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① problem 1.6 sec: 1 Bn: 2 أحمد حسن عبد الرازق

ⓐ for one sample we have that $P(v=0) = (1-\mu)^{10}$
so for $\mu = 0.05$, we get $P(v=0) = 0.5987369$
$\mu = 0.5$, " " $P(v=0) = 9.765625 \times 10^{-4}$
$\mu = 0.8$, " " $P(v=0) = 1.024 \times 10^{-7}$

ⓑ for 1000 independent samples

$$P(\text{at least one sample has } v=0) = 1 - P(v_i > 0 \ \forall i)$$
$$= 1 - \prod_{i=1}^{1000} P(v_i > 0)$$
$$= 1 - \prod_{i=1}^{1000} [1 - P(v_i = 0)]$$
$$= 1 - \prod_{i=1}^{1000} [1 - (1-\mu)^{10}]$$
$$= 1 - [1 - (1-\mu)^{10}]^{1000}$$

which give us 1 when $\mu = 0.05$
0.6235762 $\rightarrow \mu = 0.5$
$1.0239476 \times 10^{-4} \rightarrow \mu = 0.8$

ⓒ repeat (b) for 1000000 independent sample
$P(\text{at least one sample has } v=0) = 1 - [1 - (1-\mu)^{10}]^{1000000}$
which give us 1 $\rightarrow \mu = 0.05$
1 $\rightarrow \mu = 0.5$
0.0973316 $\rightarrow \mu = 0.8$

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Problem 2.5

To prove the inequality we begin with the case $D=0$

$$1=\binom{N}{0}\le N^0+1=2$$

Now we assume the result is correct for $D\,(D\ge1)$ and will prove for $D+1$

$$\sum_{i=0}^{D+1}\binom{N}{i}=\sum_{i=0}^{D}\binom{N}{i}+\binom{N}{D+1}$$

$$\le N^D+1+\binom{N}{D+1}$$

$$\le N^D+1+\frac{N!}{(D+1)!\,(N-D-1)!}$$

Now we need to prove that $\frac{N!}{(N-D-1)!}\le N^{D+1}$

$$\frac{1}{N^{D+1}}\;\frac{N!}{(N-D-1)!}\le 1$$

$$\frac{1}{N^{D+1}}\;\frac{N!}{(N-D-1)!}=\frac{1}{N^{D+1}}\prod_{i=0}^{D}(N-i)=\prod_{i=0}^{D}\frac{N-i}{N^{D+1}}\le 1$$

$$\sum_{i=0}^{D+1}\binom{N}{i}\le N^D+1+\frac{N!}{(D+1)!\,(N-D-1)!}$$

$$\le N^D+1+\frac{N^{D+1}}{(D+1)!}$$

$D\ge1$ so $(D+1)!\ge 2$

$$\frac{1}{(D+1)!}\le\frac{1}{2}$$

$$\sum_{i=0}^{D+1}\binom{N}{i}\le N^D+1+\frac{N^{D+1}}{(D+1)!}$$

$$\le N^D+1+\frac{N^{D+1}}{2}$$

$$\le\frac{N^{D+1}}{2}+1+\frac{N^{D+1}}{2}=N^{D+1}+1 \quad \#$$

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