end{array}
$$

f)
$$
\begin{array}{lcl}
A & \rightarrow & BC \\
B & \rightarrow & Bb \mid \varepsilon \\
C & \rightarrow & AC \mid \mathtt{a}
\end{array}
$$

Solution:

$$
\begin{array}{lcl}
A & \rightarrow & BC \mid C \\
B & \rightarrow & \mathtt{b}B' \\
B' & \rightarrow & \mathtt{b}B' \mid \varepsilon \\
C & \rightarrow & \mathtt{b}B'CCC' \mid \mathtt{a}C' \\
C' & \rightarrow & CC' \mid \varepsilon
\end{array}
$$

Exercise 4-2

Left Factoring

Left-factor each of the following grammars:

a)
$$
\begin{array}{lcl}
S & \rightarrow & \mathtt{0}S\mathtt{1} \mid \mathtt{01}
\end{array}
$$

Solution:

$$
\begin{array}{lcl}
S & \rightarrow & \mathtt{0}S' \\
S' & \rightarrow & S\mathtt{1} \mid \mathtt{1}
\end{array}
$$

b)
$$
\begin{array}{lcl}
S & \rightarrow & \mathtt{abx} \mid \mathtt{aby} \mid \mathtt{acx} \mid \mathtt{acy}
\end{array}
$$

Solution:

$$
\begin{array}{lcl}
S & \rightarrow & \mathtt{a}S' \\
S' & \rightarrow & \mathtt{b}S'' \mid \mathtt{c}S'' \\
S'' & \rightarrow & \mathtt{x} \mid \mathtt{y}
\end{array}
$$

Exercise 4-3

Consider the following CFG:

$$\begin{array}{lcl} S & \rightarrow & \texttt{0}T\texttt{1}S \mid \varepsilon \\ T & \rightarrow & \texttt{0}T\texttt{1} \mid \varepsilon \end{array}$$

a) Compute the FIRST and FOLLOW sets.

Solution:

$\text{FIRST}(S) = \{\texttt{0}, \varepsilon\}$

$\text{FIRST}(T) = \{\texttt{0}, \varepsilon\}$

$\text{FOLLOW}(S) = \{\$\}$

$\text{FOLLOW}(T) = \{\texttt{1}\}$

b) What language does this grammar recognize?

Solution:

$\{\texttt{0}^n\texttt{1}^n | n \geq 1\}^*$

c) Give an unambiguous grammar for this language.

Solution:

$$\begin{array}{lcl} S & \rightarrow & TS \mid \varepsilon \\ T & \rightarrow & \texttt{0}T\texttt{1} \mid \varepsilon \end{array}$$

Exercise 4-4

Consider the following CFG:

$$\begin{array}{lcl} S & \rightarrow & SAB \mid SBC \mid \varepsilon \\ A & \rightarrow & \texttt{a}A\texttt{a} \mid \varepsilon \\ B & \rightarrow & \texttt{b}B \mid \varepsilon \\ C & \rightarrow & \texttt{c}C \mid \varepsilon \end{array}$$

a) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

Non-terminal	FIRST	FOLLOW
S	$\{\texttt{a}, \texttt{b}, \texttt{c}, \varepsilon\}$	$\{\$, \texttt{a}, \texttt{b}, \texttt{c}\}$
A	$\{\texttt{a}, \varepsilon\}$	$\{\$, \texttt{a}, \texttt{b}, \texttt{c}\}$
B	$\{\texttt{b}, \varepsilon\}$	$\{\$, \texttt{a}, \texttt{b}, \texttt{c}\}$
C	$\{\texttt{c}, \varepsilon\}$	$\{\$, \texttt{a}, \texttt{b}, \texttt{c}\}$

b) Build the parsing table.

Solution:

Non-terminal	Input Symbol			
	a	b	c	$
S	SAB SBC ε	SAB SBC ε	SAB SBC ε	SAB SBC ε
A	aAa ε	ε	ε	ε
B	ε	bB ε	ε	ε
C	ε	ε	cC ε	ε

c) From the parsing table, show why the grammar is not LL(1).

Solution:

The grammar is not LL(1) because some entries of the parsing table contain more than one rule.

Exercise 4-5

Consider the following CFG:

$$\begin{aligned} S &\rightarrow \texttt{(}L\texttt{)} \mid \texttt{a} \\ L &\rightarrow L\texttt{,}S \mid S \end{aligned}$$

a) Eliminate left recursion.

Solution:

$$\begin{aligned} S &\rightarrow \texttt{(}L\texttt{)} \mid \texttt{a} \\ L &\rightarrow \texttt{(}L\texttt{)}L' \mid \texttt{a}L' \\ L' &\rightarrow \texttt{,}SL' \mid \varepsilon \end{aligned}$$

b) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

$\text{FIRST}(S) = \{\texttt{(}, \texttt{a}\}$

$\text{FIRST}(L) = \{\texttt{(}, \texttt{a}\}$

$\text{FIRST}(L') = \{\texttt{,}, \varepsilon\}$

$\text{FOLLOW}(S) = \{\texttt{)}, \texttt{,}, \$\}$

$\text{FOLLOW}(L) = \{\texttt{)}\}$

$\text{FOLLOW}(L') = \{\texttt{)}\}$

Exercise 4-6

Consider the following CFG:

$$S \rightarrow SS\texttt{+} \mid SS\texttt{*} \mid \texttt{a}$$

a) Left factor the grammar and eliminate left recursion.

Solution:

$$\begin{array}{rcl} S & \rightarrow & \mathtt{a}S' \\ S' & \rightarrow & SXS' \mid \varepsilon \\ X & \rightarrow & \mathtt{+} \mid \mathtt{*} \end{array}$$

b) Compute FIRST and FOLLOW sets for each non-terminal.

Solution:

$\text{FIRST}(S) = \{\mathtt{a}\}$

$\text{FIRST}(S') = \text{FIRST}(S) \cup \{\varepsilon\} = \{\mathtt{a}, \varepsilon\}$

$\text{FIRST}(X) = \{\mathtt{+}, \mathtt{*}\}$

$\text{FOLLOW}(S) = \text{FIRST}(X) \cup \{\$\} = \{\mathtt{+}, \mathtt{*}, \$\}$

$\text{FOLLOW}(S') = \text{FOLLOW}(S) = \{\mathtt{+}, \mathtt{*}, \$\}$

$\text{FOLLOW}(X) = \{\mathtt{a}, \mathtt{+}, \mathtt{*}, \$\}$

c) Build the parsing table.

Solution:

Non-terminal	Input Symbol			
	a	+	*	$
S	aS			
S'	SXS'	ε	ε	ε
X		+	*	